4th SYMPOSIUM OF VKI PHD RESEARCH

# Development of advanced models for transition to turbulence in hypersonic flows
## Prediction of transition under uncertainties

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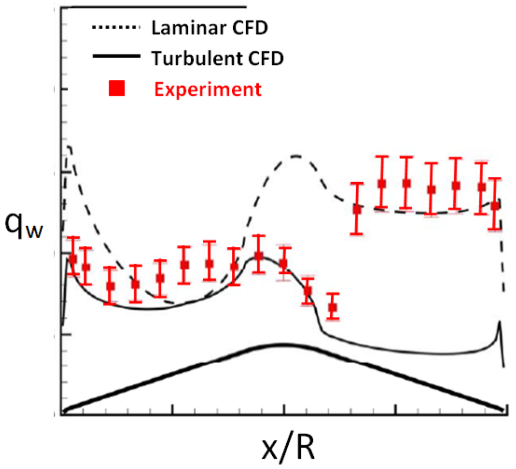
MTFC
RESEARCH GROUP

MULTIPHYSICS & TURBULENT
FLOW COMPUTATION

*Aeronautics and*
*Aerospace Department*

# Introduction & Motivation


Mars Exploration Rover (MER)



***MSL Mach 10, α = 16-deg***
***Data and Comparisons from AEDC Tunnel 9***

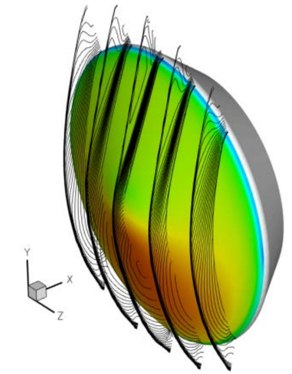
Atmospheric reentry high speed & potential energy converted into heat

Space vehicles need heat shields (TPS)

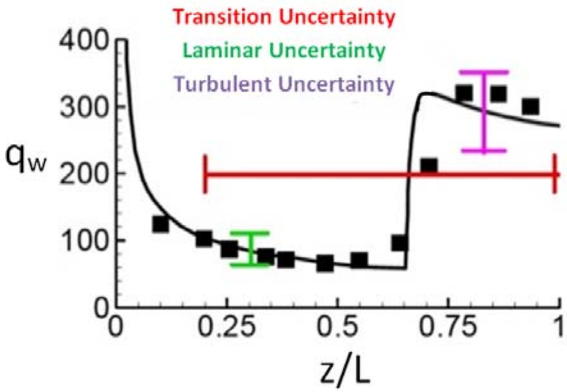
Turbulent heat rates several times higher than in the laminar regime

Safety margins are necessary for the design of the TPS

Quantify the margins for a less conservative design



***MSL CFD in reentry conditions***

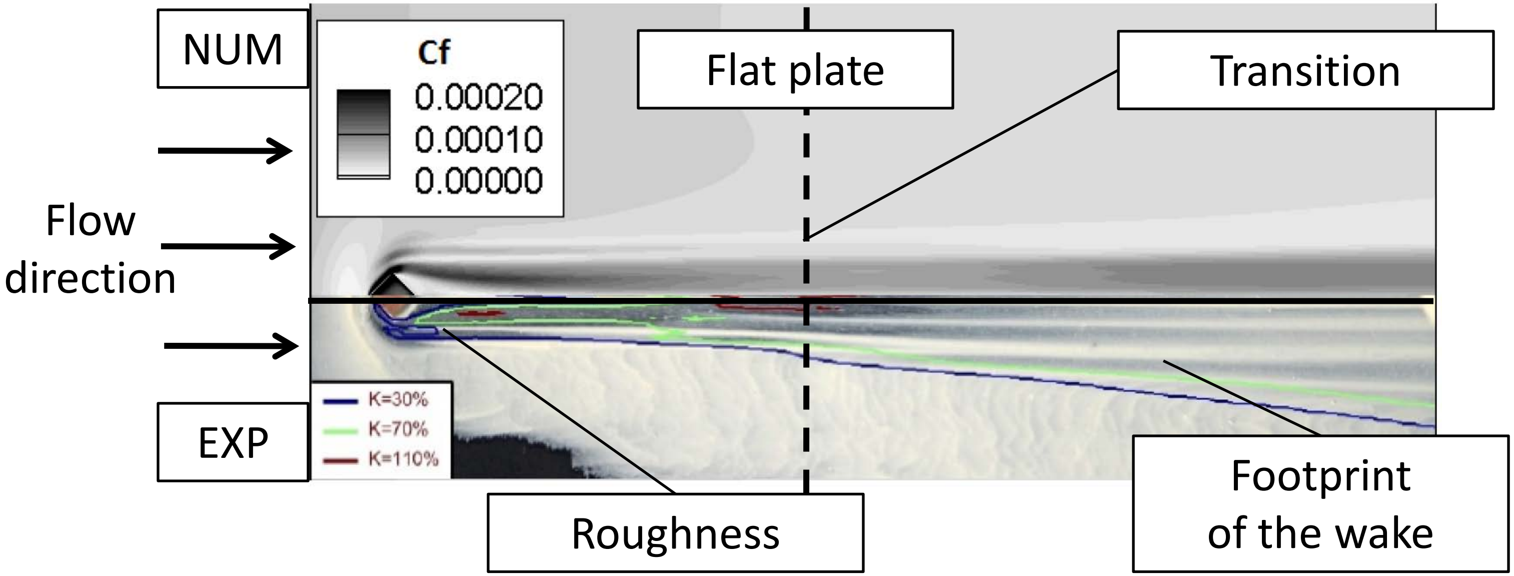


***Steven P. Schneider. Hypersonic laminar-turbulent transition on circular cones and scramjet forebodies., 2004.***

# Introduction & Motivation

**Transition prediction – The State of the Art**

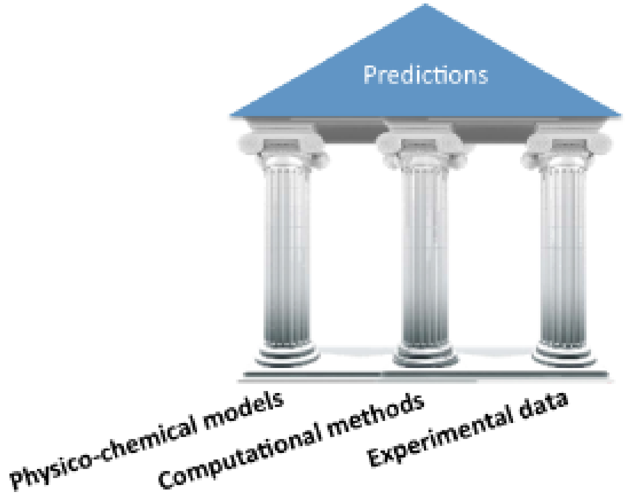
- **Experiments** : empirical criteria and correlation (Shuttle, Van Driest)
  - **Good** : successfully used (Apollo, Shuttle);
  - **Bad** : expensive, limited in time and no real operating conditions (Re, Ma);
- **CFD** : Transition models ( Menter, Goldberg, $R\text{-}\gamma$ )
  - **Good** : fast , design;
  - **Bad** : simplified physics, very sensitive to free stream conditions (Re, Ma, Tu);

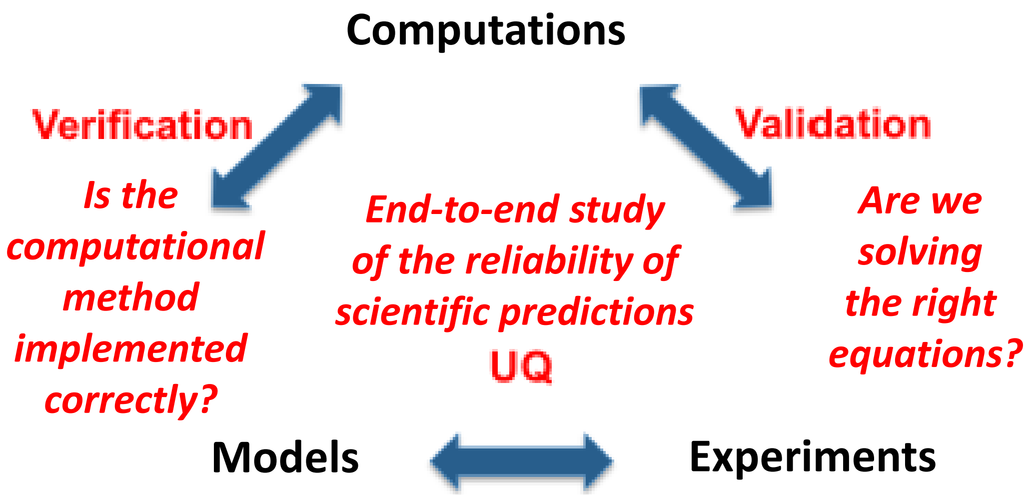


**G.Serino, F.Pinna, P.Rambaud, " Numerical computations of hypersonic boundary layer roughness induced transition on a flat plate ", 2012**

