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The Casimir Effect and the Vacuum Energy: Duality in the Physical Interpretation

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Abstract The Casimir effect is usually interpreted as arising from the modification of the zero point energy of QED when two perfectly conducting plates are put very close to each other, and as a proof of the "reality" of this zero point energy. The Dark Energy, necessary to explain the acceleration of the expansion of the Universe is sometimes viewed as another proof of the same reality. Recently, several physicists have challenged the usual interpretation, arguing that the Casimir effect should rather be considered as a "giant" van der Waals effect. All these aspects are shortly reviewed.

1 Introduction

The Casimir effect (CE) is a quantum force acting, in its simplest version, between two uncharged parallel conducting plates. It is usually considered as originating from the differences of the QED ground state energies when boundary conditions are modified. It has been calculated for the first time, along this line, by Casimir [1]. The CE appears as a tiny difference between two infinite, or at least very large, ground state energies. It is nevertheless a real effect. It has been verified experimentally during the nineties and it is encountered daily in nanotechnology.

The QED ground state energy is not directly measurable, but the CE, which corresponds to a mere change of this energy when boundary conditions are modified, is usually considered as a *proof* of the physical reality of this ground state energy, although the latter cannot be tapped, or even measured, by usual (electromagnetic) means. However, as any kind of energy, it can be subjected to gravitation. Since this energy can pervade all places of the Universe, it can contribute to the dark energy, that seems to be a firmly established feature in Cosmology. This has been proposed for the first time by Zel'dovich [2].

Viewing CE as a proof of the physical reality of the ground state of QED is shared by a large number of physicists and has entered standard textbooks. Just to quote one of them:

"That this concept [the vacuum energy] is *not a figment* of the physicist's imagination was already *demonstrated* many years ago, when Casimir predicted that by modifying boundary conditions on the vacuum state, the change of the vacuum energy would lead to a measurable force, subsequently detected and measured ..." [3].

This conventional wisdom about CE is seriously challenged nowadays by many authors. Some even claim that CE cannot really test the ground state energy and should be regarded as a limiting case of van der Waals forces, when the "molecules" become gigantic and perfectly conducting objects. It is the purpose of this paper of making a critical review of these arguments and of presenting some handwaving arguments to link the van der Waals forces in different "geometries", from molecules to conducting plates.

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2 The Casimir Effect as a Manifestation of Zero-point energy

The ground state energy of a free electromagnetic radiation field is given (in the standard Coulomb gauge) by the summation over the normal modes of one half of the corresponding quanta of energy:

$$E = g \sum_{\mathbf{k}} \frac{1}{2} \hbar \omega_{\mathbf{k}}, \tag{1}$$

where g is the degeneracy. Let us now assume two parallel perfectly conducting plates, separated by a distance d, perpendicular to the z direction and infinitely long in x and y directions. Between the plates the normal modes of the electromagnetic field are not the same as in free space. In the latter case, the modes are characterized by a wave vector $\mathbf{k}$ of any value. In the former case, the tangential component of the electric field and the normal component of the magnetic field have to vanish on the plates. As a consequence, the z-component of the wave vector takes discrete values. So, the zero-point energy is changed and a force is acting on the plates. The latter force can also be understood as arising from the fact that the pressure is not the same between the plates and outside of the plates (where the field is the same as in free space; see Ref. [4] for a discussion of this point, and of the effects of finite size of real plates). I will not describe here the normal modes (for details see Refs. [4,5]), but it is more or less evident that the modes with a small perpendicular component of the wave vector (large wavelength) are meaningfully different from the free modes, whereas those with large wave number (small wavelength) are essentially the same in both configurations, as illustrated in Fig. 1. The relevant separating value is roughly $1/d$.

