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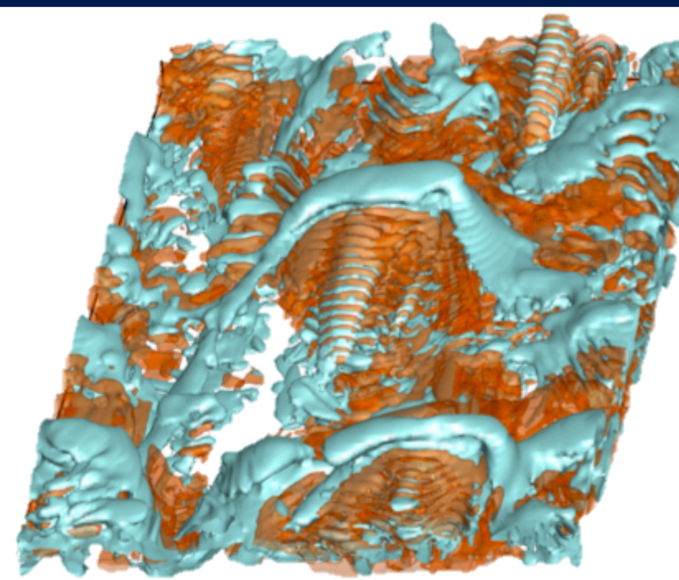
Dynamics of Elasto-Inertial Turbulence in Flows with Polymer Additives

Vincent E. Terrapon
Yves Dubief
Julio Soria



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Acknowledgements

Collaborators

- **Yves Dubief** — University of Vermont, USA
- **Julio Soria** — Monash University, Australia; King Abdulaziz University, Kingdom of Saudi Arabia

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- Australian Research Council
- Center for Turbulence Research Summer Program

Outline

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- **Context**
- Models and numerical implementation
- Polymer drag reduction
- Elasto-inertial turbulence
- Conclusions and future work

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Polymers and turbulence

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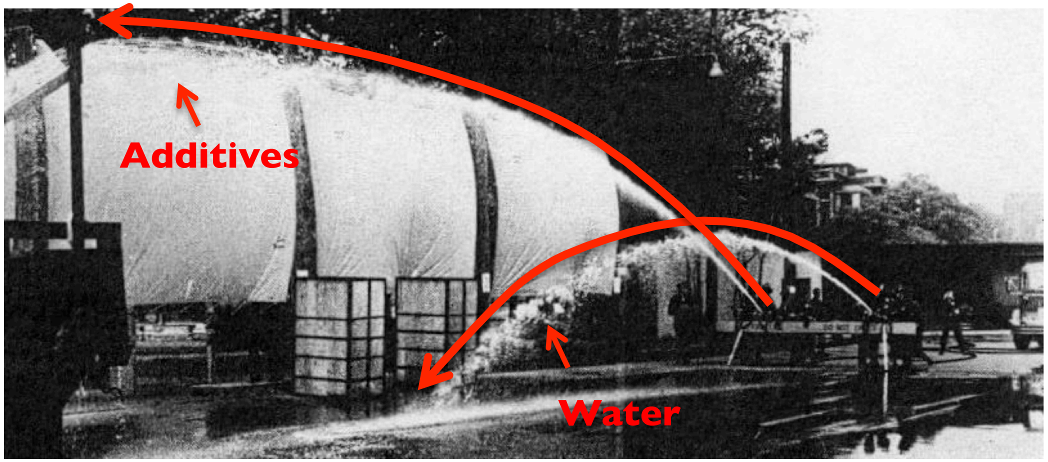
Turbulent drag reduction

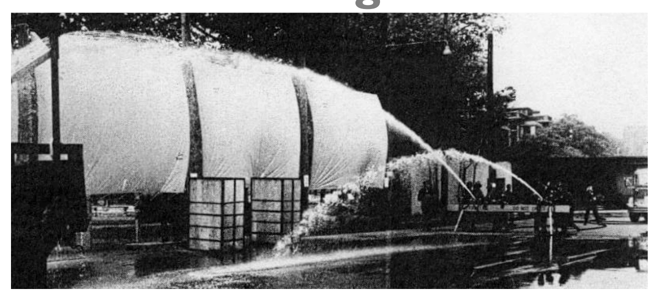


Fire hoses with and without polymer additives

- Up to 80% friction drag reduction, even at low concentration
- No significant effect on drag in laminar flows
- Bounded by Maximum Drag Reduction (MDR) asymptote

- Pipeline

Turbulent drag reduction



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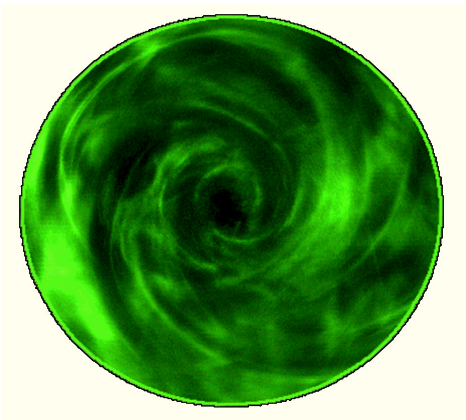
Elastic turbulence

