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Highly Transient Mixed Flows with Air/Water Interactions: Homogeneous Equilibrium Model and Friction Correlations

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Abstract: **An original 1D unified numerical model dealing with aerated mixed flow is derived and applied to the case of a gallery. The mathematical model is based on a new area-integration of the Homogeneous Equilibrium Model (HE-Model) over the cross-section of a free-surface flow and consists in a simple set of equations analogous to the Saint-Venant equations for single-phase flow. The frictional pressure drop is computed by means of three types of correlations, namely the homogeneous friction, the Lockhart-Martinelli correlation and the Müller-Steinhagen and Heck correlation. Performances of each method are compared. Finally, both free-surface and pressurized flows are mathematically modeled by means of the free-surface set of equations (Preissmann slot model). The original concept of the negative Preissmann slot is proposed to simulate sub-atmospheric pressure. This model is shown to be particularly well suited for the simulation of bubbly and intermittent flows**

Keywords: **Numerical Simulation, Mixed Flow, Two-phase Flow, Preissmann Slot, Two-phase Frictional Pressure Drop**

I. INTRODUCTION

Mixed flows, characterized by the simultaneous occurrence of free-surface and pressurized flows, are frequently encountered in rivers networks, sewer systems, storm-water storage pipes, flushing galleries, bottom outlets,… As a matter of fact, some hydraulic structures are designed to combine free-surface and pressurized sections (e.g. water intakes). In addition, dynamic pipe filling bores may occur in hydraulic structures designed only for conveying free-surface flow under an extreme water inflow or upon starting a pump. During such a transition, highly transient phenomena appear and may cause structural damages to the system [1], generate geysers through vertical shafts [2], engender flooding,… What is more, air/water interactions may arise, particularly at the transition bore [3], and alter thoroughly the flow regime and its characteristics. On account of the range of applications affected by mixed flows, a good prediction of mixed flow features is an industrial necessity. Numerical simulation of mixed flow remains however challenging for two main reasons. Dissimilarity in the pressure term arises between the classical sets of equations describing free-surface and pressurized flows. Air/water interaction has to be taken into account through a two-phase flow model.

Different mathematical approaches to describe mixed flows have been developed to date. First, the so-called *shock-tracking approach* consists in solving separately free-surface and pressurized flows through different sets of equations [4, 5]. However, such an algorithm is very complicated and case-specific so that it cannot be applied for practical applications. Second, the *Rigid Water Column Approach* [6] treats each phase (air/water) separately on the basis of a specific set of equations. The latter approach succeeds in simulating complex configurations of the transition but fails in its attempt to describe all flow regimes. What is more, using the method for practical application is not possible because of the complexity and specificity of the algorithm. Third, the so-called *shock-capturing* approach is a family of method which computes pressurized and free-surface flows by using a single set of equations [7-10]. In this paper, a shock-capturing approach is used, based on the model of the Preissmann slot [7]. Free-surface flow and pressurized flow are this way equally solved through a free surface set of equations. An original concept developed by the authors, the *negative Preissmann slot*, extends the Preissmann slot model to simulate sub-atmospheric pressurized flows.

Computing air-water interaction requires using a two-phase flow model. On the one hand, to authors' knowledge, no mixed flow model takes into account the effect of entrained air in the water flow. Only the air phase pressurization is usually modeled, as in the Rigid Water Column [6] and in the shock-capturing model of Vasconcelos [8]. On the other hand, usual multiphase flow investigations focus mainly on fully pressurized flow in small diameter pipes for chemical and mechanical engineering applications. There have been only a few attempts, often based on a transport equation [11], to simulate air entrainment in large hydraulic structures. Consequently, the current research aims at applying classical model for multiphase flow to civil engineering applications. In this paper, a Homogeneous Equilibrium Model (HE-Model) coupled with the Preissmann slot model is derived. For this purpose, the local instant formulation [12], which characterized each phase individually and the interfacial transfers, is time-averaged; By introducing macroscopic variables in terms of mixture properties and assuming equilibrium between each phases, the 3D HE-Model is established. Then, a new area-integration of the Homogeneous Equilibrium Model (HE-Model) over the cross-section of a free-surface flow gives a simple mathematical model analogous to the Saint-Venant equations for single-phase flow. Finally, the 1D HE-model derived is closed by specification of constitutive equations. The pressure constitutive equation is derived by using a non-dimensional analysis of the neglected momentum equations and the Phase change source term is derived from experimental measures. The frictional pressure drop is computed by means of three types of correlations, namely the homogeneous friction, the Lockhart-Martinelli correlation and the Müller-Steinhagen and Heck correlation. Performances of each method are compared.

These developments have been implemented in the one-dimensional module of the software package WOLF. WOLF is finite volume flow simulation modelling system developed within the Laboratory of Hydrology, Applied Hydrodynamics and Hydraulic Constructions (HACH) of the University of Liege.

Application to this new model to the case of flows in a gallery is presented in this paper. Experimental results from a physical model build in the Laboratory of Structures Hydraulics of the University of Liege are used for comparison with numerical results.

II. HOMOGENEOUS EQUILIBRIUM MODEL

A. 3D Time-averaged Governing Equations

If we assume that each sub-region bounded by interfaces in an air-water flow may be considered as a continuum, the standard single-phase Navier-Stokes equations holds for each subregion with appropriate jump and boundary conditions. This is the Local Instant Formulation (LIF) [12] which is summarized in Fig. 1.

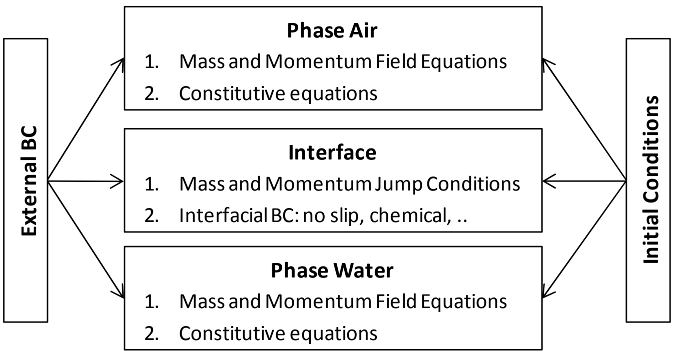


Fig. 1 : Local Instant Formulation according [12]

In principle, a two-phase flow model should solve the local instant formulation. Obtaining a solution this way is however mathematically difficult and beyond the present computational capability for many engineering applications. On account of this, practical model have been developed. Majority of them is notably derived by application of averaging procedure on the LIF. Many averaging methods have been developed to date and used to study two-phase flow systems: the Eulerian Averaging, the Lagrangian Averaging and the Boltzmann statistical averaging [13]. In the present work, the Eulerian time averaging procedure is chosen because it is proven to be particularly useful for turbulent two-phase flow. As pointed in Fig. 2, it results into different models according the choice of the macroscopic variables.

