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# An extended Drucker-Prager hardening model for cross-anisotropy of soft rocks

## Un modèle de comportement à écrouissage de type Drucker-Prager pour roches tendres à anisotropie transverse

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ABSTRACT

The mechanical behaviour of natural geomaterials is often anisotropic. Sedimentary rocks usually show a limited form of anisotropy, called cross-anisotropy. This paper presents an original constitutive model based on a hardening Drucker-Prager elasto-plastic framework that has been adapted to consider cross-anisotropic elasticity as well as an anisotropic plastic criterion. The cohesion is a function of the angle between the direction of the major compressive stress and the normal to the bedding plane. This original model consists in a relatively simple upgrading of a well-known elasto-plastic model, well-adapted for sedimentary hard soils or soft rocks. The ability of the model to reproduce the directional dependency of the elasto-plastic response of clay rocks, as observed in triaxial tests, is proved by numerical simulations of experimental tests.

RÉSUMÉ

Le comportement mécanique de géomatériaux naturels est souvent anisotrope. Les roches sédimentaires ont la plus part du temps une forme d'anisotropie limitée, appelé anisotropie transverse. Ce papier présente un modèle de comportement original basé sur un modèle élasto-plastique de type Drucker-Prager à écrouissage qui a été adapté pour considérer l'élasticité anisotrope transverse ainsi qu'un critère de plasticité anisotrope. La cohésion dépend de l'angle entre la direction de la contrainte majeure de compression et la normale au plan de stratification. Ainsi, ce modèle original constitue une adaptation relativement simple d'un modèle élasto-plastique bien connu, bien adapté pour les matériaux sédimentaires tels que les sols indurés ou les roches tendres. La capacité du modèle à reproduire la dépendance directionnelle de la réponse élasto-plastique des roches argileuses, comme observé dans les essais triaxiaux, est évaluée grâce à des simulations numériques.

Keywords: Cross-anisotropy, mechanical behaviour, hardening, elasto-plasticity, sedimentary rocks

## 1 INTRODUCTION

Anisotropy is an important factor determining the behaviour of clay soft rocks. Clay rocks, as most of the sedimentary rocks, exhibit anisotropy mainly related to their bedding plane orientation due to their depositional nature. The properties of such materials are usually independent of rotation about an axis of symmetry normal to the bedding plane. This type of anisotropy is called transverse isotropy or cross-anisotropy.

This work presents the development and the validation of a mechanical constitutive model that extends the symmetric Drucker-Prager yield criterion [1] to cross-anisotropic materials [2] and that is coupled with the classical cross-anisotropic elasticity [3]. The new criterion assumes that the strength of materials may vary ac-

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cording to the orientation of the principal compressive stress with respect to the bedding plane orientation. Such a criterion remains in agreement with cross-anisotropy in the sense that it is unaffected by the rotation around the normal of bedding plane (axe $e_3$ in Figure 1). In addition, the developed model allows a progressive hardening or softening process through the evolution of the mobilized cohesion and friction angle with plastic strain. The ability of the model to reproduce the directional dependency of the elasto-plastic response of clay rocks is proved by the means of comparison between numerical predictions and experimental results of triaxial tests.

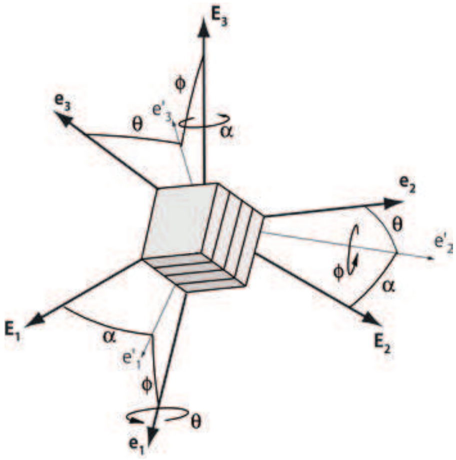


Figure 1. Transformation of the global axis ($E_1$, $E_2$, $E_3$) into anisotropic axes ($e_1$, $e_2$, $e_3$). $e_1$ and $e_2$ are orthogonal axis in the bedding plane while $e_3$ is the normal to bedding plane.

## 2 MECHANICAL CONSTITUTIVE MODEL

Because of elastic anisotropy, the elasto-plastic stress-strain relations are more convenient to be expressed in the anisotropic axis, as indicated by the star in exponent ($\sigma'^{*}_{ij}$ and $\varepsilon^{*}_{ij}$). In the more general situation, the reference axes do not coincide with the axes of anisotropy and the expression of $\sigma'^{*}_{ij}$ and $\varepsilon^{*}_{kl}$ can be obtained from $\sigma'_{ij}$ and $\varepsilon_{kl}$ expressed in the system of reference:

$$\sigma'^{*}_{ij} = R_{ki} R_{lj} \sigma'_{kl} \quad ; \quad \varepsilon^{*}_{ij} = R_{ki} R_{lj} \varepsilon_{kl} \tag{1}$$

with $R$ being the rotation matrix:

$$R = \begin{bmatrix} \cos\alpha\cos\varphi & \sin\alpha\cos\varphi & \sin\varphi \\ \begin{array}{l}-\sin\alpha\cos\theta \\ -\sin\theta\sin\varphi\cos\alpha\end{array} & \begin{array}{l}\cos\alpha\cos\theta \\ -\sin\theta\sin\varphi\sin\alpha\end{array} & \sin\theta\cos\varphi \\ \begin{array}{l}\sin\theta\sin\alpha \\ -\cos\alpha\sin\varphi\cos\theta\end{array} & \begin{array}{l}-\sin\varphi\sin\alpha\cos\theta \\ -\sin\theta\cos\alpha\end{array} & \cos\varphi\cos\theta \end{bmatrix} \tag{2}$$