# Introduction & Motivation

**Three pillars for predictive engineering simulations**





**Computations**

**Verification**

*Is the computational method implemented correctly?*

*End-to-end study of the reliability of scientific predictions*

**UQ**

**Validation**

*Are we solving the right equations?*

**Models**

**Experiments**

**Transition prediction – What we propose**

- Introduce ***Uncertainty Quantification*** (UQ) in deterministic simulations for transition prediction to :
  - Take into account the ***physical variability*** of the system to simulate
  - Transition is a ***stochastic process***
  - ***Improve*** and ***verify*** transition tools currently used in design
  - A ***stochastic model*** for transition prediction does not yet exist

# Outline

## 1. Introduction to deterministic and probabilistic tools

- **Linear Stability Theory and transition prediction**
- **Uncertainty Quantification and numerical simulations**

## 2. Formulation of the method

- **Assumptions for the deterministic simulations**
- **Description of the method**

## 3. The VKI-H3 test case

- **The forward problem**
- **The inverse problem**

## 4. Conclusions & Future works

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# 1. Introduction to deterministic and probabilistic tools

## Linear Stability Theory and transition prediction

- Small disturbances: baseline + disturbances;
- Linearization of Navier-Stokes equations + parallel flow approximation;

$$u = \bar{u} + u'$$

***baseline*** ***disturbances***

$$\bar{v} = 0 \quad \bar{u} = \bar{u}(y)$$

***parallel flow***

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0$$

$$\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x} + v'\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} + \nu\left[\frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u'}{\partial y^2}\right]$$

$$\frac{\partial v'}{\partial t} + \bar{u}\frac{\partial v'}{\partial x} = -\frac{1}{\rho}\frac{\partial p'}{\partial y} + \nu\left[\frac{\partial^2 v'}{\partial x^2} + \frac{\partial^2 v'}{\partial y^2}\right]$$

- Wave like disturbances;

$$q'(x, y, t) = \tilde{q}(y)e^{i(\alpha x - \omega t)}$$

***Propagation in time (real)***

- Space amplification theory

***Propagation in space (complex)***

**Degrez G., "*Two dimensional boundary layer*", 2012**

# 1. Introduction to deterministic and probabilistic tools

## Linear Stability Theory and transition prediction

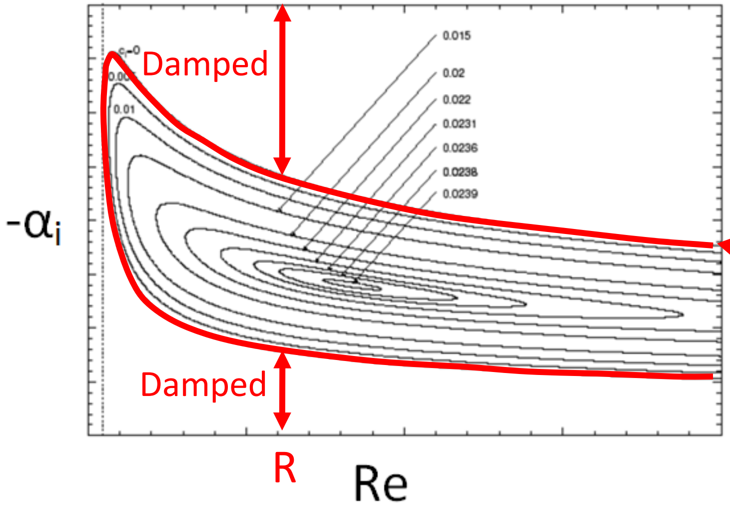
- Orr-Sommerfeld equations;

$$i\alpha(\bar{u}-c)(\tilde{v}''-\alpha^2\tilde{v})-i\alpha\tilde{v}\bar{u}''=\frac{1}{R}\left[\tilde{v}^{iv}-2\alpha^2\tilde{v}''+\alpha^4\tilde{v}\right]$$

- B.C. : disturbances vanish at the wall and in the far field;

$$\tilde{v}(0)=\tilde{v}'(0)=0 \qquad \lim_{y\to\infty}\tilde{v}(y)=\lim_{y\to\infty}\tilde{v}'(y)=0$$

- Eigenvalue problem;



Amplification rates contour lines for Blasius velocity profile plotted in the R- $\alpha$ plane

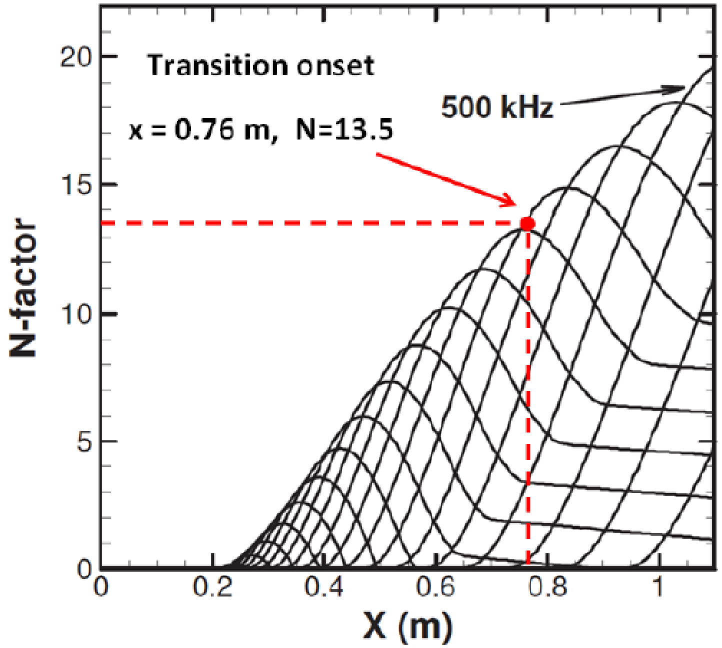
Neutral Stability curve : $c_i$ = 0
boundary between damped
and amplified disturbances

**Degrez G., “*Two dimensional boundary layer*”, Course Notes, 2012**

# 1. Introduction to deterministic and probabilistic tools

## Linear Stability Theory and transition prediction

- $e^N$ *transition prediction method;*



$$N = log\frac{A}{A_o} = \int_{x_0}^{x} -\alpha_i dx$$

N factors computed on the HIFire I reentry vehicle
Mach number = 5.28, H = 21km

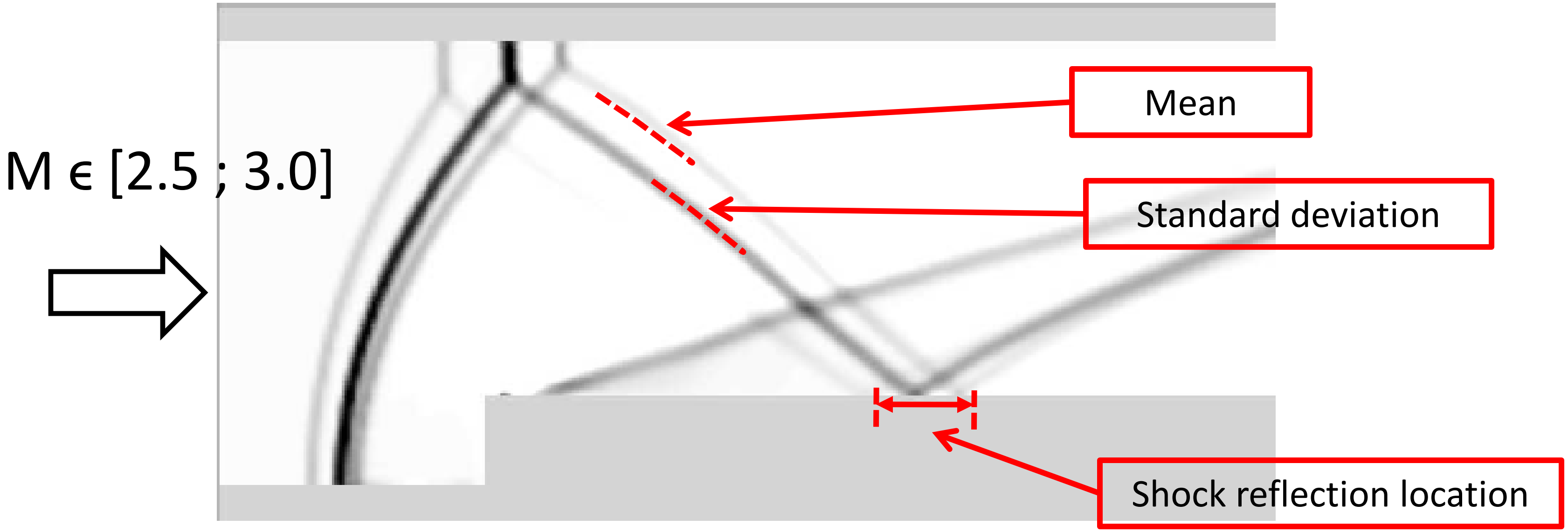
**Fei Li *et al.*,**
**“Hypersonic Transition Analysis for HIFiRE Experiments”, 2012.**