The first type of model is called two-fluid model. Each phase is treated as a separate fluid with its own set of governing equations [14]. Difficulties stem from the derivation of interfacial conditions [12]. In the second type of method, it is assumed that the multiphase flow may be described as a single phase flow of mixture variables which refers to the motion of the centre of mass. The motion of the dispersed phase is then treated in terms of diffusion through the mixture. This model is usually called drift-flux model [15]. Since the momentum equation for this phase is neglected, a constitutive equation for the relative velocity is required. In particular, if all phases are assumed to move at the same velocity (the relative velocity is negligible), it results in the Homogeneous Equilibrium Model (HE-Model).

Homogenous flow theory provides the simplest technique for analyzing multiphase flows. Using suitable averaged properties, the fluid is treated as a pseudo-fluid that obeys the usual equations of a single-component flow. This assumption is particularly suited for dispersed bubbly flow. The model is commonly used for the simulation of heat exchangers [16, 17], two-phase flow in ducts [18],…In the current section, the layout of the 3D HE-Model is presented and then the 1D free-surface HE-Model resulting from an original area-integration is introduced.

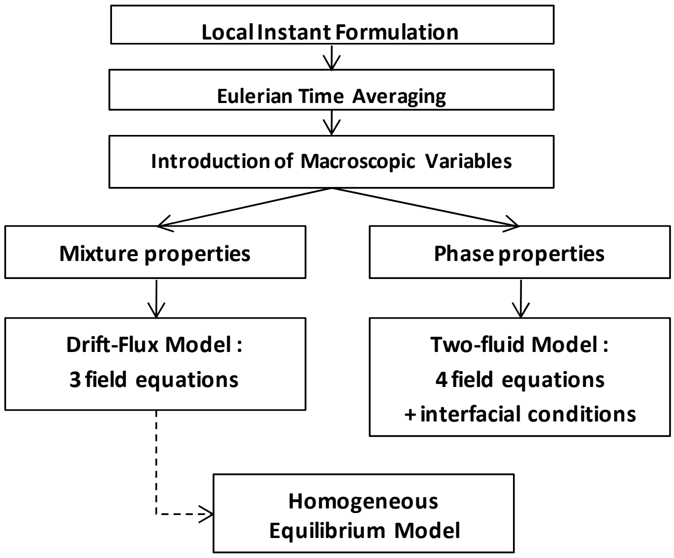


Fig. 2 : Modelling approaches

3D HE-model is derived through the time averaging of the Local Instant Formulation for multiphase flow, the introduction of suitable mixture variables and the assumption of equilibrium between phases. For further details, we refer the interested reader to the classical book of Ishii and Hibiki [12]. The resulting field equations are written as:

- The continuity equation:

$$\frac{\partial \rho_m}{\partial t} + \nabla\left(\rho_m \mathbf{v}_m\right) = 0 \tag{1}$$

- The diffusion equation:

$$\frac{\partial \alpha_g}{\partial t} + \nabla\left(\alpha_g \mathbf{v}_m\right) = \Gamma_g \tag{2}$$

- The momentum equation:

$$\frac{\partial \rho_m \mathbf{v}_m}{\partial t} + \nabla\left(\rho_m \mathbf{v}_m \mathbf{v}_m\right) = -\nabla p_m + \nabla\left(\tau_m + \tau^T\right) + \rho_m \mathbf{g} + \mathbf{M}_m \tag{3}$$

where ρ_m[kg/m³] is the mixture density, $\mathbf{v}_m$[ms^{-1}] is the mixture velocity vector (under the assumption of velocity equilibrium, $\mathbf{v}_m = \mathbf{v}_{water} = \mathbf{v}_{air}$), α_g[-] is the air void fraction, Γ_g[s^{-1}] is the phase change volume generation, p_m[Nm^{-2}] is the mixture pressure, τ_m[Nm^{-2}] and τ^T[Nm^{-2}] are the viscous and turbulent stress tensors, g[ms^{-2}] is the gravity and $\mathbf{M}_m$[kgs^{-2}m^{-}

[2]] is the interfacial momentum source. It is worthwhile noting that the simplicity of equations (1) to (3) results from the wise choice of the mixture macroscopic properties.

Closure of the HE-model requires the definition of the mixture variables and a constitutive equation. Air and water are supposed to be incompressible Newtonian fluid, and the mixture properties are written as follows:

$$\begin{aligned} \rho_m &= \alpha_g\rho_g + \left(1-\alpha_g\right)\rho_w \cong \left(1-\alpha_g\right)\rho_w \\ \tau_m &= \left[\alpha_g\mu_g + \left(1-\alpha_g\right)\mu_w\right]\left(\nabla.\mathbf{v} + \left(\nabla.\mathbf{v}\right)^T\right) \end{aligned} \tag{4}$$

At this point, no assumption is needed for the constitutive equations of the turbulent stress τ^T, the phase change volume generation Γ_g, the pressure distribution p_m and the mixture momentum source $\mathbf{M}_m$. These terms will be taken into account by means of macroscopic laws specifically derived for the 1D model.

B. 1D Area-integrated HE-Model

In many cases, the computational domain is essentially one-dimensional (both the flow depth and width are way smaller than the flow length) and the computation effort can be greatly reduced by simplifying two-equations of momentum and area-integrating the remaining equations [19]. The originality of the present paper is to consider a free-surface flow in the integration process. It is indeed shown in section III.D how the free-surface set of equations can be used to simulate pressurized flow as well. It results in the 1D free-surface HE-Model.

