The nuclear physics of OHe

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Abstract

A recent composite-dark-matter scenario assumes that the dominant fraction of dark matter consists of O-helium (OHe) dark atoms, in which a lepton-like doubly charged particle O^{--} is bound with a primordial helium nucleus. It liberates the physics of dark matter from unknown features of new physics, but it demands a deep understanding of the details of known nuclear and atomic physics, which are still unclear. Here, we consider in detail the physics of the binding of OHe to various nuclei of interest for direct dark matter searches. We show that standard quantum mechanics leads to bound states in the keV region, but does not seem to provide a simple mechanism that stabilizes them. The crucial role of a barrier in the OHe-nucleus potential is confirmed for such a stabilization.

1 Introduction

Direct searches for dark matter have produced surprising results. Since the DAMA collaboration observed a signal, several other collaborations seem to confirm an observation, while others clearly rule out any detection. We summarize the situation in Table 1, and the current experimental situation is reviewed in [1]. This apparent contradiction comes from the analysis of the data under the assumption that nuclear recoil is the source of the signal.

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Starting from 2006 it was proposed [2, 3] that the signal may be due to a different source: if dark matter has weakly bound states with normal matter, the observations could come from radiative capture of thermalized dark matter, and could depend on the detector composition and temperature. This scenario comes naturally from the consideration of composite dark matter. Indeed, one can imagine that dark matter is the result of the existence of heavy negatively charged particles that bind to primordial nuclei.

Cosmological considerations imply that such candidates for dark matter should consist of negatively doubly-charged heavy ($\sim$ 1 TeV) particles, which we call O^{--}, coupled to primordial helium. Lepton-like technibaryons, technileptons, AC-leptons or clusters of three heavy anti-U-quarks of 4th or 5th generation with strongly suppressed hadronic interactions are examples of such O^{--} particles (see [2, 3] for a review and for references).

The cosmological and astrophysical effects of such composite dark matter (dark atoms of OHe) are dominantly related to the helium shell of OHe and involve only one parameter of new physics $-$ the mass of O^{--}. The positive results of the DAMA/NaI and DAMA/LIBRA experiments are explained by annual modulations of the rate of radiative capture of OHe by sodium nuclei. Such radiative capture is possible only for intermediate-mass nuclei: this explains the negative results of the XENON100 experiment. The rate of this capture is proportional to the temperature: this leads to a suppression of this effect in cryogenic detectors, such as CDMS. OHe collisions in the central part of the Galaxy lead to OHe excitations, and de-excitations with pair production in E0 transitions can explain the excess of the positron-annihilation line, observed by INTEGRAL in the galactic bulge.

These astroparticle data can be fitted, avoiding many astrophysical uncertainties of WIMP models, for a mass of $O^{--} \sim 1$ TeV, which stimulates searches for stable doubly charged lepton-like particles at the LHC as a test of the composite-dark-matter scenario. The problem with OHe dark matter is that its constituents may interact too much with normal matter. OHe is neutral, but a priori it has an unshielded nuclear attraction to matter nuclei. To avoid the problem, it was assumed that the effective potential between OHe and a normal nucleus would have a barrier, preventing He and/or O^{--} from falling into the nucleus, allowing only one bound state, and diminishing considerably the interactions of OHe. Under these conditions elastic collisions dominate in OHe interactions with matter, which is important for many aspects of the OHe scenario.

In this paper, we show that indeed such a barrier is needed to make the model work, and we try to establish its existence through several methods. In the first section, we review the classical description of the problem [3]

Detector	nuclei	A	Z	temperature	detection
DAMA (/NaI [4] +/LIBRA [5])	Na I Tl	23 127 205	11 53 81	300 K	8.9 σ
CoGeNT[6]	Ge	70-74	32	70 K	2.8 σ
CDMS[7]	Ge (Si)	70-74 (28-30)	32 (14)	cryogenic	–
XENON100[8]	Xe	124-134	54	cryogenic	–

Table 1: Results of various dark matter searches and composition of the detectors.

and show that in fact it does not lead to a repulsive force. In section 2, we explore the spectrum of the bound states of OHe. We show that, if one considers only the screened Coulomb force, then bound states exist only for light nuclei, whereas if we consider a polarization of OHe due to a second-order Stark effect, then most nuclei have keV bound states. In the last section, we check that the description of the Stark effect that we used is reasonable via a perturbative calculation at large distances, but it is not reliable when the nucleus comes close to OHe, as one would then need to take into account a strong and inhomogeneous deformation of the ground state by the common effect of Coulomb and nuclear force.

2 Classical model

To study the polarization of the OHe atom under the influence of an approaching A nucleus, we can first treat OHe as a classical structure and neglect the effects of O^{--} and nucleus motion. The polarization of OHe is then fixed by the equilibrium of forces acting on the He nucleus. For every position of the A nucleus, we can work on the O-A axis, in the rest frame of the O^{--} particle, as shown in Fig. 1.

We take the He and A nuclei as uniformly charged spheres of radii R_{He} and R_A and of charges $Z_{He} = 2$ and Z_A. We also assume that O^{--} is point-like. We then obtain the electrostatic potential for the interactions with O^{--} :

$$\begin{aligned} V_{OA}(z_A) &= -\frac{Z_A Z_O \alpha}{z_A}, \text{ for } z_A > R_A \\ &= \frac{-Z_A Z_O \alpha}{2R_A}\left(3 - \frac{z_A^2}{R_A^2}\right), \text{ for } z_A < R_A \end{aligned} \quad (1)$$

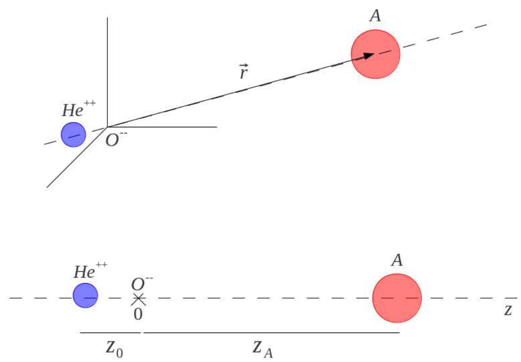

Figure 1: One-dimensional OHe atom polarized along the $O^{--}-A$ axis, denoted z. z_0 is the distance between O and He and z_A is the distance of the A nucleus along the $z-$axis.