- **Transition** : $N$-factor = $N_{exp}$
- $N = N$**(wind tunnel, free stream parameters)**
- $N$-factor = **4-5** (WT) , **13-14** (Flight);

# 1. Introduction to deterministic and probabilistic tools

## Uncertainty quantification and numerical simulations

- ***Goal*** : study how physical variability of systems affects ***Quantity of Interest***
- ***UQ*** : End-to-end study of the reliability of scientific predictions;



Mean velocity divergence field

Iaccarino G. *et al*.,
“Numerical methods for uncertainty propagation in high speed flows,”
V European Conference on Computational Fluid Dynamics,
2011

# Outline

1. Introduction to deterministic and probabilistic tools
    - Linear Stability Theory and transition prediction
    - Uncertainty Quantification and numerical simulations

2. **Formulation of the method**
    - **Assumptions for the deterministic simulations**
    - **Description of the method**

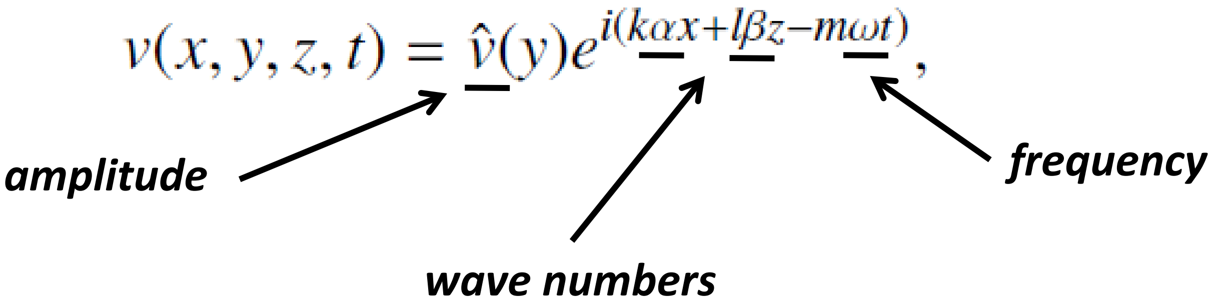
3. The VKI-H3 test case
    - The forward problem
    - The inverse problem

4. Conclusions & Future works

# 2. Formulation of the method

## Assumptions for the deterministic simulations

- Wave-like disturbances



$$v(x,y,z,t) = \underline{\hat{v}}(y)e^{i(\underline{k\alpha} x + \underline{l\beta} z - \underline{m\omega} t)},$$

***amplitude***

***wave numbers***

***frequency***

- 2D waves ( $\beta$ vanishes );
- Spatial amplification theory : $\omega$ real , $\alpha$ complex, wave propagation speed $c = \omega/\alpha_r$ , amplification rate in space $-\alpha_i$ ;
- Transition prediction with the $e^N$ method : VKI-H3 $N$-factor = 5 (Mach 6 WT);

# 2. Formulation of the method

## Description of the method

1. Linear Stability Analysis

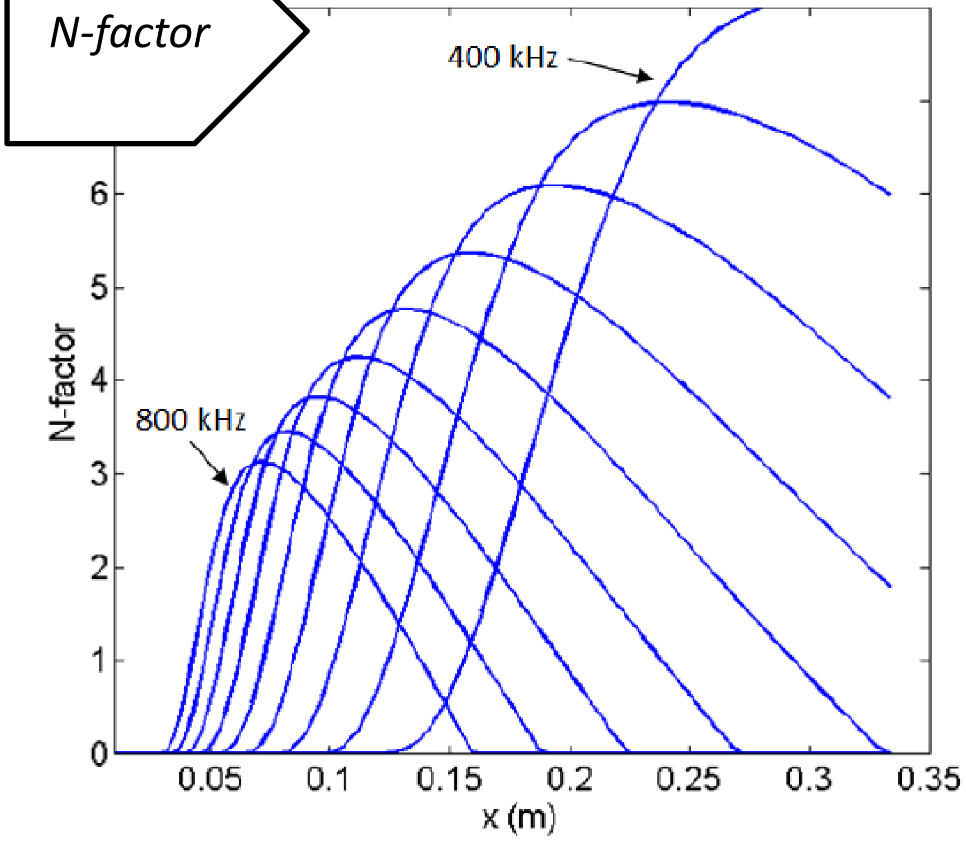
Base solution

- Free stream conditions
- B.L. profiles (CFD, SS)

LST

- F ϵ [400 - 800] kHz
- VESTA (Pinna 2012)

*N-factor*



**Masutti D., *Natural and induced transition on a 7 degree half-cone at Mach 6*, 2012.**

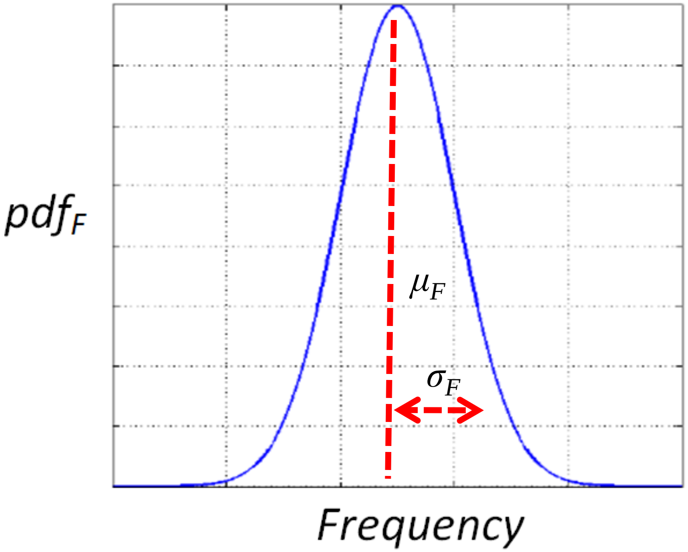
# 2. Formulation of the method

## Description of the method

2. Definition of the uncertainties

**Frequency**

- Input uncertainty
- Normal *pdf* ($\mu_F$ , $\sigma_F$)



