EDGE CHARACTERIZATION IN HIERARCHICAL SUBBAND CODING SCHEMES

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INTRODUCTION

We may consider that an image is composed of textures and edges. By combining those two, it becomes possible to derive a high level image description like a collection of objects.

Marr [1,2] has suggested that the early visual processing of high level visual systems consists of a first primitive description of the image, which is called the *primal sketch* followed by geometrical connections which may be interpreted as an object extraction procedure.

The first steps of the primal sketch and the object extraction are conjectured to be an hierarchical approach, i.e the image and especially the edges are studied through different frequency channels. As pointed out, in the different frequency channels, edges should appear at the same position and with the same orientation. If singularities appear in some channels but not in those corresponding to lower frequencies, one may consider that they come from average effects between different "physical" singularities.

Marr detects those edges by filtering with the Laplacian of a two-dimensional Gaussian distribution. Edges coincide with the zero-crossings segments of the filtered signal. Directional second order derivatives allow to determine the edge orientation. This amounts to the same thing as to search for the direction of maximum slope at a given zero-crossings point.

Marr represents the edges by linear segments of zero-crossings points and by the amplitude of the first derivative of the filtered signal taken in the direction perpendicular to the segment.

Based on Logan's theorem [3] on the completeness of zero-crossings of bandpass signals, Marr conjectured that the knowledge of his edge representation is complete, i.e. sufficient to reconstruct the initial image. Marr imposes no given number of channels provided that they are at least two.

The subband decomposition and the wavelet theory constitutes a natural frame for application and analysis of Marr's theory. Experience confirms that there is a strong correlation between the different bands of a hierarchical subband scheme, especially for edges.

In what follows, we adopt a similar approach to Marr's. The idea behind the developments is to make use of the apparent signal redundancy in the different subbands.

Starting from the highest resolution, we first characterize the signal, in this case only edges. After this step, it is possible to predict the signal in the other bands.

The correlations are easy to establish when the different subbands are not downsampled.

Downsampling introduces spectrum foldings which perturbe the general theory: the information needed for edge or singularity characterization is degraded and spread over a region which becomes wider as the downsampling is increased. All the information remains present but it becomes quite trickier to extract.

SUBBAND DECOMPOSITION SCHEME AND WAVELETS

Hierarchical subband coding [4,5] is an easy way to implement Marr's conjecture. In this approach, the original signal is split up in different frequency channels by passing through a perfect reconstruction filter bank. The resulting signals are downsampled. In the hierarchical approach, the lowest frequency channel (i.e. the DC band) is the only one which is further decomposed by repeating the same procedure. The initial signal can be recovered by interpolation (i.e. by adding the correct number of zeros between each samples) and by passing through a synthesis filter

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bank which cancels aliasing effects.
It has been shown [6,7,8] that hierarchical subband decomposition is in fact a discrete wavelet transformation. A wavelet transform is mainly a frequential and spatial local filtering. Therefore it "reacts" only on singularities which are in its neighbourhood and which live at the same scale. In fact Marr's approach is a wavelet transformation with the Mexican hat (i.e. the second derivative of the Gaussian distribution).
Marr's conjecture implies that the edges are characterized by their behaviour at different scales (at least in two different channels). M. Holschneider has proved [9], that the regularity of a function at a given point is completely characterized by the behaviour, at different scales, of the wavelet transformation, in a neighbourhood of this point. When the signal is digitalized, we may expect that such properties are still satisfied: we should be able to recognize the type of singularities by looking at them at different levels of decomposition.
This implies strong correlations between corresponding subbands at different levels of decomposition. In fact, in the different subbands, an edge seems to repeat itself. It would be extremely interesting to use the knowledge of a subband to predict the edges in the corresponding subbands of the next levels of decomposition.
If we observe the resulting signals in the three AC subbands of the first level of decomposition, we see that an edge appears in two or three of the subbands. The purpose of this paper is to check the reality of those geometrical correlations and to use those results to predict the edge orientation. The AC subbands contains the high frequencies: i) in the horizontal direction (H subband); ii) in the vertical direction (V subband); iii) in the 45^0 and -45^0 directions (D subband).

Decorrelating Algorithm

Having stated the correlation for edges through scales and between subbands of a same resolution, we detail an algorithm for coding and predicting the edge signal in subbands so to eliminate as much redundancies as possible. It combines a analysing stage devoted to the step characterization and a predicting stage:

i) The band containing the horizontal high frequencies (first level) is coded exactly.

ii) After analysis, we associate the angle to the edge necessary to recover the diagonal content of the corresponding pixels of the first subband.

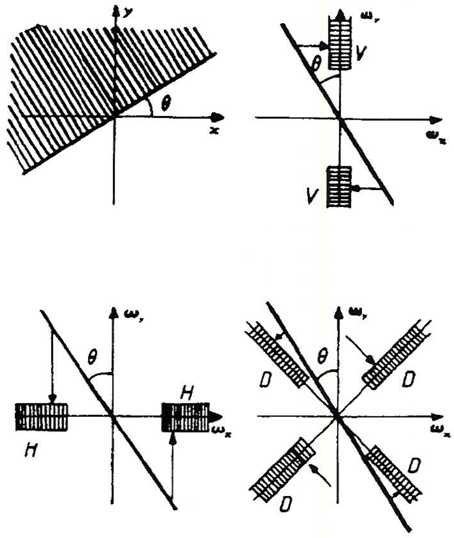

Figure 1: Model for geometrical charachterization of an edge.

iii) Then, we predict the corresponding pixels for the vertical high frequencies (first level) and we code the error (i.e. vertical edges will be completely coded in this subband).

iv) The procedure is repeated at lower resolutions (level 2 and 3). At each level, we exchange the part of the horizontal high frequencies with the one of the vertical high frequencies.

v) We keep going with this procedure for the lower frequency subband except that we try to predict also the value of the singularities in the first subband.

