MODELLING SOUND DIFFUSION IN RAY TRACING PROGRAMS

PACS REFERENCE : 43.55.Ka

EMBRECHTS Jean-Jacques
Professor at the University of Liège, Techniques du son et de l'image
Campus du Sart-Tilman, B28,
B-4000 Liège 1
Belgium
Tel : +32 4 3662650
Fax: +32 4 3662649
E-mail : jjembrechts@ulg.ac.be

ABSTRACT

The problem of the separate treatment of the specular and diffuse reflections is first addressed. It is shown that most rough surfaces effectively exhibits this duality in their scattering properties. If the surface is not alone, but included in a closed space, the same scattering directional distribution can be used if some conditions are fulfilled. Finally, some ideas are presented to help finding the scattering coefficient used in most sound ray programs.

INTRODUCTION

Sound diffusion in most ray tracing programs is taken into account by the separate treatment of what is called the "specular" energy reflected by the surface and the "diffuse" or "scattered" energy. This paper will first address the problem of the theoretical and experimental validity of such an approach.

Next, it will be examined how it is possible to derive the scattering coefficient from experimental data or mathematical models of the directional distribution of the reflected energy. The scattering coefficient (also called the diffusion factor or the diffusion coefficient) is the fraction of the reflected energy which is scattered by the (rough) surface. The discussion will be focused on a recent method proposed by E. Mommertz [1].

SPECULAR AND DIFFUSE CONTRIBUTIONS : THEORETICAL SURVEY

Reflection From A Plane Surface

It is known that a plane, rigid and infinite surface reflects an incident plane wave in one and only one direction which is called the "specular" direction. If the sound wave is created by a point source, the technique of the "mirror" or "image" source can be used to draw the specular reflected ray which contains all the energy reflected to a given receiving point.

If the infinite plane is not rigid, the existence of the image source can still be justified if the receiving point is not too close to the surface [2]. However, if the surface is still plane, but of finite size, then the directional distribution of the reflected intensity exhibits a lobe around the specular direction [3]. The angular extent of this lobe is related to the size of the surface : if the dimensions are much greater than the wavelength, then the lobe reduces to a single ray. Otherwise, the angular extent of the lobe increases as the dimensions decrease. There is a clear difference between this form of scattering (due to the finite size of the surface) and the scattering due to the roughness (see below). To avoid confusion, it will be called “specular scattering” in the following.

Reflection From A Rough Surface

Roughness creates sound diffusion. Three particular classes of rough surfaces will be examined below: periodic roughness profiles, QRD (quadratic residue diffusers) and random roughness profiles.

For an infinite surface with a periodic profile, the scattered field can be expressed as a sum of plane waves propagating in some well-defined directions (given by the grating equation) : see for example Holford [4]. The specular direction is always a solution of this equation since it corresponds to the fundamental mode. Therefore, it can be assumed that the “specular” energy corresponds to this fundamental mode, while the “diffuse” energy includes all other propagating modes. It seems quite disturbing that the energy reflected in well-defined directions (other than specular) is considered to be diffuse, but as long as we try to prove the existence of a distinct specular reflection, this can be justified. If the periodic rough surface is of finite size, there still exists modes giving rise (in the far-field) to well-defined directions of propagation. As for the plane surface, lobes appear around these directions if the size becomes small (compared to the wavelength) : see, for example, Lam [5].

QRD diffusers are a particular case of periodic rough surfaces [6]. These diffusers are designed such that the amplitudes of all propagating lobes are quite similar. The specular lobe exists, but it is certainly not dominant. Therefore, the question may be asked whether it should be extracted from the directional distribution of the reflected intensity, or included in the “diffuse” contribution.

Sound scattering by finite random rough surfaces can be studied by the Kirchhoff approximation [3,7]. The main conclusions are the following :

- a distinct specular reflection can be observed, if the surface is not too rough. For this component, the statistical averages of both the sound pressure and the intensity in the far field are different from zero. The amplitude of this component decreases as the rms height of the surface or the angle of incidence increase (something equivalent to the Rayleigh criterion);
- a distinct diffuse component is also observed. The mean amplitude of the scattered pressure is zero, but not the intensity : its angular extent increases with the rms slope of the rough surface.