Example of *pdf* of the input parameter for the UQ analysis

**Propagation**

- Monte Carlo
- Method of transformation

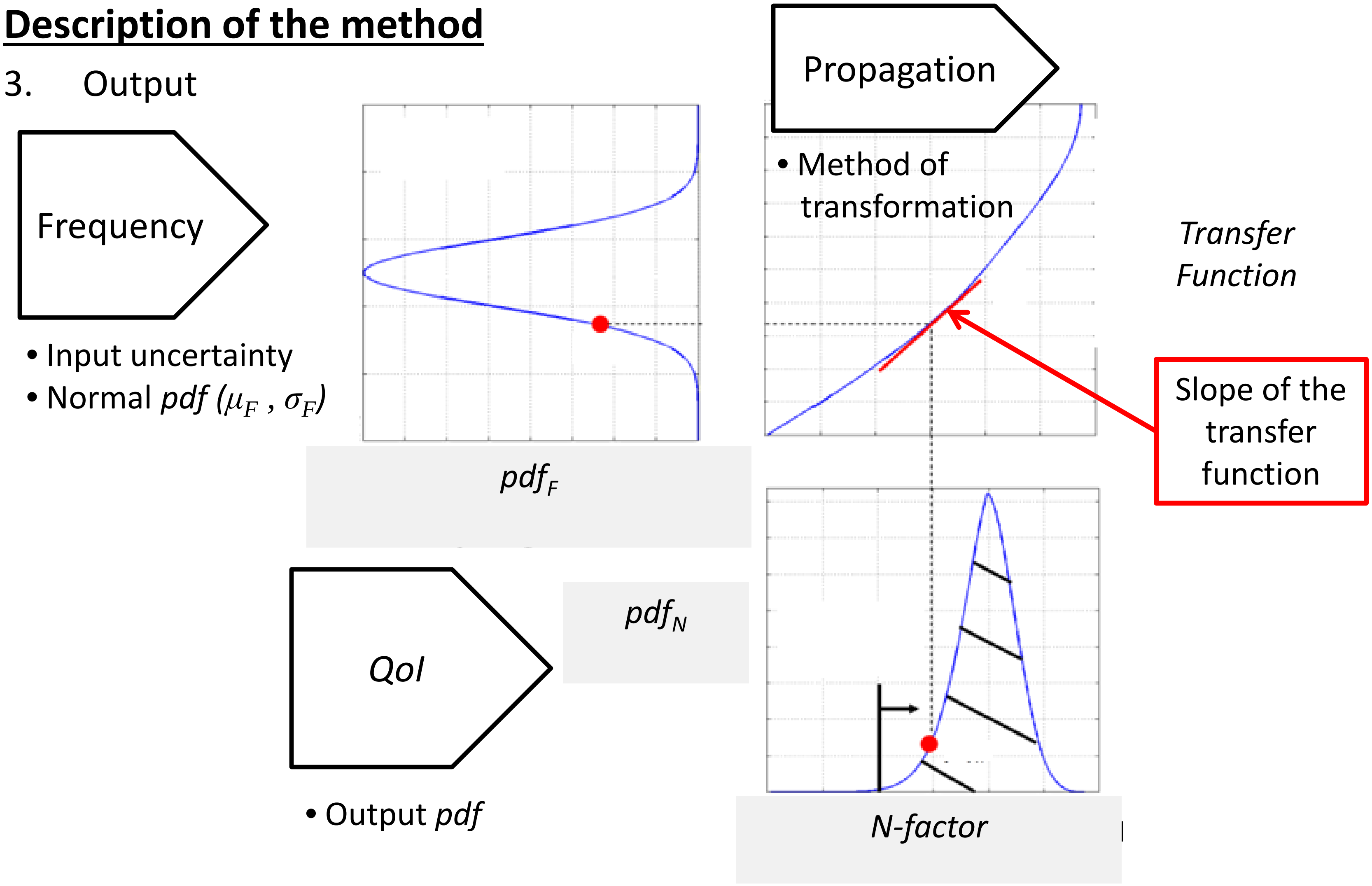
***QoI***

- Output *pdf*

# 2. Formulation of the method

## Description of the method

3. Output



Frequency

- Input uncertainty
- Normal *pdf* ($\mu_F$ , $\sigma_F$)

*$pdf_F$*

Propagation

- Method of transformation

*Transfer Function*

Slope of the transfer function

*QoI*

- Output *pdf*

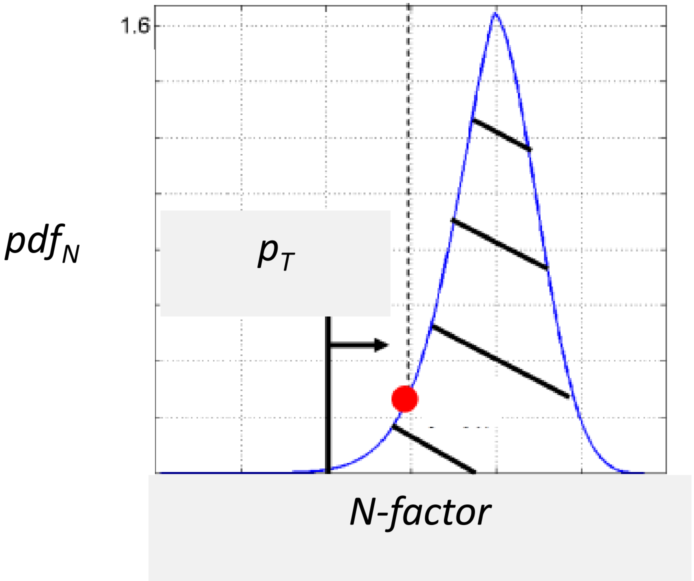
*$pdf_N$*

*N-factor*

# 2. Formulation of the method

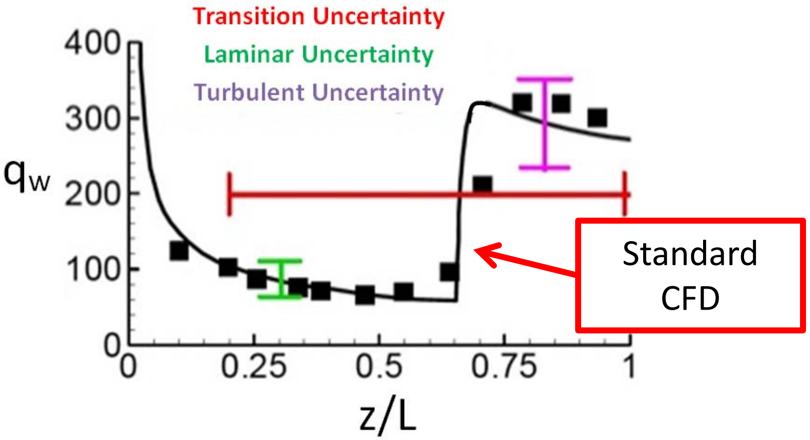
## Description of the method

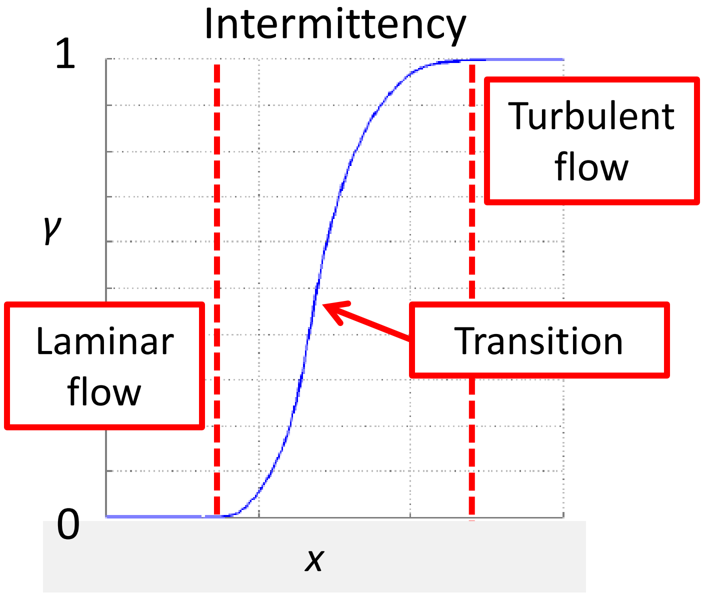
4. Probability of transition



$$p_T = 1 - \int_0^{N_{crit}} pdf(\bar{N})d\bar{N} = \gamma$$

- $p_T$ , probability of transition
- $N_{crit}$ , critical *N-factor* at the transition onset





