Chapter 7

Implications of tribimaximal lepton mixing for leptogenesis

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Abstract

In models featuring exact mixing patterns the mass matrices that define the effective light neutrino mass matrix -and the light neutrino matrix itself- are form-diagonalizable. We study leptogenesis in type I seesaw models in these contexts pointing out that the CP asymmetry in right-handed neutrino decays vanishes as a consequence of the mass matrices being form-diagonalizable. A non-vanishing CP asymmetry arises once deviations from the exact mixing scheme, induced by higher order effective operators, is allowed. Finally we discuss alternative pathways to viable leptogenesis in these kind of models.

7.1. Introduction

Leptogenesis is a scenario in which the baryon asymmetry of the Universe is dynamically generated first in the lepton sector and reprocessed into a baryon asymmetry via standard model electroweak sphaleron processes [1]. In order for this mechanism to take place lepton number must be broken ¹ thus implying models for Majorana neutrino masses provide the frameworks for leptogenesis.

The standard seesaw model (type I seesaw) [3] defines the scheme for standard leptogenesis [4]. In this model leptogenesis becomes plausible due to the fact that: (i) the Yukawa couplings of the fermionic electroweak singlets (right-handed (RH) for brevity) contain new physical CP phases; (ii) lepton number violation is provided by the RH neutrino masses; (iii) the expansion of the Universe guarantees deviations from thermodynamic equilibrium in RH neutrino decays. With these conditions satisfied the generation of a net \( B - L \) asymmetry proceeds through the decays of the lightest RH neutrino.

In standard leptogenesis the problem of calculating the baryon asymmetry depends -in first approximation- on two parameters: the washout factor \( \tilde{m} \), determined by the contribution of the lightest RH neutrino to light neutrino masses, and the CP asymmetry \( \epsilon_N \) in RH neutrino decays. Thus, two conditions must be satisfied

¹The exception being scenarios of purely flavored leptogenesis as the one discussed in [2].
in order to produce a net baryon asymmetry: overcome the washout effects and a non-vanishing $\epsilon_N$. The determination of both requires the specification of the RH neutrino Yukawa couplings and mass spectrum, however the former is more involved as it demands calculating the efficiency factor, which in turn implies solving the corresponding kinetic equations describing the RH neutrino dynamics.

The seesaw parameter space consist of 18 parameters out of which 9 are constrained by low-energy data. This implies once these restrictions are placed there is a remaining arbitrary 9 dimensional parameter space. Is indeed partially due to this arbitrariness that leptogenesis suffers from the lack of testability [5]. If further restrictions on the parameter space can be placed the arbitrariness should be reduced, and this is actually the case if a lepton flavor symmetry is present as some of these parameters will be either forced to vanish or to be correlated.

Even in the light of recent neutrino data [36] there is still a strong motivation to believe that the leptonic mixing is a result of an underlying flavor symmetry operating in the lepton sector [7]. The tri-bimaximal mixing (TBM) pattern [8] as an input ansatz remains as a viable guideline to construct lepton flavor models accounting for neutrino masses and mixing angles. We here discuss standard leptogenesis in the context of the seesaw extended with flavor symmetries \(^2\). We will study the implications that a generic flavor symmetry associated to a flavor group $G_F$ leading to the TBM pattern may have for the baryon asymmetry generated via leptogenesis. It will be proven that if leptogenesis takes place below the scale at which the flavor symmetry is broken $\epsilon_N$ vanishes in the limit of exact TBM. Viable leptogenesis becomes possible once departures -induced by higher order effective operators- are allowed. We will point out other pathways to leptogenesis in flavor models. We will closely follow references [12].

### 7.2. General considerations

With the addition of three RH neutrinos $N_{Ri=1,2,3}$ the standard model Lagrangian is extended with a new set of interactions that, in a generic basis in which the charged lepton Yukawa coupling matrix is diagonal, can be written as

$$\mathcal{L} = -i \bar{N}_{Ri} \gamma_\mu \partial^\mu N_{Ri} + \bar{\ell}_L N_{Ri} \lambda_{ij} \phi + \frac{1}{2} N_{Ri} C M_R \bar{N}_{Ri} + \text{h.c.}. \quad (7.1)$$