for the interaction between the O^{--} and the A nucleus, and a similar expression for $V_{OHe}(z_{He})$. The potential between the two nuclei has both electrostatic and nuclear contributions. In the former, we neglect the He size, and for the latter we use an experimental parametrisation of the α-nucleus potential from scattering experiments [9]:

$$\begin{aligned} V_{HeA}(z_A - z_0) &= \frac{Z_{He}Z_A\alpha}{|z_A - z_0|} + \frac{-V_0}{1+e^{(|z_A-z_0|-R_*)/a}}, \text{ for } |z_A - z_0| > R_A \\ &= \frac{Z_{He}Z_A\alpha}{2R_A}\left(3 - \frac{(z_A-z_0)^2}{R_A^2}\right) + \frac{-V_0}{1+e^{(|z_A-z_0|-R_*)/a}}, \\ &\qquad \text{for } |z_A - z_0| < R_A. \end{aligned} \tag{2}$$

The nuclear interaction is represented in a Woods-Saxon form, with parameters $V_0 = 30$ MeV, $a = 0.5$ fm and $R_*(fm) = 1.35 \times A^{1/3} + 1.3$ (fm).

The equilibrium position z_0 of the He nucleus will be at the minimum of the potential $V_{HeA} + V_{OHe}$ and will depend on z_A. At that point, the Coulomb force balances the nuclear force:

$$\vec{F}_{OHe} + \vec{F}_{HeA} = \vec{0}. \tag{3}$$

When the equilibrium position is determined, the OHe-A potential is obtained by adding the dipole potential to the Woods-Saxon one

$$V_{OHeA}(z_A) = V_{dip}(z_A) + V_{WS}(z_A - z_0), \tag{4}$$

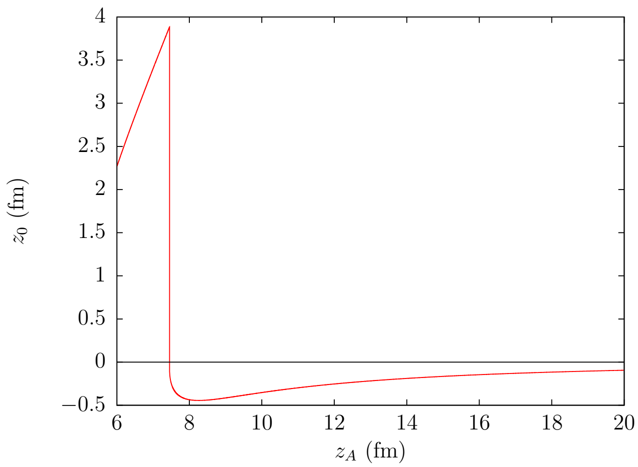

Figure 2: Polarization z_0 of the one-dimensional OHe atom as a function of the distance z_A of an approaching sodium nucleus. The break in the curve corresponds to the fact that for a distance of about 9 fm, He falls into the nuclear potential of A.

where

$$V_{dip}(z_A) = \frac{2Z_A \alpha z_0(z_A)}{z_A\,(z_A - z_0(z_A))} \tag{5}$$

is the dipole potential of the polarized OHe atom.

Fig. 2 shows the polarization z_0 of the OHe atom as a function of the position z_A along the $z-$axis for an approaching sodium nucleus with $Z_A = 11$. We see that it is negative at large distance, giving rise to an attractive dipole potential and that the nuclear force starts to reverse the dipole when the nucleus gets closer to O^{--}.

This situation corresponds to a repulsive dipole that acts against the nuclear force, but it can be seen in Fig. 3 that this repulsive force is not sufficient to overcome the nuclear force between the two nuclei and to give rise to a repulsive global potential.

When $z_A \lesssim 7.5$ fm, Equation (3) projected along the $z-$axis loses its initial solution and another one remains, that is located in the nuclear well, giving rise to a jump in the polarization and therefore in the total potential. Similar results and pictures can be obtained for other nuclei.

Hence we see that, classically, no repulsive potential appears, even if the electrostatic force of OHe repels the A nucleus. In fact one can argue that this is a generic classical result which does not depend on the details of the calculation. If the configuration of the 3 objects is He-O-A, then clearly the force is attractive. If the configuration is O-He-A, then that means that the

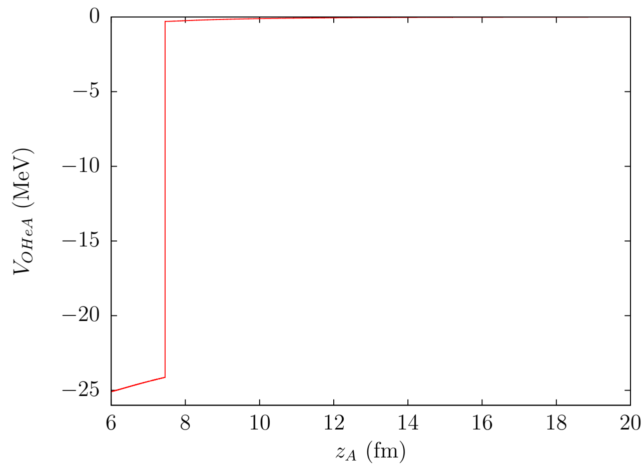

Figure 3: Total OHe-A potential for sodium.

nuclear force on He is larger than the electrostatic force from A. Again, a net attraction between OHe and A results.

To settle this classical picture of permanent attraction in the OHe-nucleus system, a proper quantum treatment of the problem is needed. On the one hand, we shall see from the following discussion that simple semiclassical and perturbative descriptions cannot solve the problem of permanent attraction in the OHe-nucleus system. On the other hand, a crucial point may be missing in such treatments: the correct description of the nuclear effects when the nucleus is close to helium and when neither semiclassical nor perturbative approaches are valid.

3 Semiclassical model

The quantum problem involves very different scales: the OHe binding is of the order of one MeV, and we are looking for bound states of about one keV. To obtain both in the same framework would imply a solution of the 3-body problem at better than one per thousand, which is clearly very hard.

