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Modelling of Joints with Clearance and Friction in Multibody Dynamic Simulation of Automotive Differentials

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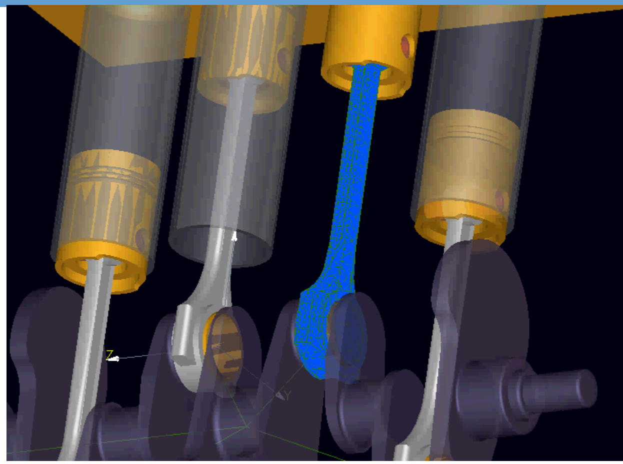
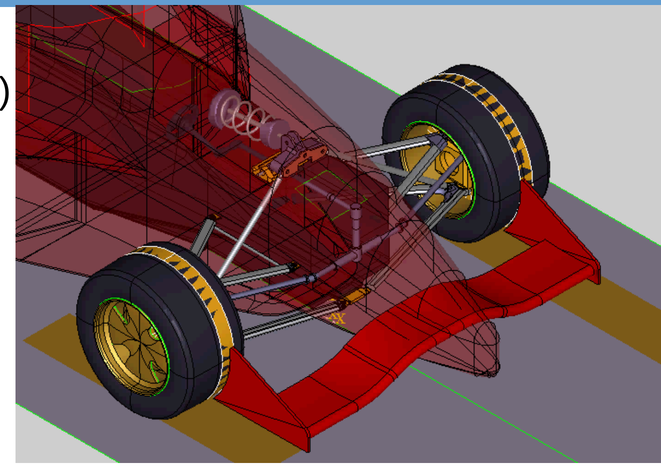
Driveline modeling

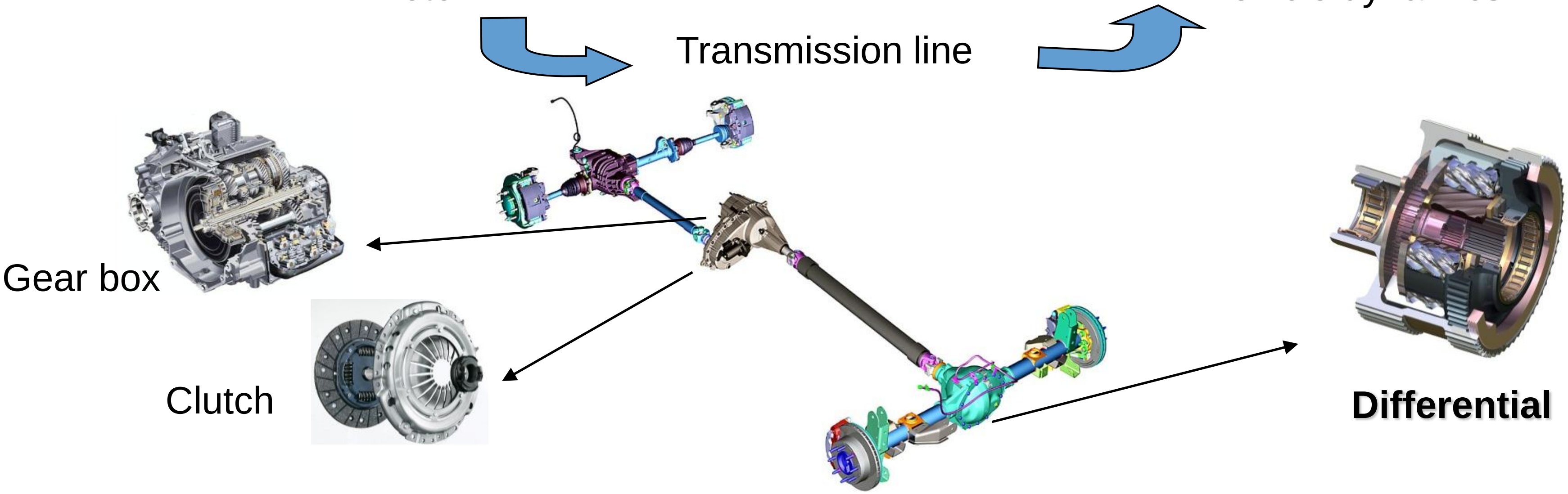


Motor

(Courtesy: LMS- SAMTECH)



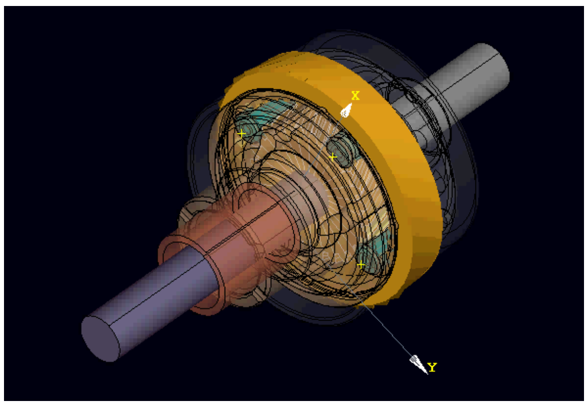
Vehicle dynamics

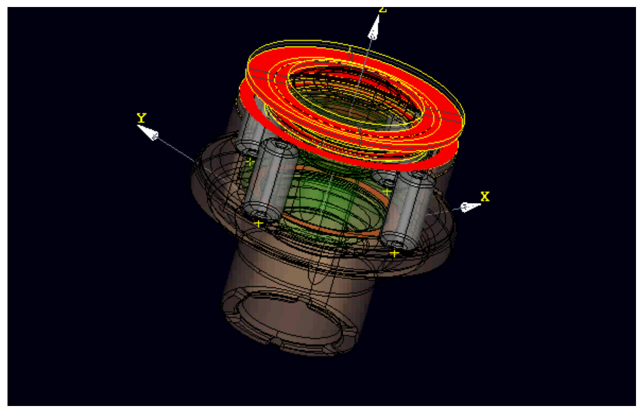


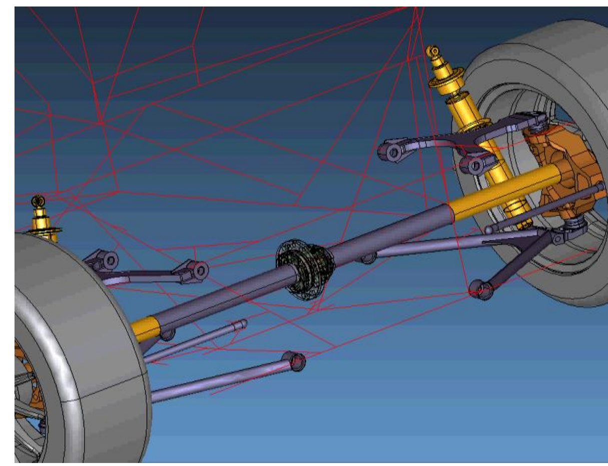
- Complex phenomena involved: backlash, stick-slip, contact, discontinuities, hysteresis, non linearities ➔ Numerical problems

