Determination of the Scattering Coefficient of Statistical Rough Surfaces

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Abstract : The scattering coefficient of a diffusing surface accounts for the part of sound power which is "non-specularly" reflected. This coefficient is an essential parameter for the description of walls and surfaces in room acoustics problems. In this paper, the scattering coefficient of random rough surfaces is calculated from the complete scattered sound pressure distribution. This distribution is evaluated using a Kirchhoff Approximation method. The results obtained for several rough surfaces are compared with theoretical expressions of the scattering coefficient. These expressions show the influence of the angle of incidence, the sound frequency and the geometrical parameters of the surface profile on the scattering coefficient. It is shown that these theoretical expressions give reliable results as long as the Kirchhoff Approximation conditions hold.

INTRODUCTION

In room acoustics, the scattering coefficient of a diffusing surface is defined as the ratio of non-specularly reflected power to the total power reflected by the surface. The value of this coefficient (and its dependence on frequency) is essential for the users of any modern room acoustics software, because most of them can now account for the effects of surface diffusion. However, rather few data have already been collected on surface scattering properties and only a small part of these data have been published. Therefore, a great amount of work must still be done in order to better understand the mechanisms of surface scattering and to measure diffusion parameters.

The random-incidence scattering coefficient of diffusing surfaces can be measured in a reverberant room, using a method developed by Vorlaender and Mommertz [1]. This method is presently under investigation by an ISO working group. However, this communication addresses the problem of finding the value of the scattering coefficient, not by measurements, but rather by theoretical developments.

METHOD SUMMARY

We consider finite size rough surfaces described by their elevation $z = \xi(x,y)$ relative to a reference plane. Assuming an incident plane wave, the distribution of the scattered pressure can be calculated with the Kirchhoff Approximation method [2]. The development of this method leads to the following expression for the complex pressure $\underline{p_1}$ in the scattering direction $\overrightarrow{k_s}$:

$$\underline{p_1}(\overrightarrow{k_s}) = \underline{K} \int_S e^{j\vec{v}.\vec{r}} (\vec{v}.\vec{\gamma})\, \underline{C_r}\, dx\, dy \qquad (1)$$

In this equation, $\underline{K}$ is a constant complex number (depending on the strength of the incident wave and the distance of the receiver), S is the area of the rough surface projected onto the reference plane, $\vec{v} = \overrightarrow{k_i} - \overrightarrow{k_s}$ where $\overrightarrow{k_i}$ is the incident vector (the magnitude of both vectors $\vec{k}$ is $2\pi/\lambda$), $\vec{r}$ is the position vector of the surface element at (x, y, ξ) and $\vec{\gamma} = (-\xi_x^{'}, -\xi_y^{'}, 1)$ is a vector perpendicular to the rough surface at this surface element. $\underline{C_r}$ is the local reflection factor.

With this expression, it is possible to calculate (within the assumptions of the Kirchhoff Approximation which are not discussed here) the complete distribution of the complex scattered pressure. The scattering coefficient of the diffusing surface defined by $z = \xi(x,y)$ can then be obtained with a formula proposed by Mommertz [3]. This scattering coefficient is called in the following $\delta_{K.A.}$, since it is computed using the more general formulation of the Kirchhoff Approximation method.

RESULTS FOR GAUSSIAN ROUGH SURFACES

To solve (1), we of course need to define the profile of a given rough surface. In this study, the analysis has been focused on a typical class of random rough surfaces, namely the gaussian surfaces.

For these particular surfaces, it is possible to derive fairly good approximations of the scattering coefficient $\delta_{K.A.}$. As will be seen in the following, these approximations can be obtained without calculating the complete distribution of the scattered pressure. They are therefore more easily computed and they also better illustrate the influences of sound frequency and geometrical parameters on the scattering coefficient. The first approximation is :

$$\delta_{C.F.} = 1 - \left|\underline{r_s}\right|^2 \qquad \underline{r_s} = \frac{1}{S}\int_S e^{jv_z\xi}\,dxdy \qquad (2)$$

where C.F. means *characteristic function*, which is the mathematical name for the average of $e^{jv_z\xi}$ if ξ is a random variable. The second approximation is :

$$\delta_{RMS} = 1 - e^{-4s^2k^2\cos^2\theta_i} \qquad s^2 = \frac{1}{S}\int_S \xi^2(x,y)\,dxdy \qquad (3)$$

In this expression, θ_i is the angle of incidence of the plane wave and *s* is the r.m.s. height of the random rough surface.

Both approximations have been compared with the *theoretically exact* value of the scattering coefficient $\delta_{K.A.}$ derived from equation (1). The *characteristic function* model (2) always leads to a very good correlation with $\delta_{K.A.}$, as long as the conditions of validity of the Kirchhoff Approximation are satisfied.

The *rms height* model (3) states that the scattering coefficient of gaussian rough surfaces only depends on the r.m.s. height of the surface (relative to the wavelength) and on the angle of incidence. The correlation with $\delta_{K.A.}$ is not as good as in the previous model, but the deviations are not really significant unless the scattering coefficient reaches high values (δ>0.8). This is illustrated for a particular class of gaussian surfaces in figure 1.

On the other hand, the *rms height* model is much easier to calculate, and it gives a very clear interpretation of the influence of all parameters affecting the scattering coefficient.

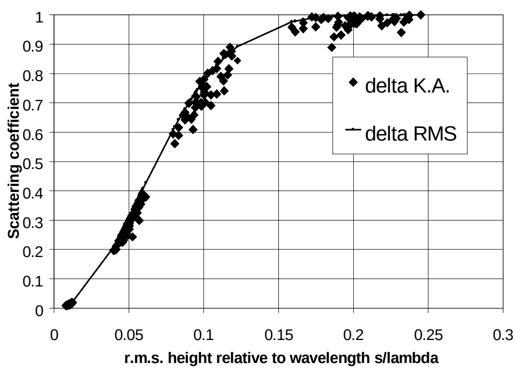

FIGURE 1. Scattering coefficients $\delta_{K.A.}$ and δ_{RMS} computed from expression (1) and by the *rms height* model (3) respectively, for 200 rigid ($\underline{C_r} = 1$) gaussian rough surfaces characterized by a correlation length of 5λ. The angle of incidence is 20°.

Figure 1 gives only some examples of the many gaussian surfaces which have been considered in this study. After analysing all these results, it has been found that the approximations (2) and (3) are in fact valid in many situations, including many angles of incidence ($\theta_i \le 60°$), surface dimensions, correlation lengths ($T \ge \lambda$) and r.m.s. heights. We even found that the approximations also hold for non-rigid surfaces.

It turns out that, at least for gaussian rough surfaces, the key parameters (concerning sound diffusion) seem to be the ratio of r.m.s. height to the wavelength and the angle of incidence. Further similar studies on other surfaces with deterministic profiles will certainly bring new information to this theory.

REFERENCES

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