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PREDICTING THE FATIGUE NOTCH EFFECT FROM PARTIAL EXPERIMENTAL DATA

J.F. Debongnie

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1 Introduction

The main difficulty of design against fatigue lies in the fact that the endurance stress of a notched part varies with the material and the scale. Theoretical models exist from long, but they are restricted to notches defined by known values of stress concentration factor and gradient. Nevertheless there exist a lot of *unpredictable* notches for which these two characteristic parameters are unknown, the most typical case being the press-fitted assembly. Our purpose is then to present a new similarity method allowing a safe evaluation of the fatigue behaviour of such notches, from partial experimental data. A comparison with further literature results shows a fair agreement between the present approach and experience.

2 The conventional method

The classical method of design against fatigue relates the endurance limit of a notched part of diameter $d, \sigma_{DN}(d)$, to the endurance limit of a smooth part of diameter $d_o, \sigma_D(d_o)$, by dividing the latter by two effect factors, namely the scale factor K_s and the notch factor K_f,

$$\sigma_{DN}(d) = \frac{\sigma_D(d_o)}{K_s.K_f} \tag{1}$$

Unfortunately, these two factors are interrelated and therefore, if the notch effect is obtained for a reference diameter of 10 mm for example, then the scale effect will also depend on the notch. A purely experimental approach would therefore require a large amount of tests, including large scale ones, which are very expensive.

In order to circumvene the abovementioned difficulties, it was tried to relate the notch factor K_f to the stress concentration factor K_t by a *notch sensitivity index* q defined by

$$q = \frac{K_f - 1}{K_t - 1}, \tag{2}$$

q depending on the material and on the notch radius. This approach is not completely satisfactory from a theoretical point of view because K_t, the ratio between the maximum stress and the nominal one, comes from an arbitrary choice of σ_{nom} and the so obtained criterium, as finally expressed in terms of the maximum stress, is not objective.

3 The gradient method

By far more attractive is the gradient method as pioneered by Siebel [1] and Petersen [2] and which is now a standard in Germany. The fundamental idea of this method is to admit that the endurance limit, as expressed in terms of the maximum stress, is a function of the *relative stress gradient*

$$\chi = \frac{1}{\sigma}\|grad\ \sigma\| \text{ at the maximum,} \tag{3}$$

whose general form is

$$\chi = \frac{B_1}{d} + \frac{B_2}{R} \tag{4}$$

d being the diameter, and R the root radius. B_1 takes the value 0 in axial loading and 2 in bending or torsion. Typical values of B_2 are 2.33 for axial loading or bending and 1 for torsion. The general endurance criterium is

$$\sigma_{DN} = \frac{\sigma_{D_o} + A\sqrt{\chi}}{K_t} \tag{5}$$

where σ_{D_o} is the endurance limit in *axial loading*, and A a material constant. Slightly different values of A are given by different authors, but in each case, this constant is approximately the same for any steel. A recent study of the compatibility of the gradient method with fracture mechanics [3] led to a confirmation of this unique value of A for steels. It also came to the conclusion that sharp notches may be treated by a limiting process. Finally, checking the gradient method on a set of 292 experimental results given by Heywood [4] conducted the author to find a fair agreement clearly confirming the validity of relation (5).

However, a severe limitation of this method lies in the fact that it is restricted to what may be called *predictable* notches, for which K_t and χ are well known.

4 Unpredictable notches

There are in fact a lot of actual notches which cannot enter in the preceeding scheme, since K_t or χ or both are unknown. Such notches will be called *unpredictable*. Are included in this category,

- sharp notches, for which K_t and χ tend to infinity. It is however well known that such notches are characterized by a non zero, even weak, endurance.
- a lot of notches for which insufficient data is available, or having not well defined root radii. Such are keyways, splined shafts, screw-threads, and so on.
- press-fitted assemblies, in which the stress state is unknown and does not depend on any radius.