Here $\ell_L$ are the lepton $SU(2)$ doublets, $\phi^T = (\phi^+, \phi^0)$ is the Higgs electroweak doublet, $M_R$, are the RH neutrino masses, $C$ is the charge conjugation operator and $\lambda$ is a $3 \times 3$ Yukawa matrix in flavor space (we will denote matrices in bold-face). In the seesaw limit $M_R \gg v$ (with $v \simeq 174$ GeV) the effective neutrino mass matrix is obtained once the heavy fields are integrated out:

$$m_\nu^{\text{eff}} = -m_D \tilde{M}_R^{-1} m_D^T, \quad (7.2)$$

with $m_D = v \lambda$. From now on we will assume the Lagrangian in (7.1) to be invariant under a flavor group $G_F$ that enforces the TBM pattern. This has two implications. Firstly, the mass matrices $m_D, M_R$ and $m_\nu^{\text{eff}}$ are form-diagonalizable [14]. Secondly, the effective neutrino mass matrix is diagonalized by the TBM leptonic mass matrix, namely

$$\hat{D} U_T \tau \text{ ) \text{ with \ } U_T = \left( \begin{array}{ccc} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{array} \right), \quad (7.3)$$

where the matrix $\hat{D} = \text{diag}(e^{i\varphi_1}, e^{i\varphi_2}, 1)$ contains the low-energy Majorana CP phases. The matrices $M_R$ and $m_D$, being in general non-diagonal, can be diagonalized according to

$$U_L m_D U_R^\dagger = \hat{m}_D \quad \text{and} \quad V_R M_R V_R^T = \hat{M}_R. \quad (7.4)$$

\(^2\)This subject has been recently analyzed in a series of papers [9,10,11,12,13].
neutrino decays is entirely determined by the CP violating asymmetry resulting from the interference between

equilibrium and CP violating decays of the lightest RH neutrino can be accurately neglected as all the

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7.3. TBM and leptogenesis

Depending on the temperature regimen at which leptogenesis takes place the lepton doublets states that

propagate in space-time can be either a superposition of flavor states or the actual flavor components. For

temperatures above $\sim 10^{13}$ GeV the flavor composition of the lepton doublets produced in the out-
of-equilibrium and CP violating decays of the lightest RH neutrino can be accurately neglected as all the

standard model lepton Yukawa reactions are slow. In that case the amount of CP violation generated in RH

neutrino decays is entirely determined by the CP violating asymmetry resulting from the interference between

\[ \begin{align*}
\ell_j & \quad \phi \\
\ell_j & \quad \phi \\
\ell_j & \quad \phi
\end{align*} \]

Figure 7.1. Tree-level and one-loop vertex and wave-function corrections responsible for the CP asymmetry

in RH neutrino decays.

with $U_{L,R}$ and $V_R$ unitary $3 \times 3$ matrices characterized in general by three rotation angles and six phases. By

means of eqs. (7.3) and (7.4) eq. (7.2) can be rewritten as

\[ \hat{m}_\nu = \hat{D} \left( U_{TB}^T U_L \right) \hat{m}_\nu \left( U_R^\dagger V_R \right) \hat{M}_R \left( V_R^T U_R^\dagger \right) \hat{m}_D \left( U_L^T U_{TB} \right) \hat{D}. \] (7.5)

Since the mass matrices $m_D$ and $M_R$ are form-diagonalizable the corresponding diagonalization matrices

$U_{L,R}$ and $V_R$ do not depend upon the corresponding eigenvalues entering in the diagonal matrices $\hat{m}_D$ and

$\hat{M}_R$ [14]. Accordingly, eq. (7.5) is satisfied if and only if the two following conditions are satisfied:

\[ U_{TB}^T U_L = \hat{P}_L O_{D_i} \quad \text{and} \quad U_R^\dagger V_R = O_{D_i} \hat{P}_R O_{R_m}. \] (7.6)

The matrices $O_{D_i}$ and $O_{R_m}$ are unitary and complex orthogonal matrices, respectively, that rotate the $i$ and

$m$ degenerate eigenvalues in $\hat{m}_D$ and $\hat{M}_R$. They are such that if there is no degeneracy in non of the two

mass matrices $O_{D_i} = 1$ and $O_{R_m} = 1$. The matrices $\hat{P}_{L,R}$ are given by $\hat{P}_{L,R} = \text{diag} ( e^{i \alpha_i^{L,R}}, e^{i \alpha_2^{L,R}}, e^{i \alpha_3^{L,R}} )$

and thus taking $\hat{M}_R$ to be real the following constraints on the CP phases must be satisfied:

\[ \varphi_i + \alpha_i^L + \gamma_i = 2k\pi \quad \text{and} \quad \alpha_3^L + \alpha_3^R + \gamma_3 = 2n\pi, \] (7.7)

with $\gamma_i$ the CP phases in $\hat{m}_D$.