Planet gear – planet carrier joints

- Previous work:
  - development of global multibody models of TORSEN differentials
  - validation by comparison with experimental data





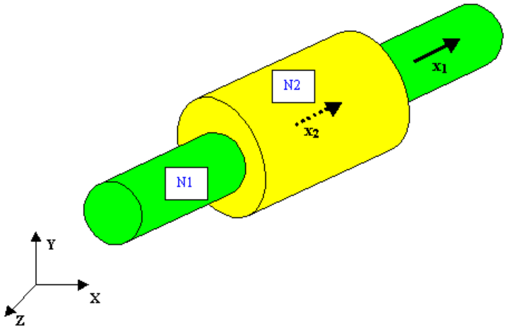


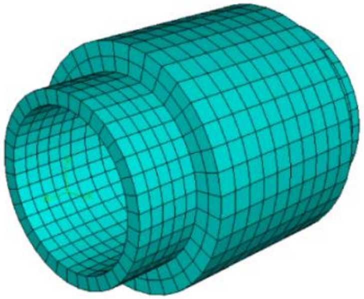
- **Modeling assumption**: joints between planet gears and differential housing have been modeled with idealized cylindrical joints.

Planet gear – planet carrier joints

- In TORSEN differentials and some other automotive transmission components composed of a planetary gear train, the planet gears are not linked to the planet carrier with a material rotation axis.

Global 3D joint which accounts for clearance, misalignment and friction





Global kinematic constraints

- Perfect joints without defects
- Easy to implement
- No CPU-time expensive

Contact condition between FE

- Real geometrical shapes ➔ Imperfections represented
- Not trivial to achieve
- Very CPU time expensive

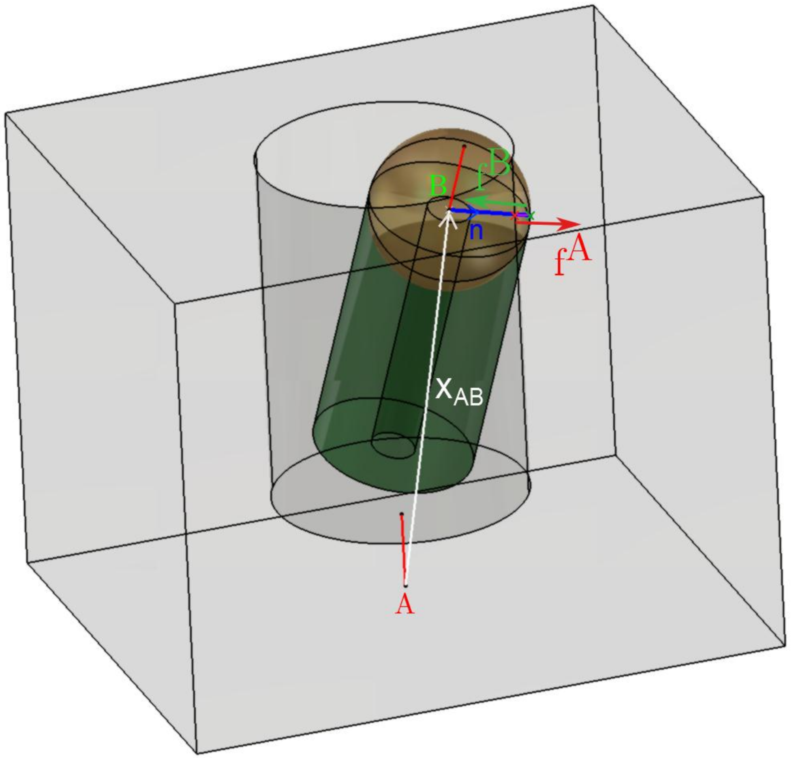
Outline

- Joint description: geometry, kinematics
- Contact law: continuous impact modelling
- Formulation of the contact and friction forces
- Description of the application: TORSEN differential
- Numerical Results
- Conclusion

Geometry of the new joint

- Global joint to model the transient behaviour
- Contact conditions, based on a penalty method, between the pin and the inner cylinder (rigid bodies)

- <u>Assumptions</u>:
  - Contact can occur on the top and/or on the bottom of the pin ➔ new element defined for one pin extremity and used twice for each cylindrical joint
  - Low clearance ➔ Spherical shape (radius r_A) considered at pin extremities
  - Joint defined between two nodes:
    - Node A on the axis of hollow cylinder (= planet carrier)
    - Node B at the face center of the pin (= planet gear)



Finite Element method in multibody systems dynamics

(Géradin and Cardona, 2001)

- Modelling of rigid and flexible bodies, kinematic joints and force elements
- Absolute nodal coordinates: no distinction between rigid and elastic coordinates ➔ nonlinear flexible effects and large deformations can be taken into account

- Equations of motion + constraints

$$\begin{aligned} \boldsymbol{M}(\boldsymbol{q})\ \ddot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) + \boldsymbol{\Phi}_{\boldsymbol{q}}^T(p\boldsymbol{\Phi} + k\boldsymbol{\lambda}) &= \mathbf{0} \\ k\ \boldsymbol{\Phi}(\boldsymbol{q}, t) &= \mathbf{0} \end{aligned}$$

- Representation of large 3D rotations: parametrization with the cartesian rotation vector + update lagrangian approach

$$\delta\boldsymbol{\Theta} = \boldsymbol{T}(\boldsymbol{\Psi}_{inc})\ \delta\boldsymbol{\Psi}_{inc}$$