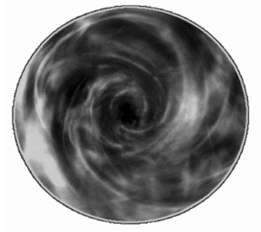


Chaotic motion of a polymer solution in micro-channel (Groisman & Steinberg, 2000)

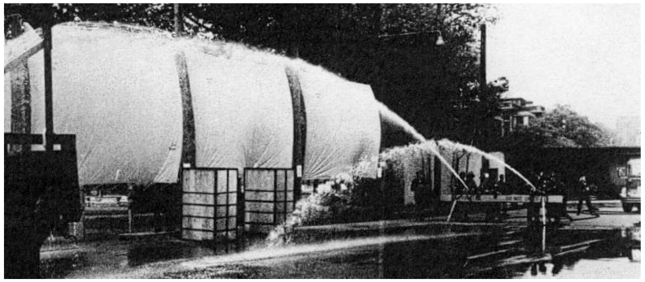
- Existence of elastic turbulence in flows with curved streamlines
- Observed at low Reynolds number
- Strong increase in mixing properties

- Blood flow
- Micro-channel flow

Elastic turbulence



Turbulent drag reduction



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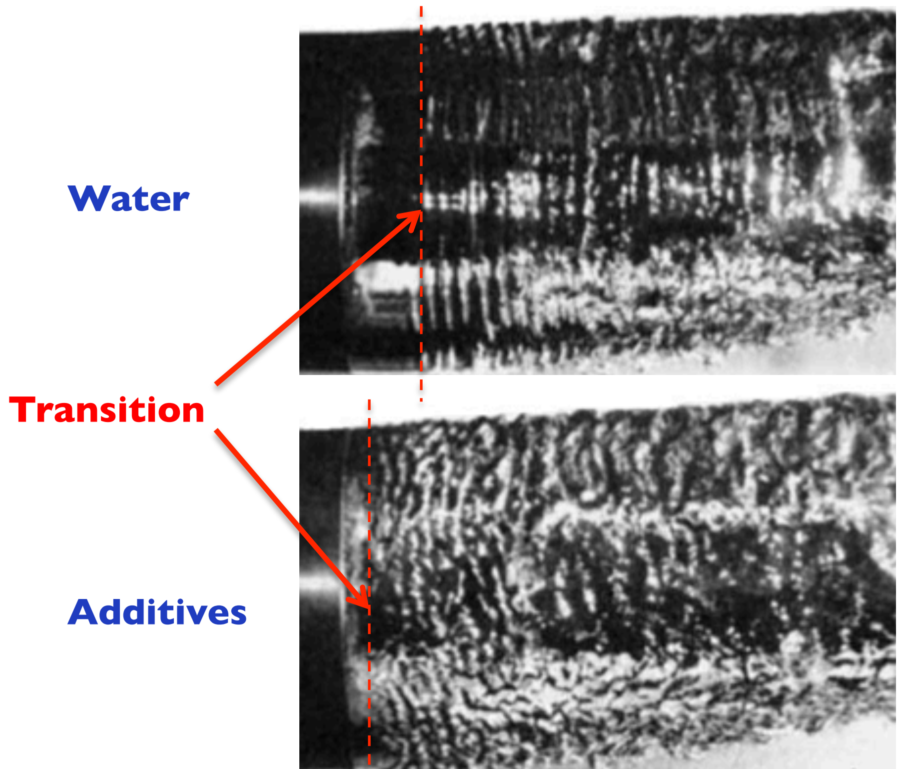
Early turbulence

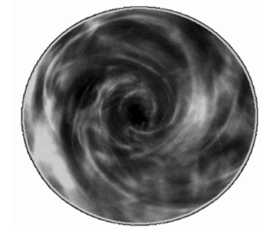


Transition to turbulence around an ogival head with ventilated cavity (Hoyt, 1977)

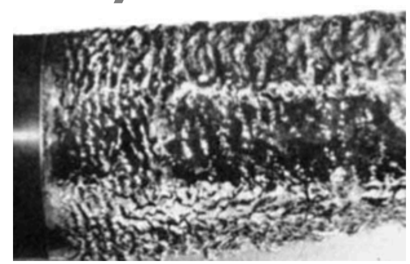
- Transition to turbulence promoted by polymers