Fortunately, for the very excited bound states, one can use a simplified method. For these states, the OHe atom will not dissociate, so we can treat that system as a whole, allowing a small polarisation in the A direction. Furthermore, the interaction potential OHe-nucleus can be taken as radial, as the polarisaion of OHe will be in the A direction. Hence we can use spherical coordinates, with the O^{--} fixed at the origin and $\overrightarrow{r}$ the position of the center of the nucleus A. We know in this case that the solutions of the Schrödinger equation take the form $\psi_{k,l,m}(r,\theta,\varphi) = \frac{u_{k,l}(r)}{r} Y_l^m(\theta,\varphi)$ where

$Y_l^m(\theta, \varphi)$ are the spherical harmonics and where the radial part $u_{k,l}(r)$ has to satisfy the radial Schrödinger equation

$$\frac{d^2 u_{k,l}(r)}{dr^2} + 2m_A \left[E_{k,l} - V(r) - \frac{l\,(l+1)}{2m_A r^2} \right] u_{k,l}(r) = 0 \tag{6}$$

where l is the relative angular momentum, $E_{k,l}$ is the total energy in the center-of-mass (O^{--}) frame, and $V(r)$ is the sum of the nuclear and of the electrostatic potentials between OHe and A.

The next simplification comes from the fact that one is looking for weakly bound states, for which the WKB method applies, and considerably simplifies the solution. Finally, we further simplify the problem by approximating the He wave function in the OHe bound state by a $1s$ hydrogenoid wave function.

The A nucleus is seen as a uniformly charged sphere of charge Z_A and of radius R_A(fm)$= 1.35A^{1/3}$[9], where A is the number of nucleons in the nucleus. Its mass m_A is corrected by the nuclear binding energy B given by the Bethe-Weizsäcker formula: $m_A = Z_A m_p + N_A m_n - B$, where N_A is the number of neutrons, and m_p and m_n are the masses of the proton and of the neutron respectively.

The interactions between the OHe atom and the nucleus take two forms: nuclear attraction between the helium and the A nucleus at distances $r \lesssim R_A$ and electrostatic interaction due to the electrical charges of the components at distances $r \gtrsim R_A$.

Out of the nuclear region, the electrostatic interaction is dominant and can be separated into two contributions : 1) the electrostatic interaction between the spherical charge distribution of the OHe atom in its ground state and the spherical charge distribution of the nucleus; 2) the electrostatic interaction between the polarized OHe atom and the nucleus due to the Stark effect. Therefore, we can write

$$V_{Elec} = V_{Coul} + V_{Stark} \tag{7}$$

where V_{Coul} corresponds to Coulomb attraction between O^{--} screened by the helium charge distribution and A, and V_{Stark} represents the interaction term of the charged nucleus and the dipole.

Outside the nucleus, i.e. for $r \geqslant R_A$, we find for the Coulomb term

$$\begin{aligned} V_{Coul}(r) &= \frac{3}{8}\left(\frac{-Z_O Z_A \alpha}{\rho^3 r}\right) e^{-2r/r_0} \left[e^{-2\rho} \left\{ \rho^2 + \frac{5}{4} + \frac{5}{2}\rho + \left(\frac{1}{2} + \rho\right)\frac{r}{r_0} \right\} \right. \\ &+ \left. e^{2\rho} \left\{ -\rho^2 - \frac{5}{4} + \frac{5}{2}\rho + \left(-\frac{1}{2} + \rho\right)\frac{r}{r_0} \right\} \right] \end{aligned} \tag{8}$$

with the Bohr radius of the OHe atom $r_0 = 1/(m_{He} Z_O Z_{He} \alpha) \simeq 1.81$ fm and $\rho = R_A/r_0$. This expression can be considered as an improvement of the form from [10] where the nucleus was assumed to be a point-like particle.

For the Stark potential, we use the formula for the quadratic effect in a constant electric field [11], taken to be the field of the nucleus at the position of O. The dipole moment of the OHe atom in its perturbed ground state can then be written:

$$< q\overrightarrow{R} >= \frac{9}{2} r_0^3 \overrightarrow{E} \tag{9}$$

so that, for $r \geqslant R_A$,

$$V_{Stark} = -q\overrightarrow{R}.\overrightarrow{E} = -\frac{9}{2} r_0^3 E^2 = -\frac{9}{2} r_0^3 \frac{Z_A^2 \alpha}{r^4}. \tag{10}$$

Expressions (8) and (10) are valid when the nuclear effects are negligible.

In the nuclear region, we take a trapezoidal nuclear well, which will simplify the WKB solution:

$$\begin{array}{lcll} V_{nucl} & = & -V_0 & \text{for } r \leq R_* \\ & = & \frac{V_{Elec}(R_*+2a)+V_0}{a}(r - R_*) - V_0 & \text{for } R_* \leq r \leq R_* + 2a \\ & = & 0 & \text{for } r > R_* + 2a \end{array} \tag{11}$$

characterized by its depth V_0 and its diffuseness parameter a representing the region of r in which it goes linearly from $-V_0$ to $V_{Elec}(R_* + a)$. From diffusion experiments of α particles on nuclei [9], one gets $V_0 \approx 30$ MeV for nuclei with $Z_A \leq 25$ and $V_0 = 45$ MeV for $Z_A > 25$, as well as $a = 0.5$ fm. In the following, we shall avoid the transition region $Z_A \in [21, 29]$, which does not contain any nucleus used for direct dark matter detection. The nuclear radius parameter $R_* = (R_A + 1.3 + r_0)$ fm, where $R_A + 1.3$ fm is taken from [9] to take the finite size of the alpha particles into account. Fig. 4 shows the form of the potential between OHe and Na.

To find the spectrum corresponding to the potential of Fig. 4, we use the approximate WKB solutions, which, once applied to each region, give a quantization condition for the energy.

For $l = 0$, we obtain

$$\rho = \left(k - \frac{1}{4}\right)\pi, \quad k = 1, 2, 3, ... \tag{12}$$

where $\rho = \int_0^b k(r)dr$, b is the turning point such that $E = V(b)$ and $k(r) = \sqrt{2m_A\left(E - V(r)\right)}$.