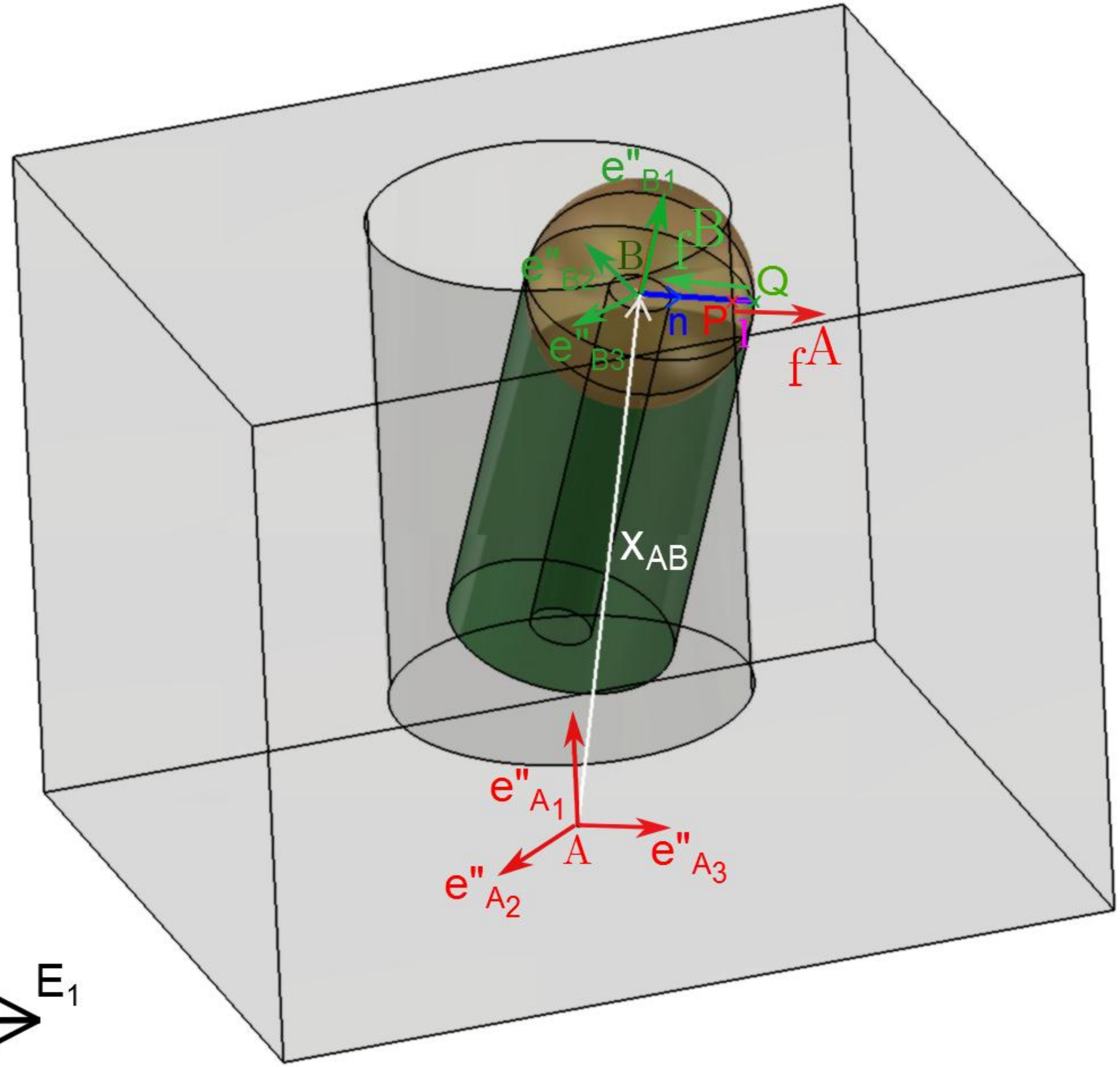
- Implemented in SAMCEF/MECANO

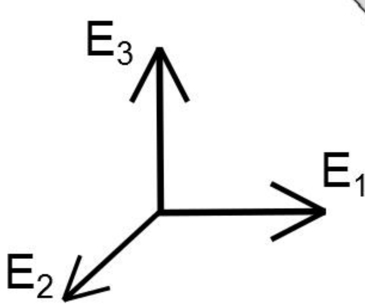
Kinematic description

- Local material frames attached to bodies A and B

$$\begin{aligned} \boldsymbol{e}''_{A_i} &= \boldsymbol{R}_A\ \boldsymbol{e}_{A_i} \\ &= \boldsymbol{R}_A\ \boldsymbol{R}_1\ \boldsymbol{E}_i \\ \boldsymbol{e}''_{B_i} &= \boldsymbol{R}_B\ \boldsymbol{e}_{B_i} \\ &= \boldsymbol{R}_B\ \boldsymbol{R}_1\ \boldsymbol{E}_i \end{aligned}$$

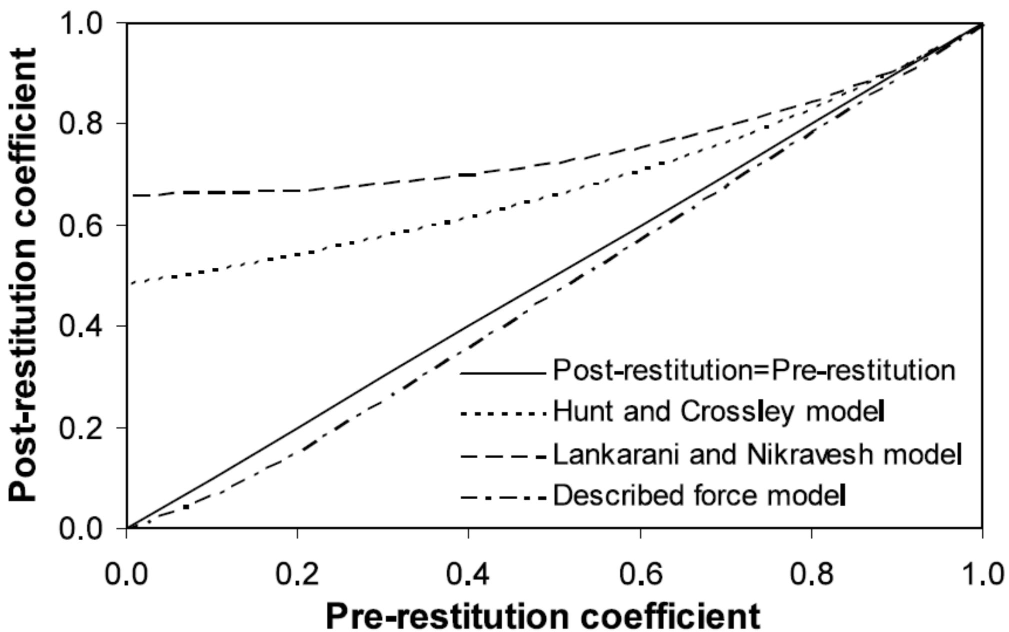
(the triads are assumed parallel at the initial configuration, t = 0 s)





Continuous impact modelling

- Restitution coefficient:
  - summarizes the kinetic energy loss
  - depends on shapes and material properties of colliding bodies and their relative velocity
  - roughly estimated by experince, determined by costly experiments or multi-scale simulations



(Flores, 2011)

- Contact force law

$$f_c(l,\dot{l}) = k\ l^n + c\ l^n\ \dot{l}$$

$$c = \frac{8\,(1-e)}{5\,e}\frac{k}{\dot{l}_s}$$

Restitution coefficient

$$0 \leq e \leq 1$$

Total energy loss

No energy loss

Contact force

- Force element: no algebraic constraint

- Virtual work of the normal contact forces

$$\delta W_n = \delta \boldsymbol{x}_P^T \ \boldsymbol{f}^A + \delta \boldsymbol{x}_Q^T \ \boldsymbol{f}^B$$