- Biofluids

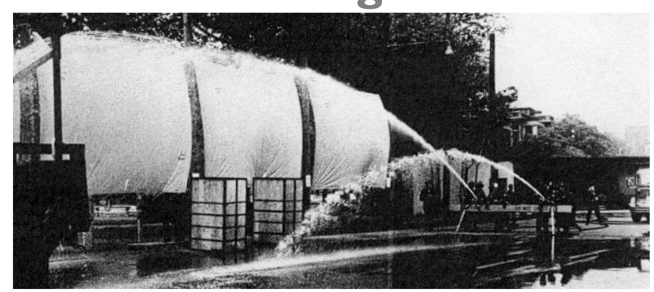
Elastic turbulence



Early turbulence



Turbulent drag reduction



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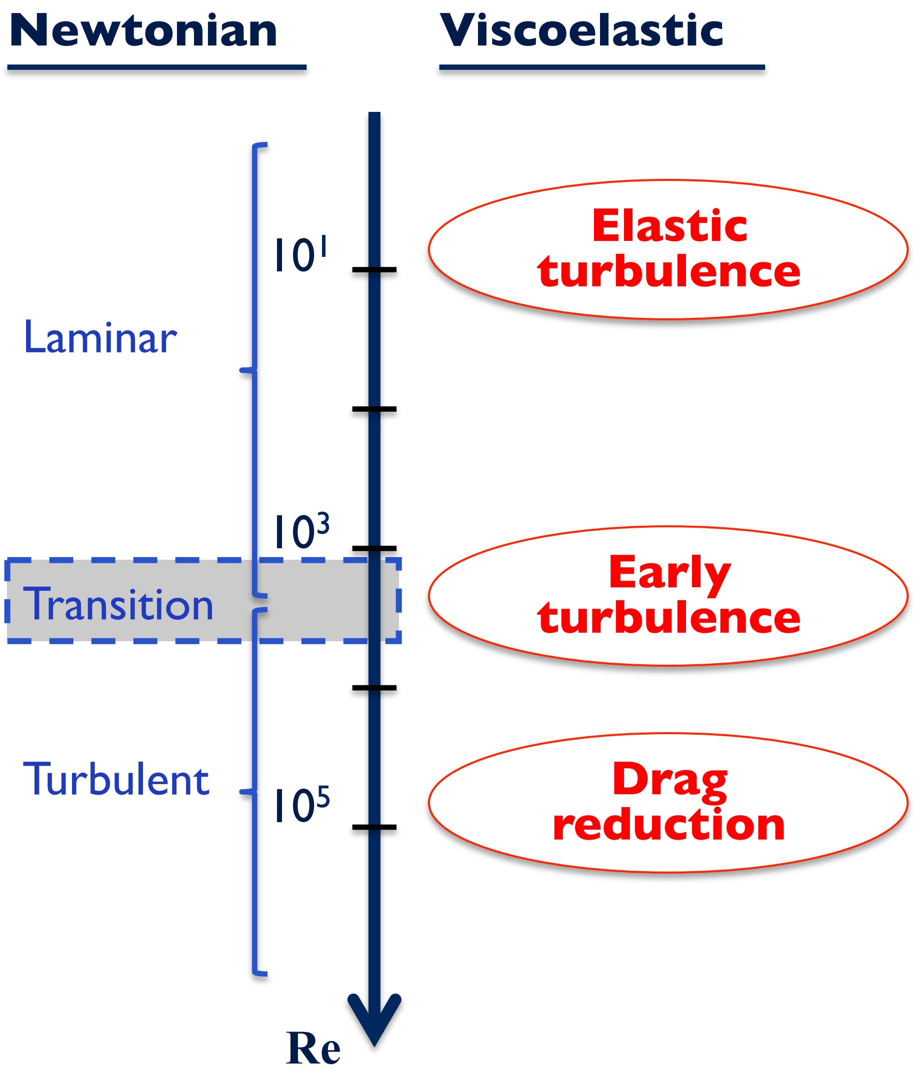
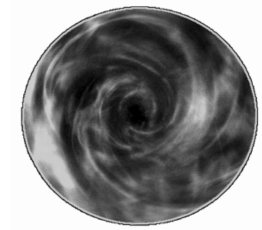
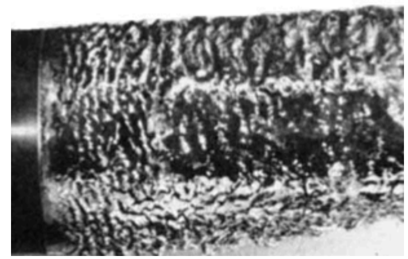


