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SEISMIC BEHAVIOUR OF STORAGE RACKS MADE OF THIN-WALLED STEEL MEMBERS

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ABSTRACT

This paper presents an example of rack structure modelling accounting for structural non-linearities and for the possible relative motion between the rack and the stored goods that occurs as soon as the inertial force exceeds the friction resistance. It evidences that the constant reduction factor proposed by the RMI should actually be modulated according to the structural typology (and in particular the first natural frequency of the system) and to the ratio a_g/μ between the design acceleration and the friction coefficient of the pallets.

1 INTRODUCTION

Despite their lightness, storage racking systems made of thin-walled cold formed steel products are able to carry very high live load many times larger than the dead load, opposite to what happens in usual civil engineering structures. These racks can also raise considerable height. For these reasons, their use is nowadays very common in warehouses (see fig. 1). However, these structures have to be carefully designed. Indeed many difficulties arise in the prediction of their structural behaviour, such as instabilities (global, local and distortional) or modelling problems (beam-upright connection stiffness, base plate anchorages) [1].
Things become even more complicated when a storage rack is installed in a seismic zone where, subjected to an earthquake, it has to withstand horizontal dynamic forces (see fig. 2-a). In that case, in addition to usual seismic global and local mechanisms, another limit state of the system is the fall of pallets with subsequent damages to goods, people and to the structure itself (see fig. 2-b). Indeed the horizontal inertial forces acting on the pallets may be sufficient

to exceed the friction resistance. Nevertheless if the amplitude of the sliding movement is not too important, in such a way that pallets remain on the rack, this effect can benefit to the structure as it limits the horizontal forces on the rack to the friction force at the interface between pallet and beams. The American recommendations RMI propose to use in all situation a reduction coefficient equal to 0.67.

Results presented in this paper are part of a wider research project "Seisracks – Storage racks in seismic area" [2] funded by the European Union (RFCS research program). This research program aims at constituting a scientific background document for the drafting of an European Standard [3] and includes therefore many items such as:

- Experimental determination of friction properties of pallets lying on rack beams;
- Statistical evaluation of the rate of occupancy of racks in order to define the design value of horizontal seismic action, which is directly related to the mass of stored goods;
- Experimental study of the cyclic behaviour of beam-to-upright joints and of base anchor-ages;
- Experimental and numerical study of the global dynamic structural behaviour of racks subjected to earthquakes including sliding of pallets.

The present paper intends to develop one of the main aspects of this research, namely the development of numerical tools dedicated to the non-linear dynamic time-history analysis of rack structures subjected to earthquake, accounting for the global geometrical non-linearities, for the non-linear material behaviour of the joints and for the possible sliding of the pallets with respect to the supporting structure. Additional comparisons with test results are also presented, as well as results of basic parameter studies.

Figure 1: Example of a storage rack

Figure 2: (a) – Collapse of a rack structure during Northridge earthquake (1994) – (b) Fall of goods during Northridge earthquake (1994) – See Ref. [4]

2 BASES OF THE NUMERICAL TOOL

2.1 General context

The advanced numerical tool has been developed in order to allow evaluating accurately the behaviour of racks subjected to seismic action with a due account for possible sliding of supported pallets. This tool has been developed in the main frame of the non linear finite element software FineLg developed at University of Liège for more than 30 years [5]. Indeed this software already included many possibilities regarding the step-by-step dynamic analysis of steel structures accounting for geometrical and material non-linearities. In particular it was already possible to study the response of strongly non-linear structures when subjected to an earthquake defined by the time-history of the ground acceleration. The main missing feature was the possibility to let the masses slide.

2.2 Stick / slip model of the pallets

The starting point of the development of the sliding-mass model is the use of the concept of “mathematical deck” already available in FineLg since its development by FH Yang [5]. The mathematical deck was originally elaborated to study the dynamic behaviour of structures subjected to moving loads or vehicles and particularly to study the bridge-vehicles interaction.

In this approach, the interactive behaviour is obtained by solving two uncoupled sets of equations, respectively for the structure and for the vehicles, and then by ensuring compatibility and equilibrium at the contact points between the structure and the vehicles with an iterative procedure. In this scheme, the so-called mathematical deck acts as an interface element to evaluate the position of the vehicles with respect to the physical deck and to perform the iterative compatibility process (Fig. 3-a).