At $l \neq 0$, we know from [12] that the behaviour of the effective potential $V_{eff}(r) = V(r) + \frac{l(l+1)}{2m_A r^2}$ at the origin requires to modify the WKB method

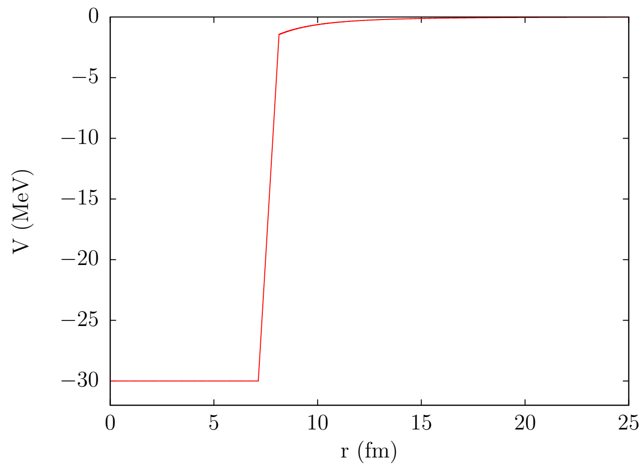

Figure 4: Shape of the interaction potential OHe-nucleus, according to Eqs. 7 and 11.

by applying its solutions to $u(r)$ after having changed $l\,(l+1)$ to $\left(l+\frac{1}{2}\right)^2$ in the radial equation : $V_{eff}(r) \to \widetilde{V}_{eff}(r) = V(r) + \frac{\left(l+\frac{1}{2}\right)^2}{2m_A r^2}$. Therefore, the quantization condition becomes

$$2e^{2\sigma}\cos\rho + \sin\rho = 0 \tag{13}$$

where $\rho = \int_a^b \widetilde{k}(r)dr$, $\sigma = \int_0^a \widetilde{\kappa}(r)dr$, a and b are the turning points such that $E = \widetilde{V}_{eff}(a) = \widetilde{V}_{eff}(b)$ and $\widetilde{k}(r) = \sqrt{2m_A\left(E - \widetilde{V}_{eff}(r)\right)}$, $\widetilde{\kappa}(r) = \sqrt{2m_A\left(\widetilde{V}_{eff}(r) - E\right)}$.

3.1 Spectra from a screened Coulomb potential

In [3], the spectrum was considered for a screened Coulomb potential at long distance, as in Eq. 8. We reanalyse this question with our WKB formalism. For small nuclei $Z_A \leq 20$, we first fix the exact value of V_0 to obtain the highest level at -3 keV for ^{23}Na from DAMA for $l = 0$. We obtain $V_0 = 31.9$ MeV, in good agreement with [9] and use this value for all nuclei with $Z_A \leq 20$. The spectrum of the OHe-^{23}Na system is shown in Fig. 5.

We see a rich spectrum with many levels in the MeV region, corresponding to nuclear levels, for which the WKB approximation may not be appropriate. The only level in the keV region is at $l = 0$. It can be considered as being due to the presence of the electrostatic potential. It is

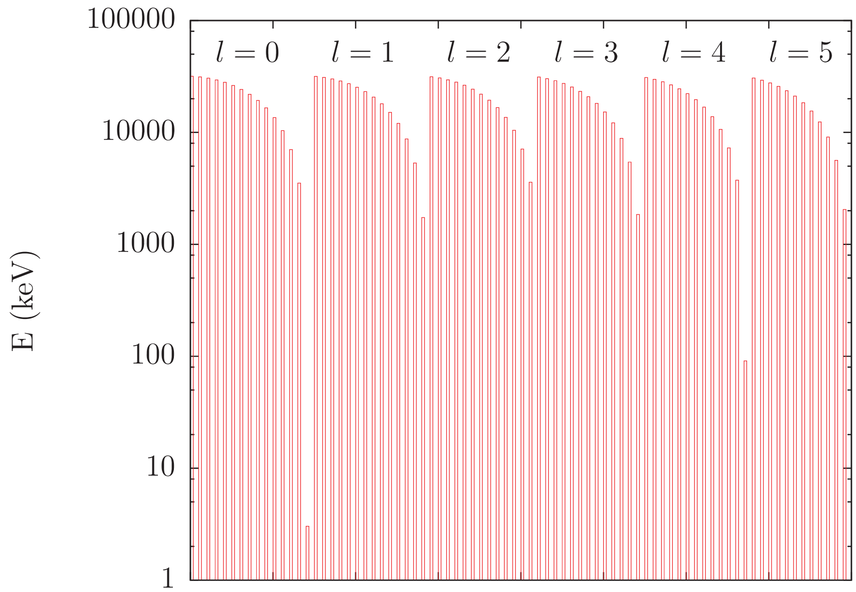

Figure 5: Spectrum of the OHe-^{23}Na system at different values of l. The energies are in absolute value. $V_0 = 31.9$ MeV and $a = 0.5$ fm.

remarkable that other nuclei such that $Z_A \leq 20$ do not have keV bound states. Fig. 6 shows the highest level at $l = 0$ for the most stable nuclei for Z_A going from 1 to 20. It turns out that only ^{23}Na at $Z_{Na} = 11$ has a level in the keV region.

For large nuclei $Z_A \geq 30$, the data indicate that the nuclear well is deeper. In this case, we take as a reference germanium $Z_{Ge} = 32$, $A = 74$ from CoGeNT, for which we find a highest level at $l = 0$ in the keV region for $V_0 = 45$ MeV, which is precisely the central value from [9] for larger nuclei. This value is used for all nuclei with $Z_A \geq 30$. Fig. 7 represents the spectrum of the OHe-^{74}Ge system. It is of the same kind as for ^{23}Na, with only one level in the keV region.

The second column of Table 2 gives the highest-energy level at $l = 0$ for the large stable nuclei involved in the experiments of interest. According to this model, iodine and thallium from DAMA each admit one level in the keV region, while xenon from XENON100 doesn't.

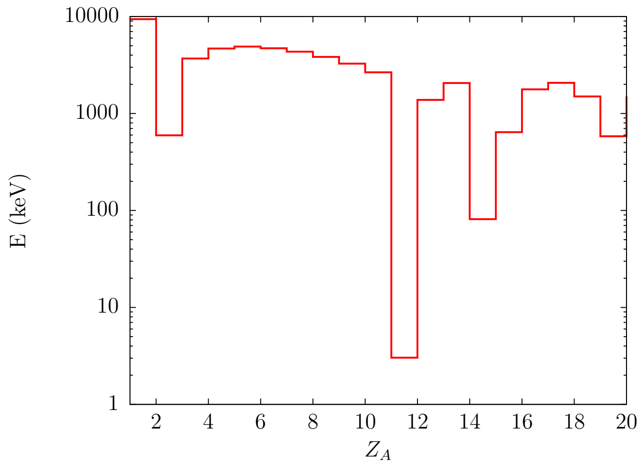

Figure 6: Highest-energy level at $l = 0$ for stable nuclei from $Z_A = 1$ to $Z_A = 20$. The energies are in absolute value. $V_0 = 31.9$ MeV and $a = 0.5$ fm.