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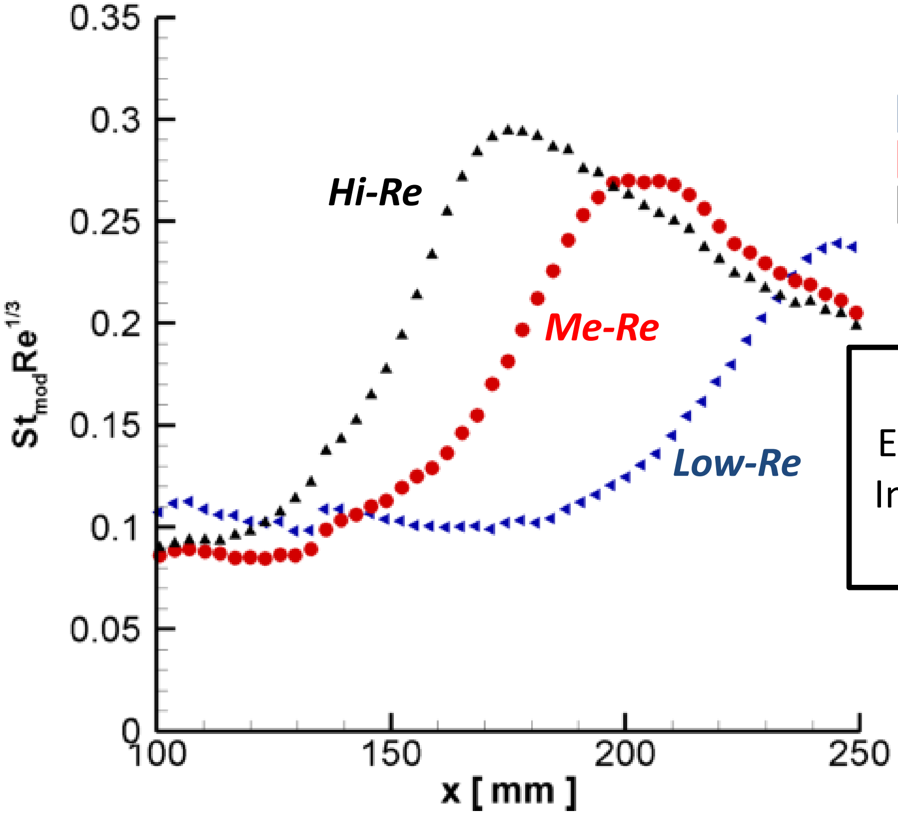
# 3. The VKI-H3 test case

## Study of natural transition on a 7° half-cone model

- Transition detected by surface measurements of the heat flux;
- Different Reynolds number conditions;



| Test case | $M_\infty$ | $T_\infty$ [K] | $Re_\infty$ [1/m] | $T_w$ [K] |
|---|---|---|---|---|
| Low Reynolds | 6.0 | 60 | $18.0 \times 10^6$ | 294 |
| Medium Reynolds | 6.0 | 60 | $22.8 \times 10^6$ | 294 |
| High Reynolds | 6.0 | 60 | $27.1 \times 10^6$ | 294 |

Experimental Intermittency

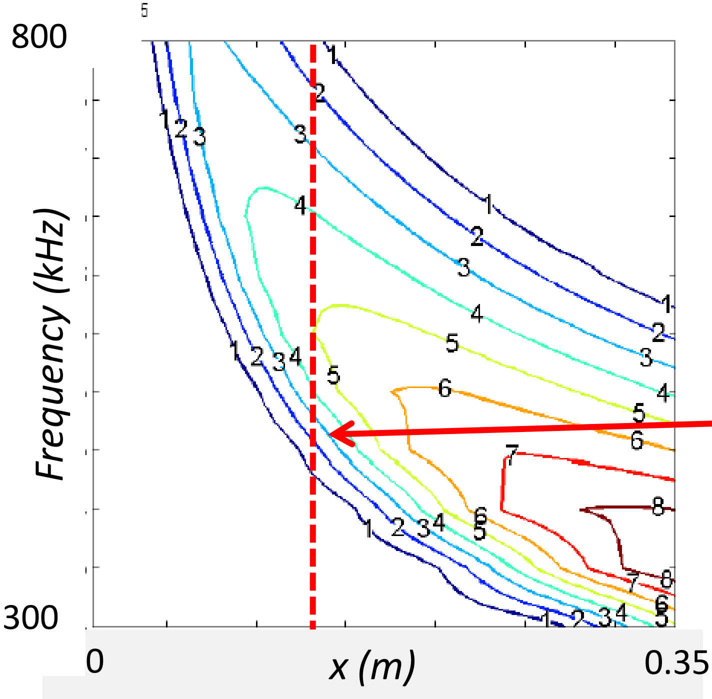
$$\gamma(x) = \frac{St(x) - St_{x=x_{onset}}}{St_{x=x_{offset}} - St_{x=x_{onset}}}$$

Masutti D., *Natural and induced transition on a 7 degree half-cone at Mach 6*, 2012.

# 3. The VKI-H3 test case

## The forward problem

- ***Goal*** : computation of the probability of transition caused by assumed freestream perturbation spectrum (Frequency distribution);
- ***Assumption*** : transition caused by perturbations in the BL upstream of the transition location;
- ***Transfer function*** : Linear Stabilitv analysis to compute *N=N(F)*;



N-factor -Frequency relation
with VESTA

Experimental transition onset
$N = 5$

# 3. The VKI-H3 test case

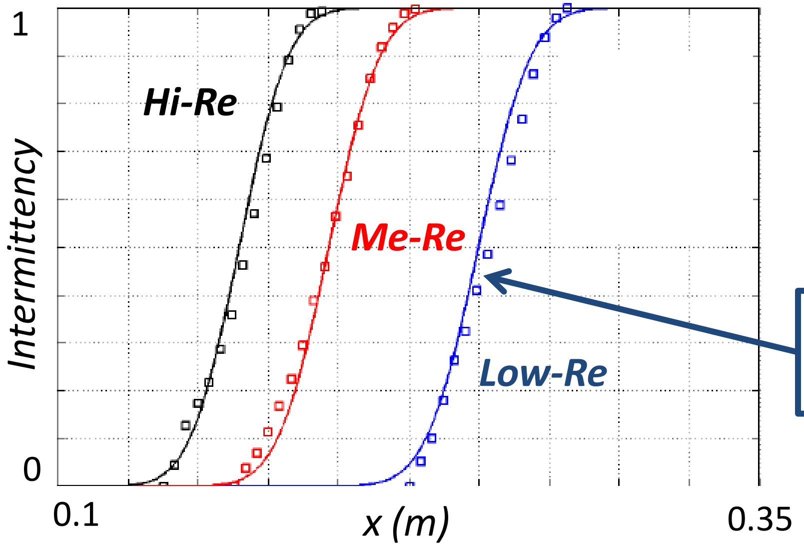
## The forward problem

- ***UQ approach*** : free stream perturbations as *pdf* of the Frequency with *normal distributions* ;

| Test case | $\mu_f$ [kHz] | $\sigma_F$ [kHz] |
|---|---|---|
| Low Reynolds | 330 | 10 |
| Medium Reynolds | 410 | 20 |
| High Reynolds | 480 | 25 |

| Range [kHz] |
|---|
| $200 \div 800$ |

- Computation of the *probability of transition* and comparison with experiments;



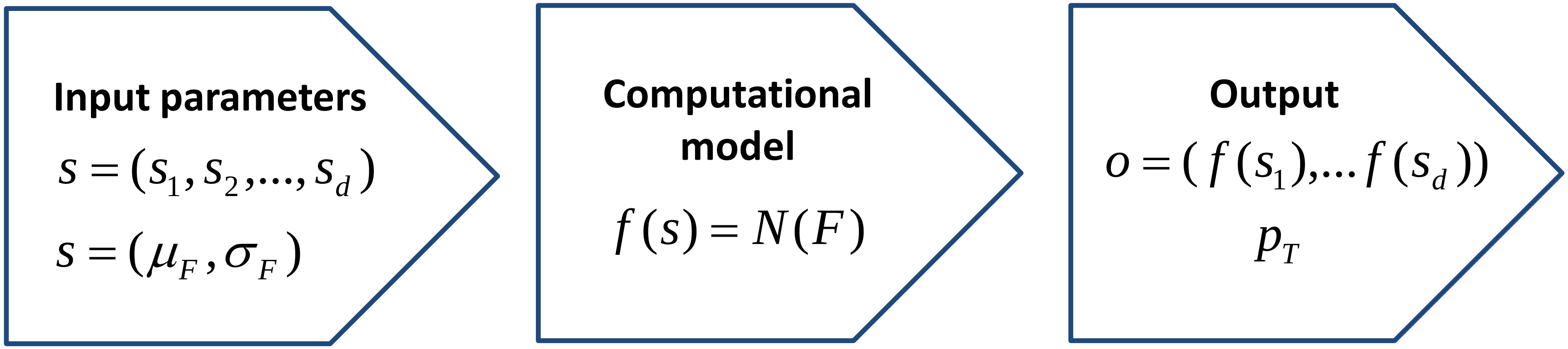
UQ analysis (-) and experimental results (□) for different conditions