The difference of zero-point energy (per unit surface) can be put in the form of the difference between the integral of a continuous function and the sum of the values of the same function at integer values of the suitably reduced wave number:

$$\frac{\Delta E}{S} = \frac{E_{cav}}{S} - \frac{E_{free}}{S} = \frac{\hbar c \pi^2}{4d^3} \left\{ \frac{1}{2} \int_0^\infty du \sqrt{u} + \sum_{n=1}^{\infty} \int_0^\infty du \left(u + n^2\right)^{1/2} - \int_0^\infty dx \int_0^\infty du \left(u + x^2\right)^{1/2} \right\}. \tag{2}$$

Each term in the rhs is infinite, but the final expression can be made meaningful by introducing a regularising factor (e.g. $exp(-\beta\sqrt{u+x^2})$). Expression 2 can be transformed by the Euler-McLaurin theorem and then depends only on the values of the function and its derivatives at the end of the integration domain [7]. As the regularizing parameter β goes to zero, only one term survives. One finally gets:

$$\frac{\Delta E}{S} = -\frac{\hbar c \pi^2}{720 d^3}. \tag{3}$$

The Casimir force can be obtained by differentiating with respect to d:

$$\frac{F}{S} = -\frac{\hbar c \pi^2}{240 d^4}. \tag{4}$$

This truly remarkable result corresponds to a quantum (signalled by $\hbar$) attractive force, independent of the nature of the plates. No surprise that the CE is considered as a genuine property of the vacuum. It is a very tiny force for ordinary values of the separation distance (with $d = 1\mu\text{m}$, $F/S \approx 4 \times 10^{-4} N/\text{m}^2$), but becomes important at the nanometer scale, due to the inverse proportionality to the fourth power of the separation distance. Expression 4 has been verified experimentally: after a few unconclusive or partially conclusive attempts Eq. 4 was verified to a accuracy of a few percent by Lamoreaux [8] and of one percent by Ederth [9], down to the micrometer scale.

In the next sections, we are going to analyze the difficulties associated with this standard interpretation.

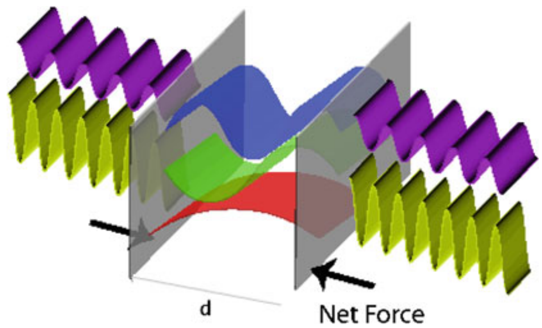

Fig. 1 Schematic representation of the normal modes of the electromagnetic field. Adapted from Ref. [6]

3 The Casimir Effect in Cosmology

It has been suggested that the Casimir effect or rather the vacuum energy could account for dark energy (DE). The first suggestion dates to Einstein who introduced a cosmological constant Λ in his fundamental equations of general relativity coupling the structure of space-time and the energy content:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{5}$$

in order to manage a static solution. After the discovery of the expansion of the Universe, this term was dropped off. Later, with the development of quantum field theory, the possibility of the existence of a vacuum energy was taken seriously and was related to Λ by $\Lambda = 8\pi G\rho_v$. This possibility was taken even more seriously when it was realized that the expansion of the Universe seems to be accelerating [10, 11]. Recent observations require a value of $\Lambda = (2.14 \pm 0.13 \times 10^{-3} eV)^4$ at the present time [12]. The name "dark energy" has been dubbed for this energy. The zero point energy of the electromagnetic field has been proposed as a candidate for DE. However, the identification of DE with zero-point energy of quantum fields faces serious difficulties. First of all, the zero point energy density is in principle infinite. Of course, one may admit that the contribution of the very high frequencies should be cut somewhere, rendering the energy density finite. The natural cut should come from gravity and can be taken as the Planck scale. One then has

$$\rho_v = \hbar c \int^{k<k_{cut}} \frac{d^3\mathbf{k}}{(2\pi)^3} k = \hbar c \pi k_{cut}^4. \tag{6}$$