Regarding the possible motion of the vehicles, the horizontal displacement is imposed according to the own speed of the vehicle and to its traffic lane. The vertical displacement, velocity and acceleration are on the contrary the result of a dynamic computation and are obtained from the behaviour of the vehicle itself, of the underlying structure and of their possible interaction.
The idea in elaborating the "sliding mass" model is to start from a "moving mass" vehicle without any user-imposed speed and to derive the horizontal behaviour of the mass through a dynamic computation according to a stick/slip model (Fig. 3-b).

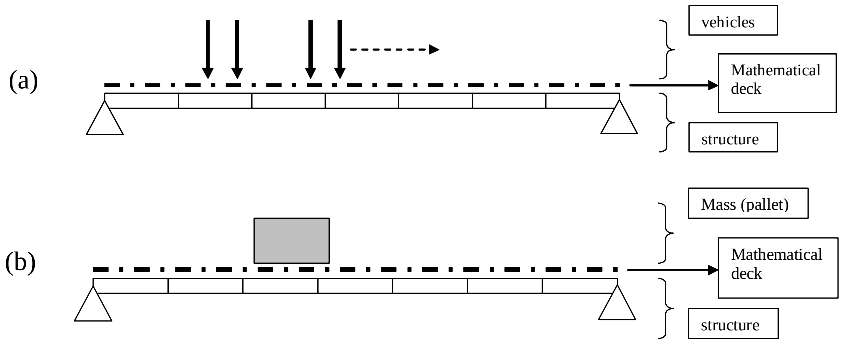


Figure 3: General scheme of the mathematical deck – (a) Original formulation – (b) Evolution for the sliding mass model

- "Stick" behaviour

The procedure for solving the global system when the masses are assumed to be fixed on the structure is the following. For each time-step:

1. Solve the structure assumed to be empty and subjected to the imposed ground acceleration. This is done by a classical Newmark procedure.
2. Thank to the mathematical deck, calculate the acceleration of the structure at the location of the contact points between the structure and the pallets. The acceleration is computed in both horizontal and vertical directions.
3. Estimate the inertial forces on the pallets corresponding to the level of acceleration computed in step 2. From these inertial forces, evaluate the contact force (horizontal and vertical) to be transferred from the pallets to the structure.
4. Solve the structure subjected to the ground motion and to the estimated contact forces. Update the acceleration of the contact points.
5. Go back to step 3 and loop until stabilization of the structural displacement. Fig. 4 presents a schematic picture of the final converged situation.
6. From the converged value of the horizontal component of the contact force, define for each pallet if the next time-step has to be treated as "stick" or "slip".

- "Slip" behaviour

As soon as the horizontal contact force computed in step 6 exceeds the static friction resistance $\boldsymbol{R_{h,st}}$, the mass starts sliding. The dynamic response of the two sub-systems (pallets and structure) are then evaluated separately under the combined effect of the imposed ground acceleration and of a constant contact force equal to the dynamic friction resistance $\boldsymbol{R_{h,dyn}}$ (Fig. 5). During this stage, the pallet moves on the mathematical deck and its position, velocity and acceleration (= $\boldsymbol{R_{h,dyn}/M}$) can be evaluated at any time step. The sliding behaviour lasts until the relative velocity between the pallet and the structure becomes equal to zero. So by estimating the relative velocity at the end of each time-step, it is possible to define if the next time-step has to be treated as "stick" or '"slip".

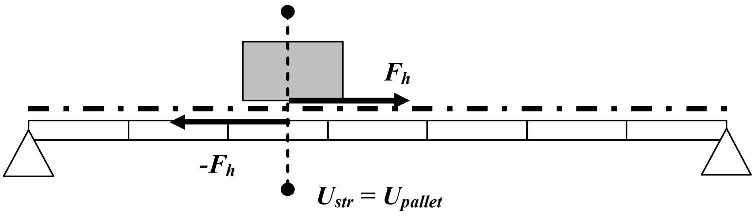


Figure 4: Sliding mass model in "stick" phase at the end of the iterative procedure (equal displacements and contact forces)