“Specular scattering” is also included in the specular component for finite size surfaces. Some directional distributions showing these two components can be found in Thorsos [8].

Conclusions For A Single Surface

The theoretical survey has shown (at least for three significant classes of diffusers) that the separate treatment of a “specular” and a “diffuse” reflected energy was justified. However :

- the specular energy is not always concentrated in a single direction, as it is assumed in a ray tracing program. This is particularly the case at low frequencies, where the “specular scattering” must be considered;
- the separation is clear for random and for plane surfaces. However, for periodic surfaces, there remains to decide if the fundamental mode is always considered as the specular component, or sometimes as part of the diffuse component.

SPECULAR AND DIFFUSE COMPONENTS FOR A SURFACE INCLUDED IN A CLOSED SPACE

Integral Equation

In a ray tracing program, once a ray hits a surface (element), it is split into a specular ray and a “diffuse” ray. If this separation can be justified for a single surface in free space, it is not evident for the same surface included in a room.

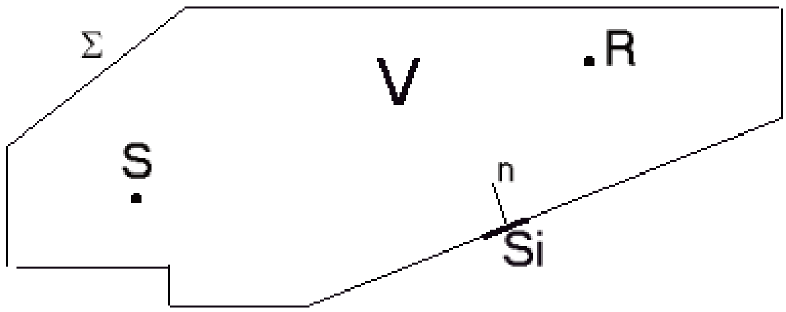

Figure 1 : Definition of symbols for the integral equation

Consider the following situation (figure 1) : the room is viewed as a volume V closed by the surface Σ. The (rough) surface S_i belongs to Σ : it is not a surface element, but a finite size surface with at least several corrugations of the roughness profile. The sound pressure p(R) at the receiving point R inside the room is given by the following integral equation [3,7] :

$$C(R)p(R) = p_0(R) + \frac{1}{4\pi}\int_{\Sigma}\left(p(d\Sigma)\frac{\partial G(R,d\Sigma)}{\partial n} - G(R,d\Sigma)\frac{\partial p(d\Sigma)}{\partial n}\right) d\Sigma \qquad (1)$$

where C(R)=0.5 if the receiving point belongs to the surface Σ and C(R)=1 otherwise, p_0 is the direct contribution from the source and G is the Green function. If we assume all surfaces to be locally reacting, then the pressure gradient can be replaced by $j\omega\rho_0\eta p$ (where η is the admittance). We can therefore discretize and write the following equation :

$$p(R) = p_0(R) + \sum_i \int_{S_i} p(dS_i)H(dS_i, R)dS_i \qquad (2)$$

Next, $p(dS_i)$ can be expressed by eq.(1) with C=0.5, giving :

$$p(R) = p_0(R) + \sum_i \int_{S_i} 2p_0(dS_i)H(dS_i,R)dS_i + \sum_i \sum_j \int_{S_i}\int_{S_j} 2p(dS_j)H(dS_j,dS_i)H(dS_i,R)dS_i \qquad (3)$$

The same operation can be repeated and repeated again, which finally leads to the following conclusion (the very long developments cannot be written in this paper) : the resulting sound pressure at point R is the sum of :

- the direct contribution p_0(R);
- the pressure at point R created by the reflection on each surface S_i, <u>as if this surface was alone with the source in free space</u> (this is not only the second term of (3), but also terms coming from the development of the double integral);
- the pressure at R created by the reflection on each surface S_i, as if this surface was alone in free space, the source being replaced by the surfaces S_j with their own distribution of pressures $p(dS_j)$.

