INTERPLAY OF PERIODIC, CYCLIC AND STOCHASTIC VARIABILITY IN SELECTED AREAS OF THE H-R DIAGRAM
ASP Conference Series, Vol. 292, 2003
C. Sterken, ed.

A new version of the moment method, optimized for mode identification in multiperiodic stars

M. Briquet

Institut d'Astrophysique et de Géophysique, Université de Liège, Belgium

C. Aerts

Instituut voor Sterrenkunde, Katholieke Universiteit Leuven, Belgium

Abstract. We present a numerical version of the moment method for the identification of non-radial pulsation modes. The new version requires less computation time than the previous one, allowing to consider all the information contained in the first three moments of a multiperiodic star and to identify multiple modes simultaneously. This, together with the use of a new discriminant which considers the moments calculated at each time of observation, increases the feasibility and the accuracy of the mode identification for multiperiodic stars. Moreover, the technique is extended to rotating pulsating stars. We apply the new version to three B stars showing multiperiodicity.

1. Introduction

In this paper, we present a new numerical version of the moment method for mode identification from spectroscopy (see Aerts 1996 for the previous version). This new version improves the efficiency of the technique by performing the simultaneous identification of all the modes that are present in the data.

2. Optimization compared to the 1996-version

2.1. Gain of computation time

In the new version, the amplitudes of the first three moments are written in a such way that functions of ℓ, m and the angle of inclination of the star i are isolated. These functions are computed and memorized in files once and for all for chosen values. This leads to an important gain of computation time in the computation of the moments for different values of the other parameters A_p, K, v_Ω and σ which are respectively the velocity amplitude, the ratio between the amplitude of the horizontal to the amplitude of the vertical motion, the projected rotational velocity and the intrinsic line profile width.

We point out that, for chosen (ℓ, m, K, i), the amplitude A_p is no longer a free parameter. We limit its range by imposing that the theoretical first moment amplitude $A_{\rm th} = A_p\ A(\ell,\ m,\ K,\ i)$ must be equal to the observed one $A_{\rm obs}$.

This condition, which was not considered by Aerts (1996), allows us to reduce greatly the grid of tested parameters. Such an approach is fully justified, as the relative standard error of $A_{\rm obs}$ is always much smaller than the ones of any of the other amplitudes of the higher-order moments.

2.2. A new discriminant

In the version of Aerts (1996), the mode identification is achieved by comparing the theoretically calculated amplitudes of $< v >$, $< v^2 >$ and $< v^3 >$ with the observed ones through a discriminant. In general, the observed amplitude of the first moment as well as the constant term of the second moment are determined accurately. However, the other observed moment amplitudes can have large uncertainties. For this reason, we prefer using the moment values calculated at each time of observation t_k $(k = 1, ..., N_{\rm obs})$ instead of the amplitudes of their fit.

We propose to choose the modes and the parameters for which the following new discriminant attains the lowest value

$$\Sigma = \Big\{\frac{1}{N_{\rm obs}}\sum_{k=1}^{N_{\rm obs}}\Big[\Big(< v > (t_k) - < v >_{\rm obs} (t_k)\Big)^2$$
$$+\Big| < v^2 > (t_k) - < v^2 >_{\rm obs} (t_k)\Big|$$
$$+\Big(< v^3 > (t_k) - < v^3 >_{\rm obs} (t_k)\Big)^{2/3}\Big]\Big\}^{1/2},$$

where $< v^n >_{\rm obs}$ denotes the nth observed moment.

2.3. Increased feasibility and accuracy for multiperiodic stars

Mode identification with the 1996-version of the moment method was still difficult for multiperiodic stars. Indeed, because of large computation time, the previous version of the moment method did not take into account coupling terms appearing in the second and third moments of a multiperiodic star so that multiple modes had to be determined independently, often leading to inconsistent values of the continuous velocity parameters (i, v_Ω, σ).

In the new numerical version, all observed terms of the first three moments can be used, in particular the constant term of the second moment which is an important constraint. The modes are then determined simultaneously, leading to only one derived value for v_Ω, i and σ. We performed a large number of tests on artificial data representing the presence of respectively one, two and three modes. It appears that the method performs very well on synthetic data sets with realistic observation times and noise.