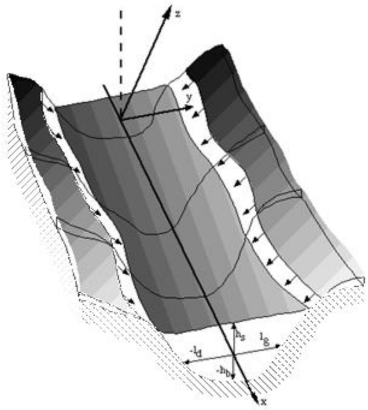


Fig. 3 : Domain of integration

For this purpose, a Cartesian coordinate system oxyz is set in such a way that x-axis is parallel to the predominating flow direction of the computational domain (Fig. 3). The whole process of integration is beyond the scope of this paper. The derivation is performed by analogy to the integration of the Saint-Venant equations for pure water flow as exposed in [19] but the basis equations are in this case a two-phase flow model. Briefly, momentum equations (3) along both the y-axis and the z-axis are simplified by means of a non-dimensional analysis and reduce to a pressure distribution over the flow section:

$$\begin{cases} \dfrac{\partial p}{\partial z} = -\rho_m g \sin\theta_z \\ \dfrac{\partial p}{\partial y} = 0 \end{cases} \tag{5}$$

Successive integration over the flow width (y-abscissa) and the flow depth (z-abscissa) are performed on the basis of the Leibniz integral rule [19] and adapted boundary conditions at the bottom, free-surface and banks of the cross-section. The success of the method relies on choosing wisely the definition of the area-average . As a consequence, the area-average of a general function f is defined as:

$$\langle f\rangle(x,t) \triangleq \frac{1}{\Omega}\int_\Omega f(x,y,z,t)\,dA \tag{6}$$

where Ω[m²] is the flow cross-section area. Likewise, the 1D mixture velocity is chosen as the mixture density weighted area-average of the 3D mixture velocity:

$$\tilde{u}_m \triangleq \frac{\langle \rho_m u_m\rangle}{\langle \rho_m\rangle} \tag{7}$$

The resulting field equations are written as:

- The area-integrated continuity equation:

$$\frac{\partial\left(1-\langle\alpha_g\rangle\right)\Omega}{\partial t} + \frac{\partial\left(1-\langle\alpha_g\rangle\right)\tilde{u}_m\Omega}{\partial x} = 0 \tag{8}$$

- The area-integrated diffusion equation:

$$\frac{\partial\langle\alpha_g\rangle\Omega}{\partial t} + \frac{\partial\langle\alpha_g\rangle\tilde{u}_m\Omega}{\partial x} = \left\langle\frac{\Gamma_g}{\rho_g}\right\rangle\Omega \tag{9}$$

- The area-integrated momentum equation:

$$\begin{aligned} &\frac{\partial\left(1-\langle\alpha_g\rangle\right)\tilde{u}_m\Omega}{\partial t} + \frac{\partial\left(1-\langle\alpha_g\rangle\right)\tilde{u}_m\tilde{u}_m\Omega}{\partial x} \\ &= \\ &g\left(1-\langle\alpha_g\rangle\right)\Omega\left(\frac{\partial Z}{\partial x} - S_F\right) - g p_w\frac{\partial\langle\alpha_g\rangle}{\partial x} \\ &-\frac{1}{\rho_w}\frac{\partial}{\partial x}\left(\sum_k \mathrm{COV}\left(\alpha_k\rho_k u_k u_k\right)\Omega\right) \end{aligned} \tag{10}$$

Where

$$p_w = \int_{-h_b}^{h_s}\left(h_s - z\right)l(z)\,dz \tag{11}$$

and Z[m] is the free surface elevation, S_F[-] is the friction slope (resulting from the integration of the viscous, turbulent shear stress and the interfacial momentum source). In equation (10), the covariance term has been introduced as the difference between the average of a product and the product of the average of two-variables [20]. Since the profile of velocity is assumed to be flat in the present paper, the covariance term reduces to zero.

C. Constitutive Equation for the Phase Change

To close the partial differential system, we still need to give an expression for the phase change volume generation Γ_g. Literature is abundant for empirical relations. To keep the generality of the model, a very fundamental relation given in [11] for air entrainment in free-surface flow is used:

$$\left\langle\frac{\Gamma_g}{\rho_g}\right\rangle\Omega = -m\Gamma\left(\alpha - \alpha_{eq}\right) \tag{12}$$

where Γ and α_g are constants calibrated with experimental results. The onset of air entrainment is controlled by the parameter m=1 or m=0.

D. Numerical Scheme

Discretization of equations (4)-(7) is performed by means of a finite volume scheme with an original flux vector splitting [21]. The scheme has been proven to be 1st order accurate and very robust. The time discretization is achieved with a classical 3-step Runge-Kutta algorithm [22]. The efficiency of such an explicit method is well known because of its low computation-cost. Moreover the coefficients have been tuned to emphasize the dissipation and the stability properties of the scheme.

III. Constitutive Equation for the Friction

Head loss in pressurized and free-surface single phase flow can be readily calculated by means of the Darcy-Weisbach equation [19] coupled with the Moody-Stanton diagram, the Blasius equation or the Colebrook implicit relation.

However, additional head-loss has to be accounted for in two-phase flow. Due to the importance of a correct evaluation of the frictional pressure drop, pressure drop and void fraction data have been collected for horizontal, vertical and inclined gas-liquid systems and many attempts have been made to develop general procedures for predicting these quantities. Thus, the literature contains a plethora of engineering correlations for pipe friction, channel friction and some data for other interesting components such as pumps. In this paper, a comparative study of the three most widespread correlations is proposed. In particular, the various formulations are applied on a practical application in civil engineering.

A. Homogeneous Friction

When the mixture is thoroughly mixed both air and water can be assumed to move at the same velocity and the frictional pressure drop can be approximated by the friction coefficient for a single phase flow calculated on the basis of suitable “mixture parameters”. This model is called homogeneous model [17, 23] or no-slip model [24]. The most thorough discussion of the model is given by Wallis [17]. The frictional pressure gradient is then calculated by means of the Darcy-Weisbach equation:

$$-\left(\frac{dp}{dx}\right)_F = \langle\rho_m\rangle g\Omega S_F = \langle\rho_m\rangle f \frac{\tilde{u}_m^2}{2D_h}\Omega \tag{13}$$

where *f* is the friction factor and D_h is the hydraulic diameter which is defined as:

$$D_h = 4R_h = 4\frac{\Omega}{\partial\Omega} \tag{14}$$

R_h is the hydraulic radius which is given by the cross-sectional area Ω divided by the wetted perimeter $\partial\Omega$.

In chemical and process engineering, the friction factor f is usually computed with an explicit Blasius-like correlation as follows:

$$f = \begin{cases} 64\,Re_f^{-1} & \text{ik} \quad Re_f \le 2500 \\ 0.3164\,Re_f^{-0.25} & \text{ik} \quad Re_f > 2500 \end{cases} \tag{15}$$

In civil engineering, the implicit Colebrook-White correlation for the friction factor is preferred as it takes into account the pipe roughness as well:

$$f = \begin{cases} 64\,Re_f^{-1} & \text{ik} \quad Re_f \le 2500 \\ \sqrt{\dfrac{1}{f}} = -2\log\left(\dfrac{k_D}{3.7D_h} + \dfrac{2.51}{Re_f\sqrt{f}}\right) & \text{ik} \quad Re_f > 2500 \end{cases} \tag{16}$$

where k_D[m] is the roughness height.

