

SU(6) $[70, 1^-]$ baryon multiplet in the $1/N_c$ expansion

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Abstract

The masses of excited states of mixed orbital symmetry of nonstrange and strange baryons belonging to the lowest $[70, 1^-]$ multiplet are calculated in the $1/N_c$ expansion to order $1/N_c$ with a new method which allows to considerably reduce the number of linearly independent operators entering the mass formula. This study represents an extension to SU(6) of our work on nonstrange baryons, the framework of which was SU(4). The conclusion regarding the role of the flavor operator, neglected in previous SU(6) studies, is reinforced. Namely, both the flavor and spin operators contribute dominantly to the flavor-spin breaking.

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I. INTRODUCTION

Already in 1974 't Hooft [1] introduced a perturbative expansion of QCD in terms of the parameter $1/N_c$ where N_c is the number of colors, to be used in a nonperturbative QCD regime. Combined with the power counting rules of Witten [2] the $1/N_c$ expansion method became a powerful tool for a systematic analysis of baryon properties, both qualitatively and quantitatively. The success of the method stems from the discovery that the ground state baryons have an exact contracted $SU(2N_f)$ symmetry when $N_c \rightarrow \infty$ [3, 4], N_f being the number of flavors. For $N_c \rightarrow \infty$ the baryon masses are degenerate. For finite N_c the contracted flavor-spin symmetry is broken by effects suppressed by powers of $1/N_c$ so that the mass splitting starts at order $1/N_c$. As a consequence a considerable amount of work has been devoted to the ground state baryons [4–10]. Operator reduction rules simplify the $1/N_c$ expansion [6, 7] and it is customary to drop higher order corrections of order $1/N_c^2$. Recently it has been shown that lattice data clearly display both the $1/N_c$ and $SU(3)$ flavor symmetry breaking hierarchy [11].

Under the assumption that 't Hooft's suggestion [1] would lead to an $1/N_c$ expansion in all QCD regimes, the applicability of the approach to excited states has been discussed since 1994 and it still remains an open problem for deriving the mass spectrum.

From the baryon spectrum, the $[70, 1^-]$ multiplet has been most extensively studied. For $N_f = 2$ there are numerous studies as for example Refs. [12–19]. To our knowledge the $N_f = 3$ case has been considered only in Ref. [20], where first order corrections in $SU(3)$ symmetry breaking were also included. In either case, the conclusion was that the splitting starts at order N_c^0 .

The above studies were based on a method where the system of N_c quarks was decoupled into a ground state core of $N_c - 1$ quarks and an excited quark [16]. This implies that each generator of $SU(2N_f)$ has to be written as a sum of two terms, one acting on the excited quark and the other on the core. As a consequence, the number of linearly independent operators O_i in the mass formula increases tremendously and the number of the coefficients c_i , encoding the quark dynamics, to be determined numerically by a fit, becomes much larger than the experimental data available, as for example for the lowest negative parity nonstrange baryons [16]. Accordingly, the choice of the most dominant operators in the mass formula becomes out of control which implies that important physical effects can be

missed, as discussed below.

A technical advantage of the decoupling scheme is to reduce the problem of the knowledge of matrix elements of $SU(2N_f)$ generators between mixed symmetric states $[N_c - 1, 1]$ to the knowledge of the matrix elements of these generators between symmetric states $[N_c - 1]$, which are easier to find than those of mixed symmetric states (see below).

As an alternative approach, we have recently proposed a new method [21] where the core + quark separation is avoided. Then we deal with $SU(2N_f)$ generators acting on the whole system. Accordingly, the number of linearly independent operators turns out to be smaller than the number of data. All these operators can be included in the fit to clearly find out the most dominant ones up to order $1/N_c$. The knowledge of matrix elements of $SU(2N_f)$ generators between mixed symmetric states $[N_c - 1, 1]$ is necessary.

In this approach we have first analyzed the nonstrange $[\mathbf{70}, 1^-]$ multiplet where the algebraic work was based on Ref. [22] which provided the matrix elements in terms of isoscalar factors of $SU(4)$, initially derived in the context of nuclear physics but quite easily applicable to a system of N_c quarks. In this way we have shown that the flavor (in this case the isospin) term becomes as dominant in Δ resonances as the spin term in N resonances. Note that the flavor operator was omitted in Ref. [16].

Due to this promise we proceeded further with the algebraic work to extend the method to $SU(6)$. We have first obtained the matrix elements of all $SU(6)$ generators between symmetric $[N_c]$ states [23] and in the next step the matrix elements between mixed symmetric states $[N_c - 1, 1]$ states [24]. According to the generalized Wigner-Eckart theorem described in Ref. [22] this amounts at finding the corresponding isoscalar factors.

Based on the knowledge of these isoscalar factors the present work can be seen as an extension of our previous analysis of nonstrange baryons to both nonstrange and strange baryons, within the method proposed in Ref. [21].

The work is organized as follows. In the next section we define the $SU(6) \times O(3)$ basis states. In Sec. III we recall the $SU(6)$ algebra and introduce the matrix elements of the $SU(6)$ generators by using a generalized Wigner-Eckart theorem. The mass operator is described in Sec. IV and the results of the fit in Sec. V. Sec. VI deals with the conclusion.

In Appendix A we recall some symmetry properties of the isoscalar factors of $SU(3)$ and $SU(6)$. In Appendix B we derive the analytic form of the matrix elements of some operators entering the mass formula, relevant for this work. In Appendix C we give the full list of

the isoscalar factors associated to the multiplets 2_8 and 4_8 by completing in this way the results derived in Ref. [24]. In Appendix D we introduce two SU(3) breaking operators, give the general formula of the matrix elements of the breaking terms in each case and exhibit tables containing the analytic expressions of these matrix elements as a function of N_c , the strangeness \mathcal{S} and the isospin I .

II. THE WAVE FUNCTION

We deal with a system of N_c quarks having one unit of orbital excitation. Then the orbital wave function must have a mixed symmetry $[N_c - 1, 1]$. Its spin-flavor part must have the same symmetry in order to obtain a totally symmetric state in the orbital-spin-flavor space. The general form of such a wave function is [25]

$$|[N_c]\rangle = \frac{1}{\sqrt{d_{[N_c-1,1]}}} \sum_Y |[N_c - 1, 1]Y\rangle_O |[N_c - 1, 1]Y\rangle_{FS} \quad (1)$$

where $d_{[N_c-1,1]} = N_c - 1$ is the dimension of the representation $[N_c - 1, 1]$ of the permutation group S_{N_c} and Y is a symbol for a Young tableau (Yamanouchi symbol). The sum is performed over all possible standard Young tableaux. In each term the first basis vector represents the orbital space (O) and the second the spin-flavor space (FS). In this sum there is only one Y (the normal Young tableau) where the last particle is in the second row and $N_c - 2$ terms where the last particle is in the first row.

If there is no decoupling, there is no need to specify Y , the matrix elements being identical for all Y 's, due to Weyl's duality between a linear group and a symmetric group in a given tensor space.

Then in SU(6) \times SO(3) the most general form of the wave function for a state of a given SU(6) symmetry $[f]$ and total angular momentum J and projection J_3 is given by

$$|\ell S; JJ_3; (\lambda\mu)YII_3\rangle = \sum_{m_\ell, S_3} \begin{pmatrix} \ell & S & J \\ m_\ell & S_3 & J_3 \end{pmatrix} |[f](\lambda\mu)YII_3; \ell S JJ_3\rangle |\ell m_\ell\rangle, \quad (2)$$

where presently we are interested in $[f] = [N_c - 1, 1]$.

III. SU(6) GENERATORS AS TENSOR OPERATORS

We recall that the group SU(6) has 35 generators S^i, T^a, G^{ia} with $i = 1, 2, 3$ and $a = 1, 2, \dots, 8$ where S^i are the generators of the spin subgroup SU(2) and T^a the generators of the flavor subgroup SU(3). The group algebra is

$$\begin{aligned} [S^i, S^j] &= i\varepsilon^{ijk} S^k, & [T^a, T^b] &= i f^{abc} T^c, \\ [S^i, G^{ja}] &= i\varepsilon^{ijk} G^{ka}, & [T^a, G^{jb}] &= i f^{abc} G^{ic}, \\ [G^{ia}, G^{jb}] &= \frac{i}{4} \delta^{ij} f^{abc} T^c + \frac{i}{2} \varepsilon^{ijk} \left(\frac{1}{3} \delta^{ab} S^k + d^{abc} G^{kc} \right), \end{aligned} \quad (3)$$

by which the normalization of the generators is fixed.

We redefine the generators forming the algebra (3) as

$$E^i = \frac{S^i}{\sqrt{3}}; \quad E^a = \frac{T^a}{\sqrt{2}}; \quad E^{ia} = \sqrt{2} G^{ia}. \quad (4)$$

Note that the generic name for every generator will also be E^{ia} [22]. Specifications will be made whenever necessary. Here we search for the matrix elements of S^i, T^a and G^{ia} between SU(6) states of symmetry $[N_c - 1, 1]$. As we shall see below, the matrix elements of S^i and T^a are straightforward. The remaining problem is to derive the matrix elements of G^{ia} .