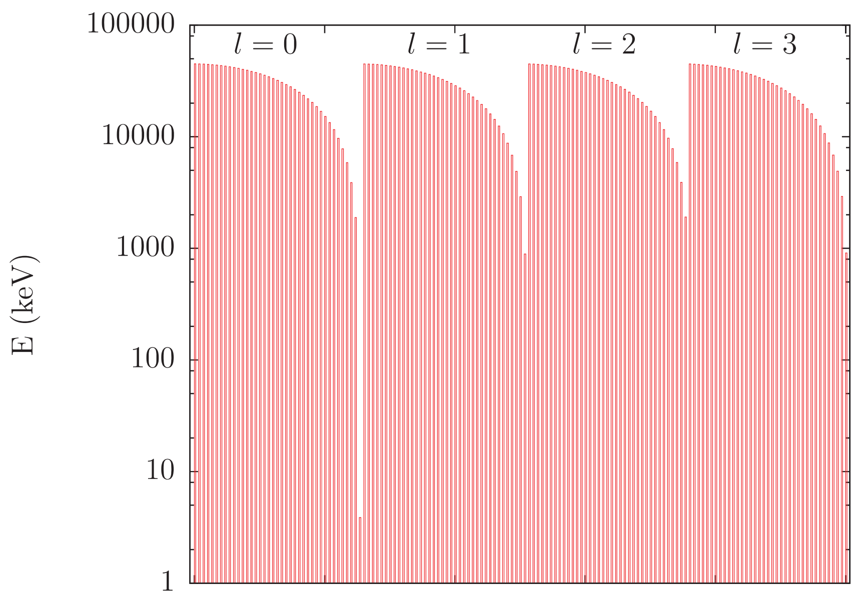

Figure 7: Spectrum of the OHe-^{74}Ge system at different values of l. The energies are in absolute value. $V_0 = 45$ MeV and $a = 0.5$ fm

Nuclei	E(keV) for a screened Coulomb potential	E(keV) for a screened Coulomb potential added to a Stark potential
^{74}Ge	3.88	1.16
^{127}I	0.500	2.31
^{132}Xe	540.	2.33
^{184}W	350.	1.86
^{201}Tl	15.6	52.7

Table 2: Highest-energy level at $l = 0$ for some heavy stable nuclei from the experiments of interest, when $V_{Elec} = V_{Coul}$ (second column), or $V_{Elec} = V_{Coul} + V_{Stark}$ (third column). The energies are in absolute value. $V_0 = 45$ MeV and $a = 0.5$ fm.

3.1.1 Spectra from a screened Coulomb potential and a Stark potential

The results can de discussed in the same way when $V_{Stark} + V_{Coul}$ is used in the calculations, and the values of V_0 are identical to the central experimental values, i.e. 30 and 45 MeV, for small and large nuclei respectively. Fig. 8 illustrates the results in the particular case of the OHe-^{23}Na system.

The major difference lies in the fact that, in this case, the levels in the keV region are obtained more easily, with sometimes several keV levels for the same nucleus, especially for large nuclei. The reason lies in the shape of V_{Elec}, that is deeper and less steep when V_{Stark} is used. Fig. 9, as well as third column of Table 2, show that most nuclei now have keV bound states. Hence the inclusion of the Stark potential seems to destroy the previous interpretation of the data, which relied on Na and Ge to be very special nuclei.

4 Perturbative analysis

The three-body OHe-nucleus bound-state problem can be simplified in another way, by noting that helium is much lighter than the A nuclei. Given this, one can simplify the total hamiltonian of the system, written in the reference frame of the O^{--} particle, and choosing the z axis in the direction of A to:

$$H \approx -\frac{1}{2m_{He}}\triangle_1 + V_{OHe}(r_1) + V_{OA}(R) + V_{HeA}(r_{12}) \tag{14}$$

in which the kinetic energy term of the A nucleus has been neglected and where $\overrightarrow{r}_1$ is the position of the He nucleus, R is the distance of the A nucleus

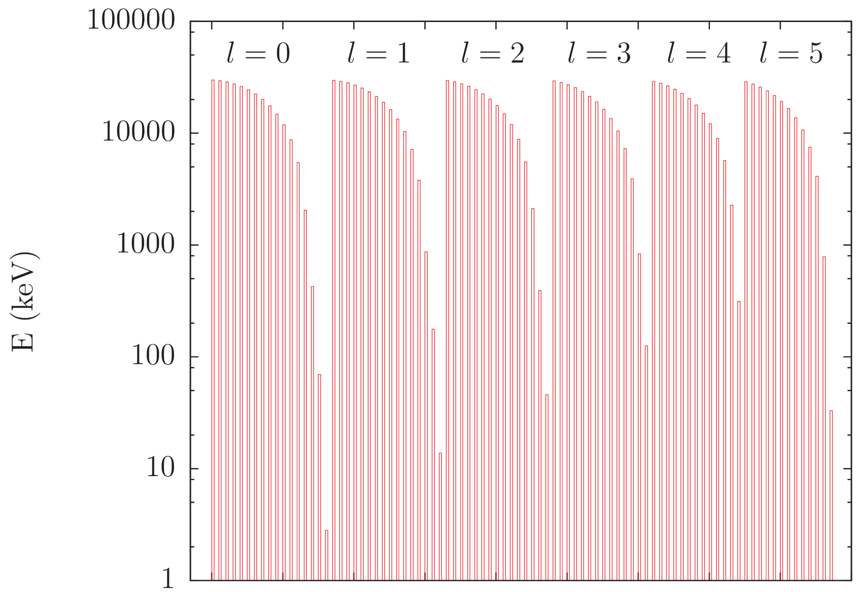

Figure 8: Spectrum of the OHe-^{23}Na system at different values of l, when $V_{Elec} = V_{Stark} + V_{Coul}$. The energies are in absolute value. $V_0 = 30$ MeV and $a = 0.5$ fm.