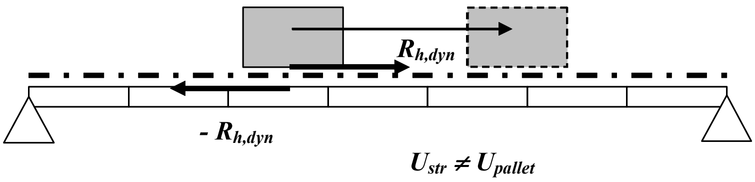


Figure 5: Sliding mass model in "slip" phase

- Note on the convergence of the iterative procedure used for the "stick" behaviour

A strict application of the procedure described above may lead to strong convergence problems. This can be illustrated on the simple example of Fig. 6, where k_s represents the stiffness of the structure and where m_s and m_p are respectively the mass of the structure and of one pallet. a_g is the imposed ground acceleration, α is a parameter of the Newmark method and Δt is the time-step.

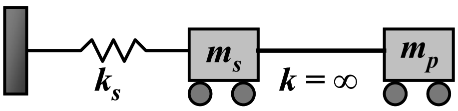


Figure 6: Simple example to illustrate the convergence problem

For this case, the steps of the iterative procedure are:

1. Calculation of the motion of the empty structure with a Newmark procedure:

$$x_s^0 = -\frac{m_s\, a_g}{k_F^{NM}} \quad with \quad k_F^{NM} = k_s + \frac{m_s}{\alpha\, \Delta t^2} \tag{1}$$

2. Structural acceleration:

$$\ddot{x}_s^0 = \frac{x_s^0}{\alpha\, \Delta t^2} \tag{2}$$

3. Contact force applied by the pallet on the structure:

$$f_{p\to s}^{1} = -m_p\left(\ddot{x}_s^0 + a_g\right) = -m_p\left(\frac{x_s^0}{\alpha\,\Delta t^2} + a_g\right) \tag{3}$$

4. Update of the estimated structural displacement:

$$x_s^1 = -\frac{1}{k_F^{NM}}\left(m_s\,a_g + m_p\,a_g + \frac{x_s^0}{\alpha\,\Delta t^2}\right) \tag{4}$$

5. Resulting global iterative process:

$$\begin{aligned} x_s^{k+1} &= \frac{-m_p}{m_s + k_s\,\alpha\,\Delta t^2}\,x_s^k + \frac{-\alpha\,\Delta t^2\left(m_s + m_p\right)a_g}{m_s + k_s\,\alpha\,\Delta t^2} \\ &= -e\,x_s^k - C \end{aligned} \tag{5}$$

The time-step being assumed small, the convergence is thus ensured if and only if:

$$e = \frac{m_p}{m_s + k_s\,\alpha\,\Delta t^2} \simeq \frac{m_p}{m_s} < 1 \tag{6}$$

This is obviously not possible for pallets on a rack structure, as the weight of the pallets is usually around 40 to 50 times the self-weight of the structure.

Therefore, the procedure is adapted and a relaxation parameter is introduced. The displacement at iteration *k+1* is defined as a linear combination of the displacement at iteration *k* and of the results of equation (5). The new iterative process is thus:

$$\begin{aligned} x_s^{k+1} &= \left(1-\eta\right)x_s^k + \eta\left(-e\,x_s^k - C\right) \\ &= \left[1-\eta\left(1+e\right)\right]x_s^k - \eta\,C \end{aligned} \tag{7}$$

Convergence is now ensured provided that the relaxation parameter η is less than η_{max} defined by:

$$\eta_{\max} = \frac{2}{1+e} \simeq \frac{2\,m_s}{m_s + m_p} \tag{8}$$

In the situation considered in this study, η_{max} can reasonably be taken equal to 0.05. The main consequent problem is that, with such a small value of the relaxation parameter, the convergence of the iterative process is relatively slow. For a fully-loaded structure, the number of iterations required to reach a precision of 10^{-5} on the structural displacement, which is necessary to manage adequately the situation of a structure with many pallets, can go up to 100 iterations in the worst situations. This iterative procedure could therefore usefully be improved in order to speed up the calculation.