By analogy to SU(4) [22] one can write the matrix elements of every SU(6) generator E^{ia} as

$$\begin{aligned} \langle [N_c - 1, 1](\lambda' \mu') Y' I' I'_3 S' S'_3 | E^{ia} | [N_c - 1, 1](\lambda \mu) Y I I_3 S S_3 \rangle = \\ \sqrt{C^{[N_c-1,1]}(\text{SU}(6))} \begin{pmatrix} S & S^i & S' \\ S_3 & S_3^i & S'_3 \end{pmatrix} \begin{pmatrix} I & I^a & I' \\ I_3 & I_3^a & I'_3 \end{pmatrix} \\ \times \sum_{\rho=1,2} \begin{pmatrix} (\lambda \mu) & (\lambda^a \mu^a) & (\lambda' \mu') \\ Y I & Y^a I^a & Y' I' \end{pmatrix}_{\rho} \begin{pmatrix} [N_c - 1, 1] & [21^4] & [N_c - 1, 1] \\ (\lambda \mu) S & (\lambda^a \mu^a) S^i & (\lambda' \mu') S' \end{pmatrix}_{\rho}, \end{aligned} \quad (5)$$

where $C^{[N_c-1,1]}(\text{SU}(6)) = N_c(5N_c + 18)/12$ is the SU(6) Casimir operator associated to the irreducible representation $[N_c - 1, 1]$, followed by Clebsch-Gordan coefficients of SU(2)-spin and SU(2)-isospin. The sum over ρ is over terms containing products of isoscalar factors of SU(3) and SU(6) respectively. In particular, T^a is an SU(3) irreducible tensor operator of components $T_{Y^a I^a}^{(11)}$, where a corresponds to $(\lambda^a \mu^a) = (11)$ in the present case. It is a scalar in SU(2) so that the index i from E^{ia} is no more necessary. The generators S^i form a rank 1 tensor in SU(2) which is a scalar in SU(3), so the index i suffices. Although we use the same

symbol for the operator S^i and its quantum numbers we hope that no confusion is created. Thus, for the generators S^i and T^a , which are elements of the $su(2)$ and $su(3)$ subalgebras of (3), the above expression simplifies considerably. In particular, as S^i acts only on the spin part of the wave function, we apply the usual Wigner-Eckart theorem for $SU(2)$ to get

$$\begin{aligned} \langle [N_c - 1, 1](\lambda' \mu') Y' I' I'_3; S' S'_3 | S^i | [N_c - 1, 1](\lambda \mu) Y I I_3; S S_3 \rangle = \\ \delta_{SS'} \delta_{\lambda\lambda'} \delta_{\mu\mu'} \delta_{Y Y'} \delta_{I I'} \delta_{I_3 I'_3} \sqrt{C(SU(2))} \begin{pmatrix} S & 1 & S' \\ S_3 & i & S'_3 \end{pmatrix}, \end{aligned} \quad (6)$$

with $C(SU(2)) = S(S+1)$. Similarly, we use the Wigner-Eckart theorem for T^a which is a generator of the subgroup $SU(3)$, so we have

$$\begin{aligned} \langle [N_c - 1, 1](\lambda' \mu') Y' I' I'_3; S' S'_3 | T^a | [N_c - 1, 1](\lambda \mu) Y I I_3; S S_3 \rangle = \\ \delta_{SS'} \delta_{S_3 S'_3} \delta_{\lambda\lambda'} \delta_{\mu\mu'} \sum_{\rho=1,2} \langle (\lambda' \mu') || T^{(11)} || (\lambda \mu) \rangle_{\rho} \begin{pmatrix} (\lambda \mu) & (11) & (\lambda' \mu') \\ Y I I_3 & Y^a I^a I_3^a & Y' I' I'_3 \end{pmatrix}_{\rho}, \end{aligned} \quad (7)$$

where the reduced matrix element is defined as [26]

$$\langle (\lambda \mu) || T^{(11)} || (\lambda \mu) \rangle_{\rho} = \begin{cases} \sqrt{C(SU(3))} & \text{for } \rho = 1 \\ 0 & \text{for } \rho = 2 \end{cases}, \quad (8)$$

in terms of the eigenvalue of the Casimir operator $C(SU(3)) = \frac{1}{3} g_{\lambda\mu}$ where

$$g_{\lambda\mu} = \lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu. \quad (9)$$

Note that the presence of the index ρ has the same origin as in Eq. (5), namely it reflects the multiplicity problem appearing in the direct product of $SU(3)$ irreducible representations

$$\begin{aligned} (\lambda \mu) \times (11) = (\lambda + 1, \mu + 1) + (\lambda + 2, \mu - 1) + (\lambda \mu)_1 + (\lambda \mu)_2 \\ + (\lambda - 1, \mu + 2) + (\lambda - 2, \mu + 1) + (\lambda + 1, \mu - 2) + (\lambda - 1, \mu - 1). \end{aligned} \quad (10)$$

Each $SU(3)$ CG coefficient factorizes into an $SU(2)$ -isospin CG coefficient and an $SU(3)$ isoscalar factor [27]

$$\begin{pmatrix} (\lambda \mu) & (11) & (\lambda' \mu') \\ Y I I_3 & Y^a I^a I_3^a & Y' I' I'_3 \end{pmatrix}_{\rho} = \begin{pmatrix} I & 1 & I' \\ I_3 & I_3^a & I'_3 \end{pmatrix} \begin{pmatrix} (\lambda \mu) & (11) & (\lambda' \mu') \\ Y I & Y^a I^a & Y' I' \end{pmatrix}_{\rho}. \quad (11)$$

The analytic expression of the isoscalar factors can be found in Table 4 of Ref. [26].

IV. THE MASS OPERATOR

When SU(3) is broken the mass operator takes the following general form as first proposed in Ref. [9] for the symmetric baryon multiplet $[N_c]$

$$M = \sum_i c_i O_i + \sum_i d_i B_i. \quad (12)$$

The operators O_i are of type (13)

$$O_i = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{SF}^{(k)}, \quad (13)$$

where $O_\ell^{(k)}$ is a k -rank tensor in SO(3) and $O_{SF}^{(k)}$ a k -rank tensor in SU(2)-spin, but invariant in SU(N_f). Thus O_i are rotational invariant. For the ground state one has $k = 0$. The excited states also require $k = 1$ and $k = 2$ terms.

The rank $k = 2$ tensor operator of SO(3) is

$$L^{(2)ij} = \frac{1}{2} \{L^i, L^j\} - \frac{1}{3} \delta_{i,-j} \vec{L} \cdot \vec{L}, \quad (14)$$

which, like L^i , acts on the orbital wave function $|\ell m_\ell\rangle$ of the whole system of N_c quarks (see Ref. [28] for the normalization of $L^{(2)ij}$).

The operators B_i are SU(6) breaking and are defined to have zero expectation values for nonstrange baryons. The values of the coefficients c_i and d_i which encode the QCD dynamics are determined from numerical fits to data. They are presented below.

Table I gives the list of all linearly independent operators of type (13) organized by powers of $1/N_c$ in their matrix (N_c , N_c^0 and N_c^{-1}). Our choice of operators entering the mass formula (12) is presented in Table II, which contains the most relevant O_i of Table I. Unlike to SU(4) (two flavors), in the SU(6) case (three flavors), the N_c order of the matrix elements of a given operator O_i is not always the same for all multiplets, as one can see from Table III.

As far as the SU(6) breaking is concerned we have first selected the most dominant operator $B_1 = \mathcal{S}$, where \mathcal{S} is the strangeness. Next we have introduced an operator named B_2 , which was found to play an important role, as discussed below.

In Table II the first nontrivial operator is the spin-orbit operator O_2 . In the spirit of the Hartree picture [2], generally adopted for the description of baryons, we identify the spin-orbit operator with the single-particle operator

$$\ell \cdot s = \sum_{i=1}^{N_c} \ell(i) \cdot s(i), \quad (15)$$

TABLE I: The linearly independent spin-singlet flavor-singlet operators for $N_f = 3$, organized by powers of $1/N_c$ in their matrix elements.

Order of matrix element	Operator
N_c	$N_c \mathbf{1}$
N_c^0	$\ell \cdot s, \frac{1}{N_c} \left[T \cdot T - \frac{1}{12} N_c(N_c + 6) \right], \frac{3}{N_c} L \cdot T \cdot G, \frac{15}{N_c} L^{(2)} \cdot G \cdot G, \frac{1}{N_c^2} L \cdot G \cdot \{S, G\}$
N_c^{-1}	$\frac{1}{N_c} S \cdot S, \frac{1}{N_c} L^{(2)} \cdot S \cdot S, \frac{1}{N_c^2} L^{(2)} \cdot T \cdot \{S, G\}, \frac{3}{N_c^2} S \cdot T \cdot G$

TABLE II: Operators and their coefficients in the mass formula obtained from numerical fits. The values of c_i and d_i are indicated under the heading Fit n ($n = 1, 2, 3$), in each case.

Operator	Fit 1 (MeV)	Fit 2 (MeV)	Fit 3 (MeV)
$O_1 = N_c \mathbf{1}$	489 ± 4	492 ± 4	492 ± 4
$O_2 = \ell^i s^i$	24 ± 6	6 ± 6	6 ± 5
$O_3 = \frac{1}{N_c} S^i S^i$	129 ± 10	123 ± 10	123 ± 10
$O_4 = \frac{1}{N_c} \left[T^a T^a - \frac{1}{12} N_c(N_c + 6) \right]$	145 ± 16	134 ± 16	135 ± 16
$O_5 = \frac{3}{N_c} L^i T^a G^{ia}$	-19 ± 7	3 ± 7	4 ± 3
$O_6 = \frac{15}{N_c} L^{(2)ij} G^{ia} G^{ja}$	9 ± 1	9 ± 1	9 ± 1
$O_7 = \frac{1}{N_c^2} L^i G^{ja} \{S^j, G^{ia}\}$	129 ± 33	6 ± 33	
$B_1 = -S$	138 ± 8	138 ± 8	137 ± 8
$B_2 = \frac{1}{N_c} \sum_{i=1}^3 T^i T^i - O_4$	-59 ± 18	-40 ± 18	-40 ± 18
χ_{dof}^2	1.7	0.9	0.84

the matrix elements of which are of order N_c^0 . For simplicity we ignore the two-body part of the spin-orbit operator, denoted by $1/N_c (\ell \cdot S_c)$ in Ref. [16], as being of a lower order (the lower case indicates operators acting on the excited quark and the subscript c is attached to those acting on the core). The spin operator O_3 and the flavor operator O_4 are two-body

TABLE III: Matrix elements of O_i for all states belonging to the $[\mathbf{70}, 1^-]$ multiplet. The vanishing off-diagonal matrix elements are not included.