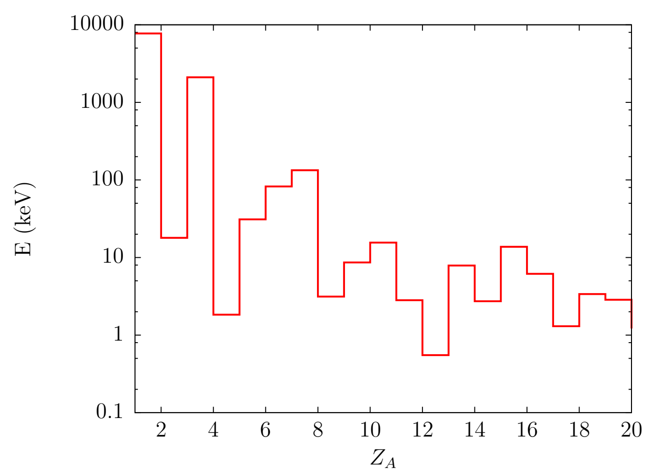

Figure 9: Highest level at $l = 0$ for stable nuclei from $Z_A = 1$ to $Z_A = 20$ due to an electromagnetic potential $V_{Stark} + V_{Coul}$. The energies are in absolute value. $V_0 = 30$ MeV and $a = 0.5$ fm.

on the positive part of the $z-$axis and r_{12} is the distance between He and the A nucleus. V_{IJ} stand for the interaction potential between I and J. We are thus left with the one-body problem of He in a total potential depending on the parameter R. We shall consider here the contribution of the external A nucleus as a perturbation to the OHe atom. The hamiltonian (14) can be rewritten as the sum of an unperturbed part H_0 and a perturbation W:

$$H = H_0 + W \tag{15}$$

where $H_0 = -\frac{1}{2m_{He}}\triangle_1 + V_{OHe}(r_1)$ corresponds to the isolated OHe atom and where $W = V_{OA}(R) + V_{HeA}(r_{12})$ is due to the presence of the external nucleus. We use here the following two-body interaction potentials :

$$\begin{aligned}
V_{OHe}(r_1) &= -\frac{Z_O Z_{He}\alpha}{r_1} \\
V_{OA}(R) &= -\frac{Z_O Z_A\alpha}{R} && , R > R_A \\
&= -\frac{Z_O Z_A\alpha}{2R_A}\left(3 - \frac{R^2}{R_A^2}\right) && , R < R_A \\
V_{HeA}(r_{12}) &= \frac{Z_{He} Z_A\alpha}{r_{12}} + \frac{-V_0}{1+e^{(r_{12}-R_A)/a}} && , r_{12} > R_A \\
&= \frac{Z_{He} Z_A\alpha}{2R_A}\left(3 - \frac{r_{12}^2}{R_A^2}\right) + \frac{-V_0}{1+e^{(r_{12}-R_A)/a}} && , r_{12} < R_A
\end{aligned}$$

where, in order to simplify the calculation, the OHe atom is treated as a hydrogenoid system, and where the A nucleus is seen as a sphere of radius $R_A(\text{fm}) = 1.35 \times A^{1/3}$ [9] and charge Z_A in the definitions of potentials V_{OA} and V_{HeA} that are therefore point (O^{--} or He^{++}) - sphere (A) interaction potentials. A Woods-Saxon potential of parameters V_0 and a has been added in V_{HeA} to take the nuclear interaction of both nuclei into account.

We are studying the perturbed ground state $E_0(R)$ of the OHe atom under the influence of the external perturbation $W(R)$. The perturbed energy is therefore an approximation of the energy of the total O-He-A system, described by the hamiltonian (15). If this energy presents a minimum for some R_b, then the system will tend to this configuration to minimize its energy, and we will get a stable OHe-nucleus bound state of length R_b and energy $E_0(R_b)$. If there is no minimum, then we will have to conclude that no stable bound state can form at those distances.

In the following, we shall go to 3rd-order perturbation theory. We assume that H_0 has a spectrum $|\psi_n^0>$ of eigenfunctions with eigenvalues E_n^0, and we assume that the unperturbed energy level is non-degenerate. The formulae for the wave function at order 2 and for the energy at order 3 are given by:

$$
\begin{aligned}
E_n &= E_n^0 \\
&+ <\psi_n^0|W|\psi_n^0> \\
&+ \sum_{i,p\neq n} \frac{|<\psi_{p,i}^0|W|\psi_n^0>|^2}{E_n^0 - E_p^0} \\
&- <\psi_n^0|W|\psi_n^0> \sum_{i,p\neq n} \frac{|<\psi_{p,i}^0|W|\psi_n^0>|^2}{(E_n^0 - E_p^0)^2} \\
&+ \sum_{i,p\neq n} \sum_{i',p'\neq n} \frac{<\psi_{p',i'}^0|W|\psi_n^0><\psi_{p,i}^0|W|\psi_{p',i'}^0><\psi_n^0|W|\psi_{p,i}^0>}{\left(E_n^0 - E_p^0\right)\left(E_n^0 - E_{p,}^0\right)},
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
|\psi_n> &= |\psi_n^0> \\
&+ \sum_{i,p\neq n} \frac{<\psi_{p,i}^0|W|\psi_n^0>}{E_n^0 - E_p^0} |\psi_{p,i}^0> \\
&- <\psi_n^0|W|\psi_n^0> \sum_{i,p\neq n} \frac{<\psi_{p,i}^0|W|\psi_n^0>}{(E_n^0 - E_p^0)^2} |\psi_{p,i}^0> \\
&- \frac{1}{2} \sum_{i,p\neq n} \frac{|<\psi_{p,i}^0|W|\psi_n^0>|^2}{(E_n^0 - E_p^0)^2} |\psi_n^0> \\
&+ \sum_{i,p\neq n} \sum_{i',p'\neq n} \frac{<\psi_{p',i'}^0|W|\psi_n^0><\psi_{p,i}^0|W|\psi_{p',i'}^0>}{\left(E_n^0 - E_p^0\right)\left(E_n^0 - E_{p'}^0\right)} |\psi_{p,i}> .
\end{aligned} \tag{17}
$$

In our case, the non-degenerate unperturbed energy E_n^0 is the ground level of the hydrogenoid OHe atom:

$$
E_n^0 = E_{OHe} = -\frac{1}{2} m_{He} \left(Z_O Z_{He} \alpha\right)^2 \simeq -1.58\,\mathrm{MeV} \tag{18}
$$

and the unperturbed eigenfunction $|\psi_{p,i}^0>$ are those of the hydrogen atom:

$$
\psi_{p,i}^0(\overrightarrow{r_1}) = \psi_{n,l,m}^0(\overrightarrow{r_1}) = R_{n,l}(r_1) Y_l^m(\theta_1, \varphi_1), \tag{19}
$$

where the Y_l^m are the normalised spherical harmonics, and where the radial part $R_{n,l}$ is given by

$$
R_{n,l}(r_1) = C_{n,l} \times r_1^l \sum_{q=0}^{n-l-1} c_q \left(\frac{r_1}{r_0}\right)^q e^{-\frac{r_1}{n r_0}}, \tag{20}
$$

$C_{n,l}$ being the normalization coefficient of $R_{n,l}$ and r_0 being the Bohr radius of the OHe atom. The coefficients c_q in (20) are recursively given by $\frac{c_q}{c_{q-1}} = -\frac{2\left(1-\frac{q+l}{n}\right)}{q(q+2l+1)}$.