All these cases, which ironically are the most current ones, necessarily require an experience-based approach. But experimental results are sparse. In fact, most references only give one value of K_f for a given steel and a given diameter. In the best case, values of K_f are proposed for a given diameter and some different steels. We only found one reference [5] giving values of the endurance limit for one steel, and various diameters, up to 290 mm. The very strong scale effect which was found shows that applying notch factors obtained from small parts to large parts may be catastrophic.

As large experimental plants are not feasible, the question is now the possibility to find a law by which partial results could be safely extrapolated. In this way, a key argument is the fact that practical notches remain approximately similar from a geometrical point of view when the scale is modified.

So, let us consider a family of geometrically similar pieces. This family may be characterized by an identical value of two characteristic numbers, which are

- The stress concentration factor K_t
- The gradient number $G = \sqrt{\chi\, d}$.

Let us define the weakening factor

$$\gamma = \frac{\sigma_{DN}(d)}{\sigma_{D_o}} \text{ (for torsion, } \gamma = \frac{\tau_{DN}(d)\sqrt{3}}{\sigma_{D_o}}\text{)} \tag{6}$$

(which is the inverse of Niemann's $\overline{\beta}_k$ factor [6]). From the gradient method, one has

$$\gamma = \frac{1}{K_t} + \frac{G}{K_t}\frac{A}{\sigma_{D_o}\sqrt{d}} = C_1 + C_2 \frac{A}{\sigma_{D_o}\sqrt{d}} \tag{7}$$

where C_1 and C_2 are two new characteristic numbers of the family, and there appears a new undimensional number

$$Z = \frac{A}{\sigma_{D_o}\sqrt{d}} \tag{8}$$

taking account of the material through A and σ_{D_o} and of the scale through d. Physically, C_1 is the weakening factor for infinite hardness or infinite diameter. It is the lowest possible value of the weakening factor. C_2 is responsible of the scale effect and of the material dependency. The higher C_2, the higher the scale effect, which here appears as an overstrength for low scales.

As all studied versions of the gradient method lead to an approximately constant value of A for steels, it is reasonable in this case to define a new constant $C_3 = C_{2*}$ A, which has the dimension of a stress intensity factor ($MPa\sqrt{mm}$) and leads to a simplified law,

$$\gamma = C_1 + \frac{C_3}{\sigma_{D_o}\sqrt{d}} \tag{9}$$

Finding constants C_1 and C_3 requires at least two experimental results with different steels or different scales. More results are of course needed to perform a regression analysis, which may be useful to increase the reliability of this approach.

5 The case of sharp notches

Sharp notches, characterized by a zero radius, generally lead to a non-zero endurance. In this case, $K_t \to \infty$, $G \to \infty$, but generally,

$$C_2 = \lim \frac{G}{K_t} = \mathit{finite}$$

As $C_1 = 0$, the strongest scale effect is obtained with such notches, that is,

$$\gamma = \frac{C_3}{\sigma_{D_o}\sqrt{d}} \tag{10}$$

However somewhat fictitious, since a true sharp notch is not possible, due to the radius of the tool, this case may be taken as a conservative approximation when only one experimental result is known.

6 A first case of study

The press-fitted assembly in bending is probably the most typical case of an unpredictable notch. In fact, no one knows how to compute such a notch. Data from Lehr [5] indicate a very strong scale effect, the endurance stress varying for a DIN St50 steel (Rm = 500 MPa) from 160 MPa for a 10 mm diameter to 70 MPa for a 290 mm diameter. From the 4 experimental values given by Lehr, the following constants were obtained,

$C_1 = 0.2373$, $C_3 = 341.4$

with a regression coefficient of 0.9976. Other values are given in the Dubbel momento [7] for 9 different steels and a constant 40 mm diameter. Applying our model with the above constants, a less than 3.6 % discrepancy is obtained. A good accordance is also obtained with data given by Niemann [6] but with a constant factor probably due to an extra-security for design values and a not precisely given scale. Other spot verifications on isolated results are very consistent, always within a range of ± 4 %. It is the only case where results concerning both diameter and material variations were available, but it clearly confirms our point of view.