- Virtual displacements of the contact points

$$\begin{aligned} \delta \boldsymbol{x}_P &= \delta \boldsymbol{x}_A - \widetilde{\boldsymbol{x}}_{AP} \boldsymbol{R}_1 \boldsymbol{R}_A \ \delta \boldsymbol{\Theta}_A \\ \delta \boldsymbol{x}_Q &= \delta \boldsymbol{x}_B - \widetilde{\boldsymbol{x}}_{BQ} \boldsymbol{R}_1 \boldsymbol{R}_B \ \delta \boldsymbol{\Theta}_B \end{aligned}$$

- Expression of contact forces

$$\boldsymbol{f}^B = -\boldsymbol{f}^A = f_c \ \boldsymbol{n}$$

Contact force

- Definition of the normal direction

$$\boldsymbol{n} = \frac{\left(\boldsymbol{I} - \boldsymbol{e}''_{A_1}\boldsymbol{e}''_{A_1}{}^T\right)\boldsymbol{x}_{AB}}{\left\|\left(\boldsymbol{I} - \boldsymbol{e}''_{A_1}\boldsymbol{e}''_{A_1}{}^T\right)\boldsymbol{x}_{AB}\right\|}$$

- Relative penetration (l) and penetration velocity

$$\begin{aligned} l &= \boldsymbol{x}_{PQ}^T\,\boldsymbol{n} = \boldsymbol{x}_{AB}^T\,\boldsymbol{n} + r_B - r_A \\ \dot{l} &= \dot{\boldsymbol{x}}_{PQ}^T\,\boldsymbol{n} + \boldsymbol{x}_{PQ}^T\,\dot{\boldsymbol{n}} \end{aligned}$$

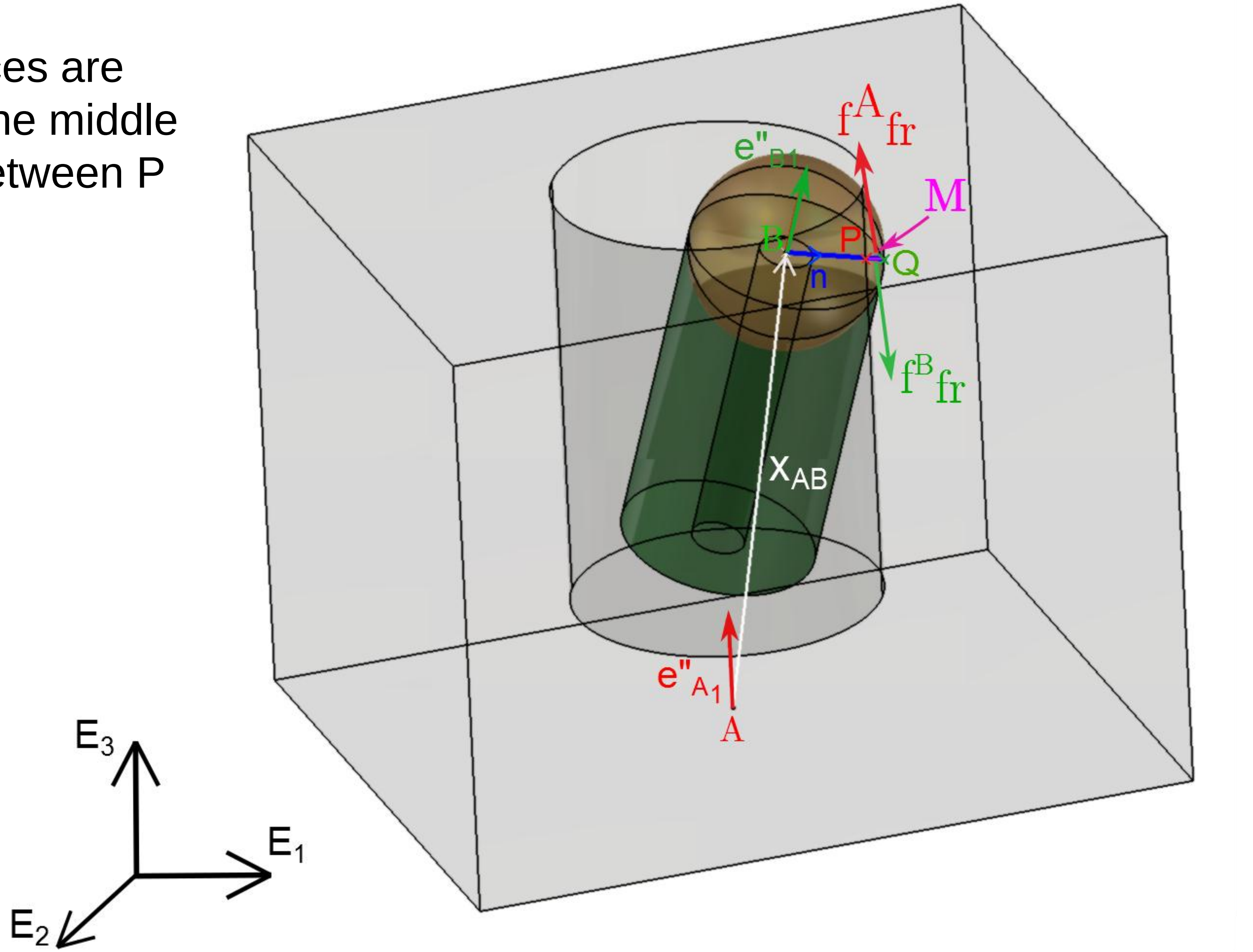
- Contribution to the internal forces of the multibody system

$$\delta W = \delta \boldsymbol{q}^T\;\boldsymbol{g}^{int}\left(\boldsymbol{q}, \dot{\boldsymbol{q}}\right)$$

$$\boldsymbol{q} = \begin{Bmatrix} \boldsymbol{x}_A \\ \boldsymbol{\Psi}_{A\ inc} \\ \boldsymbol{x}_B \\ \boldsymbol{\Psi}_{B\ inc} \end{Bmatrix} \qquad \boldsymbol{g}_n^{int}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = f_c \begin{Bmatrix} -\boldsymbol{n} \\ -\boldsymbol{T}^T(\boldsymbol{\Psi}_{A\ inc})\boldsymbol{R}_A^T\boldsymbol{R}_1^T\widetilde{\boldsymbol{x}}_{AB}\ \boldsymbol{n} \\ \boldsymbol{n} \\ \boldsymbol{0} \end{Bmatrix}$$