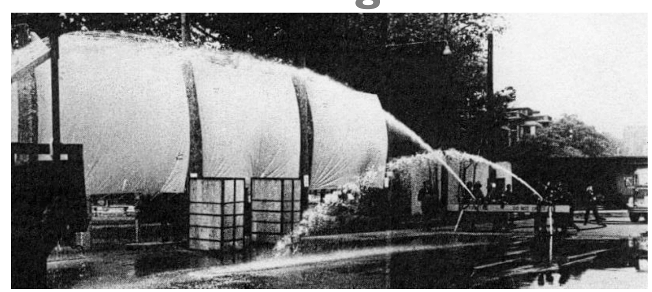
Elastic turbulence



Early turbulence



Turbulent drag reduction



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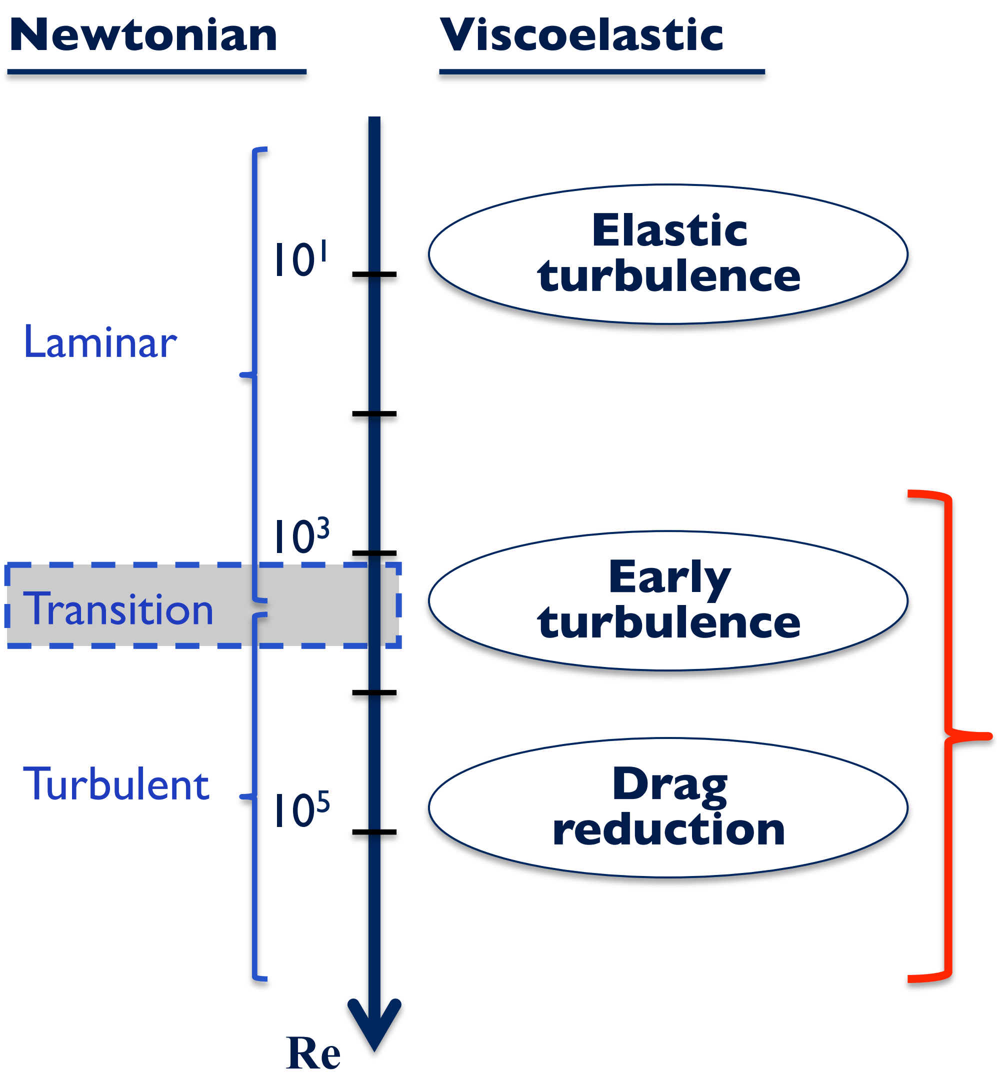

Elasto-Inertial Turbulence (EIT)

- State of small-scale turbulence
- Contributions from both **elastic** and **inertial** instabilities
- Observed over a wide range of Reynolds numbers
- Link between different phenomena

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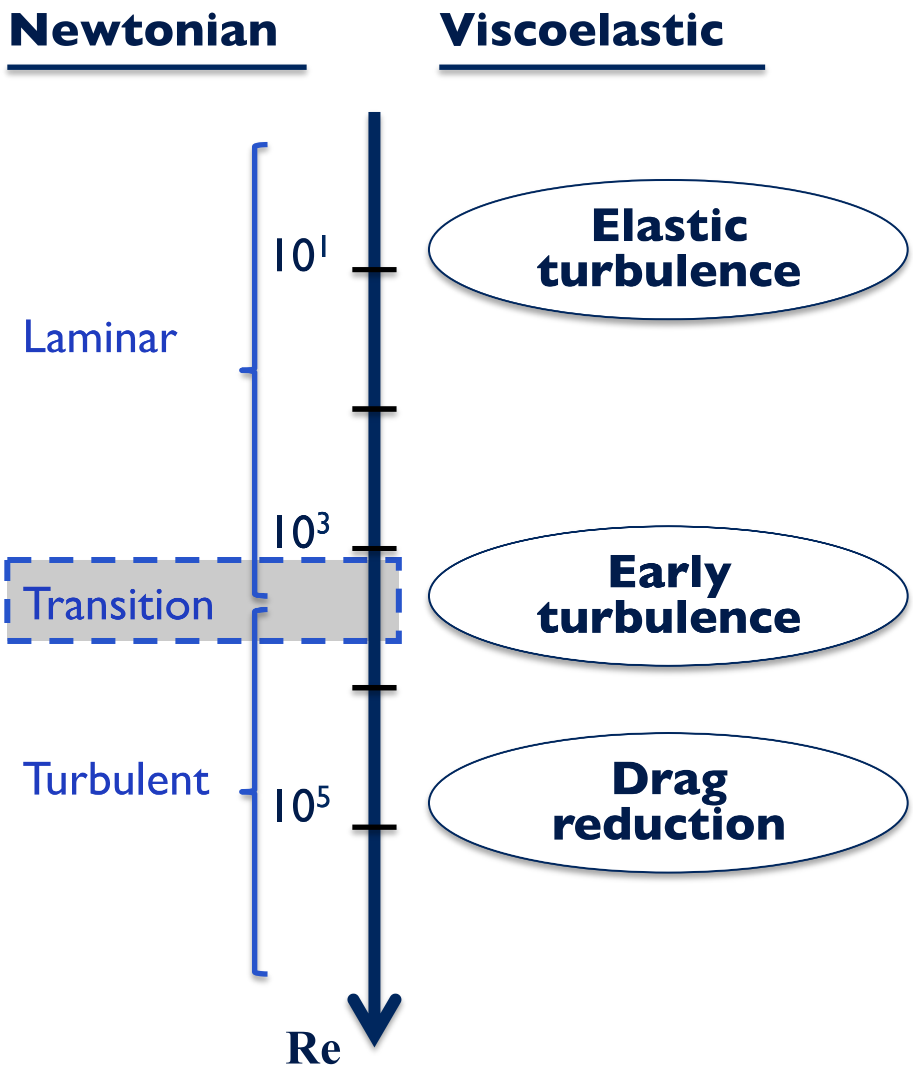