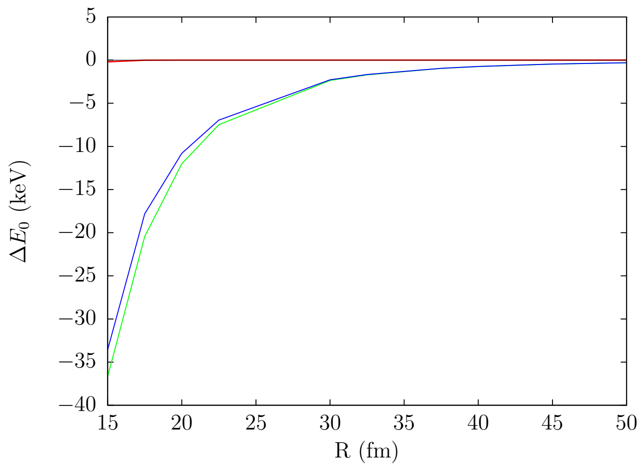

Figure 10: $\triangle E_0 = E_0(R) - E_{OHe}$ (keV) up to orders 1 (upper), 2 (lower) and 3 (middle) for an external sodium nucleus, as a function of its distance R (fm). $V_0 = 30$ MeV and $a = 0.5$ fm.

4.1 Correction to the OHe energy

First, we consider the effect of an approaching sodium nucleus on the OHe energy. Fig. 10 shows the results for $\triangle E_0 = E_0(R) - E_{OHe}$ for $V_0 = 30$ MeV and $a = 0.5$ fm. We see that order 1 doesn't bring a large modification to the unperturbed energy ($\sim 10^{-3}$ keV for R between 50 and 15 fm), while order 2 gives the largest correction ($\sim 1 - 10$ keV for R between 50 and 15 fm). This change from order 1 to order 2 justifies the inclusion of order 3 in the calculations, but it turns out that this one doesn't modify greatly the results from order 2, and that is why order 4 has not been added. We see on Fig. 10 that $\triangle E_0$ is always decreasing, in other words that there is no minimum in this curve in the region of validity of the perturbative calculation.

Similar results hold if one strengthens the nuclear potential, or if one considers different nuclei, as shown in Fig. 11 in the case of iodine.

4.2 Interaction with the incoming nucleus and polarization

In the same way, we can calculate the electrostatic and nuclear interaction energies between the perturbed charge distribution of the helium nucleus and the charge distribution of the A nucleus as a function of its distance R, as well as the mean position of He along the $z-$axis, that is the polarization of the OHe under the influence of the external nucleus.

The electrostatic interaction energy between two charge distributions is

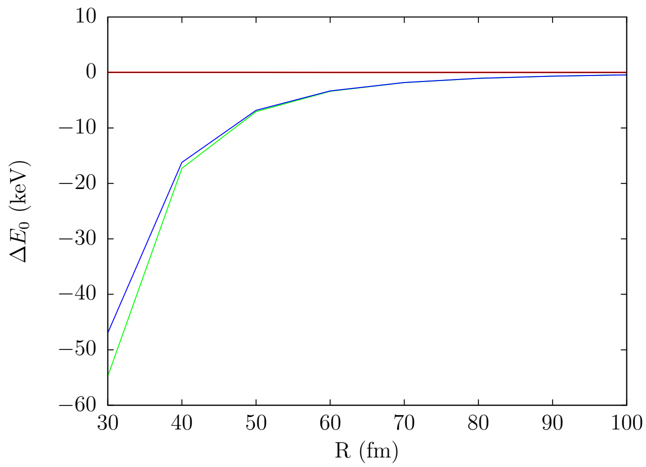

Figure 11: $\triangle E_0 = E_0(R) - E_{OHe}$ (keV) up to orders 1 (upper), 2 (lower) and 3 (middle) for an external iodine nucleus, as a function of its distance R (fm). $V_0 = 45$ MeV and $a = 0.5$ fm.

given by

$$E_{el} = \int_{V1} \int_{V2} \frac{\rho_1 \left(\overrightarrow{r_1}\right) \rho_2 \left(\overrightarrow{r_2}\right)}{|\overrightarrow{r_1} - \overrightarrow{r_2}|} d\overrightarrow{r_1} d\overrightarrow{r_2} \tag{21}$$

where each integral is performed over the extension V_1 or V_2 of the corresponding charge distribution and where ρ_1 and ρ_2 are the charge densities of each distribution. In the following, we take the first-order version of ψ for He (as it is responsible for the dominant second-order shift in energy), and $\rho_1 \left(\overrightarrow{r_1}\right) = |\psi_{He} \left(\overrightarrow{r_1}\right)|^2$, while the external nucleus is treated as a uniform sphere.

The results are compared to the Stark potential used in the previous section in Fig. 12. We see, as might be expected, that the simplifying assumption of the constant electrical field for the nucleus is reasonable at large distance, while the gap becomes more pronounced around 15 fm, because the uniform Coulomb field is always stronger than the true one. The fact that E_{el} becomes repulsive at shorter distance is due to the change of the polarization of the OHe atom under the influence of the nuclear force of the sodium nucleus, which makes the helium component turning to positive mean z_1, that is, towards the external nucleus.

We can also integrate the Woods-Saxon potential $\frac{-V_0}{1+e^{(r-R_A)/a}}$ over the distribution of the helium nucleus to get the total nuclear interaction energy E_{nucl}. Adding it to the previous contribution gives us the curve of Fig. 13, which has no sign of a potential barrier.