	O_1	O_2	O_3	O_4	O_5	O_6	O_7
${}^2 8_{\frac{1}{2}}$	N_c	$-\frac{2N_c-3}{3N_c}$	$\frac{3}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{3}{N_c}$	0	$\frac{5-N_c(3N_c+10)}{24N_c^2}$
${}^4 8_{\frac{1}{2}}$	N_c	$-\frac{5}{6}$	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{5(N_c+3)}{4N_c}$	$-\frac{25(N_c-1)}{8N_c}$	$-\frac{5(N_c-1)(3N_c+5)}{48N_c^2}$
${}^2 8_{\frac{3}{2}}$	N_c	$\frac{2N_c-3}{6N_c}$	$\frac{3}{4N_c}$	$\frac{3}{4N_c}$	$\frac{3}{2N_c}$	0	$-\frac{5-N_c(3N_c+10)}{48N_c^2}$
${}^4 8_{\frac{3}{2}}$	N_c	$-\frac{1}{3}$	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{N_c+3}{2N_c}$	$\frac{5(N_c-1)}{2N_c}$	$-\frac{(N_c-1)(3N_c+5)}{24N_c^2}$
${}^4 8_{\frac{5}{2}}$	N_c	$\frac{1}{2}$	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	$\frac{3(N_c+3)}{4N_c}$	$-\frac{5(N_c-1)}{8N_c}$	$\frac{(N_c-1)(3N_c+5)}{16N_c^2}$
${}^2 10_{\frac{1}{2}}$	N_c	$\frac{1}{3}$	$\frac{3}{4N_c}$	$\frac{15}{4N_c}$	$-\frac{3(N_c+1)}{2N_c}$	0	$\frac{17-N_c(3N_c+10)}{24N_c^2}$
${}^2 10_{\frac{3}{2}}$	N_c	$-\frac{1}{6}$	$\frac{3}{4N_c}$	$\frac{15}{4N_c}$	$\frac{3(N_c+1)}{4N_c}$	0	$-\frac{17-N_c(3N_c+10)}{48N_c^2}$
${}^2 1_{\frac{1}{2}}$	N_c	-1	$\frac{3}{4N_c}$	$-\frac{2N_c+3}{4N_c}$	$\frac{N_c-3}{2N_c}$	0	$-\frac{1+3N_c(N_c+4)}{24N_c^2}$
${}^2 1_{\frac{3}{2}}$	N_c	$\frac{1}{2}$	$\frac{3}{4N_c}$	$-\frac{2N_c+3}{4N_c}$	$-\frac{N_c-3}{4N_c}$	0	$\frac{1+3N_c(N_c+4)}{48N_c^2}$
${}^2 8_{\frac{1}{2}} - {}^4 8_{\frac{1}{2}}$	0	$-\frac{1}{3}\sqrt{\frac{N_c+3}{2N_c}}$	0	0	$\frac{1}{2N_c}\sqrt{\frac{N_c(N_c+3)}{2}}$	$-\frac{25}{4N_c}\sqrt{\frac{N_c(N_c+3)}{2}}$	$\frac{5}{12N_c^2}\sqrt{\frac{N_c(N_c+3)}{2}}$
${}^2 8_{\frac{3}{2}} - {}^4 8_{\frac{3}{2}}$	0	$-\frac{1}{6}\sqrt{\frac{5(N_c+3)}{N_c}}$	0	0	$\frac{1}{4N_c}\sqrt{5N_c(N_c+3)}$	$\frac{5}{8N_c}\sqrt{5N_c(N_c+3)}$	$\frac{5}{24N_c^2}\sqrt{5N_c(N_c+3)}$

and linearly independent. The operators O_5 and O_6 , are two-body, which means that they carry a factor $1/N_c$. But as G^{ia} sums coherently, it introduces an extra factor N_c and makes the matrix elements of O_5 and O_6 of order N_c^0 as well, except for O_5 in the ${}^2 8$ multiplet.

The operators O_5 and O_6 are normalized to allow their coefficients c_i to have a natural size [20, 29]. The normalization factors result from the matrix elements of O_i presented in Table III. We have also included the more complex operator O_7 , which contains an anticommutator and is three-body. But because it contains the coherent generator G^{ia} two times, its matrix elements turn out to be of order $\mathcal{O}(N_c^0)$. The presence of coherent factors in a composite operator leads to an enhancement of the order of the matrix elements, as

already known from the SU(4) case [16].

The matrix elements of the operators O_i have been calculated for all available states of the multiplet $[70, 1^-]$ starting from the wave function (2) and using the isoscalar factors of Ref. [24] combined with Tables V, VI, VII and VIII of the present work. For completeness the general analytic expressions of O_5 , O_6 and O_7 , are given in Appendix B, up to an obvious factor. In Table III the nonvanishing off-diagonal matrix elements are also indicated whenever the case.

One would think of also including the operator

$$O_8 = \frac{1}{N_c} L^{(2)ij} S^i S^j, \quad (16)$$

of order $1/N_c$. However, interestingly, in our basis we found a proportionality relation between the following expectation values

$$\langle L^{(2)ij} S^i S^j \rangle = -\frac{12}{N_c - 1} \langle L^{(2)ij} G^{ia} G^{ia} \rangle, \quad (17)$$

for all states belonging to the $[70, 1^-]$ multiplet. This implies that we cannot include O_8 independently in the fit to the experimental spectrum, because its expectation values are proportional to those of O_6 and we ignore the off-diagonal matrix elements of $\langle O_8 \rangle$.

The operator B_2 was introduced in order to obtain a splitting between Σ and Λ baryons, like in the Gell-Mann - Okubo mass formula. Its matrix elements are given by

$$B_2 = \frac{1}{N_c} I(I + 1) - O_4 \quad (18)$$

where O_4 can be found in column 5 of Table III.

There are two other SU(3) breaking operators defined in Appendix D together with their matrix elements given in Tables X and XI. We found that their contribution is negligible in improving the fit. An account of such findings can be found in Ref. [30].

V. RESULTS

We have performed three distinct fits of the mass formula (12) using the experimental masses from PDG [31]. There are 17 resonances available, with a status of three or four stars and two mixing angles. The latter are defined by the following equations

$$\begin{aligned} |N'_J\rangle &= \cos \theta_J |^4 N_J\rangle + \sin \theta_J |^2 N_J\rangle, \\ |N_J\rangle &= \cos \theta_J |^2 N_J\rangle - \sin \theta_J |^4 N_J\rangle. \end{aligned} \quad (19)$$

TABLE IV: The partial contribution and the total mass (MeV) predicted by the $1/N_c$ expansion obtained from the Fit 1. The last two columns give the empirically known masses [31] and the resonance name and status.

	Part. contrib. (MeV)									Total (MeV)	Exp. (MeV)	Name, status
	c_1O_1	c_2O_2	c_3O_3	c_4O_4	c_5O_5	c_6O_6	c_7O_7	d_1B_1	d_2B_2			
$N_{\frac{1}{2}}$	1467	-8	32	36	19	0	-31	0	0	1499 ± 10	1538 ± 18	$S_{11}(1535)^{****}$
$\Lambda_{\frac{1}{2}}$								138	15	1668 ± 9	1670 ± 10	$S_{01}(1670)^{****}$
$\Sigma_{\frac{1}{2}}$								138	-25	1628 ± 10		
$\Xi_{\frac{1}{2}}$								276	0	1791 ± 13		
$N_{\frac{3}{2}}$	1467	4	32	36	-10	0	16	0	0	1542 ± 10	1523 ± 8	$D_{13}(1520)^{****}$
$\Lambda_{\frac{3}{2}}$								138	15	1698 ± 8	1690 ± 5	$D_{03}(1690)^{****}$
$\Sigma_{\frac{3}{2}}$								138	-25	1658 ± 9	1675 ± 10	$D_{13}(1670)^{****}$
$\Xi_{\frac{3}{2}}$								276	0	1821 ± 11	1823 ± 5	$D_{13}(1820)^{***}$
$N'_{\frac{1}{2}}$	1467	-20	162	36	48	-18	42	0	0	1648 ± 11	1660 ± 20	$S_{11}(1650)^{****}$
$\Lambda'_{\frac{1}{2}}$								138	15	1784 ± 16	1785 ± 65	$S_{01}(1800)^{***}$
$\Sigma'_{\frac{1}{2}}$								138	-25	1745 ± 17	1765 ± 35	$S_{11}(1750)^{***}$
$\Xi'_{\frac{1}{2}}$								276	0	1907 ± 20		
$N'_{\frac{3}{2}}$	1467	-8	162	36	19	15	-17	0	0	1675 ± 10	1700 ± 50	$D_{13}(1700)^{***}$
$\Lambda'_{\frac{3}{2}}$								138	15	1826 ± 12		
$\Sigma'_{\frac{3}{2}}$								138	-25	1787 ± 13		
$\Xi'_{\frac{3}{2}}$								276	0	1949 ± 16		
$N_{\frac{5}{2}}$	1467	12	162	36	-29	-4	25	0	0	1669 ± 10	1678 ± 8	$D_{15}(1675)^{****}$
$\Lambda_{\frac{5}{2}}$								138	15	1822 ± 10	1820 ± 10	$D_{05}(1830)^{****}$
$\Sigma_{\frac{5}{2}}$								138	-25	1782 ± 11	1775 ± 5	$D_{15}(1775)^{****}$
$\Xi_{\frac{5}{2}}$								276	0	1945 ± 14		
$\Delta_{\frac{1}{2}}$	1467	8	32	181	38	0	-24	0	0	1702 ± 18	1645 ± 30	$S_{31}(1620)^{****}$
$\Sigma''_{\frac{1}{2}}$								138	34	1875 ± 16		
$\Xi''_{\frac{1}{2}}$								276	59	2037 ± 22		
$\Omega_{\frac{1}{2}}$								413	74	2190 ± 29		
$\Delta_{\frac{3}{2}}$	1467	-4	32	181	-19	0	12	0	0	1668 ± 20	1720 ± 50	$D_{33}(1700)^{****}$
$\Sigma''_{\frac{3}{2}}$								138	34	1841 ± 16		
$\Xi''_{\frac{3}{2}}$								276	59	2003 ± 21		
$\Omega_{\frac{3}{2}}$								413	74	2156 ± 27		
$\Lambda''_{\frac{1}{2}}$	1467	-24	32	-108	0	0	-38	138	-44	1421 ± 14	1407 ± 4	$S_{01}(1405)^{****}$
$\Lambda''_{\frac{3}{2}}$	1467	12	32	-108	0	0	19	138	-44	1515 ± 14	1520 ± 1	$D_{03}(1520)^{****}$
$N_{1/2} - N'_{1/2}$	0	-8	0	0	-10	-55	18	0	0	-55		
$N_{3/2} - N'_{3/2}$	0	-12	0	0	-15	17	28	0	0	18		