Key questions

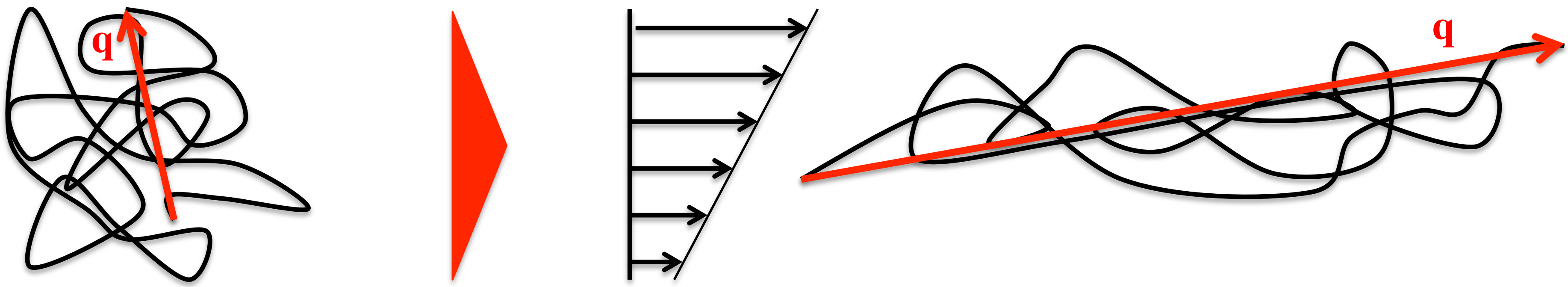
- Is drag reduction
  - a viscous and large-scale effect (Lumley)
  - an elastic and small-scale effect (de Gennes)
- What is the nature of EIT?
  - Relative contributions of elastic and inertial instabilities?
  - Characteristics of MDR?
  - Dynamical interactions between flow and polymers?

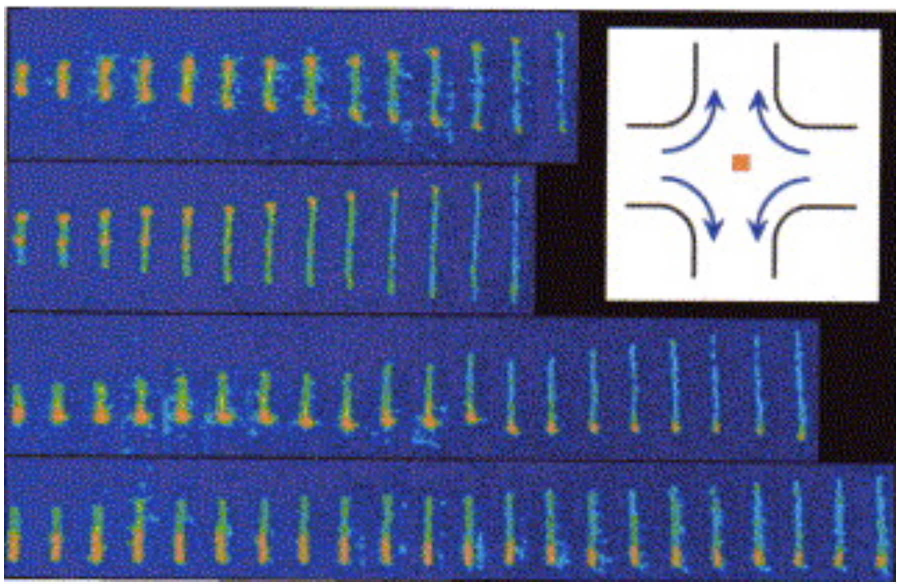
Outline

- Context
- **Models and numerical implementation**
- Polymer drag reduction
- Elasto-inertial turbulence
- Conclusions and future work