Computed probability of transition

# 3. The VKI-H3 test case

## The inverse problem

### FORWARD PROBLEM



**Input parameters**

$s = (s_1, s_2, \dots, s_d)$

$s = (\mu_F, \sigma_F)$

**Computational model**

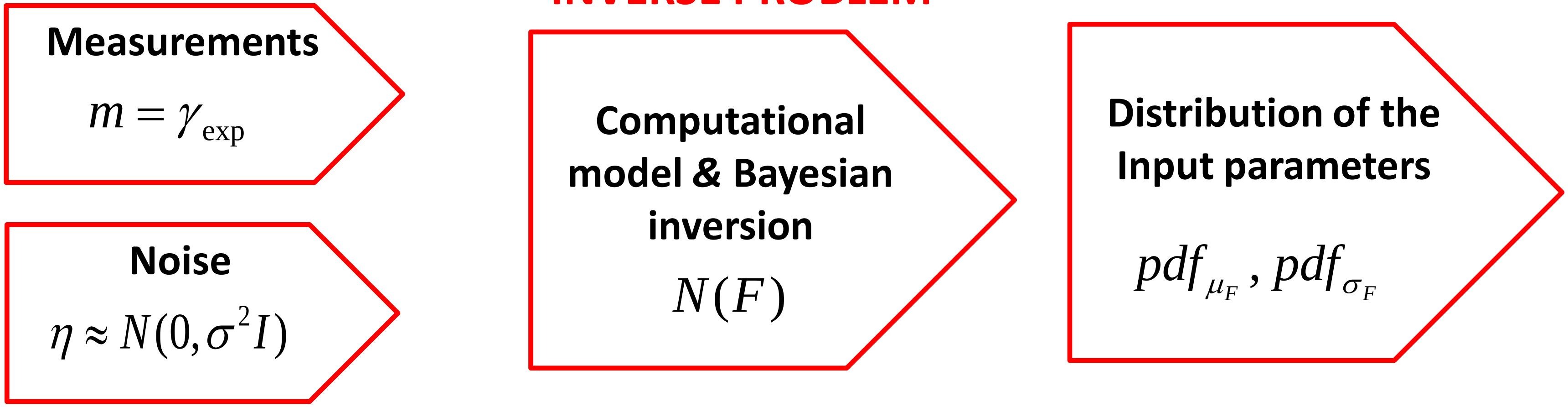
$f(s) = N(F)$

**Output**

$o = (f(s_1), \dots f(s_d))$

$p_T$

### INVERSE PROBLEM



**Measurements**

$m = \gamma_{\exp}$

**Noise**

$\eta \approx N(0, \sigma^2 I)$

**Computational model & Bayesian inversion**

$N(F)$

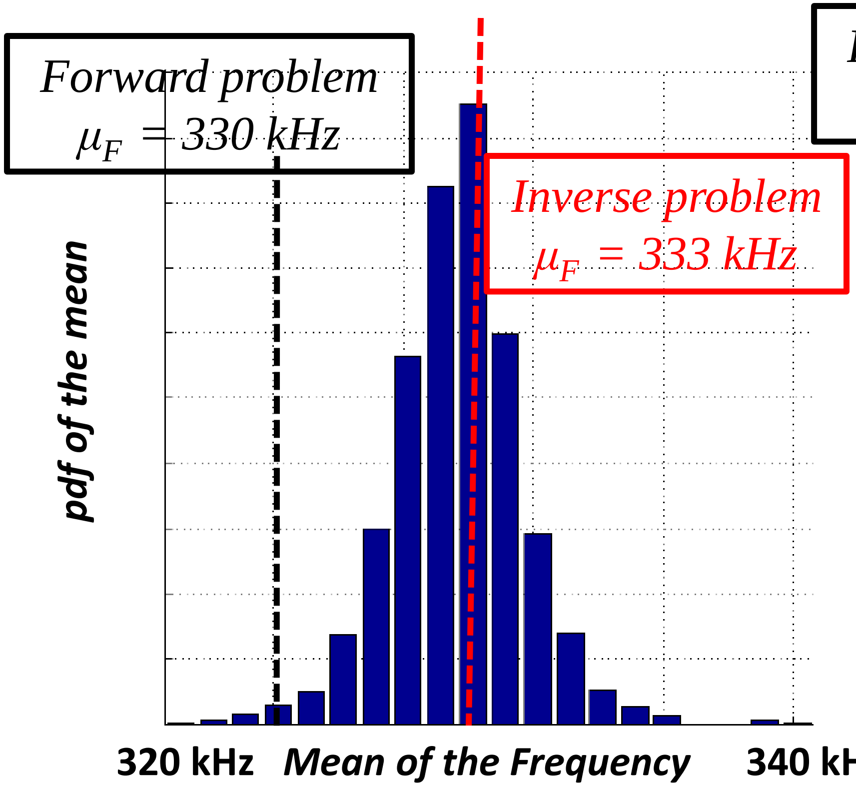
**Distribution of the Input parameters**

$pdf_{\mu_F}, pdf_{\sigma_F}$

# 3. The VKI-H3 test case

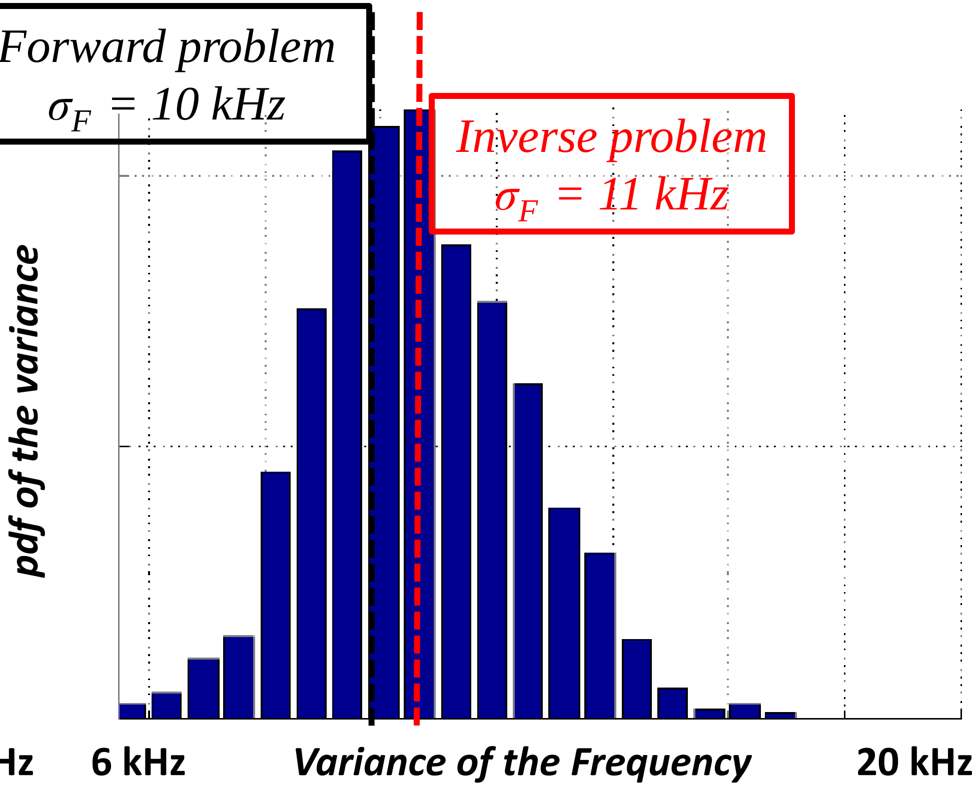
## The inverse problem

- ***VKI-H3 Low-Re***: MCMC to obtain the posterior *pdf* of the mean and the variance of the frequency distribution (Geweke's test for convergence)