In both equations (15) and (16), the Reynolds number Re is the mixture Reynolds defined as:

$$Re_{f,m} \triangleq \frac{\langle\rho_m\rangle\tilde{u}_m D_h}{\langle\mu_m\rangle} \tag{17}$$

The mixture viscosity μ_m is approximated with rheological models that take into account the void fraction. Many correlations are available but the authors found that the McAdams formulation [25] gives the most reliable results:

$$\frac{1}{\langle\mu_m\rangle} = \frac{\langle x_g\rangle}{\mu_g} + \frac{1-\langle x_g\rangle}{\mu_w} \tag{18}$$

where the quality x_g is defined as:

$$\langle x_g\rangle = \frac{\langle\alpha_g\rangle\rho_{air}}{\langle\rho_m\rangle} \tag{19}$$

B. Lockhart-Martinelli Correlation (LM)

Two-phase friction pressure drop are still nowadays often modeled on the basis of the classical theory established by Lockhart and Martinelli [26]. Two-phase flow is considered to be divided into liquid and gas streams. Correlations are constructed with the results for the frictional pressure gradient in single-phase pipe flows of each of the two fluids. They are calculated on the basis of the Darcy-Weisbach equation applied to each single-phase stream:

- For the water flow:

$$-\left(\frac{dp}{dx}\right)_{F,w} = \rho_w f_{f,w} \frac{\left(\tilde{u}_m\left(1-\langle\alpha_g\rangle\right)\right)^2}{2D_h}\Omega \tag{20}$$

- For the gas flow:

$$-\left(\frac{dp}{dx}\right)_{F,g} = \rho_g f_{f,g} \frac{\left(\tilde{u}_m\langle\alpha_g\rangle\right)^2}{2D_h}\Omega \tag{21}$$

The friction factors are calculated by means of the Blasius-like equation (15). In the original paper of Lockhart and Martinelli [26] they found that the strict determination of X^2 using the exact Blasius equation does not fit adequately when compared with experimental data. They achieve agreement when setting $f = 0.184\,Re_f^{-0.2}$ for turbulent flow. These values are normally used nowadays in chemical engineering and have been validated for civil engineering applications in [27].

The pressure drops computed this way are then correlated with the Lockhart-Martinelli parameter defined as:

$$X^2 \triangleq \left(\frac{dp}{dx}\right)_{F,w} \Big/ \left(\frac{dp}{dx}\right)_{F,g} \tag{22}$$

X^2 gives a measure of the degree to which the two-phase mixture behaves as the water rather than as the gas.

In addition, the two-phase frictional pressure drop is expressed in terms of two-phase multipliers defined as:

$$\Phi_{f,w}^2 \triangleq \left(\frac{dp}{dx}\right)_F \Big/ \left(\frac{dp}{dx}\right)_{F,w} \quad \text{and} \quad \Phi_{f,g}^2 \triangleq \left(\frac{dp}{dx}\right)_F \Big/ \left(\frac{dp}{dx}\right)_{F,g} \qquad (23)$$

In the initial paper of Martinelli and Lockhart [26], the relations of $\Phi_{f,w}^2$ and $\Phi_{f,g}^2$ as a function of X^2 was presented in graphical forms for the 4 flow regimes: turbulent-turbulent, viscous-turbulent, turbulent-viscous and viscous-viscous. For sake of easier numerical application, Chisholm [28] develop simplified equations:

$$\Phi_{f,w}^2 = 1 + \frac{N}{X} + \frac{1}{X^2} \qquad (24)$$

$$\Phi_{f,g}^2 = 1 + N.X + X^2 \qquad (25)$$

The coefficient N can thereby be set according to the flow regime defined previously according to table 1.

Tableau 1 : Coefficient N according to[28]

Liquid	**Gas**	**N**
Turbulent	Turbulent	20
Viscous	Turbulent	12
Turbulent	Laminar	10
Viscous	Laminar	5

C. Approach of Muller-Steinhagen and Heck (MSM)

Müller-Steinhagen and Heck [29] suggested a new correlation for the prediction of the frictional pressure gradient in two-phase flow in pipes. The effort was explicitly aimed at developing an approach which is simpler in application but still reliable in terms of accuracy. According to them, the pressure drops of the respective single-phase flows are calculated as follows:

$$\left(\frac{dp}{dx}\right)_{F,w0} = f_{f,w0} \frac{\left(\langle\rho_m\rangle\tilde{u}_m\right)^2}{2\rho_w D_h} = A_{MSH} \qquad (26)$$

And

$$\left(\frac{dp}{dx}\right)_{F,g0} = f_{f,g0} \frac{\left(\langle\rho_m\rangle\tilde{u}_m\right)^2}{2\rho_g D_h} = B_{MSH} \qquad (27)$$

And the friction factors are computed with Blasius-like correlation (15) where the Reynolds numbers used are given by the two following relations:

$$Re_{f,g0} = \frac{\langle\rho_m\rangle\tilde{u}_m D_h}{\mu_g} \quad \text{and} \quad Re_{f,w0} = \frac{\langle\rho_m\rangle\tilde{u}_m D_h}{\mu_w} \qquad (28)$$

The equation developed for the roughly linear increase of the pressure drop with increasing quality for x<0.7 can be written:

$$G_{MSH} = A_{MSH} + 2\left(B_{MSH} - A_{MSH}\right)\langle x_g\rangle \qquad (29)$$

To cover the full range of flow quality $0 \le \langle x_g\rangle \le 1$, a superimposition of equations (27) and (29) is used:

$$\left(\frac{dp}{dx}\right)_F = G_{MSH}\left(1 - \langle x_g\rangle\right)^{1/C} + B_{MSH}\langle x_g\rangle^C \qquad (30)$$

A value of C=3 was found by curve fitting measured data.

To determine the reliability of the method, Müller-Steinhagen and Heck [29] assessed their correlation against a data bank containing 9313 measurements of pressure gradient for different fluids, different pipe diameter and different flow conditions. They reported accuracy similar to the more complicated methods. However, for engineering applications, Keller [27] shows this method does not reach the same degree of accuracy than the Lockhart-Martinelli correlation when compared to measurement on scale model.