<u>Discussion About The Results Of The Integral Equation</u>

This fundamental development proves that when a sound ray (coming from the source or from another surface S_j) reaches the surface S_i, it creates a reflected intensity distributed like the distribution measured in an anechoic room, as if S_i was a single surface in free space. Provided that :

- the intensity is measured in the same conditions. In particular, the distance of the receiving point to the surface is significant if far-field conditions cannot be assumed;
- the same remark holds for the distance between the source and the sample;
- the size of S_i must be the same as the sample measured in the laboratory conditions, if “specular scattering” is significant.

For example, let us consider a QRD diffuser. The sample given by Schroeder in [6] has a spatial period of 0.53m (the design frequency is 1.5 kHz) and the mean height of the corrugations is about

0.11m (half the design wavelength). The sample must be sufficiently wide to be representative of a periodic surface, i.e. it must include at least 4 or 5 periods (crude estimation). The total length of the sample is therefore L=2m65. We suppose that the directional distribution of the reflected pressure is known in the far field and for free field conditions. This would here require a large anechoic room since the distance between the centre of the surface and the microphone should be "much greater" than the sample's size. But, anyway, the far field pressured is assumed to be known (for example by simulation). Can we apply this diagram in a sound ray program ?

Consider a sound ray hitting a scattering surface at a given point. We can imagine this point to be the centre of a finite sample having the same size (2m65) as the test sample. In the previous section, we concluded that the sound ray can be split into a specular and diffuse components, according to the anechoic scattering diagram. However :

- there are geometrical conditions to be in the far field, i.e. on the distances d_R and d_S between the source, the surface and the receiving point. In this example, these conditions will be fulfilled only in large rooms (both distances >> 2m65);
- besides the geometrical conditions, the wavelength must be much lower than d_R and d_S to be in the far field. A reasonable limit is λ<0.5m (f>680Hz). At these frequencies, the anechoic scattering diagram can be used;
- for f<680 Hz, the roughness is supposed to be very weak (rms height<<λ), and the surface is considered as a (nearly) plane area. At these frequencies, the QRD sample reflects specularly, which means in the well-defined specular direction if the size of the surface is much greater than the wavelength. If the dimensions of the surface are comparable to (or less than) the wavelength, then the "specular scattering" must be taken into account.

So, for this QRD diffuser, there seems to be justified to use a model partly specular and partly diffuse, if we except the geometrical conditions in small rooms. Here follows another example for which the application is not so evident. Let us now consider a seating area (period 1m, rms height 1m) and suppose that we again know the anechoic scattering diagram. The geometrical conditions are here more stringent since d_R and d_S must be much greater than 5m ! Moreover :

- the roughness is now supposed very weak for λ>5m (f<68Hz). At these frequencies, the seating area is considered to be a specular reflector. But, remember that this model is valid if the receiving point is not too close to the surface if this surface is not rigid [2];
- as above, for f> 680 Hz, the anechoic scattering diagram can be used;
- and what about the interval 68 Hz<f<680 Hz ? The only possiblity seems to divide the seating area into smaller even area and to apply the scattering diagram of plane surfaces. But this would of course be much more complicated.

These examples show that if theory can justify the application of measured or calculated scattering diagrams, in practice the conditions are not always fulfilled and we must be very careful when we draw conclusions about the validity of the technique.

DETERMINATION OF THE SCATTERING COEFFICIENT

Two methods are presently proposed to determine this coefficient. The first one is for random incidence and relies upon measurements in a reverberation room [9]. The second technique consists in calculating the scattering coefficient from the directional distribution of the reflected pressure, measured (or computed) in free field conditions and for a given angle of incidence. This technique will be addressed in the following.

E. Mommertz [1] has developed an interesting method to derive the scattering coefficient δ. Starting from the values of the (reflected) sound pressure $p_r(\theta_i)$ at discrete angles θ_i, he postulates the statistical independence of the specular and the diffuse components to write the following expression :

$$\delta = 1 - \frac{\left| \sum_{i=1}^{n} p_r(\theta_i)\, p_0^*(\theta_i) \right|^2}{\sum_{i=1}^{n} \left| p_r(\theta_i) \right|^2 \sum_{i=1}^{n} \left| p_0^*(\theta_i) \right|^2} \tag{4}$$

where p_0 is the sound pressure measured for an even reference plate of the same dimensions as the sample. We recently decided to apply this method, not to measured data, but to sound pressure distributions computed by the Kirchhoff approximation model [7]. In the following are summarized the first results of our investigations.