$\alpha$ is the rotation angle around the axes $\underline{E}_3$ (rotation in the $(\underline{E}_1, \underline{E}_2)$ plane), the angles $\varphi$ and $\theta$ defines the rotation around the axes $\underline{e}'_2$ and $\underline{e}_1$, respectively (Figure 1). The positive direction of rotation is counter-clockwise. $(\underline{E}_1, \underline{E}_2, \underline{E}_3)$ and $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ are the reference axes and the anisotropic axes, respectively.

The elastic component of the strain rate $\dot{\varepsilon}^{*e}_{ij}$ is linked to stress rate through the Hooke law :

$$\dot{\varepsilon}^{*e}_{ij} = D^{e}_{ijkl} \dot{\sigma}'^{*}_{kl} \tag{3}$$

The $D^{e}_{ijkl}$ matrix considers cross-anisotropic elasticity which requires 5 independent parameters [3]. The elastic compliance matrix is:

$$D^{e}_{ijkl} = \begin{bmatrix} \frac{1}{E_{//}} & \frac{-\nu_{////}}{E_{//}} & \frac{-\nu_{\perp//}}{E_{\perp}} & & & \\ \frac{-\nu_{////}}{E_{//}} & \frac{1}{E_{//}} & \frac{-\nu_{\perp//}}{E_{\perp}} & & 0 & \\ \frac{-\nu_{//\perp}}{E_{//}} & \frac{-\nu_{//\perp}}{E_{//}} & \frac{1}{E_{\perp}} & & & \\ & & & \frac{1+\nu_{////}}{E_{//}} & & \\ & 0 & & & \frac{1}{2G_{//\perp}} & \\ & & & & & \frac{1}{2G_{//\perp}} \end{bmatrix} \tag{4}$$

where the subscripts // and $\perp$ indicates, respectively, the direction parallel to bedding (directions 1 and 2) and perpendicular to bedding (direction 3). The symmetry of the stiffness matrix imposes that

$$\frac{\nu_{\perp//}}{E_{\perp}} = \frac{\nu_{//\perp}}{E_{//}} \tag{5}$$

The limit between the elastic and the plastic domain is represented by the Drucker-Prager yield surface $f$ [1]:

$$f = II_{\hat{\sigma}} - \frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)}\left(I_{\sigma} - \frac{3c}{\tan\phi}\right) = 0 \tag{6}$$

where $c$ and $\phi$ are the cohesion and the friction angle. $I_{\sigma}$ and $II_{\hat{\sigma}}$ are the first stress tensor invariant and the second deviatoric stress tensor invariant, respectively.

The material cohesion depends on the angle between major principal stress and the normal to the bedding plane ($\alpha_{\sigma_1}$). Three cohesion values are defined, for major principal stress parallel ($\alpha_{\sigma_1} = 0°$), perpendicular ($\alpha_{\sigma_1} = 90°$) and with an angle of 45° ($\alpha_{\sigma_1} = 45°$) with respect to the normal to bedding plane. Between those values, cohesion varies linearly with $\alpha_{\sigma_1}$ (Figure 2).

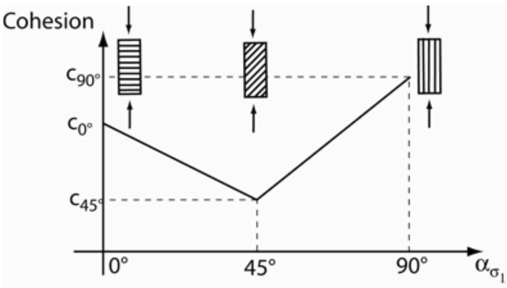


Figure 2. Schematic view of the cohesion evolution as a function of the angle between the normal to bedding plane and the direction of major principal stress.

A general non-associated plasticity framework is considered with the plastic potential $g$ :

$$g = II_{\hat{\sigma}} + \frac{2\sin\psi}{\sqrt{3}(3-\sin\psi)} I_{\sigma} = 0 \tag{7}$$

where $\psi$ is the dilatancy angle.