If k_{cut} is taken as the inverse of Planck length $\lambda_{PL} = 1.2 \times 10^{19}$ GeV, one gets a vacuum energy of the order of $10^{121}\mathrm{GeV\,m^{-3}}$. This should be compared to the critical energy density $\rho_c \approx 5\mathrm{GeV\,m^{-3}}$ and the part of about 75% taken by DE. The alleged electromagnetic vacuum energy is thus enormously too large and there is no clear mechanism to reduce it. Furthermore, one should add in principle the contribution of the vacuum energy for the other fundamental fields. This has led to the crisis of the cosmological constant and to a serious questioning about the reality of the vacuum energy of the fields. Furthermore, there are plenty of condensates in the standard model which may also contribute to dark energy in principle. The conclusion is that the (perturbative) vacuum energy (of the fields) has probably no real physical meaning or at least that our understanding of its properties, especially concerning its coupling to gravity, has to be clarified. See Ref. [13] for an interesting discussion.

4 Dependence of the Fine Structure Constant

Expression (3) looks universal, independent of the properties of matter and, surprisingly, of the fine structure constant α (in contrast with all other elementary electromagnetic effects). In fact, the dependence upon α is hidden by the implicit hypothesis of a perfect conductor made in the derivation above (through the boundary conditions on the plates). Simple considerations help to understand the role of coupling of the field to the plates. Here we closely follow Ref. [14] and quote only the important results.

Real conductors are roughly characterized by two parameters: the plasma frequency ω_{pl} and the skin depth δ (or the conductivity σ, the two quantities being related by $\delta = c/(2\pi|\sigma|)$). There is basically no propagation of electromagnetic waves inside the plasma for frequencies lower than ω_{pl} and the skin depth gives the penetration length for incident waves into the conductor. Ideal conductors correspond to infinite ω_{pl} and vanishing δ. The Drude model, in spite of its simplicity, exhibits the effect of these two parameters. In this model, the conducting electrons are free except that they are subject to a friction force of the simplest type $\mathbf{f} = -\gamma \mathbf{v}$. It is a simple exercise to find the expressions for ω_{pl} and δ (n is the electron density):

$$\omega_{pl} = \sqrt{\frac{4\pi n e^2}{m_e}}, \quad \delta^{-1} = \frac{1}{2c} \frac{\omega_{pl}^2}{\sqrt{(\gamma/m_e)^2 + \omega^2}} \tag{7}$$

The limit of a perfect conductor requires that the typical frequencies of the conductor are much smaller than the plasma frequency. In the case of CE, the typical frequencies are smaller than or of the order of c/d. Thus the approximation of a perfect conductor is satisfied when $c/d \ll \omega_{pl}$, i.e. when

$$\alpha \gg \frac{m_e}{4\pi \hbar n d^2}. \tag{8}$$

For typical cases ($Cu,\ d = 1\,\mu\text{m}$), the rhs is equal to $\sim 10^{-6}$. Condition (8) is comfortably satisfied by the actual value of the fine structure constant. In fact, Casimir's result is the $\alpha \to \infty$ limit of the effect. The exact result contains corrections involving negative powers of the fine structure constant. This can be viewed naively from the following considerations. Due to the meaning of the skin depth, the effective distance between the two plates is roughly $d + 2\delta$. Then expression (3) becomes

$$\frac{\Delta E}{S} \approx -\frac{\hbar c \pi^2}{720(d+2\delta)^3} = -\frac{\hbar c \pi^2}{720 d^3}\left(1 - 6\frac{\delta}{d} + \ldots\right). \tag{9}$$

The correction appears indeed as negative powers of α.

It is also interesting to discuss the $\alpha \to 0$ limit. The latter is a bit tricky: in this limit, the Bohr radius $a_B = \hbar^2/(m_e e^2)$ becomes infinite. The atoms are much larger than in the real world. Actually, $n \propto \alpha^3$, $\omega_{pl} \propto \alpha^2$ and $\delta \propto 1/\alpha$. Thus δ becomes very large, the plates are transparent to the radiation and the CE goes away. As all other electromagnetic effects, the CE vanishes when $\alpha \to 0$. The only distinctive feature is that it has a finite contribution as $\alpha \to \infty$ and that the asymptotic behaviour is reached for the actual value of α.