Forward problem
$\mu_F = 330$ kHz

Inverse problem
$\mu_F = 333$ kHz



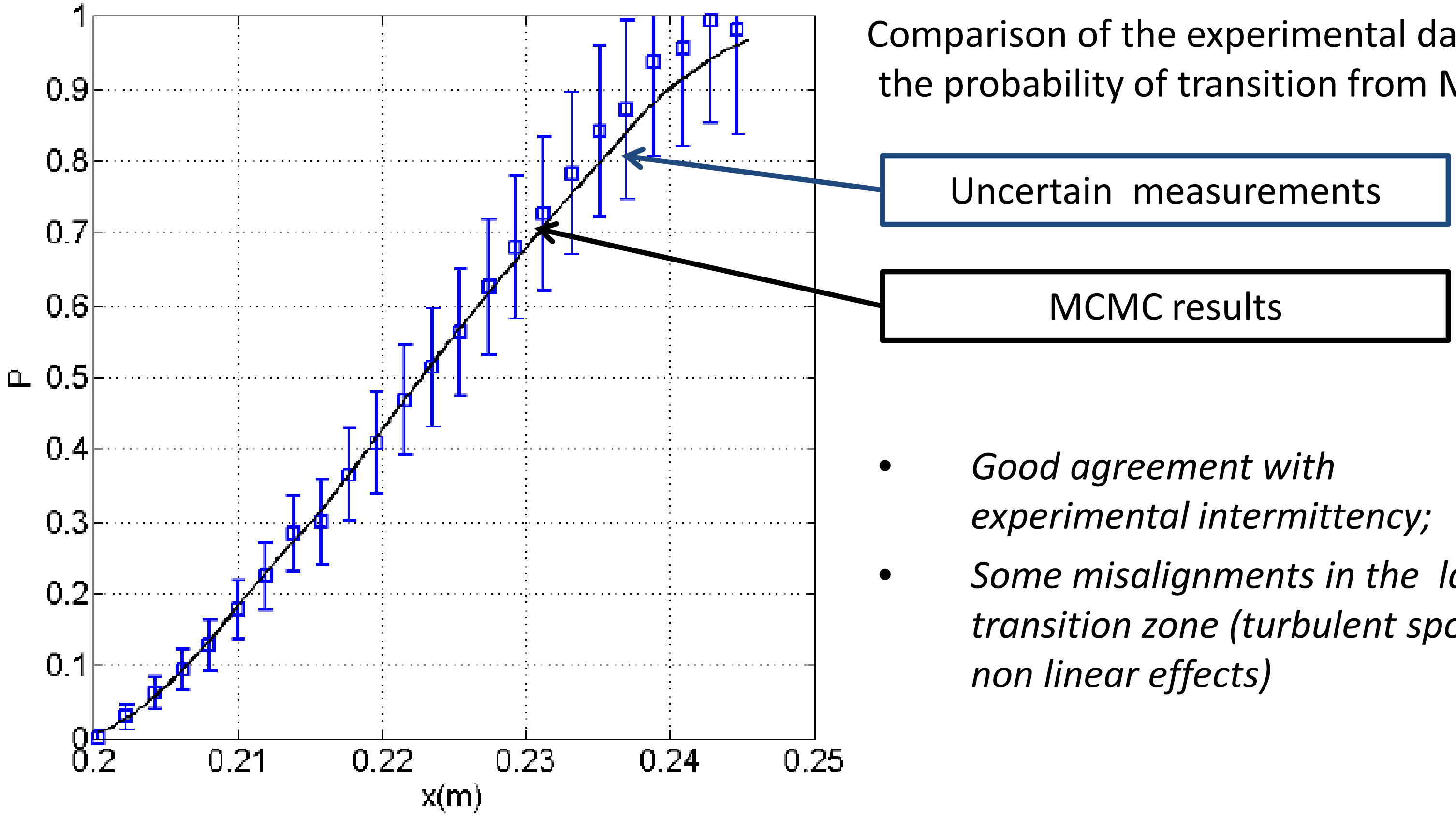
Forward problem
$\sigma_F = 10$ kHz

Inverse problem
$\sigma_F = 11$ kHz

# 3. The VKI-H3 test case

## The inverse problem

- ***Intermittency distribution*** *: VKI-Low Reynolds*



Comparison of the experimental data with the probability of transition from MCMC.

Uncertain measurements

MCMC results

- *Good agreement with experimental intermittency;*
- *Some misalignments in the late transition zone (turbulent spots, non linear effects)*

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4. **Conclusions & Future works**

# 4. Conclusions & Future works

## Conclusions

- ***Goal*** : combination of deterministic and probabilistic tools for transition prediction in high speed flow;
- ***Method*** : forward problem (intermittency distribution for given conditions) and inverse problem (frequency distribution for given measurements);
- ***Added value***
    - ***Forward problem*** –intermittency distributions resembling experimental data with fast and reliable computations (LST + $e^N$ method);
    - ***Inverse problem*** – inferring perturbation spectrum for given conditions;

## Future works

- ***RANS model for transition prediction*** : using the forward model to build a look-up table to obtain intermittency distributions at different conditions (Stanford SU2 code);
- ***New stochastic transition prediction method***;
- ***Comparison with experimental data*** : assessment of the assumptions for the inverse problem (frequency distributions) and comparison with experimental data;

# Thanks,

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I would like to thank Dr. Olaf **Marxen**, Prof. Gianluca **Iaccarino**, Dr. Catherine **Gorle** and Dr. Paul **Constantine** for their fundamental contribution.

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# 3. The VKI-H3 test case

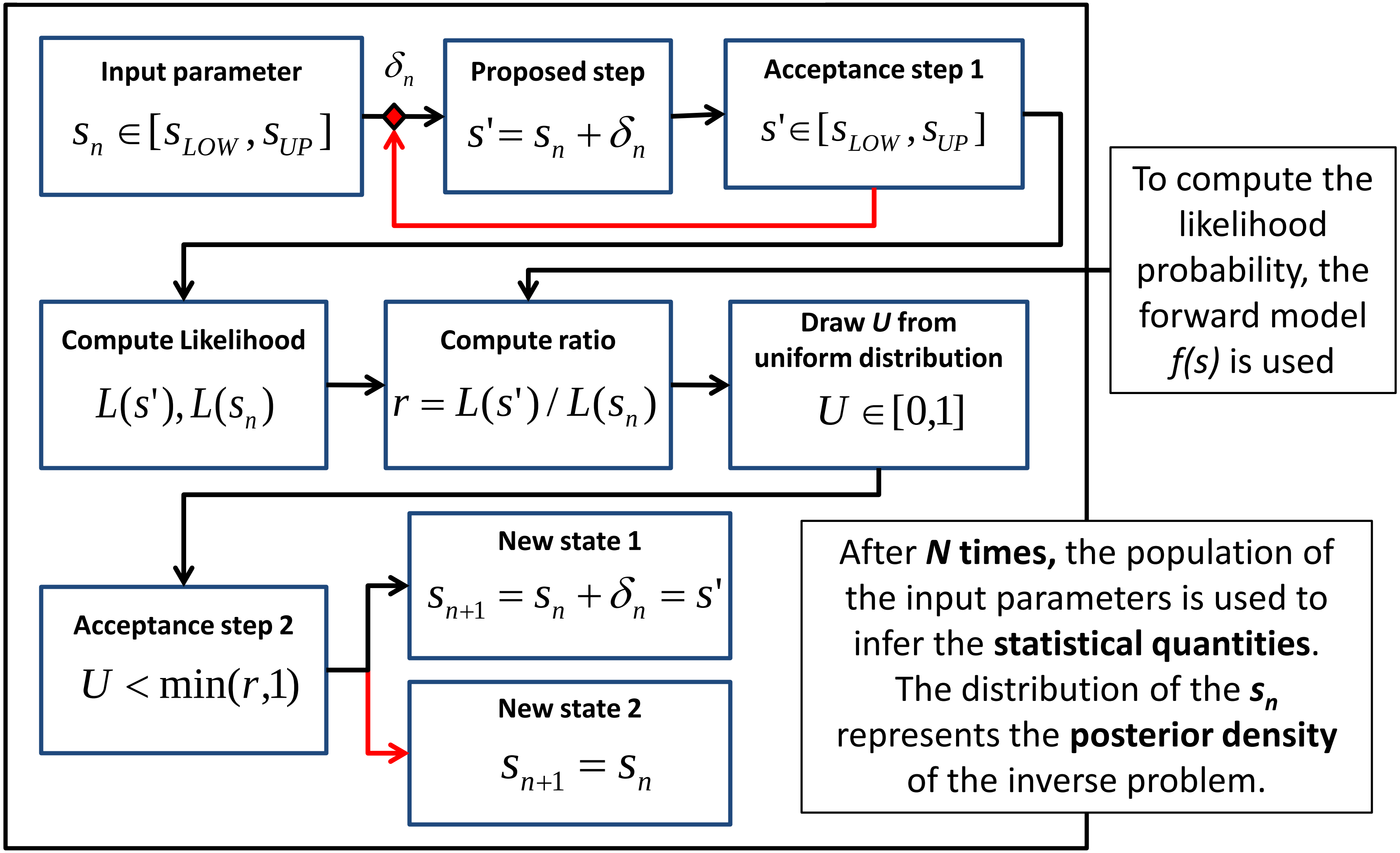
## The inverse problem

- $D$ input parameters $s = ( s_1 , s_2 , \ldots s_D )$ through the computational model $f(s)$ to give K outputs $m = ( g_1 (f(s_1 , r)), g_2 (f(s_2 , r)), \ldots g_k (f(s_k , r)) )$ with $r$ auxiliary parameters $s = ( r_1, r_2, \ldots r_N )$ ;
- ***The forward problem*** : solving $m$ with given $s$ and $r$;
- ***The inverse problem*** : inferring $s$ given the measurements of $m$ for given $r$;
- ***Parameters*** : input $s = ( s_1 , s_2 ) = ( \mu_F , \sigma_F )$ , auxiliary $r$ (conditions for the test cases), output $m = ( \gamma_1 , \gamma_2 \ldots, \gamma_k )$ at $x_1 , x_2 \ldots, x_k$ ;
- ***The strategy*** : given set of noisy measurements $m = m + \eta = ( \gamma_1 , \gamma_2 \ldots, \gamma_k )$ to seek for the input parameters $s = ( \mu_F , \sigma_F )$ using the computational model $f(s)$ ;
- ***The Bayesian inversion*** :

$$p(s|m) = \frac{p(m|s) \times p(s)}{p(m)} \propto p(m|s) \times p(s)$$

  - $p(s|m)$ = posterior *pdf* (probability of the input given the measurements)
  - $p(m|s)$ = likelihood *pdf* (probability of the measurements given the input)
  - $p(s)$ = prior *pdf* (information on the input parameters)