7 Other experimental results

Adjustments were made for a lot of results coming from literature, generally for one diameter and various steels. They are summarized in table 1. As can be seen, the regression coefficient is always greater than 0.98. This leads to a valuable chart of notches for which up to now, no effective computational method covering both steel and scale variations was known.

8 Conclusion

The proposed similarity method is an attempt to give a rational solution for unpredictable notches which, up to now, resisted to any analysis. Very simple to use, our chart of table 1 could of course be improved by inclusion of other experimental results. The exceptionally good regression coefficients which were obtained tend to prove that the proposed way is not so bad. The applicability to other materials than classical steels remains an open way of investigation.

Table 1. $\gamma = C_1 + \dfrac{C_3}{\sigma_{D_o}\sqrt{d}}$

C_3 in $MPa\sqrt{mm}$

Notch	Ref.	Variation	Regression	C_1	C_3
1. Keyway Bending $\sigma_n = M_f / W_f$ $W_f = \dfrac{\pi d^3}{32} - \dfrac{bt(d-t)^2}{2d}$ b = width, t = depth	[8]	6 steels d_o = 10 mm	0.9933	0.2853	346.5
2. Idem, torsion $\tau_n = M_t / W_t$ $W_t = \dfrac{\pi d^3}{16} - \dfrac{bt(d-t)^2}{2d}$	[8]	6 steels d_o = 10 mm	0.9892	0.2826	389.6
3. Press-fitted assembly, bending	[5] [6] [7]	4 sizes, St50 6 steels, d_o=10mm 9 steels, d_o=40mm	0.9976	0.2373	341.4
4. Press-fitted assembly, torsion	[7]	9 steels d_o = 40 mm	0.9978	0.4006	456.2
5. Metric bolted assembly, direct loading	[8]	4 steels d_o = 12 mm	0.9917	0.08538	154.6
6. Withworth bolted assembly Direct loading	[8]	4 steels d_o = 12 mm	0.9967	0.1202	206.6
7. Withworth screw-thread on a shaft, direct loading	[9]	4 steels d_o = 10 mm	0.9983	0.1556	176.8
8. Metric screw-thread on a shaft, direct loading	[9]	4 steels d_o = 10 mm	0.9982	0.1446	158.4
9. Withworth screw-thread on a shaft, Bending	[9]	5 steels d_o = 10 mm	0.9831	0.1610	437.3

Notch	Ref.	Variation	Regression	C_1	C_3
10. Metric screw-thread on a shaft, Bending	[9]	5 steels $d_o = 10$ mm	0.9827	0.1436	429.9
11. Splined shaft, bending $\sigma_n = M_f / W_f$ $W_f = \frac{\pi d^3_{prim}}{32}$ (involute) $W_f = \xi \frac{\pi d^3_{\text{int}}}{32}$ (square) $\xi = 9/8$ light series $6/5$ mean series $5/4$ hard series	[8]	8 steels $d_o = 10$ mm	0.9994	0.4508	235.3
12. Square spline, torsion $\tau_n = M_t / 2W_f$ See 11	[8]	8 steels $d_o = 10$ mm	0.9989	0.2736	167.4
13. Involute spline, torsion $\tau_n = M_t / 2W_f$ See 11	[8]	8 steels $d_o = 10$ mm	0.9993	0.5578	170.4
14. Circlips groove, bending R assumed =0	[6]	1 value, Rm = 500 Mpa $d_o = 10$ mm	-	0	368.1
15. Circlips groove, torsion R assumed = 0	[6]	1 value, Rm = 500 Mpa $d_o = 10$ mm	-	0	449.7
16. Serrated shaft, torsion. τ computed from the gross section.		Analytic approx. from [10]	-	0.3638	283.8

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J.F. Debongnie
Université de Liège, Institut de Mécanique, Laboratoire de Méthodes de Fabrication, Chemin des Chevreuils, 1, B-4000 Liège 1, Belgique, Tel. : 32/4/366.92.71, Fax : 32/4/366.92.72, E-mail : JF.Debongnie@ulg.ac.be