Traffic-induced Vibration TD-3

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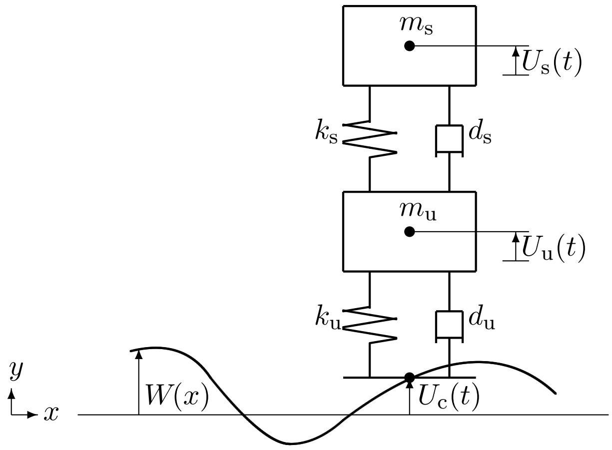

- We consider a 2-DOF model of a vehicle that moves over an uneven support at a constant horizontal velocity of v.
- Let the support unevenness be modeled by a mean-square stationary zero-mean second-order stochastic process $\{W(x),\ x \in \mathbb{R}\}$ with power spectral density function s_W.
- Let $\{U_\mathrm{c}(t),\ t \in \mathbb{R}\}$ be the vertical displacement of the contact point caused by the support unevenness.
- Let $\{U_\mathrm{s}(t),\ t \in \mathbb{R}\}$ and $\{U_\mathrm{u}(t),\ t \in \mathbb{R}\}$ be the vertical displacement of the sprung mass and the unsprung mass, respectively, with respect to the static equilibrium configuration on a flat support.

1. Express dynamical equilibrium and deduce the system of two stochastic differential equations that governs the displacements $\{U_\mathrm{s}(t),\ t \in \mathbb{R}\}$ and $\{U_\mathrm{u}(t),\ t \in \mathbb{R}\}$ of the sprung mass and the unsprung mass.
2. Consider the system of two stochastic differential equations that you obtained under 1 as a convolution transformation whose input is the stochastic process $\{U_\mathrm{c}(t),\ t \in \mathbb{R}\}$ and whose output is the pair of stochastic processes $\{U_\mathrm{s}(t),\ t \in \mathbb{R}\}$ and $\{U_\mathrm{u}(t),\ t \in \mathbb{R}\}$. Write the expression for the FRF $\hat{\boldsymbol{h}} = (\hat{h}_\mathrm{s}, \hat{h}_\mathrm{u})$ associated with this convolution filter.
3. Write the equations that relate the mean function and the power spectral density function of $\{U_\mathrm{s}(t),\ t \in \mathbb{R}\}$ and $\{U_\mathrm{u}(t),\ t \in \mathbb{R}\}$ to those of $\{W(x),\ x \in \mathbb{R}\}$.
4. Consider $s_W(\xi) = s_0/\big(1 + \frac{|\xi|}{\xi_0}\big)^\alpha$ and the numerical values of $\xi_0 = 0.5\,\mathrm{m}^{-1}$, $m_\mathrm{s} = 470\,\mathrm{kg}$, $k_\mathrm{s} = 36 \times 10^3\,\mathrm{N/m}$, $d_\mathrm{s} = 0.10 \times 2 \times \sqrt{m_\mathrm{s} k_\mathrm{s}}$, $m_\mathrm{u} = 39\,\mathrm{kg}$, $k_\mathrm{u} = 160 \times 10^3\,\mathrm{N/m}$, $d_\mathrm{u} = 0.05 \times 2 \times \sqrt{m_\mathrm{u} k_\mathrm{u}}$. Plot s_W and $|\hat{h}|$, first on a linear scale and then on a loglog scale. First using the values of s_0 and α that you obtained under TD 2 question 1(d) for the deteriorated jointed plain concrete pavement (w1.mat), and then using the values of s_0 and α that you obtained under TD 2 question 1(d) for the concrete block pavement (w2.mat), plot s_{U_s} and s_{U_u} for $v = 40\,\mathrm{m/s}$, first on a linear scale and then on a loglog scale. Interpret your results.

5. The ISO 2631-1 guide indicates that the comfort passengers perceive can be predicted on the basis of the root-mean-square value of the acceleration of the vehicle body, that is,

$$\sqrt{\int_{\Theta} |\ddot{U}_{\mathrm{s}}(t)|^2 dP} = \sqrt{\frac{1}{2\pi}\int_{\mathbb{R}} s_{\ddot{U}_{\mathrm{s}}}(\omega) d\omega} = \sqrt{\frac{1}{2\pi}\int_{\mathbb{R}} \omega^4 s_{U_{\mathrm{s}}}(\omega) d\omega}.$$

Specifically, the ISO 2631-1 guide suggests that passengers perceive comfort as follows:

root-mean-square acceleration [m/s^2]	comfort perception
Less than 0.315	not uncomfortable
0.315 to 0.63	a little uncomfortable
0.5 to 1	fairly uncomfortable
0.8 to 1.6	uncomfortable
1.25 to 2.5	very uncomfortable
greater than 2	extremely uncomfortable

Use the power spectral density function you obtained under question 4 to predict comfort perception, first when driving over the deteriorated jointed plain concrete pavement (w1.mat) and then when driving over the concrete block pavement (w2.mat).

(Note that for the sake of simplicity, we neglected here the frequency weighing suggested by the ISO 2631-1 guide to account for the fact that not all frequencies are perceived equally.)