2.4. Generalization to rotating pulsating stars

The technique described above is no more restricted to slow rotators as was the case for the method by Aerts (1996). Here we extend the application to rotating pulsating stars by using the velocity field derived by Lee & Saio (1987). We implemented this version of the moment method by using Townsend's code (1997) BRUCE, which computes the pulsation velocity field for this theory. A nice feature of our current version of the method is that this numerical version can be easily generalized to an improved formalism for the pulsational velocity, e.g. one that would take into account the effects of the centrifugal forces, should this become available.

3. Application to β Ceps and SPBs

3.1. β Crucis

Aerts et al. (1998) presented numerous high signal-to-noise spectroscopic data of the β Cephei star β Crucis. They found three frequencies in the moments of the Si III 4553 Å line: f_1=5.2305468 d^{-1}, f_2=5.958666 d^{-1} and f_3=5.472165 d^{-1}. A mode identification with the 1996-version pointed towards non-axisymmetric and non-radial modes. The mode corresponding to f_1 was found to be a low-degree sectoral mode with ℓ=1 while f_2 and f_3 clearly correspond to higher degrees (ℓ=3 or ℓ=4), explaining why the two newly-found frequencies were not detected photometrically. In order to validate and/or improve this mode identification, we performed a new one. We recover the $\ell = 1$ nature of the main mode and the higher-degree nature of the two lower-amplitude modes.

3.2. EN (16) Lacertae

Lehmann et al. (2001) made for the first time a detailed spectroscopic study of EN (16) Lacertae. They recovered and refined the three intrinsic frequencies known for this star from photometry in their radial-velocity data: f_1=5.91128 d^{-1}, f_2=5.85290 d^{-1} and f_3=5.50279 d^{-1}. Our mode identification from their data is $(\ell_1, m_1) = (0,0)$, $(\ell_2, m_2) = (2,0)$ and ℓ_3=1, which is fully compatible with an improved photometric mode identification by Dupret et al. (2002). We also refer to Aerts et al. (2002) for an asteroseismological modelling of the star.

3.3. HD 74195

De Cat (2001) studied the slowly pulsating B star HD 74195, among 12 other such stars. Based on multicolour Geneva photometry and high-resolution spectroscopy, he found 4 frequencies: f_1=0.35475 d^{-1}, f_2=0.35033 d^{-1}, f_3=0.34630 d^{-1} and f_4=0.39864 d^{-1}. The mode identification with the 1996-version attributed the first three frequencies to ℓ=2 modes and f_4 to an ℓ=1 mode. The identification from photometry, however, was found to be incompatible with this result since it pointed towards ℓ=1 modes for f_1, f_2 and f_3 and an ℓ=6 mode for f_4. In an attempt to resolve the issue, we performed a new identification. We point out that the values of the observed first moment amplitudes and of the observed constant term of the second moment clearly impose $\ell \leq 3$ for the four frequencies, directly excluding a high degree mode for f_4. By eliminating degrees greater than 3 in the photometric outcome, the new candidate degree from Geneva data is also ℓ=1 for the fourth frequency. The result with the new moment method is not too different from the spectroscopic one found by De Cat (2001). Our calculations clearly point towards an ℓ=1 sectoral mode for f_4. However, the identification for the other frequencies still needs to be confirmed.

4. Conclusion

The new version of the moment method we presented is a significant improvement over the previous one for the case of multiperiodic non-radial oscillators. It performs very well on synthetic as well as real data sets. We will use it in the near future to several data sets of pulsating B stars of different kinds. Our new method is also relevant to obtain reliable mode identification in multiperiodic bright δ Scuti and γ Doradus stars.

References

Aerts C. 1996, A&A 314, 115

Aerts C., Lehmann H., Scuflaire R. et al. 2002, in *Asteroseismology across the HR diagram*, Kluwer Academic Publishers, in press

Aerts C. De Cat P., Cuypers J. et al. 1998, A&A 329, 137

De Cat P., 2001, *An observational study of bright southern slowly pulsating B stars*, PhD Thesis, Katholieke Universiteit Leuven, Belgium

Dupret M.A., Scuflaire R., Noels A. et al. 2002, in *Asteroseismology across the HR diagram*, Kluwer Academic Publishers, in press

Lee U., Saio H. 1987, MNRAS 224, 513

Lehmann H., Harmanec P., Aerts C. et al. 2001, A&A 367, 236

Townsend R.H.D. 1997, MNRAS 284, 839

Discussion

Schwarzenberg-Czerny: The moment method is convenient for analytic work but is excessively sensitive to wings of the distribution. For your numerical work you may consider mean, median and quantities of the distribution histogram, instead.