Friction force

- Friction forces are applied at the middle point (M) between P and Q



Friction force

- Virtual work of friction forces

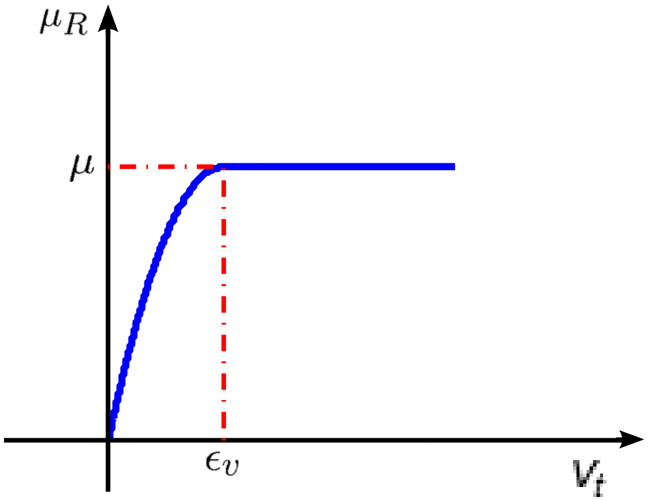
$$\delta W_{fr} = \delta \boldsymbol{x}_M^{A\,T} \; \boldsymbol{f}_{fr}^A + \delta \boldsymbol{x}_M^{B\,T} \; \boldsymbol{f}_{fr}^B$$

- Expression of the friction force

$$\boldsymbol{f}_{fr} = -\mu_R(v_t) \; f_c \; \boldsymbol{t}$$

  - with the regularized friction coefficient

$$\mu_R(v_t) = \begin{cases} \mu \left(2 \, \dfrac{v_t}{\epsilon_v} - \left(\dfrac{v_t}{\epsilon_v} \right)^2 \right) & v_t < \epsilon_v \\ \mu & v_t \geq \epsilon_v \end{cases}$$

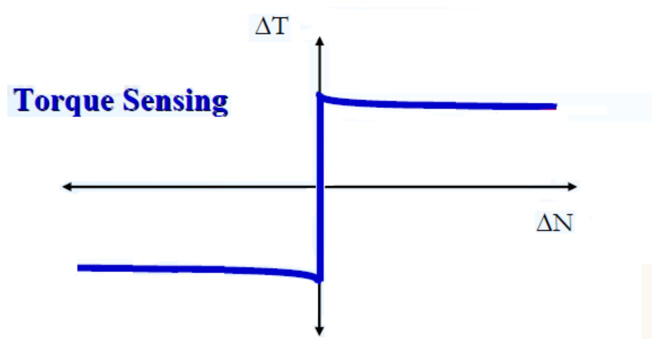


- Contribution to the internal forces of the multibody system

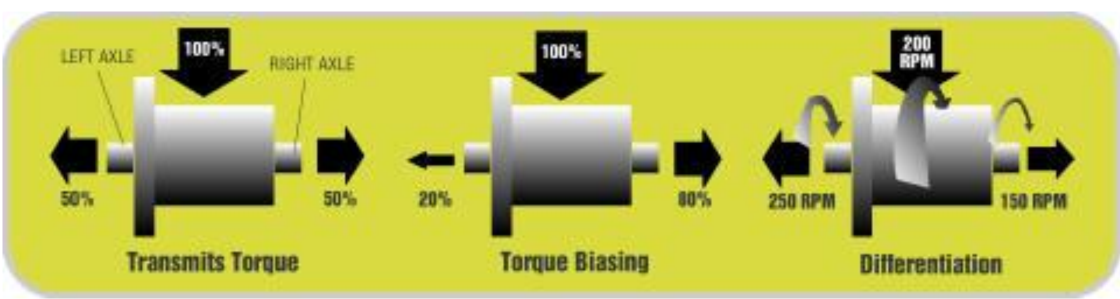
$$\boldsymbol{q} = \begin{Bmatrix} \boldsymbol{x}_A \\ \boldsymbol{\Psi}_{A\ inc} \\ \boldsymbol{x}_B \\ \boldsymbol{\Psi}_{B\ inc} \end{Bmatrix} \qquad \boldsymbol{g}_{fr}^{int}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{Bmatrix} -\boldsymbol{f}_{fr} \\ -\boldsymbol{T}^T(\boldsymbol{\Psi}_{A\ inc}) \boldsymbol{R}_A^T \boldsymbol{R}_1^T \widetilde{\boldsymbol{x}}_{AM} \; \boldsymbol{f}_{fr} \\ \boldsymbol{f}_{fr} \\ \boldsymbol{T}^T(\boldsymbol{\Psi}_{B\ inc}) \boldsymbol{R}_B^T \boldsymbol{R}_1^T \widetilde{\boldsymbol{x}}_{BM} \; \boldsymbol{f}_{fr} \end{Bmatrix}$$

TORSEN differential

- Limited slip differential
  - Allow a variable torque distribution between the output shafts ➔ avoid spinning when ground adherence is not sufficient on one driving wheel
- Torque transfer before differentiation (torque sensing)
- Full mechanical system