5 The Casimir Effect as a van der Waals Force Between Macroscopic Neutral Objects

5.1 The London Treatment of the van der Waals Force

The previous Section points to the CE not as a property of the vacuum, but as the electromagnetic interaction between neutral conducting pieces of matter in the strong coupling limit (nevertheless realized in ordinary conductors). To investigate this point further, it is interesting to look at another quantum force between neutral objects, namely the van der Waals force between atoms or molecules.

The total hamiltonian for the atoms+field system can be written as:

$$H = H_0 + H_C + H_{int} + H_{rad} = H_0 + H' + H_{rad}, \tag{10}$$

where H_0 is the hamiltonian of the atoms, H_C is the Coulomb interaction between the two atoms, H_{int} is the interaction of the atoms with the radiation field and where H_{rad} is the hamiltonian of the radiation field, quantized in the standard way (quantized oscillators in every normal mode). In the Coulomb gauge, H_{int} is given by

$$H_{int} = \sum_i \left(\frac{e}{m_e c} \mathbf{p_i}.\mathbf{A}(\mathbf{r_i}) + \frac{e^2}{2 m_e c^2} \mathbf{A}^2(\mathbf{r_i}) \right), \tag{11}$$

where the sum runs over all the electrons. The nuclei of the atoms are fixed at a separation distance r. Formally, the interaction energy is given by:

$$\Delta E = \langle \Psi_0 \mid H_0 + H_C + H_{int} + H_{rad} \mid \Psi_0 \rangle - \langle \Phi_0 \mid H_0 + H_{rad} \mid \Phi_0 \rangle, \tag{12}$$

where $| \Phi_0\rangle$ is the ground state for both the atoms and the field, and where $| \Psi_0\rangle$ is the ground state of the full hamiltonian.

The first quantum calculation of the vdW interaction has been done by London [15], in second order in H_C (neglecting H_{int}) and in second order in R_{at}/r, R_{at} being the size of the atom. The result is:

$$\Delta E^{(2)} = -\frac{e^4}{r^6}\sum_{k\neq 0}\sum_{l\neq 0}\frac{| a^1_{k0} |^2 | a^2_{l0} |^2}{E^1_k - E^1_0 + E^2_l - E^2_0}, \tag{13}$$

where the superscripts 1 and 2 refer to the respective atoms and where the quantities $a_{k0} = \langle k | \sum z_i | 0\rangle$ are the matrix elements of the dipole operator between the excited state $| k\rangle$ and the ground state $| 0\rangle$. Quantum mechanically, the atoms are spherical on the average only and the force arises from the fluctuations of the electric dipoles of the atoms around zero. The $1/r^6$ behaviour is typical of the interaction between two dipoles.

5.2 The Long Range Behaviour of the van der Waals Force

London's calculation has been improved by Casimir and Polder [16]. They include the term H_{int} in the perturbation and use perturbed atomic states (in first order) in the calculation of the matrix elements of vector potential **A**, which amounts to consider excited radiation field states in intermediate states. The calculation is technically difficult: the system should be enclosed in a box with conducting walls and the dimensions of the box should be extended to infinity. The results are remarkable. For small distances r, compared to the absolute value of all the elements a_{k0}, a_{l0} (basically the sizes of the electron orbitals), the London result is recovered. For large distances, the result is given by

$$\Delta E^{(2)}_{CP1} = -\frac{23\hbar c}{4\pi r^7}\alpha_1\alpha_2, \tag{14}$$

where the α_i's are the static polarisabilities of the atoms (in second order, they are $\alpha = e^2 \sum_{k\neq 0} | a_{k0} |^2 (E_k - E_0)^{-1}$). The force indeed softens at large r, due to the appearance of the 7th power of r and factorizes with respect to the atom properties, contrarily to London's result. The softening may be interpreted as coming from the retardation effects in the interaction, through the radiation field, of the fluctuating dipoles of the two atoms [16].