EDGE ORIENTATION DETECTION (Resolution of problem ii)

We now present a method to extract the edge orientation based only on the geometrical correlation between subbands at a given level of decomposition.
In the AC subbands, the edges may be easily detected by thresholding.
Figure 1 illustrates the simple model that we propose to extract the edges orientations. The H subband may be considered as the result of the

orthogonal projection of the high frequency content of the edge on the horizontal frequency axis. The same holds for the V subband. The D subband results from two orthogonal projections in the 45^0 and -45^0 directions. We denote $\hat{F}(\vec{\omega})$ the Fourier transform of the signal $F(\vec{x})$. $\hat{f}_\alpha(\vec{\omega})$ stands for the Fourier transform of the α subband. In first approximation, we obtain the following relationships:

$$\hat{f}_{(H)}(\vec{\omega}) = \hat{F}(\vec{\omega}) sin\theta \tag{1}$$

$$\hat{f}_{(V)}(\vec{\omega}) = \hat{F}(\vec{\omega}) cos\theta \tag{2}$$

$$\hat{f}_{(D)}(\vec{\omega}) = \hat{F}(\vec{\omega})[cos(\theta + 45^0) + cos(\theta - 45^0)] \tag{3}$$

$$\hat{f}_{(D)}(\vec{\omega}) = \sqrt{2}\hat{F}(\vec{\omega}) cos\theta \tag{4}$$

As an edge is spread in the AC subbands over a few pixels, we compute the energy content (E) in a small box centered on the point under study. We have the following predictors for the edge orientation:

$$|\theta_{pred\,HV}| = tan^{-1}\left(\frac{E_H}{E_V}\right)^{1/2} \tag{5}$$

$$|\theta_{pred\,HD}| = tan^{-1}(\left(\sqrt{2}\frac{E_H}{E_D}\right)^{1/2}) \tag{6}$$

$$|\theta_{pred}| = \frac{|\theta_{pred\,HV}| + |\theta_{pred\,HD}|}{2} \tag{7}$$

Numerical results are given for different orientations in table 1. It appears that for $\theta < 30^0$ and $\theta > 60^0$, the better prediction is given by $|(\theta)_{pred\,HV}|$.

It is also possible to compute the sum of the absolute values in the small box. $|(\theta)_{pred\,HV}|$ still give excellent predictions. In that case, the prediction of $(\theta)_{pred\,HD}$ becomes quite less accurate.

The sum of the signal values in the small boxes is not significant because of the oscillations of the wavelet functions which lead to negative interferences and perturbe the summation.

Equations 5 - 7 give predictions of the absolute value of θ. When $|\theta_{pred\,HV}|$ and $|\theta_{pred\,HD}|$ are too different, we may consider that the point under study is an isolate singularity or a corner.

It is possible to determine sign(θ) by the following technique. Let us consider the horizontal line which contains the point under study. Let Ω be the intersection of this line with the spatial region of influence of the edge. We have:

$$sign(\theta) = sign(\sum_{x \in \Omega} sign(f_H(\vec{x}))sign(f_V(\vec{x})) \tag{8}$$

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Figure 2: Signs of a subband decomposition signal (with Johnston's filters of rank 8) after thresholding. The band order is: DC band, H band, V band and D band. Character o is due to the threshold.

This is clearly illustrated with the help of figure 2 and 3.

The algorithm presented hereabove can be used to easily segment the different subbands and the original image. This is interesting for object recognition.

CONCLUSION

This paper presents a general decorrelating algorithm for signals like edges in a hierarchical subband coding scheme. It consists in first analyzing the highest frequency subbands and in predicting the signal in the other bands.

The analyzing step should determine the edge orientation and collect information about the

TABLE 1 - Angle prediction for a step edge.

Real angle in degree	45	49.84	55.49	62.02	69.44	77.66	86.42
Horizontal energy E_H	89.18	153.35	215.21	285.36	343.73	389.59	409.36
Vertical energy E_V	89.18	89.24	86.67	84.88	79.77	43.64	10.03
Diagonal energy E_D	315.96	265.50	207.39	135.81	78.14	40.68	10.03
$\lvert \theta_{pred\,HV} \rvert$	45	52.66	57.6	61.47	64.28	71.49	81.1
$\lvert \theta_{pred\,HD} \rvert$	37.04	47.06	55.23	64	71.37	77.13	83.68
$\lvert \theta_{pred} \rvert$	41.02	49.86	56.42	62.73	67.82	74.31	82.39

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Figure 3: Determination of $sign(\theta)$ by observing the product (third table) of the H and V bands.

evolution in different bands of the signal perpendicular to the edge orientation. A numerical example shows the efficiency of the orientation analysis.

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