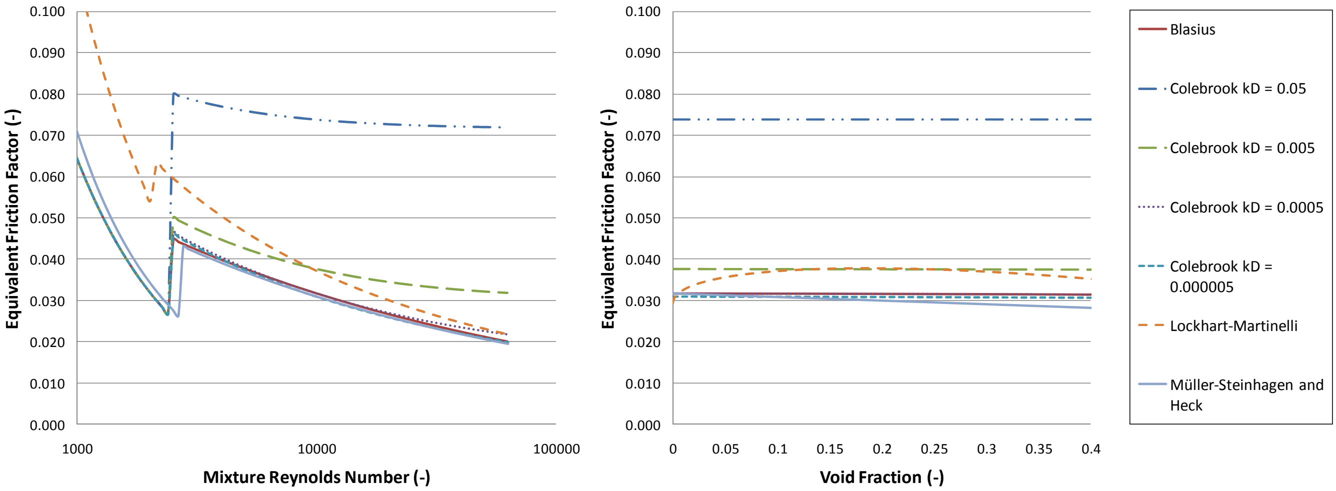


Fig. 4 : Comparison between various friction correlations

D.Comparison of the Methods

In view of the previous description of the various friction correlations, it is clear that not only the friction factor but also the kinetic term is affected by the presence of air. As a result, correlations cannot be compared in all generally by means of a Moody-like diagram. A particular case is hence specified for sake of comparison. We consider a pressurized flow in a circular pipe of 0.5m of diameter. Fig. 4 gives then the equivalent friction factor (defined as the pressure drop divided by the mixture kinetic energy) plotted against the mixture Reynolds number (for a local void fraction of 10%) and the local void fraction (for a discharge of 5m³/s). Similar analyses have been made with various cross-section shapes, with various hydraulic diameter as well as with free-surface flow. The following conclusions stay consistent. We conclude from Fig. 4 that homogeneous theory and MSM theory gives analogous results for smooth pipes and LM method gives slightly bigger friction factor, especially for laminar flow. However, if the pipe roughness becomes important, all the method based on Blasius-like formulation underestimate the friction factor.

Under the assumption that a small void fraction (α_g<5%) does not affect drastically the onset of a boundary layer at the pipe walls, homogeneous Colebrook-White correlation is consequently preferred since it takes into account the pipe roughness, which is a determinant parameter in civil engineering.

IV. Preissmann Slot Model

Pressurized flows are commonly described through the Water Hammer equations [30] derived from the equations of continuity and motion in closed pipe. According to the Preissmann slot model [7], pressurized flow can be equally calculated through the free-surface equations by adding a conceptual slot at the top of a closed pipe (Fig. 5b). When the water elevation is above the pipe crown, it provides a conceptual free-surface flow, of which the gravity wavespeed is given by $c=\sqrt{g\Omega/T_s}$ (T_s is the slot width). Strictly speaking, the pressure wave celerity of a flow in a full pipe, referred by a[m/s], depends on the properties of the fluid, the pipe, and its means of support. In first approximation, its value is not dependant of the pressure value and may be computed on the basis of solid mechanics relations [30]. It is then easy to choose a slot width T_s which equalizes the gravity wavespeed c to the water hammer wavespeed a:

$$T_s \triangleq \frac{g\Omega}{a^2} \quad \text{with} \quad a^2 \triangleq \Omega\frac{dp}{d(\rho\Omega)} \tag{31}$$

From a hydraulic point of view, all the relevant information is summarized in the relation linking the water height and the flow area (H-A). A specific relation corresponds to each geometry of the cross section (Fig. 5a). Adding the Preissmann slot leads to linearly extend the relation beyond the pipe crown head. In order to simulate pressurized flows with a piezometric head below the pipe crown, the authors propose a new concept, called *negative Preissmann slot*. It consists in extending the Preissmann straight line for water height below the pipe crown (Fig. 5c). To each water level below the pipe crown corresponds two values of the flow area: one for the free surface flow and one for the pressurized flow. The choice between the two relations is done according to the local aeration conditions (closed pipe or presence of an air vent). For further details, we refer the interested reader to the following paper [31] totally dedicated to this mathematical model.

For steady flow applications, the choice of the slot width may be arbitrary. On the one hand, the wave celerity does not affect the steady state of a flow. On the other hand, explicit numerical schemes are characterized by a time step Δt that is limited by a CFL condition of the form:

$$NbC \le 1 \quad \text{with} \quad NbC \triangleq \max\left(|u_m|+c\right)*\frac{\Delta x}{\Delta t} \tag{32}$$

It seems then reasonable to impose a wider slot than the width calculated with equation (31) in order to decrease the number of computation steps.