Experimentally one finds $\theta_{1/2}^{exp} \approx -0.56$ rad and $\theta_{3/2}^{exp} \approx 0.10$ rad [32].

The resulting dynamical coefficients c_i and d_i and the values of χ_{dof}^2 are indicated in Table II in each case. Actually only the Fit 1 is based on the experimental value of $M(\Lambda(1405)) = 1407$ MeV. The corresponding χ_{dof}^2 is 1.7. We have tried to understand the somewhat large χ_{dof}^2 and found out that it is rather difficult to accommodate the experimentally low mass of $\Lambda(1405)$. The situation is entirely similar to all types of quark models, as for example, [33–35]. The experimental mixing matrix of the $\Lambda(S01)$ resonances in terms of the flavor singlet 2_1 and 2_8 and 4_8 components [32] could not help in lowering the mass of $\Lambda(1405)$ [36].

To see indeed that the experimentally low mass of $\Lambda(1405)$ is responsible for the χ_{dof}^2 of the Fit 1, we have performed the Fits 2 and 3 with an arbitrarily larger value than the experimental mass. We took $M(\Lambda(1405)) = 1500$ MeV, inspired by quark model calculations. With this value the χ_{dof}^2 goes down to about 0.9 for these latter fits. By making the Fit 3 we explored the role of the operator O_7 . Removing it from the mass formula the result remained practically unchanged.

However we have to point out the important role of the operator O_7 in the Fit 1. Without it the χ_{dof}^2 is about 2.95, while including, it goes down to 1.7. Actually the operator O_7 contains the operators O_9 and O_{11} of Ref. [20] and similarly it plays a role in the $\Lambda(1520)$ - $\Lambda(1405)$ splitting, enhancing the effect of the spin-orbit operator O_2 and leading to a splitting quite close to the experiment.

As a common feature with the SU(4) case we found that the isospin operator O_4 contributes to the mass with a coefficient c_4 very close to that of the spin operators O_3 .

In Table IV we present the partial contributions $c_i O_i$ and $d_i B_i$ for all operator included in the mass formula (12) together with the total mass (MeV) predicted by the $1/N_c$ expansion in SU(6).

Similar to the SU(4) case [21] we found that spin operator O_3 is dominant in N resonances while the flavor operator O_4 is dominant in Δ resonances, with an even larger positive contribution.

This implies that flavor operator is as important as the spin operator, a result consistent with that obtained for nonstrange baryons. Thus these two operators bring the basic contribution to the spin-flavor breaking. Note that the operator O_4 is also dominant in the flavor singlets $\Lambda''_{\frac{1}{2}}$ and $\Lambda''_{\frac{3}{2}}$. Its negative contribution compensates to a large extent the positive

contribution of the SU(3) flavor breaking operator B_1 . Together with the operator B_2 , used here for the first time, we obtain the required splitting between Λ and Σ resonances in a given multiplet.

The terms containing the angular momentum components, O_2 , O_5 and O_6 are dynamically suppressed, as suggested by the very small values of their coefficients, c_2 , c_5 and c_6 respectively. Although the coefficient c_7 is large the contribution $c_7 O_7$ to the mass is always moderate as one can see from Table IV.

The flavor breaking operator B_1 is important to all strange baryons.

In Ref. [20] $\Lambda(1405)$ acquired a mass very close to experiment. Our result for this resonance, 1421 ± 14 MeV is not far from the experimental interval. There is however some difference between the dynamics of our approach and that of Ref. [20]. In Ref. [20] only the wave function component with $S_c = 0$ is taken into account and this component brings no contribution to the spin term in flavor singlets, which means that the operators $\frac{1}{N_c} S^c \cdot S^c$ and $\frac{1}{N_c} s \cdot S^c$ have zero expectation values. For this basic reason the mass of $\Lambda(1405)$ is not affected by the spin contribution and remains low, while the other masses are moved upwards. In our case, where we use the exact wave function, both $S_c = 0$ and $S_c = 1$ parts of the wave function contribute to the spin term. This makes the spin term expectation value identical for all states of a given J ($\langle O_3 \rangle = 3/(4N_c)$ for $J = 1/2$ and $\langle O_3 \rangle = 15/(4N_c)$ for $J = 3/2$, see Table III) irrespective of the flavor, which seems to us natural. Then, in our case, with a non vanishing spin term in flavour singlets as well, the mass formula accommodates a heavier $\Lambda(1405)$ than the experiment, like in quark models, as mentioned above (for a review on the controversial nature of $\Lambda(1405)$ see, for example, Ref. [37]). Interestingly, the isospin operator, absent in Ref. [20], although of order $\mathcal{O}(N_c^0)$ has a negative expectation value $\langle O_4 \rangle = -\frac{2N_c + 3}{4N_c}$ (Table III) for flavour singlets, which lowers the total mass, but not enough. The fit considerably improves by the introduction of the operator B_2 , considered in this work for the first time. As one can see, this operator helps in lowering the flavor singlets with respect to the rest of the spectrum.

We have made some other fits which are not presented here. For example we have included the operators $\frac{1}{N_c^2} L^{(2)} \cdot T \cdot \{S, G\}$ and $\frac{3}{N_c^2} S \cdot T \cdot G$. As we found that their contributions to the global fit are negligible we have omitted them.

Finally, our results are compatible with those of Cohen and Lebed [38] where for SU(6) mixed symmetric spin-flavor multiplets five towers of states are predicted based on five

independent operators: the O_1 operator of order $\mathcal{O}(N_c)$ and four $\mathcal{O}(N_c^0)$ operators written in the core+excited quark scheme [16]. In our case the latter operators correspond to O_2 , O_4 , O_5 and O_6 . The operator O_7 , also of order $\mathcal{O}(N_c^0)$, has not been considered in [38], being more complex. All these are called quark-picture operators. It would be useful to reanalyze the connection established by Cohen and Lebed between the quark-picture operators and the K-matrix poles of their approach.

VI. CONCLUSIONS

We have analyzed the spectrum of nonstrange and strange baryons belonging to the lowest negative parity 70-plet, to first order in SU(3) symmetry breaking by using a new method which in the $1/N_c$ expansion simplifies the mass formula, reducing it to a considerably smaller number of terms, namely 7 operators of type O_i as compared to 11 operators in Ref. [16]. This allows us to easier find the most dominant operators to order $1/N_c$. We have shown that the isospin operator O_4 , neglected in previous studies, contributes to decuplets with a coefficient of the same order of magnitude as the spin operator O_3 in octets. In addition the role of the operator O_4 is crucial in describing the flavor singlets.

Actually the remaining difficulty in perfectly fitting $\Lambda(1405)$ is consistent with the view that this resonance has a more complex nature as, for example, having a coupling to a $\bar{K}N$ system, which might survive in the large N_c limit [39, 40]. We remind that the meson-baryon coupling is a long standing problem discussed first in the Skyrme model [41] and later, in the resonant picture of the meson-nucleon scattering [42].

The isoscalar factors found in this work can be used in further SU(6) studies, formally or in physical applications.

Appendix A

We recall that the isoscalar factors of SU(3) obey the following orthogonality relation

$$\sum_{Y''I''Y^aI^a} \left(\begin{array}{c} (\lambda''\mu'') \quad (11) \\ Y''I'' \quad Y^aI^a \end{array} \parallel \begin{array}{c} (\lambda'\mu') \\ YI \end{array} \right)_\rho \left(\begin{array}{c} (\lambda''\mu'') \quad (11) \\ Y''I'' \quad Y^aI^a \end{array} \parallel \begin{array}{c} (\lambda\mu) \\ Y'I' \end{array} \right)_\rho = \delta_{\lambda'\lambda} \delta_{\mu'\mu} \delta_{Y'Y} \delta_{I'I}, \quad (\text{A1})$$

TABLE V: Isoscalar factors of the SU(6) generators Eqs. (4) and (5), corresponding to the 2_8 multiplet of $N_c = 3$.