Finally, we can calculate the mean value of the position z_1 of the helium

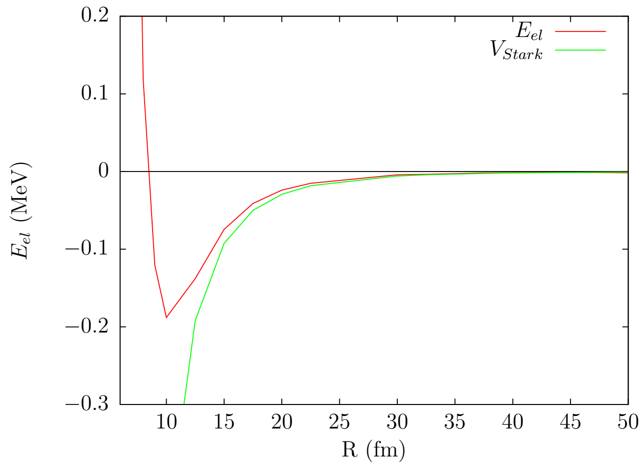

Figure 12: Electrostatic interaction energy E_{el} (MeV) at order 1 (upper) compared to the Stark interaction energy V_{Stark} (MeV) (lower) as a function of the distance R (fm) of an external sodium nucleus; $V_0 = 30$ MeV and $a = 0.5$ fm.

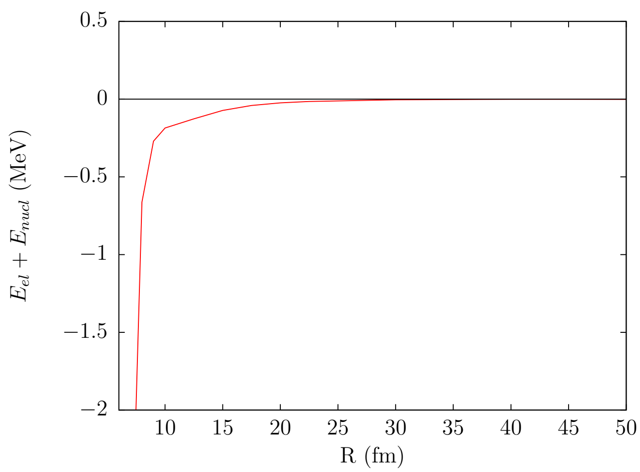

Figure 13: Total interaction energy $E_{el} + E_{nucl}$ (MeV) at order 1 as a function of the distance R (fm) of an external sodium nucleus; $V_0 = 30$ MeV and $a = 0.5$ fm.

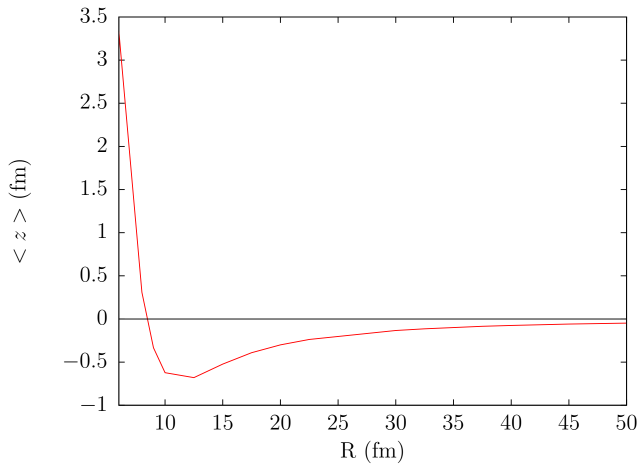

Figure 14: Polarization $< z >$ (fm) at order 1 as a function of the distance R (fm) of an external sodium nucleus; $V_0 = 30$ MeV and $a = 0.5$ fm.

nucleus along the $z-$axis, that is the polarization of the OHe atom, which is simply obtained by

$$< z >=< \psi_{He}|z_1|\psi_{He} >= \int d\overrightarrow{r_1}\, z_1 |\psi_{He}(\overrightarrow{r_1})|^2 \qquad (22)$$

Fig. 14 represents the evolution of this polarization as a function of R. It can be seen that, at large distance, the polarization is negative due to Coulomb repulsion between nuclei, as expected. Thus, Fig. 14 shows that the OHe atom gets polarized, for $R \lesssim 10$ fm, in a direction that could allow repulsion, provided that the nuclear force is not already too strong at such distance. The addition of the nuclear interaction with $V_0 = 30$ MeV and $a = 0.5$ fm in Fig. 13 shows that this condition is in fact not satisfied, giving rise to an attractive force at all distances. The modification of the nuclear parameters V_0 and a (for example $V_0 = 10$, 100, 200 MeV, $a = 0.5$, 1.5 fm), as well as the external nucleus, doesn't radically change the results, modifying only the distance from which the potential falls to nuclear values.

5 Conclusion

The advantages of the OHe composite-dark-matter scenario is that it is minimally related to the parameters of new physics and is dominantly based on the effects of known atomic and nuclear physics. However, the proper quantum treatment of this problem turns out to be rather complicated and involves several open questions.

We have presented here the state of the art of our studies of the nuclear physics of the OHe atoms, and found a difficulty in proving the original assumption of a potential barrier developing between OHe and the nucleus A, both classically and in perturbation theory.

Open questions for further analysis nevertheless remain:

(a) for the distances under consideration the size of the He nucleus may not be negligible and it may not be sufficient to treat it as a point-like particle;

(b) beyond the nucleus the nuclear force falls down exponentially but it may be strong enough to cause a non-homogeneous perturbation of the OHe atomic ground state;

(c) the nuclear force indeed leads to a change of the OHe polarization that might result in the creation of a dipole Coulomb barrier, as shown in the perturbative calculation, but this happens when the perturbative approach is no longer valid, and one should thus solve the Schrödinger equation numerically in this regime.

The answer to these open questions may be crucial for asserting the nuclear-physics basis of the OHe model. If there is no dipole Coulomb barrier between OHe and nucleus, one gets a spectrum of states, which could have transitions to each other. Although the spectra we showed in the third section are not reliable in the nuclear region, it is clear that α particles will have nuclear bound states. Without a barrier, their transitions to them will be fast and dramatic.

Hence, the model cannot work if no repulsive interaction appears at some distance between OHe and the nucleus, and the solution to the open questions of OHe nuclear physics is vital for the composite-dark-matter scenario.

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