Polymer dynamics in flows

Coil-stretch dynamics





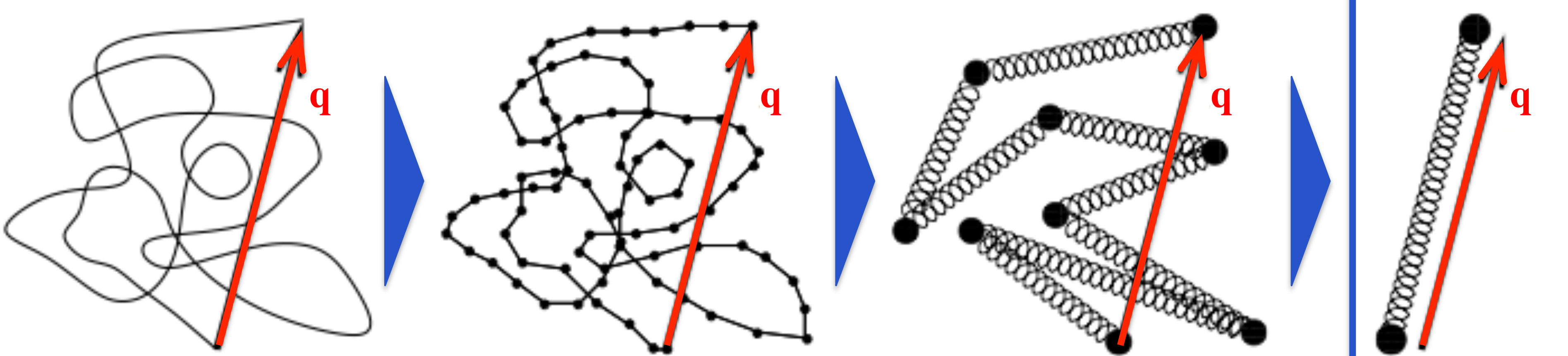
λ-DNA unraveling in extensional flow (Perkins *et al.*, 1997)

Non-Newtonian rheology

- Long and flexible macromolecules (PAM, PEO)
- Flow deforms molecules
- Polymers exert stress onto flow
- Viscoelastic behavior (memory effect)
- Extended polymers relax over a time λ
- Typically large increase in extensional viscosity

Modeling polymer dynamics

Coarse graining



Real molecule | Bead-rod chain | Bead-spring chain | Bead-spring dumbbell

- Only dynamics of end-to-end vector **q** modeled
- Two beads connected by (nonlinear) spring
- Hydrodynamic and Brownian forces concentrated on beads
- Entropic restoring force modeled by spring

Modeling polymer dynamics

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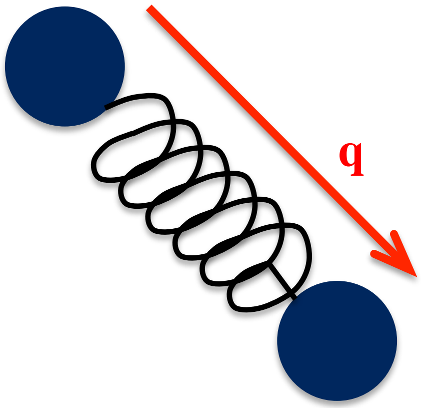
FENE model



$$\mathbf{F}^{\text{Drag}} + \mathbf{F}^{\text{Spring}} + \mathbf{F}^{\text{Brownian}} = \mathbf{0}$$

Evolution of end-to-end vector

$$\underbrace{}_{}\frac{d\mathbf{q}}{dt} = \underbrace{\nabla\mathbf{u}\cdot\mathbf{q}}_{\text{Drag}} - \underbrace{\frac{1}{\text{Wi}}\frac{\mathbf{q}}{2(1-q^2/L^2)}}_{\text{Spring}} - \underbrace{\frac{1}{\sqrt{\text{Wi}}}\frac{dW}{dt}}_{\text{Brownian}}$$

Polymer stress

$$\mathbf{T} = \frac{1}{\text{Wi}}\left\langle \frac{\mathbf{q}\mathbf{q}}{1-q^2/L^2} - \mathbf{I} \right\rangle$$

Parameters

- Wi Weissenberg number (polymer relaxation time / flow time)
- L maximum extensibility of polymer

- Stochastic
- Lagrangian
- No closure model

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Modeling polymer dynamics

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FENE-P model

Conformation tensor

$$\mathbf{C} = \langle \mathbf{qq} \rangle$$

Evolution of conformation tensor

$$\frac{D\mathbf{C}}{Dt} = \underbrace{\nabla\mathbf{u}\cdot\mathbf{C} + \mathbf{C}\cdot\nabla\mathbf{u}^{T}}_{\text{Stretching}} - \underbrace{\mathbf{T}}_{\text{Restoring}}$$

Polymer stress

$$\mathbf{T} = \frac{1}{\mathrm{Wi}}\left(\frac{\mathbf{C}}{1 - \mathrm{tr}C/L^{2}} - \mathbf{I}\right)$$

Parameters

- Wi Weissenberg number (polymer relaxation time / flow time)
- L maximum extensibility of polymer