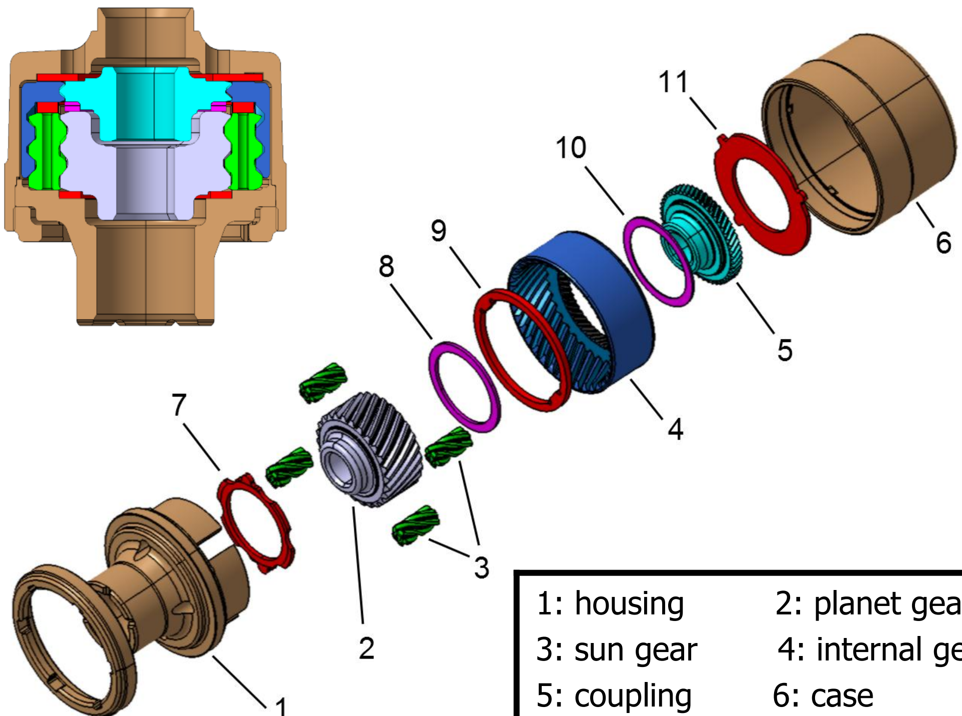


JTEKT



Type C TORSEN differential

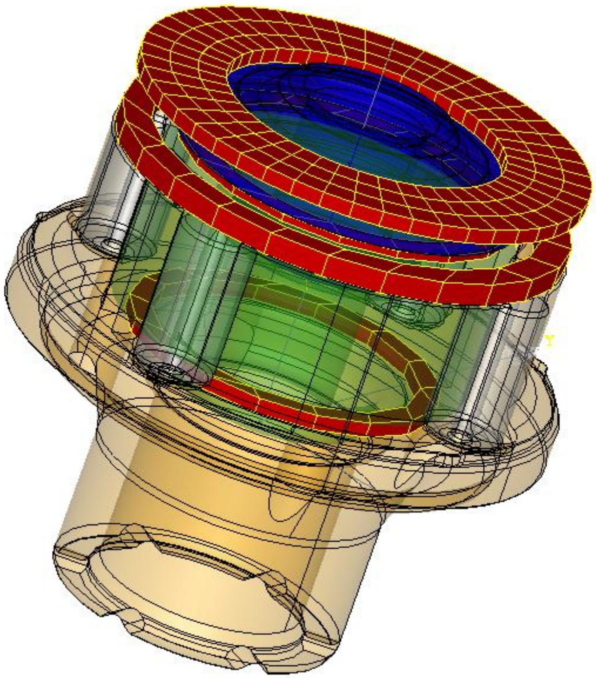
- Housing, helical gear pairs and thrust washers
- Locking due to relative friction gears ↔ washers & gears ↔ housing
- 4 working modes
- Central differential

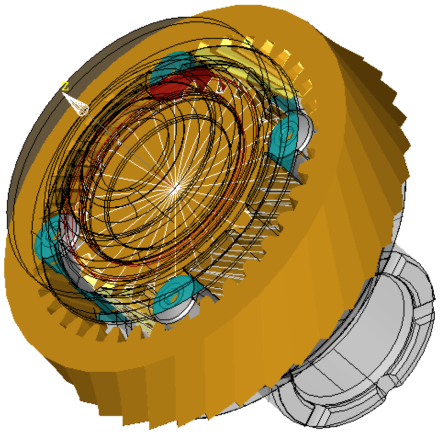


1: housing 2: planet gears
3: sun gear 4: internal gear
5: coupling 6: case
7,8,9,10,11: thrust washers

Model description

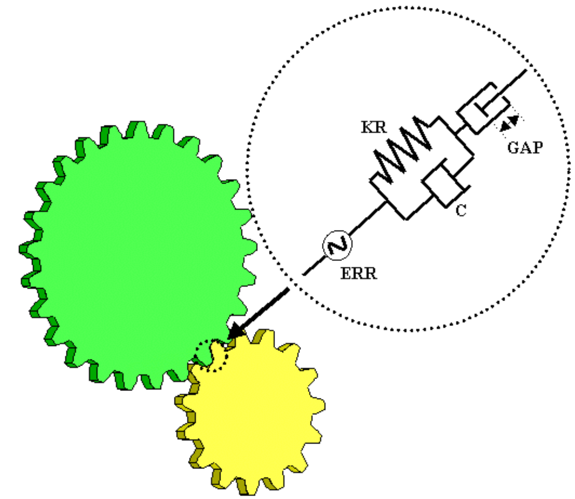
- 15 bodies:
  - *10 rigid: gear wheels, housing*
  - *5 flexible: thrust washers*
- ≈ 43000 generalized coordinates
- Kinematic constraints :
  - 8 gear pair elements
  - 5 contact relations
  - 1 screw joint
  - 4 PG/housing joints



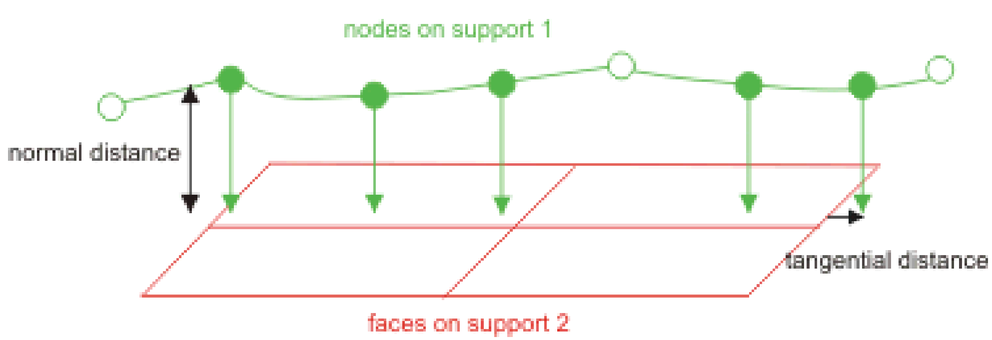


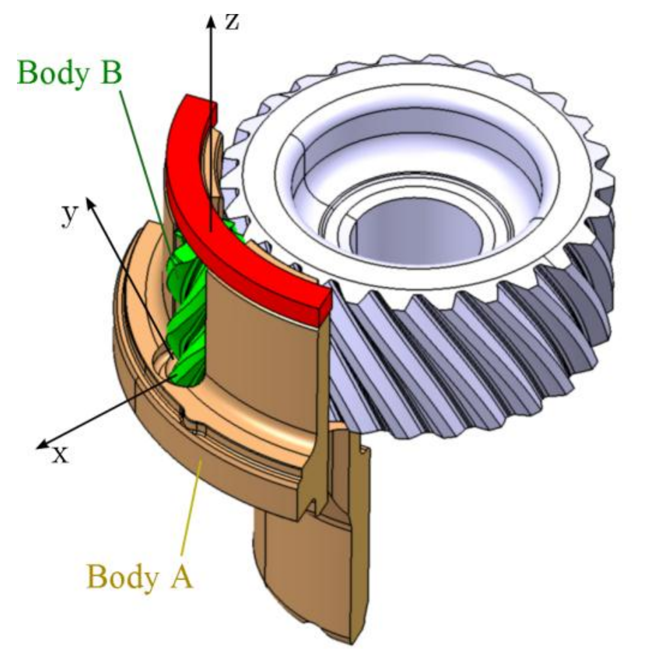
TORSEN differential modelling

- Gear pair element:
  - Global kinematic joint defined between 2 nodes: one on each gear wheels (rigid body)
  - Spring, damper, backlash, load transmission error, friction,…