First of all, we think that the Mommertz formula (4) is valid for 1D surfaces (also called cylindrical surfaces), i.e. surfaces presenting a roughness profile in only one direction. For 2D surfaces, we first have to express the total power W_r reflected by the surface (in the classical spherical coordinate system) :

$$W_r = A^2 \int_0^{2\pi} d\phi \int_0^{\pi/2} \left| p_r(\theta,\phi) \right|^2 \sin\theta \, d\theta \qquad (5)$$

The reflected pressure is evaluated at a distance R from the center of the surface and A^2 is a constant depending on R. In practice, p_r is only evaluated along some discrete directions : even if the pressure is derived from a mathematical model, eq.(5) cannot be integrated analytically and most numerical integration methods require the sound pressure values at some discrete angles θ and ϕ. The simplest way consists in taking evenly spaced values of θ and ϕ and to replace the double integral in (5) by a double sum. However, this simple method leads to problems related to the angle θ=0. The pressure is often measured (or calculated) in this direction, because it is the normal to the surface. This is particularly true for perpendicular incidence since significant energy is reflected around this normal direction. The problem is that sinθ=0, which deletes its contribution in a simple sum. We propose therefore to have a slightly different approach, consisting in dividing the half space above the surface into :

- a cone around the direction θ=0 ($0<\phi<2\pi$ and $0<\theta<\Delta\theta$);
- and classical discrete solid angles elsewhere ($\phi_i-\Delta\phi < \phi < \phi_i + \Delta\phi$ and $\theta_j - \Delta\theta < \theta < \theta_j + \Delta\theta$).

We are then able to transform (5) into a double sum in which the value representative of the pressure in the cone is of course taken at θ=0 :

$$W_r = A^2 \sum_i \sum_j \left| p_r(\theta_j,\phi_i) \right|^2 \Omega_{ij} \qquad (6)$$

For 2D surfaces, the Mommertz formula is then retrieved if the angles θ_j and ϕ_i are not evenly spaced, but arranged in such a way that the discrete solid angles Ω_{ij} are all equal. This could be quite cumbersome, but we can of course correct Mommertz formula to take into account the values of the different solid angles and still use evenly spaced angles.

We now consider 1D surfaces. This case is interesting, at least because the even reference surface is one of them. To compute the power reflected by this reference surface (an essential part of the denominator of (4)), it could be tempting to avoid measuring outside the plane of incidence since the pressure is negligible in those directions (this is of course valid if the dimensions are much greater than the wavelength, otherwise “specular scattering” occurs). It is possible to directly apply (6) and keep only the contributions in the plane of incidence, but we should then make some assumptions on $\Delta\theta$ and $\Delta\phi$, which seems quite unnatural. There’s a more elegant way to solve the problem.

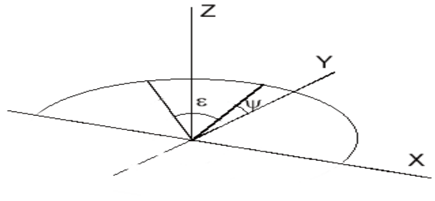

Figure 2 : New coordinate system.

First, we change the coordinate system (figure 2) : XY is the (mean) plane of the surface, Z is the normal, Ψ defines the half-plane containing X and the direction of interest (Ψ is comprised between 0 and π) and ε defines the direction in this half plane (ε is comprised between $-\pi/2$ and $\pi/2$). Not shown in this figure is the direction of incidence along X. The plane of incidence is therefore $\Psi = \pi/2$. Next, we observe that the main variation along Ψ in the integrand of (5) is given (for a 1D surface) by [3] :

$$p_r(\varepsilon,\psi) = p'_r(\varepsilon,\frac{\pi}{2})\, SINC(\pi K \cos\varepsilon \cos\psi) \tag{7}$$

where K is a constant depending on the wavelength and the dimension of the surface along Y. Integrating (5) for the variable Ψ only and using the assumption (7) gives the following result :

$$W_r = K^{-1} A^2 \int_{-\pi/2}^{\pi/2} \left| p_r(\varepsilon,\frac{\pi}{2}) \right|^2 d\varepsilon \tag{8}$$

Clearly, the Mommertz formula is strictly in accordance with eq.(8), if the reflected pressure is measured at evenly spaced angles in the plane of incidence for 1D surfaces. The only restriction to establish (8) is that the reflected pressure must be negligible at grazing angles of reflection ($|\varepsilon|=\pi/2$).