$(\lambda_1 \mu_1) S_1$	$(\lambda_2 \mu_2) S_2$	ρ	$\left(\begin{array}{cc c} [N_c - 1, 1] & [21^4] & [N_c - 1, 1] \\ (\lambda_1 \mu_1) S_1 & (\lambda_2 \mu_2) S_2 & (\lambda \mu) S \end{array} \right)_\rho$
$(\lambda \mu) S + 1$	(11)1	1	$-\frac{3\sqrt{2S(2S+3)(N_c+2S+2)}}{\sqrt{(S+1)(2S+1)[N_c(N_c+6)+12S(S+1)](5N_c+18)}}$
$(\lambda \mu) S + 1$	(11)1	2	$\frac{N_c}{S+1} \sqrt{\frac{3(2S+3)(N_c-2S+4)(N_c+2S+6)}{2(2S+1)(N_c-2S)[N_c(N_c+6)+12S(S+1)](5N_c+18)}}$
$(\lambda \mu) S$	(11)1	1	$\{12S(S+1) + N_c[4S(S+1) - 3]\} \sqrt{\frac{2}{S(S+1)[N_c(N_c+6)+12S(S+1)]N_c(5N_c+18)}}$
$(\lambda \mu) S$	(11)1	2	$\frac{4S^2(S+1)^2 - 2N_cS(S+1) - (S^2+S-1)N_c^2}{2S(S+1)} \sqrt{\frac{6(N_c-2S+4)(N_c+2S+6)}{(N_c-2S)(N_c+2S+2)[N_c(N_c+6)+12S(S+1)]N_c(5N_c+18)}}$
$(\lambda \mu) S - 1$	(11)1	1	$-3\sqrt{\frac{2(S+1)(2S-1)(N_c-2S)}{S(2S+1)(N_c(N_c+6)+12S(S+1))(5N_c+18)}}$
$(\lambda \mu) S - 1$	(11)1	2	$\frac{N_c}{S} \sqrt{\frac{3(2S-1)(N_c-2S+4)(N_c+2S+6)}{2(2S+1)(N_c+2S+2)(N_c(N_c+6)+12S(S+1))(5N_c+18)}}$
$(\lambda + 2, \mu - 1) S + 1$	(11)1	/	$-\frac{1}{S+1} \sqrt{\frac{3S(S+2)(2S+3)(N_c-2S-2)(N_c+2S+2)(N_c+2S+6)}{2(2S+1)(N_c+2S+4)N_c(5N_c+18)}}$
$(\lambda + 2, \mu - 1) S$	(11)1	/	$\frac{1}{S+1} \sqrt{\frac{3(2S+3)(N_c+2S+2)(N_c+2S+6)}{2(2S+1)(N_c+2S+4)(5N_c+18)}}$
$(\lambda + 1, \mu - 2) S + 1$	(11)1	/	$-2\sqrt{\frac{3S(2S+3)(N_c-2S-2)}{(S+1)(2S+1)(N_c-2S)(N_c+2S+4)(5N_c+18)}}$
$(\lambda + 1, \mu - 2) S$	(11)1	/	$-2\sqrt{\frac{3(N_c-2S-2)}{(S+1)(2S+1)(N_c-2S)(N_c+2S+4)(5N_c+18)}}$
$(\lambda - 1, \mu - 1) S$	(11)1	/	$\sqrt{\frac{12(N_c+2S)}{S(2S+1)(N_c-2S+2)(N_c+2S+2)(5N_c+18)}}$
$(\lambda - 1, \mu - 1) S - 1$	(11)1	/	$-2\sqrt{\frac{3(S+1)(N_c+2S)(2S-1)}{S(2S+1)(N_c-2S+2)(N_c+2S+2)(5N_c+18)}}$
$(\lambda - 2, \mu + 1) S$	(11)1	/	$\frac{1}{S} \sqrt{\frac{3(2S-1)(N_c-2S)(N_c-2S+4)}{2(2S+1)(N_c-2S+2)(5N_c+18)}}$
$(\lambda - 2, \mu + 1) S - 1$	(11)1	/	$-\frac{1}{S} \sqrt{\frac{3(S-1)(S+1)(2S-1)(N_c-2S)(N_c+2S)(N_c-2S+4)}{2(2S+1)(N_c-2S+2)N_c(5N_c+18)}}$
$(\lambda \mu) S$	(11)0	1	$\sqrt{\frac{N_c(N_c+6)+12S(S+1)}{2N_c(5N_c+18)}}$
$(\lambda \mu) S$	(11)0	2	0
$(\lambda \mu) S$	(00)1	/	$\sqrt{\frac{4S(S+1)}{N_c(5N_c+18)}}$

TABLE VI: Isoscalar factors of the SU(6) generators, corresponding to the 4_8 multiplet of $N_c = 3$.

$(\lambda_1 \mu_1) S_1$	$(\lambda_2 \mu_2) S_2$	ρ	$\left(\begin{array}{cc c} [N_c - 1, 1] & [21^4] & [N_c - 1, 1] \\ (\lambda_1 \mu_1) S_1 & (\lambda_2 \mu_2) S_2 & (\lambda - 2, \mu + 1) S \end{array} \right)_\rho$
$(\lambda - 2, \mu + 1) S$	(11)1	1	$[N_c(4S - 3) + 6S] \sqrt{\frac{2(S + 1)}{S [N_c(N_c + 6) + 12(S - 1)S] N_c(5N_c + 18)}}$
$(\lambda - 2, \mu + 1) S$	(11)1	2	$-\frac{N_c - 2S}{S} \sqrt{\frac{3(S - 1)(S + 1)(N_c - 2S + 6)(N_c + 2S)(N_c + 2S + 4)}{2(N_c - 2S + 2) [N_c(N_c + 6) + 12(S - 1)S] N_c(5N_c + 18)}}$
$(\lambda \mu) S + 1$	(11)1	/	$-\sqrt{\frac{3}{2}} \sqrt{\frac{2S + 3}{2S + 1}} \sqrt{\frac{(N_c - 2S)(N_c + 2S + 4)}{N_c(5N_c + 18)}}$
$(\lambda \mu) S$	(11)1	/	$-\frac{1}{S} \sqrt{\frac{3}{2}} \sqrt{\frac{(N_c - 2S)(N_c + 2S + 4)}{(N_c + 2S + 2)(5N_c + 18)}}$
$(\lambda \mu) S - 1$	(11)1	/	$\frac{N_c + 4S^2}{S} \sqrt{\frac{3(N_c + 2S + 4)}{2(2S - 1)(2S + 1)(N_c + 2S + 2)N_c(5N_c + 18)}}$
$(\lambda - 2, \mu + 1) S - 1$	(11)1	1	$\frac{3\sqrt{2(S - 1)(N_c + 2S)}}{\sqrt{S [N_c(N_c + 6) + 12(S - 1)S] (5N_c + 18)}}$
$(\lambda - 2, \mu + 1) S - 1$	(11)1	2	$-\frac{N_c}{S} \sqrt{\frac{3(N_c - 2S + 6)(N_c + 2S + 4)}{2(N_c - 2S + 2) [N_c(N_c + 6) + 12(S - 1)S] (5N_c + 18)}}$
$(\lambda - 1, \mu - 1) S$	(11)1	/	$-2\sqrt{\frac{3(S + 1)(N_c - 2S)(N_c + 2S)}{S(N_c - 2S + 2)(N_c + 2S + 2)N_c(5N_c + 18)}}$
$(\lambda - 1, \mu - 1) S - 1$	(11)1	/	$2(S - 1) \sqrt{\frac{3(N_c - 2S)(N_c + 2S)}{S(2S - 1)(N_c - 2S + 2)(N_c + 2S + 2)N_c(5N_c + 18)}}$
$(\lambda - 3, \mu) S - 1$	(11)1	/	$-2\sqrt{\frac{3(S - 1)(N_c + 2S - 2)}{(2S - 1)(N_c - 2S + 4)N_c(5N_c + 18)}}$
$(\lambda - 4, \mu + 2) S - 1$	(11)1	/	$-\sqrt{\frac{3}{2}} \sqrt{\frac{2S - 3}{2S - 1}} \sqrt{\frac{(N_c + 2S)(N_c - 2S + 2)(N_c - 2S + 6)}{(N_c - 2S + 4)N_c(5N_c + 18)}}$
$(\lambda - 2, \mu + 1) S$	(11)0	1	$\sqrt{\frac{N_c(N_c + 6) + 12(S - 1)S}{2N_c(5N_c + 18)}}$
$(\lambda - 2, \mu + 1) S$	(11)0	2	0
$(\lambda - 2, \mu + 1) S$	(00)1	/	$\sqrt{\frac{4S(S + 1)}{N_c(5N_c + 18)}}$

TABLE VII: Isoscalar factors of the SU(6) generators, corresponding to the 210 multiplet of $N_c = 3$.