Nevertheless, the main advantage of the proposed approach is to separate completely the resolution of the equations of motion for the structure and for the pallets. A same model can thus be used for both the "stick" and the "slip" behaviour, without any modification of the stiffness, mass and damping matrices characterizing the structure and the pallets. The only information required for computing the coupled effect is the relationship between the acceleration imposed at the base of the pallet and the reaction force applied by the pallet on its support [i.e. $f_{p\to s} = fct\left(acc_{pallet}\right)$]. This also allows a due account of the structural non-linearities, since these latter only implies a modification of the structural stiffness matrix without additional consequences on the resolution procedure.

2.3 Simple validation examples

In order to validate the sliding mass model, a series of very simple systems has been studied with FineLg and compared to equivalent MDOF systems solved with a semi-analytical approach (see ref. [7]). Some of the considered examples are presented in Fig. 7.

The results obtained with FineLg and with the reference semi-analytical procedure are found in very good agreement. As illustration, results obtained with FineLg for case (c) are plotted in Fig. 8 for μ/α = 1.00 (no sliding) and μ/α = 0.5 (μ is the friction coefficient and α is the maximum imposed acceleration referred to gravity). In this second configuration, four sliding phases are observed, during which the relative displacement between M2 and M3 varies (see the green curve in Fig. 8).

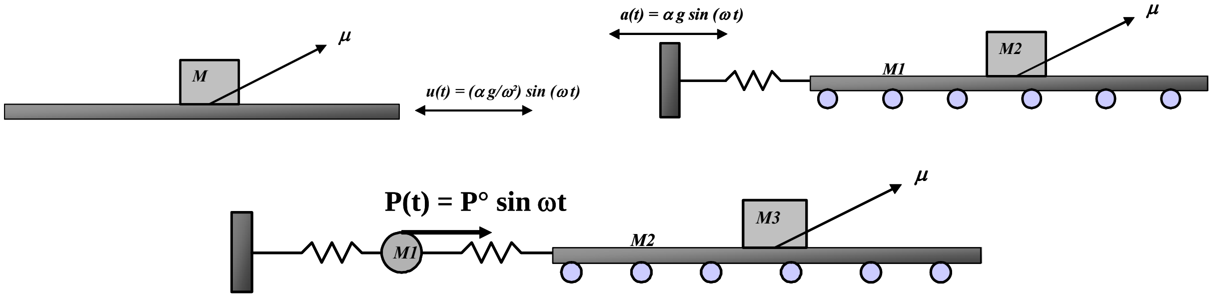


Figure 7: Validation examples (a) 1DOF – (b) 2DOF – (c) 3DOF

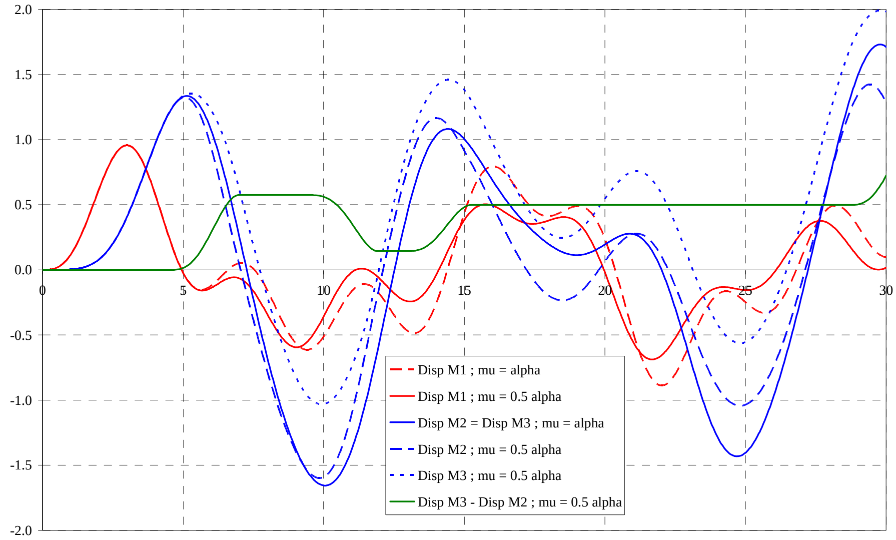


Figure 8: Time-history of the displacements obtained for case (c)