# The MCMC algorithm



**Input parameter**

$s_n \in [s_{LOW}, s_{UP}]$

$\delta_n$

**Proposed step**

$s' = s_n + \delta_n$

**Acceptance step 1**

$s' \in [s_{LOW}, s_{UP}]$

**Compute Likelihood**

$L(s'), L(s_n)$

**Compute ratio**

$r = L(s') / L(s_n)$

**Draw *U* from uniform distribution**

$U \in [0,1]$

To compute the likelihood probability, the forward model *f(s)* is used

**Acceptance step 2**

$U < \min(r,1)$

**New state 1**

$s_{n+1} = s_n + \delta_n = s'$

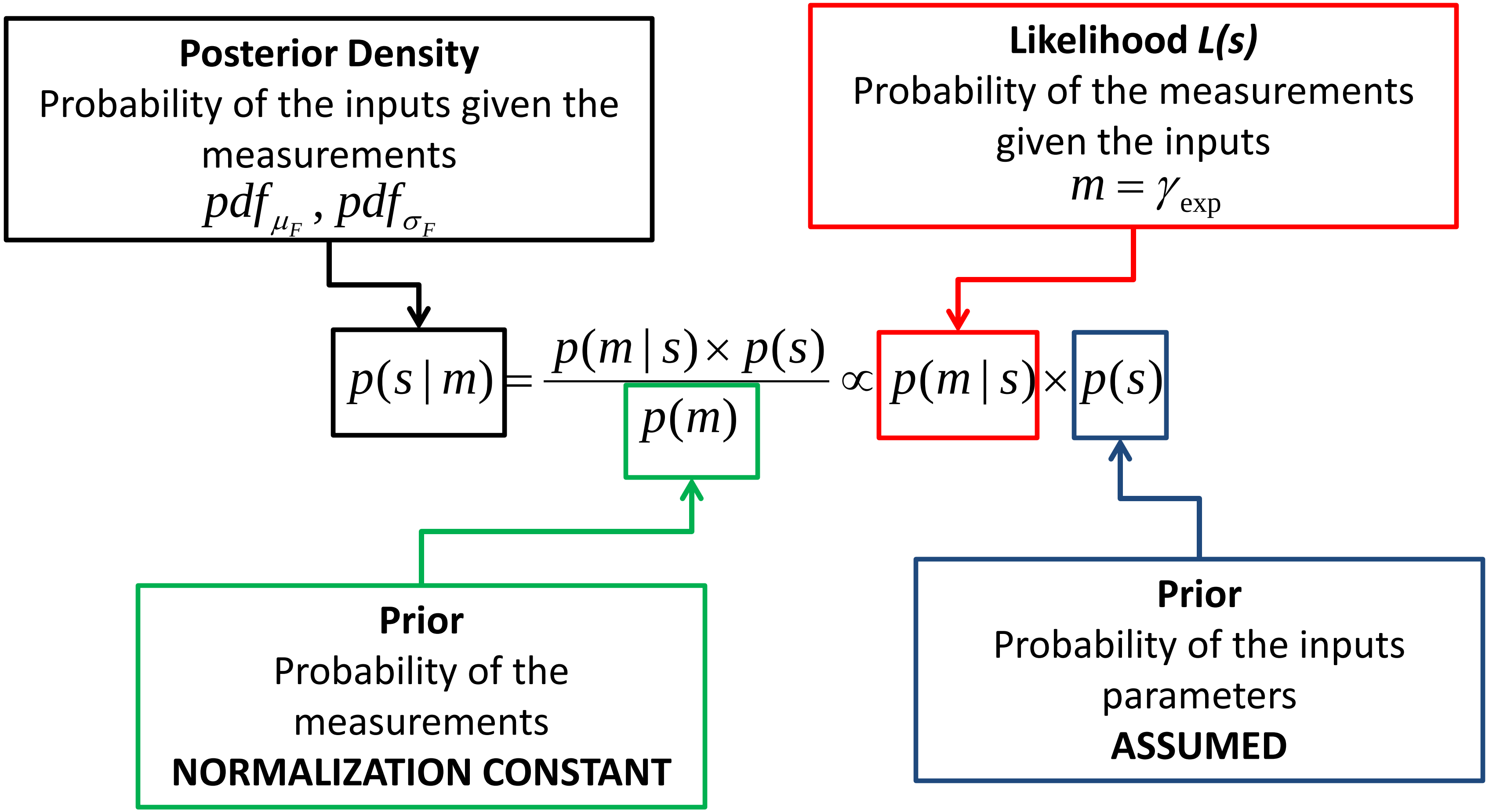
**New state 2**

$s_{n+1} = s_n$

After ***N* times,** the population of the input parameters is used to infer the **statistical quantities**. The distribution of the $\boldsymbol{s_n}$ represents the **posterior density** of the inverse problem.

# 3. The VKI-H3 test case

## The Bayesian inversion



**Posterior Density**
Probability of the inputs given the measurements
$pdf_{\mu_F}, pdf_{\sigma_F}$

**Likelihood *L(s)***
Probability of the measurements given the inputs
$m = \gamma_{\text{exp}}$

$$\boxed{p(s\,|\,m)} = \frac{p(m\,|\,s)\times p(s)}{\boxed{p(m)}} \propto \boxed{p(m\,|\,s)}\times\boxed{p(s)}$$

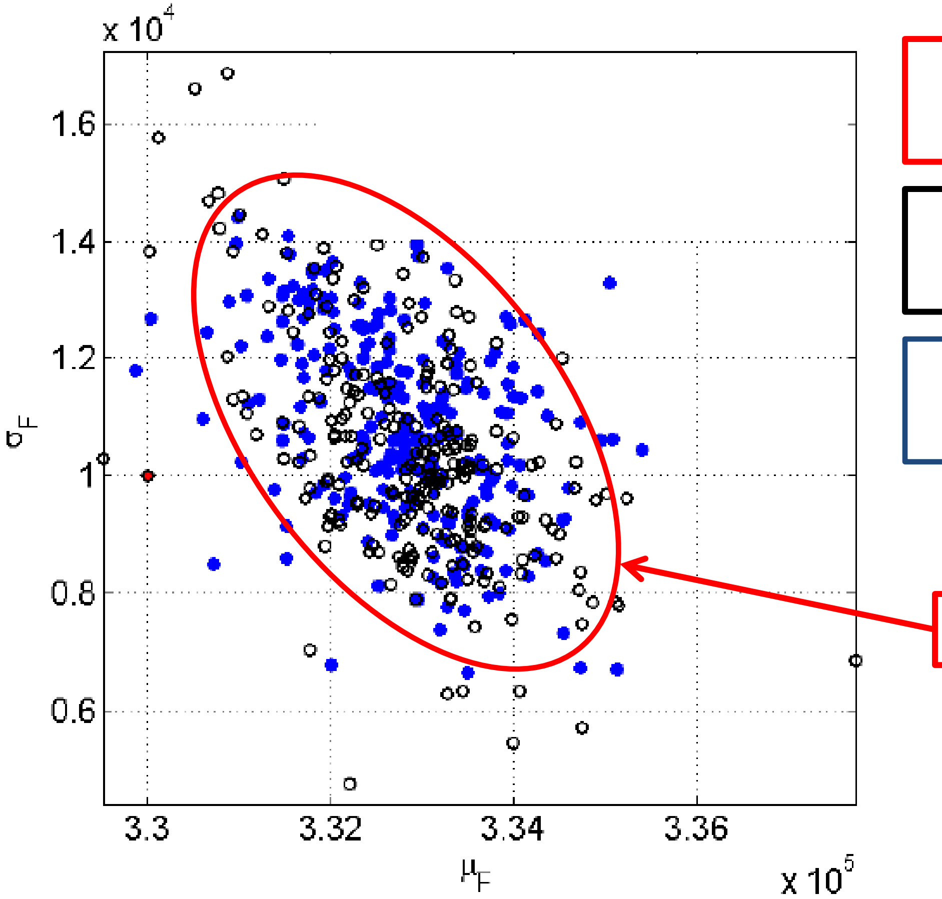
**Prior**
Probability of the measurements
**NORMALIZATION CONSTANT**

**Prior**
Probability of the inputs parameters
**ASSUMED**

# 3. The VKI-H3 test case

## The inverse problem

- ***MCMC algorithm*** : Markov Chain Monte Carlo to obtain the posterior *pdf*



Starting point
$\mu_0$ , $\sigma_0$

Samples within the burn-in period

Samples for the final distributions

Final distribution