$(\lambda_1 \mu_1) S_1$	$(\lambda_2 \mu_2) S_2$	ρ	$\left(\begin{array}{cc c} [N_c - 1, 1] & [21^4] & [N_c - 1, 1] \\ (\lambda_1 \mu_1) S_1 & (\lambda_2 \mu_2) S_2 & (\lambda + 2, \mu - 1) S \end{array} \right)_\rho$
$(\lambda + 4, \mu - 2) S + 1$	(11)1	/	$-\sqrt{\frac{3(2S+5)(N_c+2S+4)(N_c+2S+8)(N_c-2S-2)}{2(2S+3)(N_c+2S+6)N_c(5N_c+18)}}$
$(\lambda + 3, \mu - 3) S + 1$	(11)1	/	$2\sqrt{\frac{3(S+2)(N_c-2S-4)}{(2S+3)(N_c+2S+6)N_c(5N_c+18)}}$
$(\lambda + 1, \mu - 2) S + 1$	(11)1	/	$-2(S+2)\sqrt{\frac{3(N_c+2S+2)(N_c-2S-2)}{(S+1)(2S+3)(N_c-2S)(N_c+2S+4)N_c(5N_c+18)}}$
$(\lambda + 2, \mu - 1) S + 1$	(11)1	1	$3\sqrt{\frac{2(S+2)(N_c-2S-2)}{(S+1)(N_c(N_c+6)+12(S+1)(S+2))(5N_c+18)}}$
$(\lambda + 2, \mu - 1) S + 1$	(11)1	2	$-\frac{N_c}{S+1}\sqrt{\frac{3(N_c-2S+2)(N_c+2S+8)}{2(N_c+2S+4)(N_c(N_c+6)+12(S+1)(S+2))(5N_c+18)}}$
$(\lambda + 2, \mu - 1) S$	(11)1	1	$[N_c(4S+7)+6(S+1)]\sqrt{\frac{2S}{(S+1)[N_c(N_c+6)+12(S+1)(S+2)]N_c(5N_c+18)}}$
$(\lambda + 2, \mu - 1) S$	(11)1	2	$-\frac{N_c+2S+2}{S+1}\sqrt{\frac{3S(S+2)(N_c-2S-2)(N_c-2S+2)(N_c+2S+8)}{2(N_c+2S+4)[N_c(N_c+6)+12(S+1)(S+2)]N_c(5N_c+18)}}$
$(\lambda + 1, \mu - 2) S$	(11)1	/	$2\sqrt{\frac{3S(N_c+2S+2)(N_c-2S-2)}{(S+1)(N_c-2S)(N_c+2S+4)N_c(5N_c+18)}}$
$(\lambda \mu) S + 1$	(11)1	/	$\frac{N_c+4(S+1)^2}{S+1}\sqrt{\frac{3(N_c-2S+2)}{2(2S+1)(2S+3)(N_c-2S)N_c(5N_c+18)}}$
$(\lambda \mu) S$	(11)1	/	$-\frac{1}{S+1}\sqrt{\frac{3(N_c+2S+2)(N_c-2S+2)}{2(N_c-2S)(5N_c+18)}}$
$(\lambda \mu) S - 1$	(11)1	/	$-\sqrt{\frac{3(2S-1)(N_c+2S+2)(N_c-2S+2)}{2(2S+1)N_c(5N_c+18)}}$
$(\lambda + 2, \mu - 1) S$	(11)0	1	$\sqrt{\frac{N_c(N_c+6)+12(S+1)(S+2)}{2N_c(5N_c+18)}}$
$(\lambda + 2, \mu - 1) S$	(11)0	2	0
$(\lambda + 2, \mu - 1) S$	(00)1	/	$\sqrt{\frac{4S(S+1)}{N_c(5N_c+18)}}$

TABLE VIII: Isoscalar factors of the SU(6) generators, corresponding to the 2_1 multiplet of $N_c = 3$.

$(\lambda_1 \mu_1) S_1$	$(\lambda_2 \mu_2) S_2$	ρ	$\left(\begin{array}{cc c} [N_c - 1, 1] & [21^4] & [N_c - 1, 1] \\ (\lambda_1 \mu_1) S_1 & (\lambda_2 \mu_2) S_2 & (\lambda - 1, \mu - 1) S \end{array} \right)_\rho$
$(\lambda + 1, \mu - 2) S + 1$	(11)1	/	$-\sqrt{\frac{3(2S+3)(N_c-2S-2)(N_c-2S+2)(N_c+2S+4)}{2(N_c-2S)(2S+1)N_c(5N_c+18)}}$
$(\lambda + 1, \mu - 2) S$	(11)1	/	$-\sqrt{\frac{3(N_c-2S-2)(N_c-2S+2)(N_c+2S+4)}{2S(2S+1)(N_c-2S)N_c(5N_c+18)}}$
$(\lambda - 1, \mu - 1) S$	(11)1	1	$[N_c(4S-3)+6S] \sqrt{\frac{2(S+1)}{S[N_c^2+12(S^2-1)]N_c(5N_c+18)}}$
$(\lambda - 1, \mu - 1) S$	(11)1	2	$-\{N_c(N_c+6)-4[S(S-1)-3]\} \sqrt{\frac{3(2S-1)(S+1)(N_c-2S-2)(N_c+2S-2)}{2S(2S+1)(N_c-2S+2)(N_c+2S+2)[N_c^2+12(S^2-1)]N_c(5N_c+18)}}$
$(\lambda - 1, \mu - 1) S - 1$	(11)1	1	$3\sqrt{\frac{2N_c(2S-1)}{S[N_c^2+12(S^2-1)](5N_c+18)}}$
$(\lambda - 1, \mu - 1) S - 1$	(11)1	2	0 if $S = 1/2$
$(\lambda - 1, \mu - 1) S - 1$	(11)1	2	$-\{N_c(N_c+6)-12(S^2-1)\} \sqrt{\frac{3(N_c-2S-2)(N_c+2S-2)}{2S(2S+1)(N_c-2S+2)(N_c+2S+2)[N_c^2+12(S^2-1)]N_c(5N_c+18)}}$ if $S \geq 1$
$(\lambda \mu) S + 1$	(11)1	/	$\sqrt{\frac{6(2S+3)(N_c+2S+4)}{(2S+1)(N_c-2S)N_c(5N_c+18)}}$
$(\lambda \mu) S$	(11)1	/	$\frac{1}{S} \sqrt{\frac{6(N_c+2S+4)}{(N_c-2S)(N_c+2S+2)(5N_c+18)}}$
$(\lambda \mu) S - 1$	(11)1	/	$\frac{S+1}{S} \sqrt{\frac{6(2S-1)(N_c+2S+4)}{(2S+1)(N_c+2S+2)N_c(5N_c+18)}}$
$(\lambda - 2, \mu + 1) S$	(11)1	/	$\frac{1}{S} \sqrt{\frac{6(S+1)(N_c-2S+4)(2S-1)}{(N_c-2S+2)N_c(5N_c+18)}}$
$(\lambda - 2, \mu + 1) S - 1$	(11)1	/	$\frac{1}{S} \sqrt{\frac{6(N_c-2S+4)(S-1)(2S-1)}{(N_c-2S+2)(N_c+2S)(5N_c+18)}}$
$(\lambda - 3, \mu) S - 1$	(11)1	/	0 if $S = 1/2$
$(\lambda - 3, \mu) S - 1$	(11)1	/	$-\sqrt{\frac{3(N_c+2S-2)(N_c+2S+2)(N_c-2S+4)(S-1)}{2S(N_c+2S)N_c(5N_c+18)}}$ if $S \geq 1$
$(\lambda - 1, \mu - 1) S$	(11)0	1	$\sqrt{\frac{N_c^2+12(S^2-1)}{2N_c(5N_c+18)}}$
$(\lambda - 1, \mu - 1) S$	(11)0	2	0
$(\lambda - 1, \mu - 1) S$	(00)1	/	$\sqrt{\frac{4S(S+1)}{N_c(5N_c+18)}}$

which can be easily checked. For completeness also note that the isoscalar factors obey the following symmetry property

$$\begin{aligned} & \left(\begin{array}{cc} (\lambda\mu) & (11) \\ YI & -Y^a I^a \end{array} \left\| \begin{array}{c} (\lambda'\mu') \\ Y'I' \end{array} \right. \right) = \\ & (-1)^{\frac{1}{3}(\mu' - \mu - \lambda' + \lambda + \frac{3}{2}Y^a) + I' - I} \sqrt{\frac{\dim(\lambda'\mu')(2I+1)}{\dim(\lambda\mu)(2I'+1)}} \left(\begin{array}{cc} (\lambda'\mu') & (11) \\ Y'I' & Y^a I^a \end{array} \left\| \begin{array}{c} (\lambda\mu) \\ YI \end{array} \right. \right). \end{aligned} \quad (\text{A2})$$

where $\dim(\lambda\mu) = \frac{1}{2}(\lambda+1)(\mu+1)(\lambda+\mu+2)$ is the dimension of the irrep $(\lambda\mu)$ of $SU(3)$. The $SU(6)$ isoscalar factors satisfy to the following symmetry property:

$$\begin{aligned} & \left(\begin{array}{cc} [f] & [21^4] \\ (\lambda_1\mu_1)S_1 & (\lambda_2\mu_2)S_2 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda\mu)S \end{array} \right. \right) = \\ & (-1)^{1/3(\mu_1 - \mu - \lambda_1 + \lambda)} (-1)^{S_1 - S} \sqrt{\frac{\dim(\lambda_1\mu_1)(2S_1+1)}{\dim(\lambda\mu)(2S+1)}} \left(\begin{array}{cc} [f] & [21^4] \\ (\lambda\mu)S & (\lambda_2\mu_2)S_2 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda_1\mu_1)S_1 \end{array} \right. \right) \end{aligned} \quad (\text{A3})$$