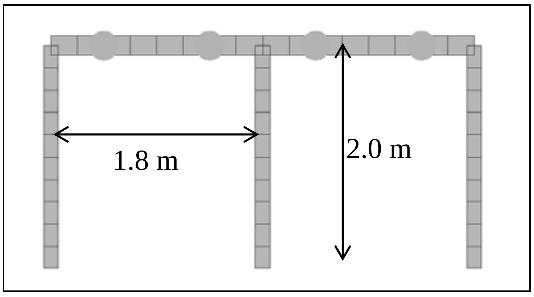
3 DOWN-AISLE BEHAVIOUR OF A RACK STRUCTURE

3.1 Elastic behaviour with sliding

This section intends to show an application of the numerical tool for the step-by-step analysis dynamic analysis of a very simple rack structure subjected to an imposed acceleration of the ground. At this stage, the structure is assumed to behave linearly (neither second-order geometrical effects nor yielding of any structural elements is taken into account). The main

practical objective is to evidence the consequences on the global seismic behaviour of the structure when the pallets are likely to slide on the beams.
The example comprises two spans and a number of levels equal to 1 or 3 with typical dimensions of rack structures (span = 1.8 m; height of a level = 2.0 m – see Fig. 9). The cross section properties of the structural elements (beams and uprights) are also typical of real rack structures. The beam-to-upright joints and base-anchorages are modelled by springs with appropriate rotational stiffness. Four masses of 750 kg are placed at each level. The damping ratio is considered equal to 3%.

CROSS SECTION PROPERTIES OF THE STRUCTURAL COMPONENTS OF THE RACK		
Element	**Area**	**Inertia**
Beams	605.8 mm²	93.8 cm^4
Uprights	488.9 mm²	41.3 cm^4
	Equivalent spring stiffness	
Beam-to-upright connection	160 kNm	
Base anchorage	160 kNm	



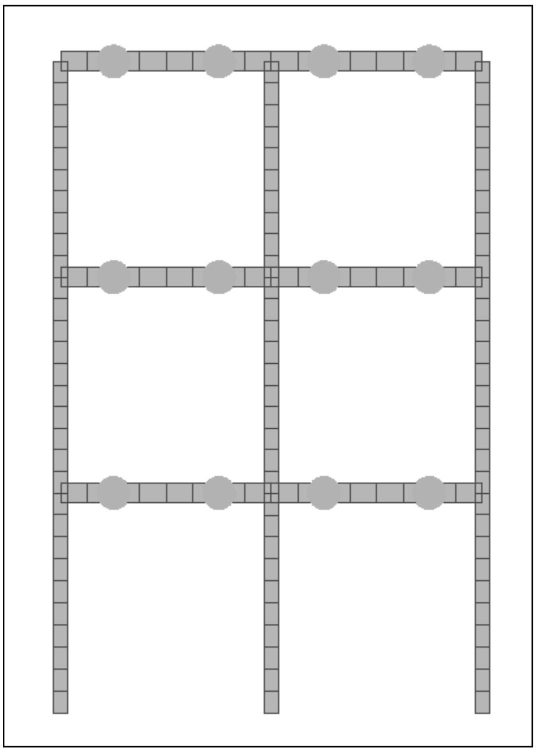
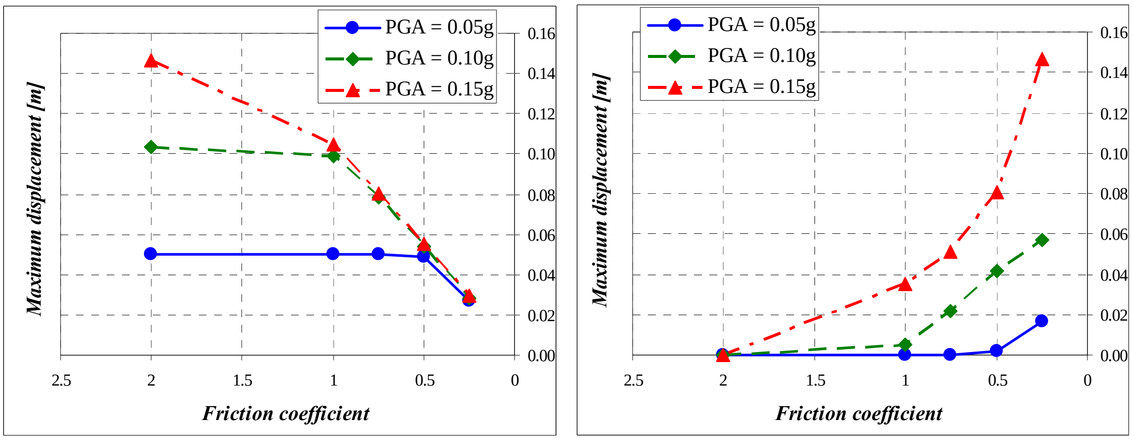