Casimir and Polder also calculated, by the same method, the interaction of an atom with a conducting plane, separated by a distance d. They arrived at the following results

$$\Delta E^{(2)}_{atom-wall} = -\frac{e^2}{4d^3}\sum_{k\neq 0} | a_{k0} |^2, \qquad \Delta E^{(2)}_{CP2} = -\frac{3\hbar c}{8\pi d^4}\alpha_1, \tag{15}$$

for small and large distances, respectively. Again there is a softening of the force at large distance. Note that the interaction energy is in first order in α.

5.3 The van der Waals Force as a Modification of the Zero Point Energy of the Electromagnetic Field

Casimir, particularly amazed by the simplicity of his results (Eqs. 14 and 15, 2d part), wondered whether they can be trusted (results are obtained in second order of standard perturbation theory). Following an advice by Niels Bohr ("Why don't you calculate the effect by evaluating the differences of zero point energies of the electromagnetic field?" [17]), Casimir evaluated the vdW force by calculating the normal modes in presence of two atoms (neglecting the change in the field in the interior of the atoms and considering the source as given by the matrix elements of the respective current operator for the atoms with respect to the unperturbed state of these atoms), and in the presence of one atom and a wall. He so succeeded in recovering his results, basically Eqs. 14 and 14 [18]. Then he realised that the calculation is even simpler for two parallel plates (the normal modes are indeed much simpler). And he published the calculation sketched in Sect. 2 in Ref. [1].

Two conclusions arise from these considerations: (i) there is a kind of duality for calculating the interaction between two atoms, (ii) when going from two atoms to two plates, it seems unavoidable to consider the zero point energy of the field with boundary conditions. We are going to argue below that there is a continuity in going from one system to the other.

5.4 The Casimir Effect as a Macroscopic van der Waals Force

First the duality observed above may perhaps be understood qualitatively. As we have seen, the Casimir–Polder approach to the interaction between two atoms consists in evaluating the first term of Eq. 12 perturbatively, considering H' as the perturbation. Casimir's calculation based on the evaluation of the normal modes in presence of the two atoms amounts to the diagonalization of the hamiltonian $H' + H_{rad} + H_0$, with some approximation for the H_0 part. In short, the two calculations are aiming at calculating the same quantity. The fact that they arrive at the same results, although both calculations are approximate, may perhaps be traced back in the fact that used approximations can be viewed as neglecting higher order terms in R_{at}/r. This point remains speculative and a direct proof of the equivalence is still to be done.

Extending the vdW interactions from atoms to plates in the London's spirit is not obvious, because vdW forces are not additive. This problem has been solved by Lifshitz who calculated the interaction between two semi-infinite blocks of dielectric by using the general theory of fluctuations of the electromagnetic field [19–21]. The CE can be found by taking the limit of very large dielectric constants. The arguments are very complex and will not be discussed here. We present an heuristic version that, in spite of being not exact, nevertheless embodies the main qualitative features. Let us consider a semi-infinite piece of dielectric medium made of atoms of type 2 occupying the $z > 0$ part of the space and an atom of type 1 situated at a distance d of the free surface of the dielectric. Suppose for the moment that the vdW forces are simply additive. From Eq. 14, the interaction between the atom 1 and the dielectric is given by

$$\Delta E = -\frac{23\hbar c}{4\pi}\alpha_1\alpha_2 n_2 \int_0^\infty dz \int_0^\infty \frac{2\pi\rho d\rho}{[(d+z)^2+\rho^2]^{7/2}}, \tag{16}$$

where n_2 is the density of atoms of type 2. A little algebra leads to

$$\Delta E = -\frac{23\hbar c}{40}\frac{\alpha_1\alpha_2 n_2}{d^4}. \tag{17}$$

We introduce the dielectric constant $\epsilon_2 = 1 + 4\pi\alpha_2 n_2$. We now take account of the non-additivity of the interaction between atoms in a rough way. This basically leads to a reduction of the local electric field compared to the macroscopic field by a factor equal to the dielectric constant [22]. One then gets:

$$\Delta E = -\frac{23\hbar c}{160\pi}\frac{\alpha_1}{d^4}\frac{\epsilon - 1}{\epsilon}. \tag{18}$$