The plastic multiplier $\dot{\lambda}$ is obtained from the consistency condition:

$$df = \frac{\partial f}{\partial \sigma'^{*}_{ij}}\dot{\sigma}'^{*}_{ij} + \left(\frac{\partial f}{\partial \phi}\frac{d\phi}{d\varepsilon^{p}_{eq}} + \frac{\partial f}{\partial c}\frac{dc}{d\varepsilon^{p}_{eq}}\right)\frac{\sqrt{3}}{3}\dot{\lambda} = 0 \tag{8}$$

The used model is a hardening Drucker-Prager model that allows hardening/softening processes during plastic flow. This is introduced via an hyperbolic variation of the friction angle and the cohesion between initial ($\phi_0$ and $c_0$) and critical state ($\phi_f$ and $c_f$) values as a function of the Von Mises equivalent plastic strain $\varepsilon^{p}_{eq}$ [4]:

$$\phi = \phi_0 + \frac{(\phi_f - \phi_0)\varepsilon^{p}_{eq}}{B_p + \varepsilon^{p}_{eq}} \; ; \; c = c_0 + \frac{(c_f - c_0)\varepsilon^{p}_{eq}}{B_c + \varepsilon^{p}_{eq}} \tag{9}$$

where $B_p$ and $B_c$ are materials parameters.

## 3 CONSTITUTIVE BEHAVIOUR

In the following, two series of triaxial tests are analysed and numerically simulated: (i) on Tournemire shale [5] and (ii) on Opalinus clay [6]. Both series of tests clearly underline that the mechanical response of the clay is highly affected by the direction of loading with respect to the bedding plane. Figure 3 shows the evolution of the peak strength of Tournemire shale as a function of the confining pressure and the loading orientation as observed experimentally [5] and compared them with the predictions of the model. Figures 4 and 5 compare the result of numerical simulations with the experimental results at 1 MPa of confining pressure for Tournemire shale and 15 MPa for Opalinus clay, respectively. For both materials, the elastic rigidity of the sample is affected by the direction of loading. The samples loaded parallel to the bedding ($\alpha_{\sigma_1} = 90°$) exhibit the highest rigidity but they are more brittle. For the Tournemire shale, the shear strengths of samples loaded in the direction of bedding ($\alpha_{\sigma_1} = 90°$) and perpendicular to bedding ($\alpha_{\sigma_1} = 0°$) are approximately equal. However, for Opalinus clay, the maximum shear strength is observed for $\alpha_{\sigma_1} = 90°$. For both materials, the minimum strength is obtained for loading directions around 45° with respect to the bedding plane orientation. The model reproduces well the anisotropic response of the materials, at least before the peak. In the post-peak behaviour, instability phenomena make the processes much more complex.

## 4 CONCLUSIONS

In many applications the anisotropic character of the natural hard soil or soft rock must be carefully considered. Consequently, in addition to cross-anisotropic elasticity the Drucker-Prager plastic criterion has been upgraded considering that the cohesion depends on the angle between the direction of the major compressive stress and the normal to the bedding plane. Numerical simulations of triaxial tests of sedimentary rocks have shown that the model is able to reproduce the strong elastic and plastic anisotropy of sedimentary soft rocks.

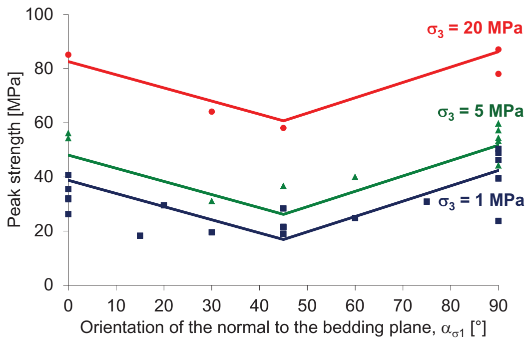


Figure 3. Evolution of the peak strength of Tournemire shale as a function of the confining pressure and the orientation of the bedding plane. Experimental results from [5] (points) compared with model predictions (lines).

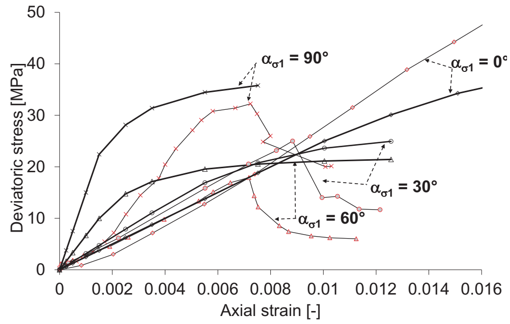


Figure 4. Numerical modelling of triaxial compression tests with a confining pressure of 1 MPa on Tournemire shale. Thin lines: Experiment; Bold lines: Modelling.

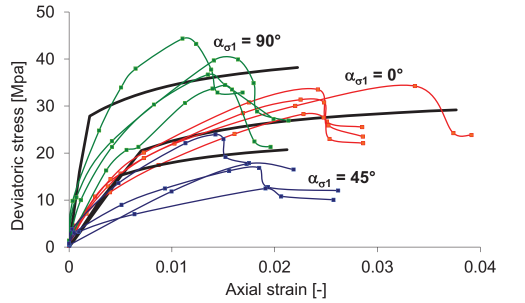


Figure 5. Numerical modelling of triaxial compression tests with a confining pressure of 15 MPa on Opalinus clay. Lines with points: Experiment; Bold lines: Modelling.

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