Figure 9: Simple rack structures

The structure is subjected to 7 artificial accelerograms with spectrum compatible with a reference spectrum having the following characteristics:

- EC8 type I spectrum;
- PGA varying from 0.05 g, 0.10 g or 0.15 g;
- Soil type C;
- Duration = 15 s.

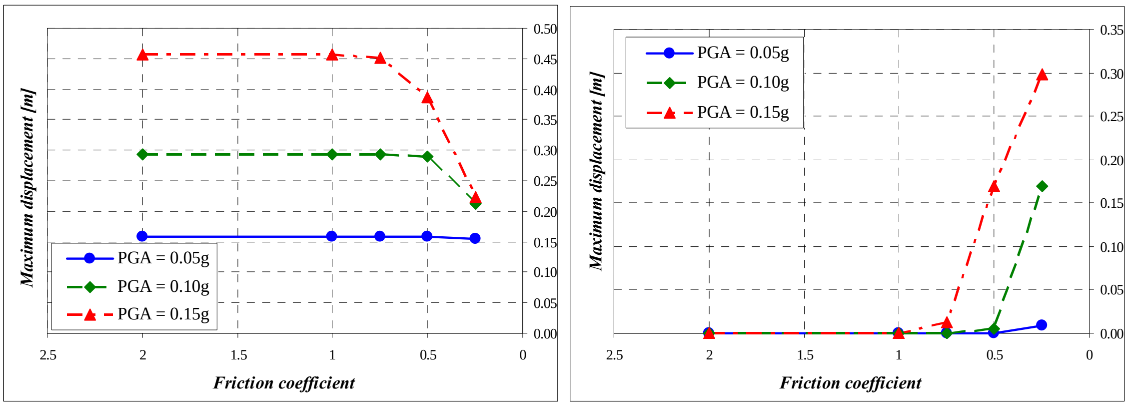
The friction coefficient of the pallets is varied from 2 (which is not physically relevant but corresponds to pallets fully fixed on the beams) to 0.25.
Figures 10 and 11 present respectively the results obtained for 1 and 3 levels. Figure 10-a shows the evolution of the maximum transverse displacement of the structure when the friction coefficient decreases. This displacement is the average of the maximum displacements obtained from the 7 considered ground motions. Figure 10-b shows in parallel the maximum relative displacement of the pallets with respect to the supporting beam. Figures 11-a and 11-b present similar results for the displacement of the top of the 3-level structure.



(a) Maximum global displacement

(b) Maximum local sliding displacement

Figure 10: Effect of sliding on the down-aisle behaviour of the one-level rack



(a) Maximum global top displacement

(b) Maximum local sliding displacement

Figure 11: Effect of sliding on the down-aisle behaviour of the 3-level rack

The main observations that can be drawn from these results are the following:

- Sliding of the masses and subsequent limitation of the inertial forces to the friction force between pallets and beams is likely to significantly reduce the global displacements of the structure and hence the internal forces in the structure and the support reactions. For low friction coefficients, these displacements can be reduced to 20% of the values obtained if the pallets are supposed to be fully fixed on the beams.
- However, this reduction cannot be considered as a general rule. Indeed it is strongly related to the PGA level and to the structural typology (i.e. the number of levels). For example, no reduction is observed for a 3-level structure subjected to an earthquake with PGA equal to 0.05 g, even for a friction coefficient equal to 0.25, while the reduction is about 50 % for the same structure but with a PGA equal to 0.15 g. Further, for a same PGA of 0.15 g and a same friction coefficient of 0.5, global displacements are reduced to about 35 % of their fixed value for a one-level structure, while they are only reduced to 80 % for a 3-level structure.
- Moreover the local sliding displacement of the pallets in the case of low friction coefficients can be very important and therefore non compatible with real conditions. For example, for a 3-level structure subjected to an earthquake with a PGA equal to 0.15 g, the local displacement of the pallets with respect to the structure can go up to 30 cm for a

friction coefficient equal to 0.25. Such a value of the displacement is obviously non admissible, since it corresponds either to a fall of the pallet or at least to an impact of the pallet against an upright. It is important to remind that a friction coefficient as small as 0.25 can be rather frequent for usual practical situations (see ref. [2]).