- Deterministic (continuum)
- Eulerian
- Closure approximation

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Viscoelastic NSE

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Continuity	$\nabla \cdot \mathbf{u} = 0$
Momentum	$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \frac{\beta}{\mathrm{Re}} \nabla^2 \mathbf{u} + \frac{1-\beta}{\mathrm{Re}} \nabla \cdot \mathbf{T} + \frac{\mathrm{d}P}{\mathrm{d}x} \mathbf{e}_x$
Polymer stress	$\mathbf{T} = \frac{1}{\mathrm{Wi}} \left(\frac{\mathbf{C}}{1 - \mathrm{tr}C / L^2} - \mathbf{I} \right)$
Conformation tensor	$\frac{\partial \mathbf{C}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{C} = \nabla \mathbf{u} \cdot \mathbf{C} + \mathbf{C} \cdot \nabla \mathbf{u}^{\mathrm{T}} - \mathbf{T}$

β	Ratio of solvent viscosity to zero-shear viscosity of solution
$\mathrm{Re} = \frac{U_b H}{\nu}$	Reynolds number
$\mathrm{Wi} = \frac{\lambda U_b}{h}$	Weissenberg number

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Hyperbolic C equation

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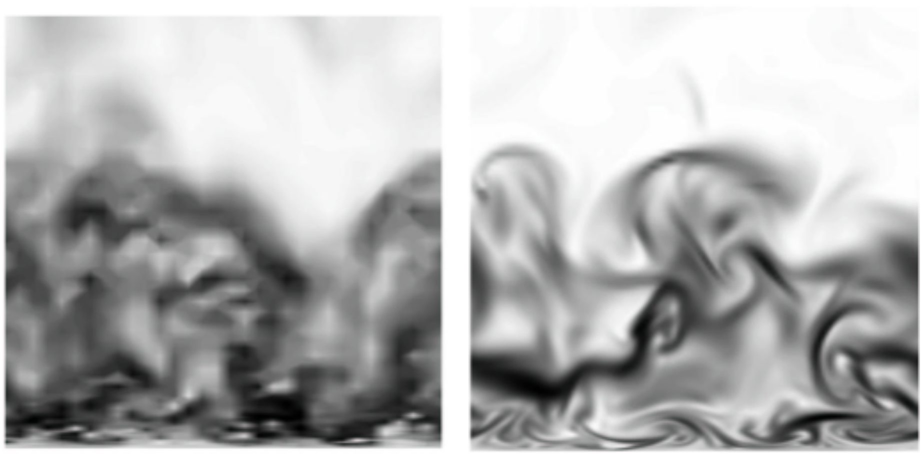
Equation for C is hyperbolic

$$\frac{\partial \mathbf{C}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{C} = \nabla\mathbf{u}\cdot\mathbf{C} + \mathbf{C}\cdot\nabla\mathbf{u}^{T} - \mathbf{T}$$

- No spatial diffusion ($Sc \sim 10^5 - 10^6$)
- Sub-Kolmogorov scales of polymer stress
- Slow decay of energy spectrum
- Numerical instabilities



Trace of C for different grid resolutions (Dubief, 2002)

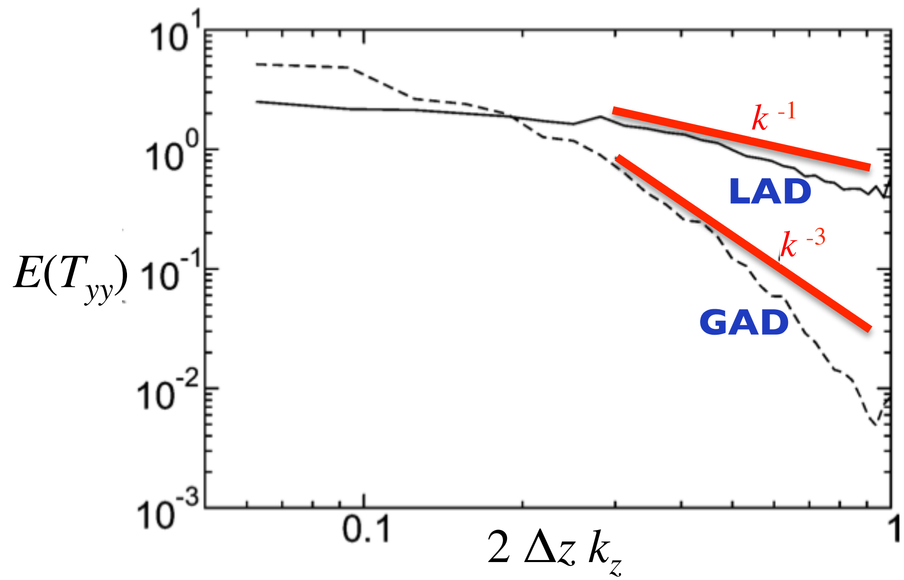
Analogy with passive scalar (Batchelor, 1959)

At $Sc > 1$, scalar energy spectrum decreases as

$$E \sim k^{-1}$$

from Kolmogorov scale η_K to Batchelor scale

$$\eta_B \sim \eta_K / Sc^{1/2}$$



Spectrum of polymer stress T_{yy} at $y^{+}=15$ (Dubief *et al.*, 2005)

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Numerical approach

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Space

- Structured grid
- 2^{nd} order FD for velocity
- Non-dissipative 4^{th} order compact scheme for polymer stress
- Compact upwind scheme for advection terms of conformation tensor