In the basis in which the RH Majorana neutrino mass matrix is diagonal the Dirac mass matrix is given by

\[ m_D^R = m_D V_R, \] (7.8)

therefore, taking into account the results in (7.6) and (7.4), the Dirac mass matrix can be rewritten according to

\[ m_D^R = U_{TB} \hat{P}_L \hat{m}_D \hat{P}_R O_{R_m}. \] (7.9)
the tree-level and one-loop vertex and wave-function Feynman diagrams depicted in figure 7.1. In the limit of a strongly hierarchical RH neutrino mass spectrum the result reads [15]

$$
\epsilon_{N_i} = \frac{3}{8 v^2 \pi} \frac{1}{(m_{D_i}^R m_{D_i}^R)_{ii}} \sum_{i \neq k} \text{Im} \left[ (m_{D_i}^R m_{D_i}^R)_{ki} \right]^2 \frac{M_{R_i}}{M_{R_k}}. \tag{7.10}
$$

From the result in eq. (7.9) the quantity $m_{D_i}^R m_{D_i}^R$ can be calculated, namely

$$
m_{D_i}^R m_{D_i}^R = O_{Rm}^T \hat{m}_D^2 O_{Rm}. \tag{7.11}
$$

From this expression it becomes clear that as long as the $G_F$ flavor group enforces the mass matrices to be form-diagonalizable the CP violating asymmetry vanishes thus implying in the limit of TBM leptogenesis is not viable. Though we have stucked to the concrete TBM pattern this result remains valid regardless of the mixing scheme. As it has been stressed it is a consequence of the form-diagonalizable form of the mass matrices, which as long as we deal with an exact mixing pattern it is always the case.

A vanishing CP asymmetry, however, can be accommodated in several ways that we now briefly discuss in turn:

- **Inclusion of higher order effective operators [11,12]:**
  Flavor models involve effective operators that result from integrating out heavy fields that account for quark masses and mixings. These effective operators, arising from example from a quark flavor model à la Froggatt-Nielsen [1], involve different powers of the ratio $\delta = \langle S \rangle / M_F$, where $\langle S \rangle$ is the vacuum expectation value of an electroweak singlet flavon that triggers the breaking of the corresponding quark flavor symmetry (a $U(1)_X$ in the case of Froggatt-Nielsen models) and $M_F$ is the mass scale of the heavy vectorlike fields.

  Whenever only leading order effective terms are included, that is to say order $\delta$ terms, there are no deviations from the exact mixing pattern. However, once next-to-leading order terms are included departures from this pattern are induced (the mass matrices deviate from there form-diagonalizable form) and thus the CP violating asymmetry becomes non-zero.

- **Presence of new physical degrees of freedom [12,18]:**
  In models in which the effective neutrino mass matrix receives contributions from other degrees of freedom, as for example in models featuring an interplay between type I and type II seesaw, the CP asymmetry typically contains additional contributions. The additional terms can be -in principle- also constrained by the flavor symmetry thus leading to a vanishing $\epsilon_N$. However, in general, these constraints are not so strong as in models entirely based in type I seesaw. Accordingly, realizations exhibiting type I and II seesaw models with a non-vanishing $\epsilon_N$ -even at order $\delta$- can be constructed.

- **The role of flavons [6]:**
  The analysis leading to the conclusion that the CP asymmetry vanishes in the limit of exact TBM has been done assuming leptogenesis takes place below the scale at which $G_F$ is broken. A new twist occurs if the generation of the lepton asymmetry happens at energy scales at which $G_F$ is still an exact symmetry (flavor symmetric phase). In that case the conventional contributions to the CP asymmetry (those given by the interference of the Feynman diagrams shown in fig. 7.1 are still zero but if: (a) some of the RH neutrinos lie in different representations of $G_F$; (b) the flavons are lighter than one of the RH neutrino representations, new non-zero contributions to the CP asymmetry can be built thus allowing leptogenesis to proceed even in the flavor symmetric phase.
7.4. Conclusions

We have analyzed the viability of leptogenesis in seesaw I models extended with flavor symmetries and assuming the lepton asymmetry is generated in the flavor broken phase. For concreteness we have taken the TBM pattern and have shown that in the limit in which this pattern is exact the CP asymmetry in RH neutrino decays vanishes. We have also discussed several pathways to leptogenesis in this stage (flavor broken phase) which include the addition of next-to-leading order corrections (higher order effective operators) or the addition of new degrees of freedom, as would be the case in models with an interplay between type I and II seesaw. Finally we have commented on scenarios for leptogenesis taking place in the flavor symmetric phase pointing out the viability of leptogenesis relies –in these cases– on the role played by the scalar flavons.

REFERENCES