Now, the limit of perfect conducting body (the thickness of this body is irrelevant) is obtained when $\epsilon \to \infty$, which yields

$$\Delta E = -\frac{23\hbar c}{160\pi}\frac{\alpha_1}{d^4}. \tag{19}$$

Except for the numerical factor, one recovers Eq. 15. It is interesting to see that the whole operation has removed a power of α (hidden in α_2), owing to the non-additivity of the vdW forces and has multiplied the expression of the force by a factor with the dimension of a volume (r^3). Going from the atom-wall system to the double wall system is even simpler. Suppose a conducting plane $z = 0$ and filling the domain $z < -d$ with atoms of type 1. Using expression (15, 2d part) and the same trick as above, one readily gets (S is the free surface of the dielectric 1, taken as tending to infinity):

$$\Delta E = -S\frac{3\hbar c}{8\pi}\frac{\alpha_1 n_1}{\epsilon_1}\int_{-\infty}^{-d}\frac{dz}{z^4}. \tag{20}$$

Taking the limit $\epsilon_1 \to \infty$, one finally has

$$\frac{\Delta E}{S} = -\frac{\hbar c}{32\pi^2}\frac{1}{d^3}, \tag{21}$$

Again, a power of α has disappeared, a factor with the dimension of a volume (Sd) has been introduced and expression (3) is recovered, up to a numerical factor.

In summary, these considerations are leading to understanding atom–atom, atom–wall and wall–wall interactions as vdW forces, i.e. resulting from the interactions between charges in the atoms. The geometry and the hypothesis of prefect conductors have eliminated the dependence of the fine structure constant.

6 The "reality" of the Zero Point Energy of the Vacuum

It results from the previous chapters that the CE is a real effect and can be understood as resulting from the electromagnetic interactions between "giant molecules" as well as the change of the vacuum energy when these "giant molecules" apply different boundary conditions on the field. However, the question remains to know whether the zero point energy itself has a physical meaning which may be relevant when gravitation comes into play. The fact that this zero point energy is infinite and needs some unknown renormalisation (see Sect. 3) has shed some reservation about this question. Several recent developments are arguing even against attaching a physical meaning to the zero point energy. Due to lack of space, we refer to Refs. [14,23] for a discussion. We only quote three points here:

1. In standard (Hamiltonian) field quantization, the zero-point energy comes from the normal ordering of fields in the classical lagrangian. There is no a priori reason for doing so. It is usually justified by the link with quantization of 1D harmonic oscillators. It should be mentioned that for fermion fields, the zero-point energy is negative!

2. Interaction between neutral objects gives "no more and no less evidence of quantum fluctuations than any other one-loop effects" [14], as they vanish as $\alpha \to 0$.

3. The CE can be derived without reference to zero-point motion. For instance, the Lifshitz approach mentioned above does not need the quantization of the electromagnetic field.

7 Conclusion

This paper tries to explain that the CE, whose reality is testified daily by micro- and the nano-technology, and which is often advocated as a manifestation of the quantum fluctuations of the vacuum or, rather, of its zero-point energy, can, and perhaps, should be viewed rather as a vdW force between gigantic conducting molecules, which, as all other electromagnetic effects, disappears in the weak coupling limit. It can thus hardly be taken as a property of the quantum vacuum. Furthermore, the Casimir force can be derived without reference to zero-point energy. The reality of the vacuum energy remains an open question. Also, the fact that the CE can be explained with or without recourse to the zero-point fluctuations introduces a puzzling "duality", which presumably covers some deeper truth.

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