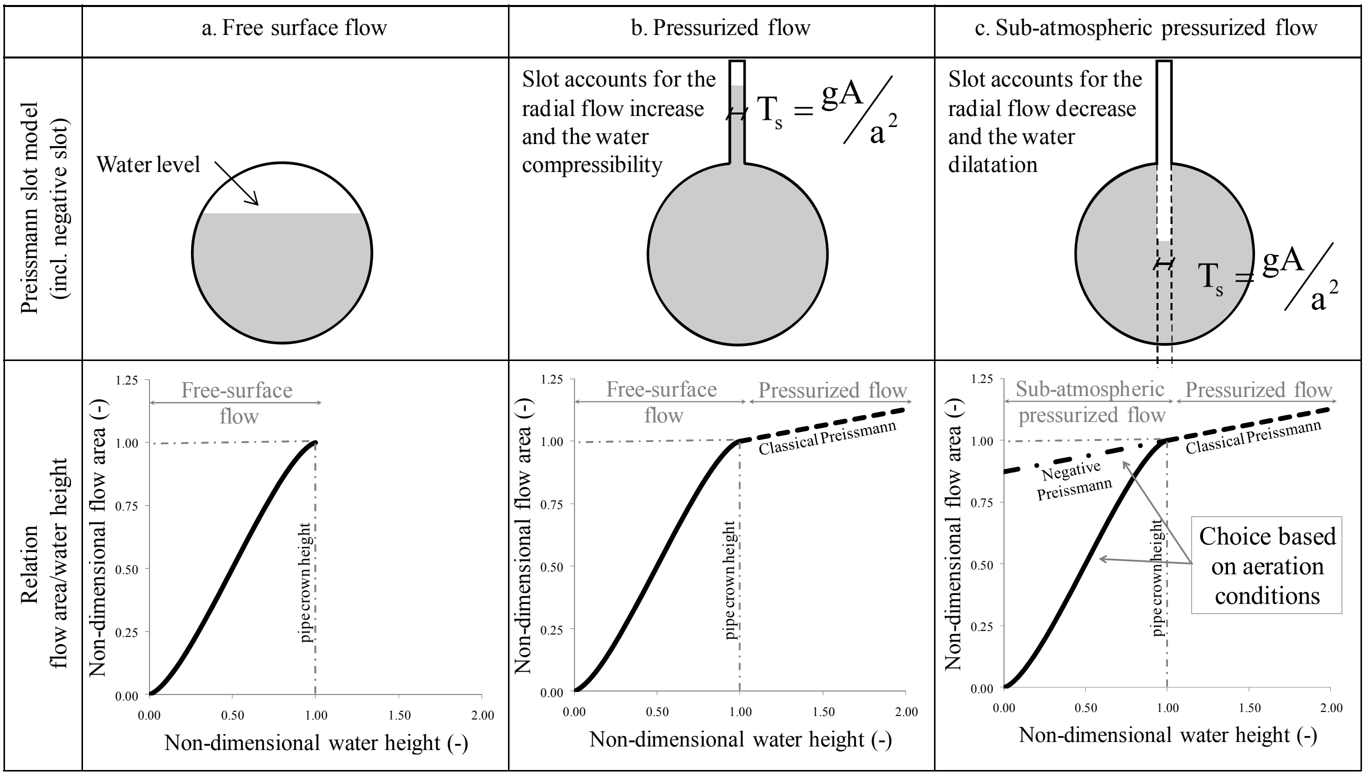


Fig. 5 : The Preissmann slot method under different flow conditions

V. Steady Flow Application

This section outlines the application of the 1D HE-Model for simulating stationary mixed flows taking place in a gallery. Numerical results are compared with experimental results provided by experimental investigations carried out in the Laboratory of Structures Hydraulics (HACH) of the University of Liege. The model (Fig. 6) includes a plexiglas circular pipe linking two tanks. Topography of the upstream and downstream tanks has been built regarding realistic in-situ natural conditions. The gallery inlet and outlet structures are also represented. Experimental apparatus, measurement systems and results are described in details in [32].

A. Experimental Investigations

Investigations focus mainly on stationary flows and aims at determining the flow discharge through the gallery as a function of the upstream pressure head. Strong air/water interactions may alter the flow behaviour. In particular, the flow discharge through the gallery is strongly influenced by air/water interaction, and consequently depends of the aeration rate as well.

Various two-phase flow patterns are observed according to the flow discharge through the gallery. Fig. 7 shows the experimental relation between the flow discharge and the upstream pressure head (zero level is set at the upstream reservoir bottom level). The curve defines 5 areas corresponding to the 5 flow patterns (Fig. 7) traditionally mentioned in the literature [17]:

1. A *smooth stratified flow.*
2. A *wavy stratified flow.*
3. An *intermittent flow* that includes *slug flow* as well as *plug flow.*
4. A *bubbly flow.*
5. A pure water *pressurized flow.*