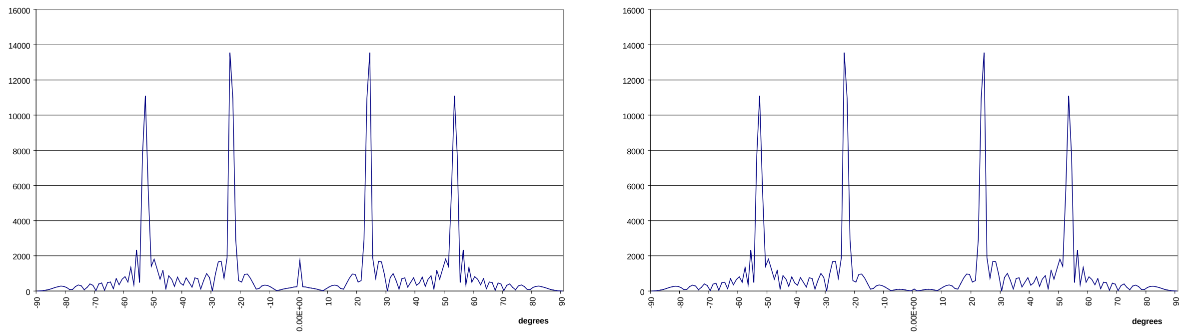

Figure 3 : Sound pressure (relative units) reflected in the far field by a 1D sinusoidal profile and calculated by the Kirchhoff approximation (left). Dimensions relative to the wavelength : height 0.2, period 2.5, length and width : 50. Normal incidence. Right : extraction of the specular contribution with the procedure recommended by Mommertz [1].

This formula has been applied to the model of the Kirchhoff approximation, and more particularly to 1D surfaces with a sinusoidal profile. Figure 3 is an example, showing the judicious extraction of the specular term by Mommertz's method. There remains of course to decide if this specular energy has to be extracted in this case or included in the diffuse component for the application in ray tracing. In the future, random profile surfaces will be tested. In particular, the statistical independence of the specular and diffuse contributions (which is an essential condition for Mommertz) will be analysed.

REFERENCES

[1] Mommertz, E. : Determination of scattering coefficients from the reflection directivity of architectural surfaces, to be published in Applied Acoustics **60**, 2000.

[2] Morse, P.M. and Ingard, K.U. : Theoretical acoustics, Mc Graw-Hill (New York), 1968, 366-371.

[3] Beckmann, P. and Spizzichino, A. : The scattering of electromagnetic waves from rough surfaces, Macmillan (New York), 1963.

[4] Holford, R.L. : Scattering of sound waves at a periodic pressure-release surface : An exact solution, Jnl. Acoust. Soc. Amer. **70**(4) , 1981, 1116-1128.

[5] Lam, Y.W. : A boundary integral formulation for the prediction of acoustic scattering from periodic structures, Jnl. Acoust. Soc. Amer. **105**(2), Pt.1, 1999, 762-769.

[6] Schroeder, M.R. : Binaural dissimilarity and optimum ceilings for concert halls : More lateral sound diffusion, Jnl. Acoust. Soc. Amer. **65**(4), 1979, 958-963.

[7] Embrechts, J.J. : Simulation of first and second order scattering by rough surfaces with a sound ray formalism, Jnl. Sound and Vib. **229**(1), 2000, 65-87.

[8] Thorsos, E.I. : The validity of the Kirchhoff approximation for rough surfaces scattering using a gaussian roughness spectrum, Jnl. Acoust. Soc. Amer. **83**, 1988, 78-92.

[9] ISO/TC43/SC 2/ WG 25 : Acoustics : Measurement of the random-incidence scattering coefficient of surfaces, draft 1997.