With this perspective, the most advantageous situation would be a structure with only one loaded level, with high design acceleration and with a very low friction coefficient. For example, if the ground acceleration a_g is equal to 0.5g, the design acceleration of the structure may rise up to 1.25g. If only one level is loaded and if the friction coefficient μ is equal to 0.25, the force acting on the structure is exactly limited to the friction force, which corresponds to an equivalent acceleration of 0.25g. The reduction factor is thus equal to 0.25g/1.25g = 0.2. On the other hand, for structures with many loaded levels and for low values of a_g, it can happen that no sliding occurs. The reduction factor is consequently equal to 1.0. The practical range of E_D is therefore rather wide, between 0.2 and 1.0. It is interesting to note that the value proposed by the American RMI is equal to 0.67, which is somewhere in the middle of this interval. Some additional studies are still required to calibrate an expression of the reduction coefficient that would depend on the structural typology and on the ratio a_g/μ.

3.2 Hysteretic behaviour of the connections

The numerical model has then been used to simulate test results obtained on the shaking table of the Laboratory of Earthquake Engineering of the NTU Athens. Figure 12 shows the tested specimen (2 bays – 3 levels non-braced structure) and the corresponding numerical model.

It is obviously not possible to describe in this paper the whole series of test results and the numerous variations of the different parameters of the model that have been taken into consideration. Results for one intermediate level of acceleration are presented (i.e. peak ground acceleration of the table equal to 0.45g) and only the impact of the parameter having the main influence on the behaviour of the rack is commented (i.e. the resistance of the connections). It may be noted that even if the imposed acceleration is much higher than in the example of section 3.1, the resulting structural acceleration is lower due to different frequency content. The influence of the friction coefficient is therefore less important and not considered in this section. The extensive comparison between numerical and experimental results can be found in [2].

In this section, geometrical non-linearities and materially non-linear behaviour of beam-to-upright joints and of base-anchorages are considered. Figure 13 presents a comparison of the displacement at the top level measured during the test on one hand and obtained with the numerical model (with different assumptions on the base resistance) on the other hand. In the numerical model, the friction coefficient was assumed equal to 0.5. No sliding was predicted by the numerical model, while very small sliding displacements were measured during the tests (about 1 mm). The input signal is the time-history of the acceleration recorded on the table during the test. Table 1 summarizes some interesting numerical values for comparison purposes.

From these results, the following observations can be drawn:

- The general shape of the time-history response of the structure is correctly predicted by the model.
- It is necessary to account for the non-linear behaviour of the joints otherwise the residual displacements of the structure cannot be explained.
- Displacements are slightly overestimated by the model (about 15%). However no sliding was accounted for in the numerical model, while some sliding actually occurred during the test and brought some additional damping to the structure.