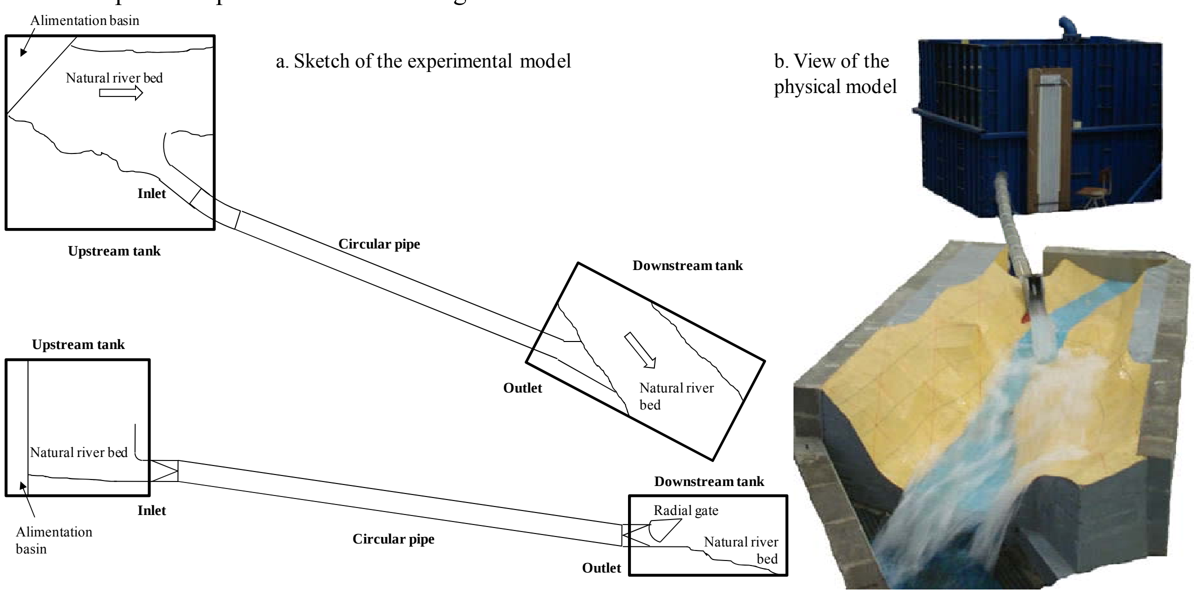


Fig. 6 : Experimental setup

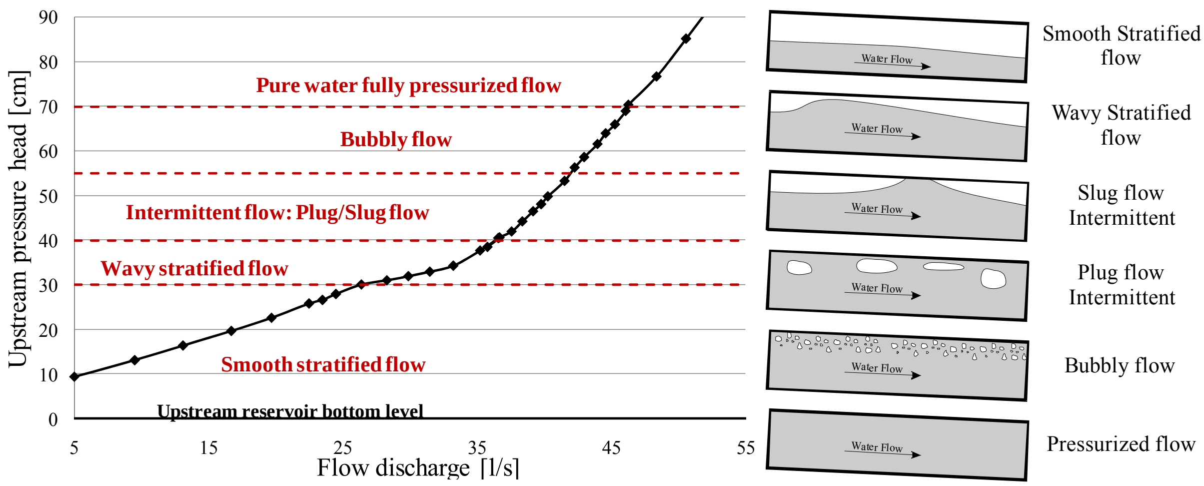


Fig. 7 : Experimental discharge curve (upstream pressure head-flow discharge) and observed flow patterns

B.Pure Water Simulation

In this section, simulations are performed under the assumption of a pure water flow (void fraction is equal to zero), with a spatial discretization step Δx=3.33cm and a CFL number limited to 0.5. As exposed in section III, the Homogenous Colebrook-White correlation is used with the McAdam formulation for the mixture viscosity and a roughness height k_D = 2.10^{-5}m. Comparison of results computed with other two-phase friction correlations is provided in section V.D. The flow discharge varies between 5l/s and 55l/s. A first head/discharge relation (dotted line in Fig. 10) is computed with the HE-Model and assuming a free surface appears in each mesh if the water height is below the pipe crown (air phase above the free surface is at atmospheric pressure). The second head/discharge relation (continuous line) is computed by activating the negative Preissmann slot (sub-atmospheric pressurized flow).

Numerical results are in good accordance with experimental data for smooth stratified flows and fully pressurized flows. Bubbly and intermittent flows show a similar behavior to the sub-atmospheric pressurized flows. A periodic instability between two unstable steady flow regimes occurs in the area of wavy stratified flows. The instability induces large period (10s to 60s) oscillations of the water level in the upstream reservoir. For further details over this regime, we refer the interested reader to the paper of Erpicum and al. [32].

Experimental and numerical data for the distribution of the total head and the pressure head (water level for free surface flow) along the gallery length are given in Fig. 8 for a smooth stratified flow (discharge of 9.5l/s) and a fully pressurized flow (discharge of 48.4l/s). In the latter case, results are in full agreement. In the former case, a slight discrepancy is observed in the total head curve. It results from the effect of the air phase flowing above the free surface that is not taken into account in the computation.

A comparison of the results given by the computation for an intermittent flow of 38.4l/s discharge is shown in Fig. 9. Pressure distribution along the gallery is computed in Fig. 9b under the assumption of a free surface flow. Large discrepancies of the results are observed. In Fig. 9a, activation of the negative Preissmann slot gives the curve corresponding to a pressurized flow. We consequently identify a large area of sub-atmospheric pressure in the upstream part of the pipe. Results are now in better accordance and it has been concluded that the aeration rate of the pipe is not sufficient to induce the apparition of a free surface flow. However, some differences still remain due to the air-water interactions.

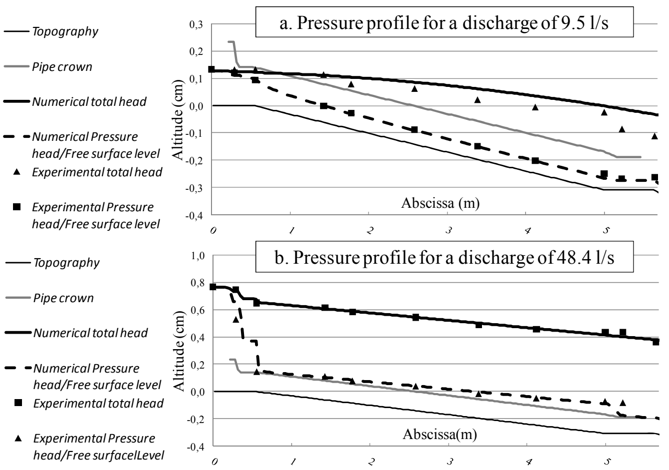


Fig. 8 : Computed total head and pressure head distribution for a smooth stratified flow and a pressurized flow

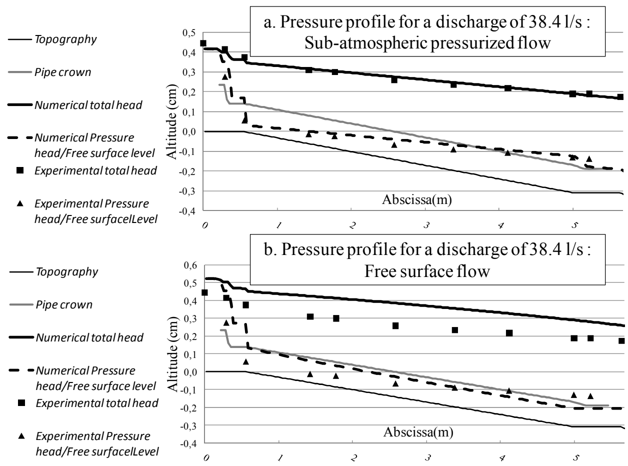


Fig. 9 : Computed total head and pressure head distribution for an intermittent flow sub-atmospheric pressurized flow and free-surface flow computation