Appendix B

Here we present the analytic form of the matrix elements of operators proportional to O_5 , O_6 . They have been obtained following the approach described in Sec. III. We recall that we have the following notations $C_{(\lambda\mu)}^{SU(3)} = \frac{1}{3}(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu)$ is the Casimir operator of the $SU(3)$ irrep $(\lambda\mu)$ and $C_{[f]}^{SU(6)}$ is the Casimir operator of the $SU(6)$ irrep $[f]$. In the present case we have $[f] = [N_c - 1, 1]$ so that

$$C_{[f]}^{SU(6)} = \frac{N_c(5N_c + 18)}{12}. \quad (\text{B1})$$

Up to the factor $\frac{3}{N_c}$ the matrix elements of the operator O_5 are obtained from the matrix elements of $L \cdot T \cdot G$ for which we have obtained the following form

$$\begin{aligned} & \langle (\lambda'\mu')Y'I'I'_3; \ell'S'JJ_3 | \sum_i (-1)^{i+a} L^i T^a G^{-(ia)} | (\lambda\mu)YII_3; \ell SJJ_3 \rangle = \\ & \delta_{\ell'\ell} \delta_{\lambda\lambda'} \delta_{\mu\mu'} \delta_{Y'Y} \delta_{I'I} \delta_{I'_3 I_3} (-1)^{J+\ell+S'} C_{[f]}^{SU(6)} \sqrt{\ell(\ell+1)(2\ell+1)(2S'+1)} \\ & \times \left\{ \begin{array}{ccc} \ell & \ell & 1 \\ S' & S & J \end{array} \right\} \left(\begin{array}{cc} [f] & [21^4] \\ (\lambda\mu)S & (11)1 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda\mu)S' \end{array} \right. \right)_{\rho=1} \left(\begin{array}{cc} [f] & [21^4] \\ (\lambda\mu)S & (11)0 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda\mu)S \end{array} \right. \right)_{\rho=1}. \end{aligned} \quad (\text{B2})$$

where we have used the short-hand notation

$$(-)^a = (-)^{I_3^a + \frac{Y^a}{2}}. \quad (\text{B3})$$

In this way one can easily define the values of λ and μ to be used in the tables for the octets, the decuplets and the singlets. The reason is that when calculating the isoscalar factors we took $\lambda = 2S$ and $\mu = \frac{N_c - 2S}{2}$. These are definitions consistent with the inner products in the flavor-spin space and provided the advantage of expressing the isoscalar factors in terms of the spin S of a given state and N_c only. Let us take the example of the two octets 28 and 48 . They represent the same flavor state but of different spin. Then in using Table 1 of Ref. [24] one has to take $\lambda = 1$ and $\mu = \frac{N_c - 1}{2}$. On the other hand in Table 2 or its extended form Table VI from the next appendix, the flavor octet 48 will be described by the irrep $(\lambda - 2, \mu + 1)$ with $\lambda = 2S = 3$ and $\mu = \frac{N_c - 3}{2}$ which gives the irrep (11) when $N_c = 3$, as it should be.

For the expectation value of $L^2 \cdot G \cdot G$ we have obtained the following expression

$$\begin{aligned} & \langle (\lambda' \mu') Y' I' I'_3; \ell' S' J J_3 | (-1)^{i+j+a} L^{(2)ij} G^{-ia} G^{-j,-a} | (\lambda \mu) Y I I_3; \ell S J J_3 \rangle = \quad (\text{B4}) \\ & \delta_{\ell' \ell} \delta_{\lambda \lambda'} \delta_{\mu \mu'} \delta_{Y' Y} \delta_{I' I} \delta_{I'_3 I_3} (-1)^{J+\ell-S} \frac{1}{2} C_{[f]}^{SU(6)} \\ & \times \sqrt{\frac{5\ell(\ell+1)(2\ell-1)(2\ell+1)(2\ell+3)}{6}} \begin{Bmatrix} \ell & \ell & 2 \\ S & S' & J \end{Bmatrix} \\ & \times \sqrt{(2S+1)(2S'+1)} \sum_{S''} (-1)^{(S-S'')} \begin{Bmatrix} 1 & 1 & 2 \\ S & S' & S'' \end{Bmatrix} \\ & \times \sum_{\rho, \lambda'', \mu''} \left(\begin{array}{c} [f] \quad [21^4] \\ (\lambda'' \mu'') S'' \quad (11)1 \end{array} \parallel \begin{array}{c} [f] \\ (\lambda \mu) S \end{array} \right)_{\rho} \left(\begin{array}{c} [f] \quad [21^4] \\ (\lambda'' \mu'') S'' \quad (11)1 \end{array} \parallel \begin{array}{c} [f] \\ (\lambda \mu) S' \end{array} \right)_{\rho}. \quad (\text{B5}) \end{aligned}$$

which multiplied by the factor $\frac{15}{N_c}$ gives the expectation value of O_6 .

Concerning O_7 , one has

$$\begin{aligned} & \langle (\lambda' \mu') Y' I' I'_3; \ell' S' J J_3 | (-1)^{i+j+a} L^i G^{ja} G^{i,-a} S^{-j} | (\lambda \mu) Y I I_3; \ell S J J_3 \rangle = \\ & (-1)^{J+\ell-S'} \frac{C_{[f]}^{SU(6)}}{2} (2S+1) \sqrt{\ell(\ell+1)(2\ell+1)} \sqrt{S(S+1)(2S'+1)} \\ & \times \begin{Bmatrix} 1 & \ell & \ell \\ J & S & S' \end{Bmatrix} \sum_{S''} \begin{Bmatrix} S' & 1 & S \\ S & 1 & S'' \end{Bmatrix} \end{aligned}$$

TABLE IX: Matrix elements of $S^i T^a G^{ia}$ for all states belonging to the $[70, 1^-]$ multiplet. the vanishing off-diagonal matrix elements are not indicated

$S^i T^a G^{ia}$	
${}^2 8_J$	$\frac{3}{4}$
${}^4 8_J$	$\frac{5(N_c + 3)}{8}$
${}^2 10_J$	$\frac{3(N_c + 1)}{8}$
${}^2 1_J$	0

$$\times \sum_{\lambda'', \mu'', \rho} \left(\begin{array}{c} [f] \quad [21^4] \\ (\lambda'' \mu'') S'' \quad (11)1 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda \mu) S \end{array} \right\rangle_{\rho} \right) \left(\begin{array}{c} [f] \quad [21^4] \\ (\lambda'' \mu'') S'' \quad (11)1 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda \mu) S' \end{array} \right\rangle_{\rho} \right), \quad (\text{B6})$$

and

$$\begin{aligned} & \langle (\lambda' \mu') Y' I' I'_3; \ell' S' J J_3 | (-1)^{i+j+a} L^i G^{ja} S^{-j} G^{-(ia)} | (\lambda \mu) Y I I_3; \ell S J J_3 \rangle = \\ & (-1)^{\ell+S+J} \frac{C_{[f]}^{SU(6)}}{2} \sqrt{\ell(\ell+1)(2\ell+1)} \sqrt{S'(S'+1)(2S+1)} \\ & \times \left\{ \begin{array}{ccc} 1 & \ell & \ell \\ J & S & S' \end{array} \right\} \sum_{\lambda'', \mu'', \rho} \left(\begin{array}{c} [f] \quad [21^4] \\ (\lambda'' \mu'') S' \quad (11)1 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda \mu) S \end{array} \right\rangle_{\rho} \right) \left(\begin{array}{c} [f] \quad [21^4] \\ (\lambda'' \mu'') S' \quad (11)1 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda \mu) S' \end{array} \right\rangle_{\rho} \right). \quad (\text{B7}) \end{aligned}$$

For completeness, we also give the general analytic form of the matrix elements of $S \cdot T \cdot G$. This is given by

$$\begin{aligned} & \langle (\lambda' \mu') Y' I' I'_3; \ell' S' J J_3 | (-1)^{i+a} S^i T^a G^{-(ia)} | (\lambda \mu) Y I I_3; \ell S J J_3 \rangle = \\ & \delta_{\ell' \ell} \delta_{S' S} \delta_{\lambda' \lambda} \delta_{\mu' \mu} \delta_{Y' Y} \delta_{I' I} \delta_{I'_3 I_3} \sqrt{S(S+1)} C_{[f]}^{SU(6)} \\ & \times \sum_{\rho} \left(\begin{array}{c} [f] \quad [21^4] \\ (\lambda \mu) S \quad (11)0 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda \mu) S \end{array} \right\rangle_{\rho} \right) \left(\begin{array}{c} [f] \quad [21^4] \\ (\lambda \mu) S \quad (11)1 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda' \mu') S' \end{array} \right\rangle_{\rho} \right). \quad (\text{B8}) \end{aligned}$$

The matrix elements of this operator are presented in Table IX. This operator is not considered in the analysis because it does not improve the fit.

Appendix C

Here we have completed the calculation of SU(6) isoscalar factors corresponding to the performed in Ref. [24]. Table V contains eight more cases of $(\lambda_1, \mu_1)S_1$ corresponding to $(\lambda, \mu)S - 1$ with $\rho = 1, 2, (\lambda + 2, \mu - 1)S + 1, (\lambda + 1, \mu - 2)S + 1, (\lambda + 1, \mu - 2)S, (\lambda - 1, \mu - 1)S - 1, (\lambda - 2, \mu + 1)S$ and $(\lambda - 2, \mu + 1)S - 1$ respectively. Table VI contains five more cases corresponding to $(\lambda_1, \mu_1)S_1 = (\lambda, \mu)S + 1, (\lambda, \mu)S, (\lambda - 1, \mu - 1)S, (\lambda - 1, \mu - 1)S - 1$ and $(\lambda - 4, \mu + 2)S - 1$. The latter case is not applicable to our physical problem. It has been derived for testing the orthonormalization properties. In Table VII the rows 1-5, 8 and 11 are new. In Table VIII the rows 1, 2, 5 - 7 and 10 - 14 are new.