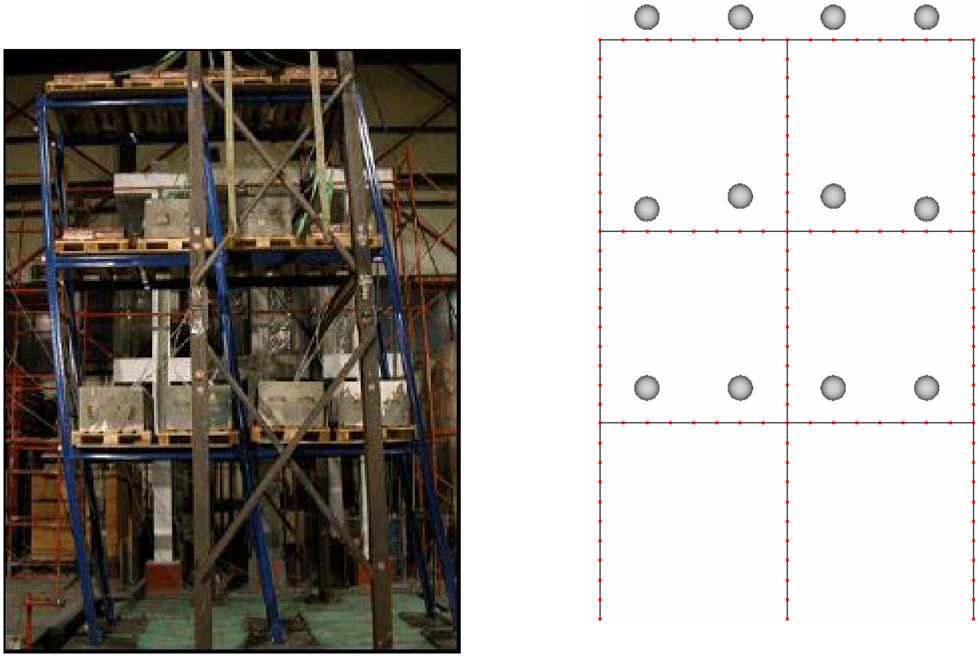


Figure 12: test specimen and corresponding numerical model

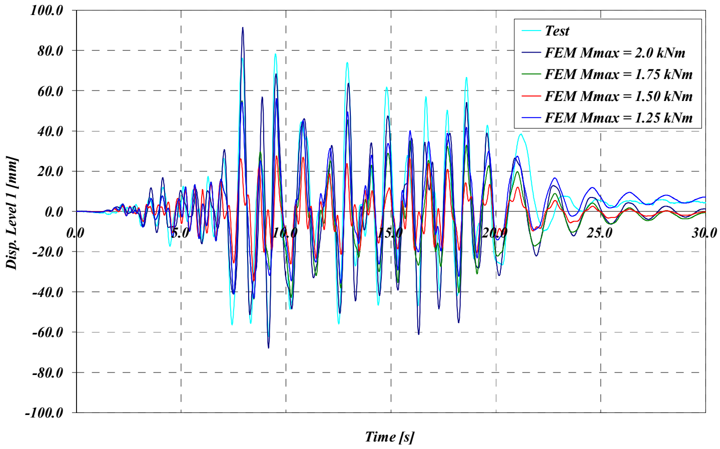


Figure 13: Effect of the column-base resistance on the structural response (time-histories)

	Test	Model (Base resistance = 2.00 kNm)	Model (Base resistance = 1.75 kNm)	Model (Base resistance = 1.50 kNm)	Model (Base resistance = 1.25 kNm)
Maximum displacement [mm]	78.3	91.4	91.4	91.4	90.3
Minimum displacement [mm]	62.2	68.0	71.0	75.9	75.0
Residual displacement [mm]	4.7	0.4	1.5	0.9	7.1

Table 1: Effect of the column-base resistance on the structural response (numerical values)

4 CONCLUSIONS AND PERSPECTIVE

In this paper, a numerical model able to reproduce in a satisfactory manner the behaviour of storage racks subjected to earthquakes has been presented and compared with analytical examples and test results. The numerical model is base on a stick / slip model of the pallets solved by using an iterative procedure with relaxation. The method has been shown as robust, even if the numerical efficiency could be improved in the perspective of an acceleration of the resolution.

The paper has then presented a short parameter study that shows the influence of the sliding of pallets on the global behaviour of the racks. The main outcome of this study is that the effect of the sliding can be very important (reduction of 80 % of the internal forces in the structure in some cases) but that this effect is also strongly related with the seismic intensity and the structural typology, contrary to what is proposed by the American RMI. Further the associated local displacements of the pallets may be unacceptable for practical reasons (fall of pallets or impacts against uprights). Finally a focus is put on the main parameters on which it is possible to act for calibrating the numerical model with respect to experimental tests, i.e. the behaviour of the connections (and in particular of the base anchorages), the friction coefficient of the pallets and the viscous damping (even if this last aspect was not commented into details in the paper).

It will thus be possible in a next research step to consider extensive parameter studies with variations of all these parameters, in the perspective of developing backgrounds for design recommendations, and in particular to calibrate the reduction coefficient in case of sliding.

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