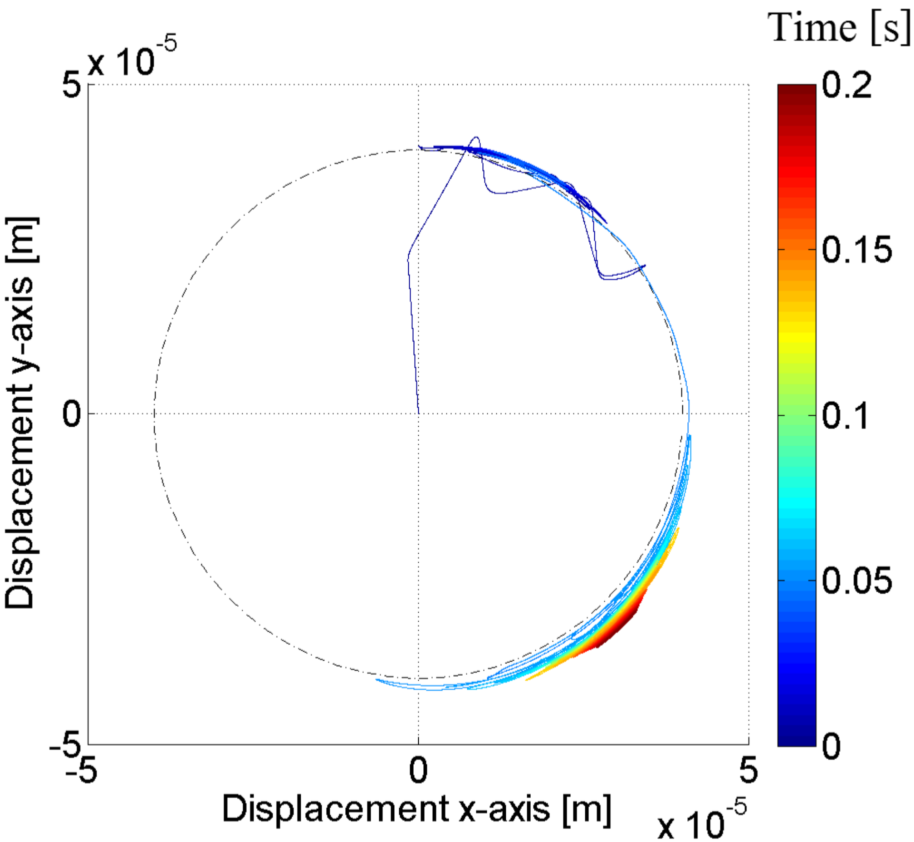


- Contact condition:
  - Flexible/flexible or rigid/flexible
  - Augmented lagrangian or penalty method