Appendix D

There are two SU(3) breaking operators which are neglected in the fit because their contributions are negligible. They are defined to have zero expectation values for non strange baryons, as follows

$$B_3 = \frac{1}{N_c} S^i G^{i8} - \frac{1}{2\sqrt{3}} O_3 \quad (D1)$$

and

$$B_4 = \frac{1}{N_c} L^i G^{i8} - \sqrt{\frac{3}{8}} O_2 \quad (D2)$$

The general form of their matrix elements is obtained from the matrix elements of the following operators

$$\begin{aligned} & \langle (\lambda' \mu') Y' I' I'_3; \ell' S' J J_3 | (-1)^i S^i G^{-i8} | (\lambda \mu) Y I I_3; \ell S J J_3 \rangle = \delta_{\ell' \ell} \delta_{S' S} \\ & \times \sqrt{\frac{C_{[f]}^{SU(6)}}{2}} \sqrt{S(S+1)} \sum_{\rho} \left(\begin{array}{cc} (\lambda \mu) & (11) \\ Y I & 00 \end{array} \left\| \begin{array}{c} (\lambda' \mu') \\ Y I \end{array} \right\rangle_{\rho} \right) \left(\begin{array}{cc} [f] & [21^4] \\ (\lambda \mu) S & (11) 1 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda' \mu') S \end{array} \right\rangle_{\rho} \right). \end{aligned} \quad (D3)$$

and

$$\begin{aligned} & \langle (\lambda' \mu') Y' I' I'_3; \ell' S' J J_3 | (-1)^i L^i G^{-i8} | (\lambda \mu) Y I I_3; \ell S J J_3 \rangle = \\ & \delta_{\ell' \ell} (-1)^{\ell + S' + J} \sqrt{\frac{C_{[f]}^{SU(6)}}{2}} \sqrt{(2L+1)(2S'+1)} \left\{ \begin{array}{ccc} S' & \ell & J \\ \ell & S & 1 \end{array} \right\} \\ & \times \sum_{\rho} \left(\begin{array}{cc} (\lambda \mu) & (11) \\ Y I & 00 \end{array} \left\| \begin{array}{c} (\lambda' \mu') \\ Y I \end{array} \right\rangle_{\rho} \right) \left(\begin{array}{cc} [f] & [21^4] \\ (\lambda \mu) S & (11) 1 \end{array} \left\| \begin{array}{c} [f] \\ (\lambda' \mu') S' \end{array} \right\rangle_{\rho} \right). \end{aligned} \quad (D4)$$

TABLE X: Matrix elements of $S^i G^{i8}$ for all states belonging to the $[\mathbf{70}, 1^-]$ multiplet.

$S^i G^{i8}$		
${}^2_8 J$	$\frac{9 - 36I(I+1) + \mathcal{S}^2(N_c - 3)^2 - 6N_c + N_c[-4I(I+1)(N_c - 6) + 9N_c] - 2\mathcal{S}[9 + N_c(N_c - 18)]}{16\sqrt{3}(N_c - 1)(N_c + 3)}$	
${}^4_8 J$	$\frac{5[-\mathcal{S}^2(N_c - 3) + 4I(I+1)(N_c - 3) + 3(N_c + 1) + 2\mathcal{S}(N_c + 3)]}{16\sqrt{3}(N_c - 1)}$	
${}^2_{10} J$	$\frac{\sqrt{3}(1 + \mathcal{S})}{8}$	$N_c = 3$
${}^2_{10} J$	$\frac{2\mathcal{S}(N_c - 3) - \mathcal{S}^2(N_c + 3) + 4I(I+1)(N_c + 3) - 3(3N_c + 5)}{16\sqrt{3}(N_c + 5)}$	$N_c > 3$
${}^2_{1J}$	$-\frac{\sqrt{3}(N_c - 3)}{4(N_c + 3)}$	
${}^2_8 J - {}^4_8 J$	0	
${}^2_8 J - {}^2_{10} J$	$-\frac{1}{16\sqrt{3}} \sqrt{\frac{(5 - \mathcal{S} + 2I)(1 + \mathcal{S} - 2I)(-3 + \mathcal{S} + 2I)(3 + \mathcal{S} + 2I)N_c(N_c + 3)}{(N_c - 1)(N_c + 5)}}$	
${}^4_8 J - {}^2_{10} J$	0	
${}^2_8 J - {}^2_{1J}$	$\frac{3\sqrt{N_c}}{2(N_c + 3)}$	
${}^4_8 J - {}^2_{1J}$	0	
${}^2_{10} J - {}^2_{1J}$	0	

For general interest the explicit form of these matrix elements as a function of N_c are given in Tables X and XI respectively. Here and elsewhere the notations ${}^{2S+1}8_J$, etc. represents the multiplicity $2J + 1$ of the spin, the SU(3) multiplet in dimensional notation and J is the total spin. Note that the matrix elements depend on N_c , the strangeness \mathcal{S} and the isospin I .

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TABLE XI: Matrix elements of $L^i G^{i8}$ for all states belonging to the $[70, 1^-]$ multiplet.

	$L^i G^{i8}$
${}^2_8_{1/2}$	$-\frac{72(N_c - 1)(N_c + 3S) + (N_c - 3)^2[3 - 2S(N_c - 9) + 9N_c + S^2(N_c + 3) - 4I(1 + I)(N_c + 3)]}{12\sqrt{6}(N_c - 1)(N_c + 3)^2}$
${}^4_8_{1/2}$	$-\frac{5(-S^2(N_c - 3) + 4I(1 + I)(N_c - 3) + 3(N_c + 1) + 2S(N_c + 3))}{24\sqrt{6}(N_c - 1)}$
${}^2_8_{3/2}$	$\frac{72(N_c - 1)(N_c + 3S) + (N_c - 3)^2[3 - 2S(N_c - 9) + 9N_c + S^2(N_c + 3) - 4I(1 + I)(N_c + 3)]}{24\sqrt{6}(N_c - 1)(N_c + 3)^2}$
${}^4_8_{3/2}$	$-\frac{-S^2(N_c - 3) + 4I(1 + I)(N_c - 3) + 3(N_c + 1) + 2S(N_c + 3)}{12\sqrt{6}(N_c - 1)}$
${}^4_8_{5/2}$	$-\frac{-S^2(N_c - 3) + 4I(1 + I)(N_c - 3) + 3(N_c + 1) + 2S(N_c + 3)}{8\sqrt{6}(N_c - 1)}$
${}^2_{10}_{1/2}$	$\frac{15 - 2S(N_c - 3) + 9N_c + S^2(N_c + 3) - 4I(1 + I)(N_c + 3)}{12\sqrt{6}(N_c + 5)}$
${}^2_{10}_{3/2}$	$\frac{2S(N_c - 3) - S^2(N_c + 3) + 4I(1 + I)(N_c + 3) - 3(3N_c + 5)}{24\sqrt{6}(N_c + 5)}$
${}^2_{11/2}$	$\frac{N_c - 3}{\sqrt{6}(N_c + 3)}$
${}^2_{13/2}$	$-\frac{N_c - 3}{2\sqrt{6}(N_c + 3)}$
${}^2_8_{1/2} - {}^4_8_{1/2}$	$\frac{6S + [3 + (S - 2)S - 4I(1 + I)]N_c}{6(N_c - 1)} \sqrt{\frac{N_c}{6(N_c + 3)}}$
${}^2_8_{3/2} - {}^4_8_{3/2}$	$\frac{6S + [3 + (S - 2)S - 4I(1 + I)]N_c}{12(N_c - 1)} \sqrt{\frac{5N_c}{3(N_c + 3)}}$
${}^2_8_{1/2} - {}^2_{10}_{1/2}$	$\frac{1}{12} \sqrt{\frac{((2I + 5 - S)(1 + S - 2I)(-3 + S + 2I)(3 + S + 2I)N_c(N_c + 3))}{6(N_c - 1)(N_c + 5)}}$
${}^2_8_{3/2} - {}^2_{10}_{3/2}$	$-\frac{1}{24} \sqrt{\frac{((2I + 5 - S)(1 + S - 2I)(-3 + S + 2I)(3 + S + 2I)N_c(N_c + 3))}{6(N_c - 1)(N_c + 5)}}$
${}^4_8_{1/2} - {}^2_{10}_{1/2}$	$-\frac{N_c + 9}{96} \sqrt{\frac{(2I + 5 - S)(1 + S - 2I)(-3 + S + 2I)(3 + S + 2I)}{3(N_c - 1)(N_c + 5)}}$
${}^4_8_{3/2} - {}^2_{10}_{3/2}$	$-\frac{N_c + 9}{96} \sqrt{\frac{5(2I + 5 - S)(1 + S - 2I)(-3 + S + 2I)(3 + S + 2I)}{6(N_c - 1)(N_c + 5)}}$
${}^2_8_{1/2} - {}^2_{11/2}$	$-\frac{\sqrt{2N_c}}{N_c + 3}$
${}^2_8_{3/2} - {}^2_{13/2}$	$\frac{\sqrt{N_c}}{\sqrt{2}(N_c + 3)}$
${}^4_8_{1/2} - {}^2_{11/2}$	$\frac{1}{2\sqrt{N_c + 3}}$
${}^4_8_{3/2} - {}^2_{13/2}$	$\frac{1}{2} \sqrt{\frac{5}{2(N_c + 3)}}$
${}^2_{10}_{1/2} - {}^2_{11/2}$	0
${}^2_{10}_{3/2} - {}^2_{13/2}$	0

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