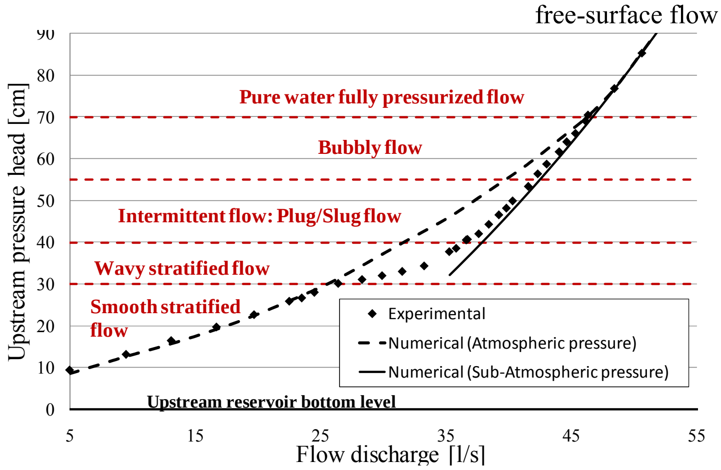


Fig. 10 : Computed flow discharge relation for pure water simulations

C. Air-water Mixture Simulation

Application of the HE-Model enables to overcome the results discrepancy observed in section V.B for bubbly and intermittent flows (Fig. 9). The effect of the entrained air on the water flow is accurately computed by using the equation (12) for the phase change volume generation Γ_g. The parameter Γ is set at 25 and α_g is calibrated according to the flow pattern observed. For bubbly flows, as bubbles arise from the air dissolved in water, equilibrium void fraction is chosen between 0.5% and 2%. For intermittent flows, an additional air supply is provided through a vertical vortex appearing at the water intake. Equilibrium void fraction is then chosen between 2% and 4.5%.

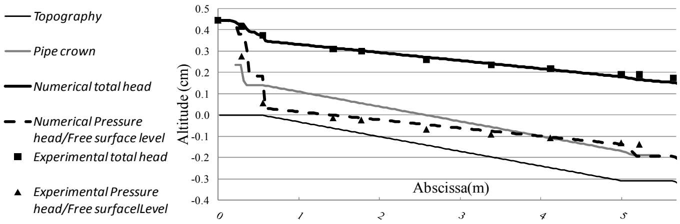


Fig. 11 : Computed total head and pressure head distribution for a bubbly (flow discharge of 38.4 l/s and void fraction of 4.5%)

Fig. 12a shows a comparison between experimental and numerical data for the discharge curve. Taking into account air/water interactions in the computation obviously gives more accurate results for bubbly and intermittent flows. The void fraction relation corresponding to this new relation is given in Fig. 12b.

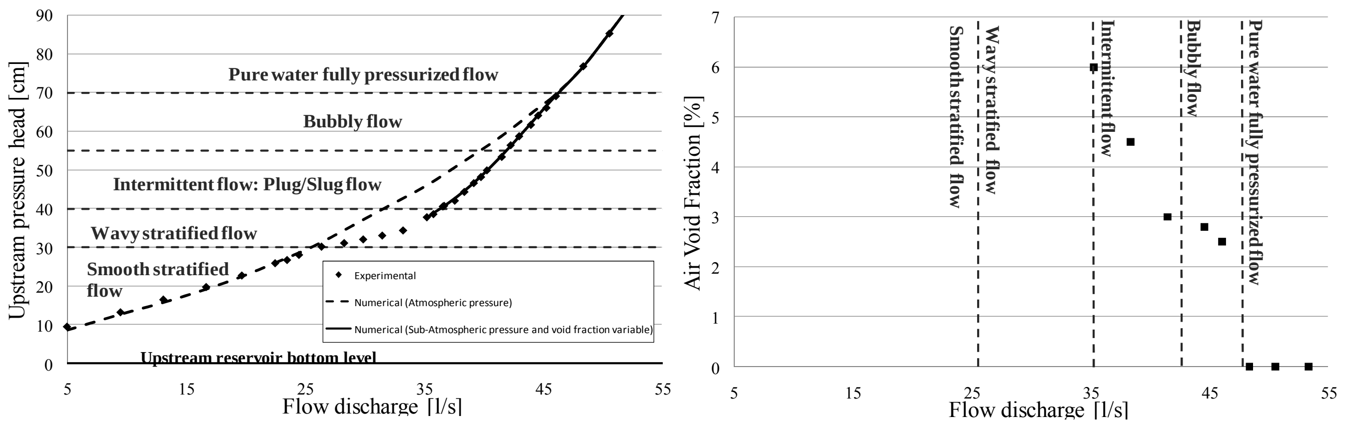
A comparison between experimental data and numerical results computed with the HE-Model is drawn on Fig. 11. Computation is performed with a flow discharge of 38.4l/s and a void fraction of 4.5%. Results are shown in full agreement.

D. Influence of the Friction Law

In this section, computation is performed for a bubbly flow of 36.5 l/s and a void fraction of 4.5%. The 4 friction correlations introduced above are considered: Homogeneous Colebrook-White ($k_D = 2.10^{-5}$m), Homogeneous Blasius, Lockhart-Martinelli and Müller-Steinhagen and Heck. Results in terms of the upstream total head, which is the parameter the most affected by the friction, are given in Tab. 1. Obviously, accuracy of the results is only slightly affected by the choice of the friction correlation. It is worthwhile noting the Homogeneous Colebrook-White gives the most conservative results (it gives the biggest head-loss). Again, it results from the fact that the Homogeneous Colebrook-White correlation is the only one that considers the pipe roughness in the calculation. As a result, this method seems the most reliable for civil engineering application for which pipe roughness is clearly a major parameter.

Tab. 1 : Comparison of friction correlations

	Upstream Total Head [cm]	Error [%]
Experimental	40.53	2.5%
Homogeneous Colebrook	39.5036	2.5%
Homogeneous Blasius	39.5027	2.5%
Lockhart-Martinelli	39.5004	2.5%
Müller-Steinhagen and Heck	39.5018	2.5%



a. Head/Discharge relation computed with variable void fraction

b. Void fraction distribution in bubbly and intermittent flows

Fig. 12 : Results of air-water mixture simulation

CONCLUSIONS

The original mathematical model derived in this paper is a first step towards a completely unified model for the simulation of highly transient mixed flow in multi-scale hydraulic structures. Thanks to the Preissmann slot method, both free-surface and pressurized flow are calculated through the free-surface set of equation by adding a narrow slot at the top of the pressurized sections. In addition, an original negative Preissmann slot has been added to simulate sub-atmospheric pressure. Area-integration of the Homogeneous Equilibrium Model over the cross section give a simple set of equations, analogous to the Saint-Venant equations, for analyzing air-water flows. This assumption has been shown to be particularly well-suited for the simulation of bubbly and intermittent flows.

The fundamental concepts introduced in the previous pages pave the way for further research. Experimental research is required to develop appropriate source terms as phase change volume generation and friction correlation. Development of a stratified air/water model would give us insight into wavy stratified flows. All results should be then easily extended to multidimensional problems.

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