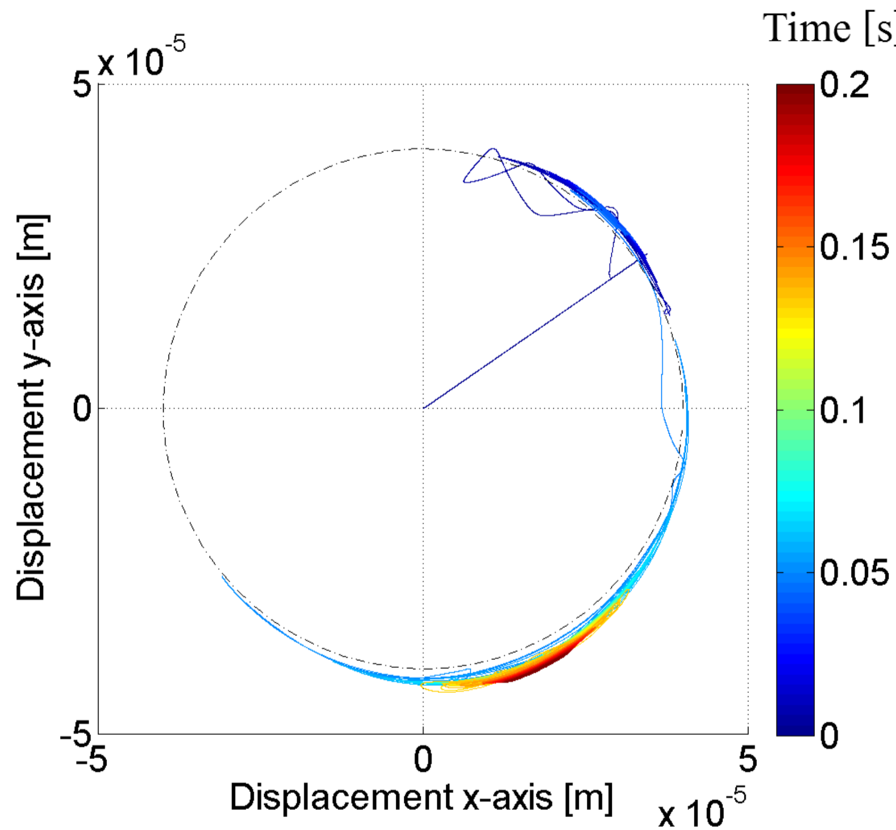




Trajectory of the planet gears

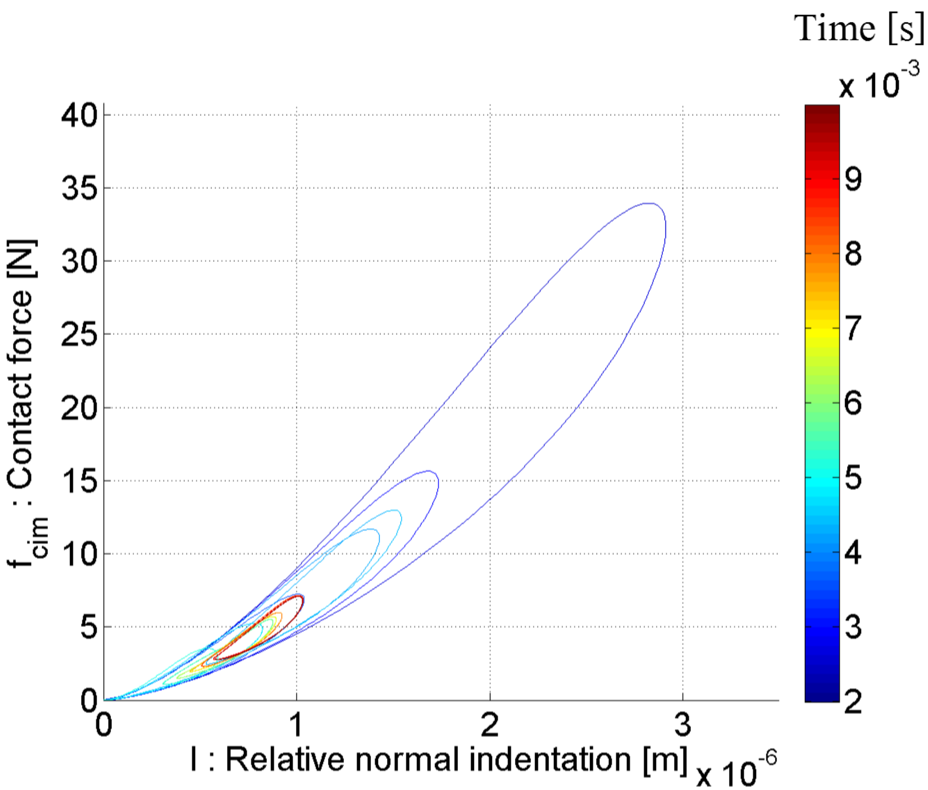


(a) Bottom face center

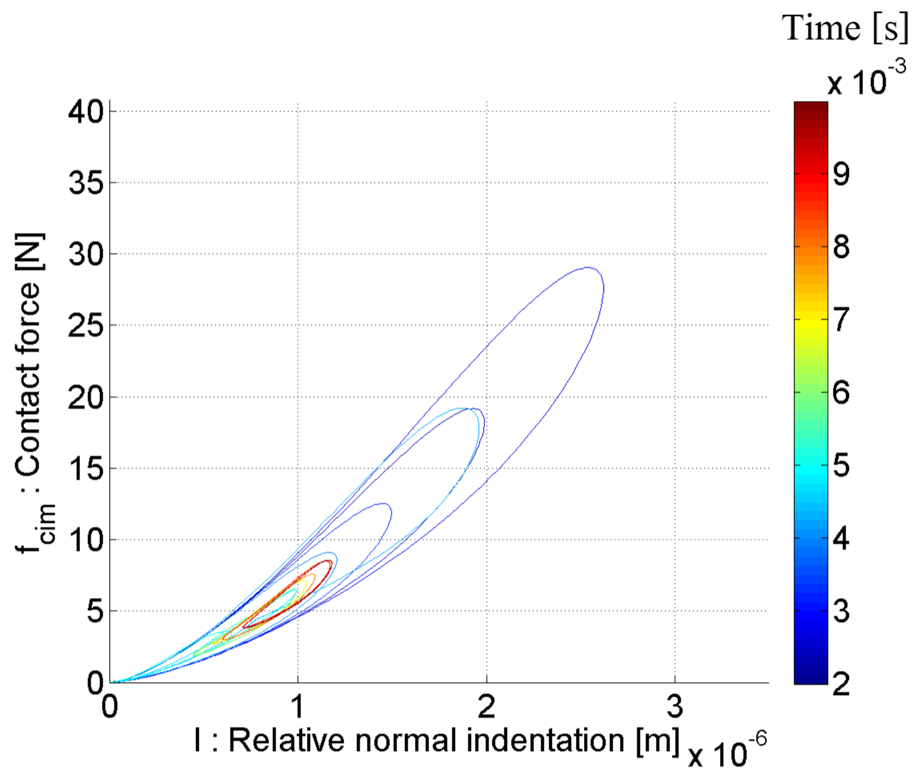


(b) Top face center

Hysteresis loop of the contact forces

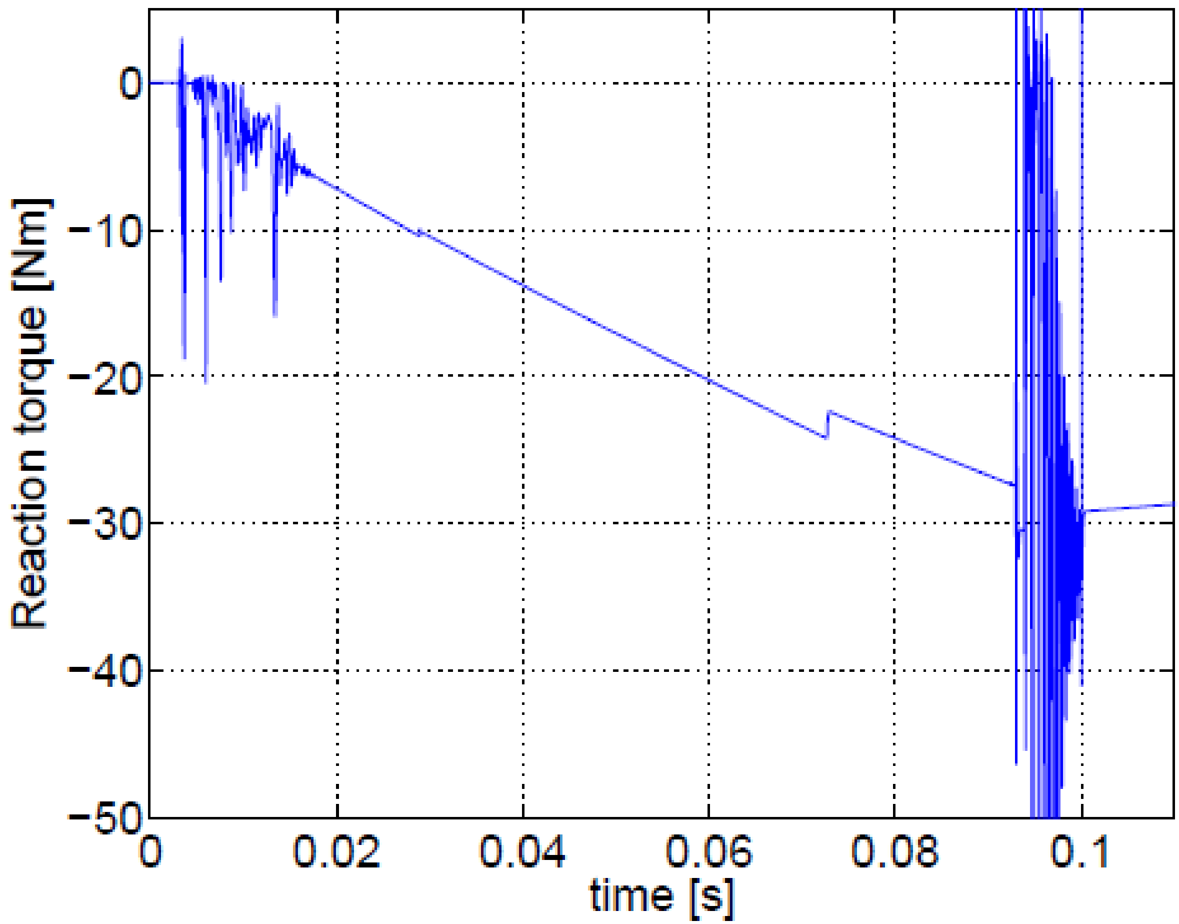


(a) Planet gear bottom



(a) Planet gear top

Reaction torque on output shaft



Conclusion & outlook

- Development of a non-ideal cylindrical joint
  - clearance, misalignment, friction forces, impact forces
  - continuous contact law with restitution coefficient
  - Test for the joint *planet gear – planet carrier* in TORSEN differentials

- Small type steps needed (h < 10^{-6} s) to allow convergence

  ➔ new contact formulation
- Difficulties to determined some parameters (friction coefficient, restitution coefficient)

- New gear pair element to account for any kind of misalignment

Thank you for your attention !

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Contact stiffness computation

- Contact condition between flexible finite element models of the two contacting bodies

  ➔ accurate but not trivial to elaborated and very CPU time expensive

- Analitycal formula:

  ➔ approximated value

$$k = \frac{2\,\pi}{3(\sigma_1 + \sigma_2)} \left(\frac{-\frac{1}{e}\frac{\mathrm{d}E}{\mathrm{d}e}}{A} \right)^{\frac{1}{2}} K^{-\frac{3}{2}}$$

with $$\sigma_i = \frac{1 - \nu_i^2}{E_i} \qquad A = \frac{1}{D_1} - \frac{1}{D_2}$$