Time

- Semi-implicit fractional step
- 2^{nd} order Crank-Nicolson/3^{rd} order Runge-Kutta
- Implicit scheme for trace of $\mathbf{C}$ to ensure bounded trace

Artificial dissipation

- Local artificial dissipation (LAD)
- Only used when determinant of tensor $\mathbf{C}$ becomes negative

- Important to rely on accurate numerical method
- Global dissipation ($Sc_{eff} \sim 1$) damps all small scales
- Capturing small polymer scales is critical to represent the correct physics

Min *et al.* (2001), Vaithianathan & Collins (2003), Dubief *et al.* (2005), Dallas *et al.* (2010)

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Configuration

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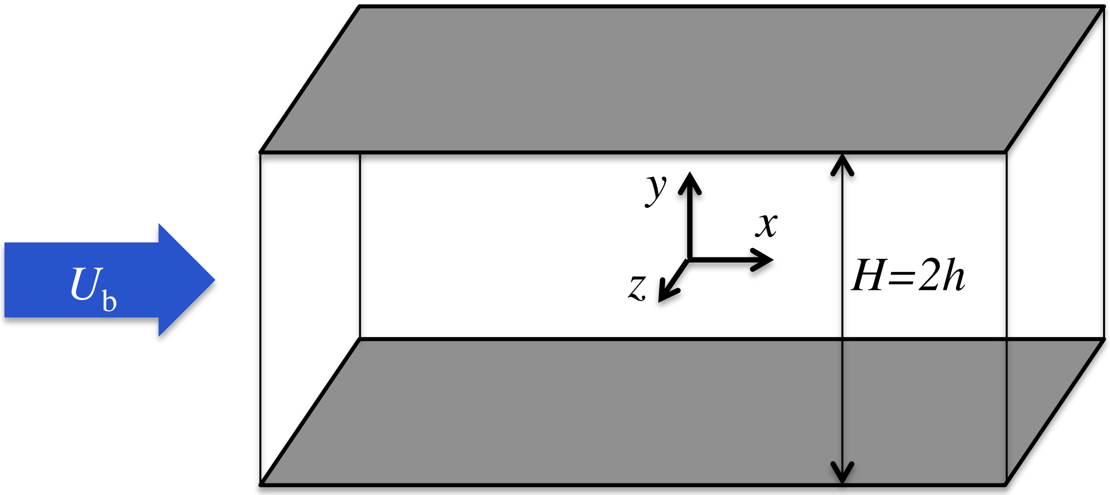
Periodic channel flow



- Mean pressure gradient in x
- Periodic in x and z
- Wall (no-slip) at $y=\pm h$

Typical parameters

• Reynolds number	$Re_b = U_b H/\nu$ = 1'000 – 10'000
• Size:	$10h \times 2h \times 5h$
• Grid:	$256 \times 151 \times 256$
• Polymers:	L = 50 – 200
	Wi = 3 – 40
	$\beta = 0.9$

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Transition to turbulence

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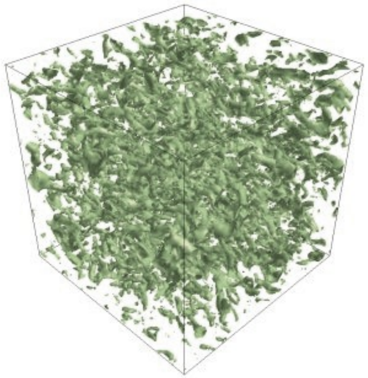
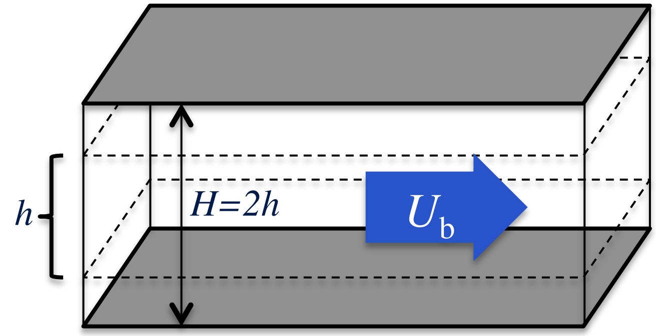
Simulated by-pass transition

Forced homogeneous isotropic turbulence





- $t < 0$:
  - Free-slip
- $t = 0$:
  - No-slip

Simulated roughness-induced transition

Weak initial wall perturbation (blowing)

$$v(x, y=\pm h, z, t) = H(t)\left[A\sin\left(\frac{8\pi}{L_x}x\right)\sin\left(\frac{8\pi}{L_z}z\right)+\varepsilon(t)\right]$$

Random noise

Designed to trigger transition in Newtonian flow at Re=6000

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Realism of the DNS

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- Hyperbolicity of $\mathbf{C}$ transport equation respected as best as numerically possible (Dubief *et al.*, 2005)

- Polymer parameters (for low polymer concentrations)
  - shear thinning effect small
  - extensional viscosity large (increasing with Wi)

- Reproduce evolution of friction factor as function of Reynolds numbers observed in pipe flow experiments (Samanta, Dubief, Holzner, Shäfer, Morozov, Wagner & Hof, submitted)

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Changes in mean velocity

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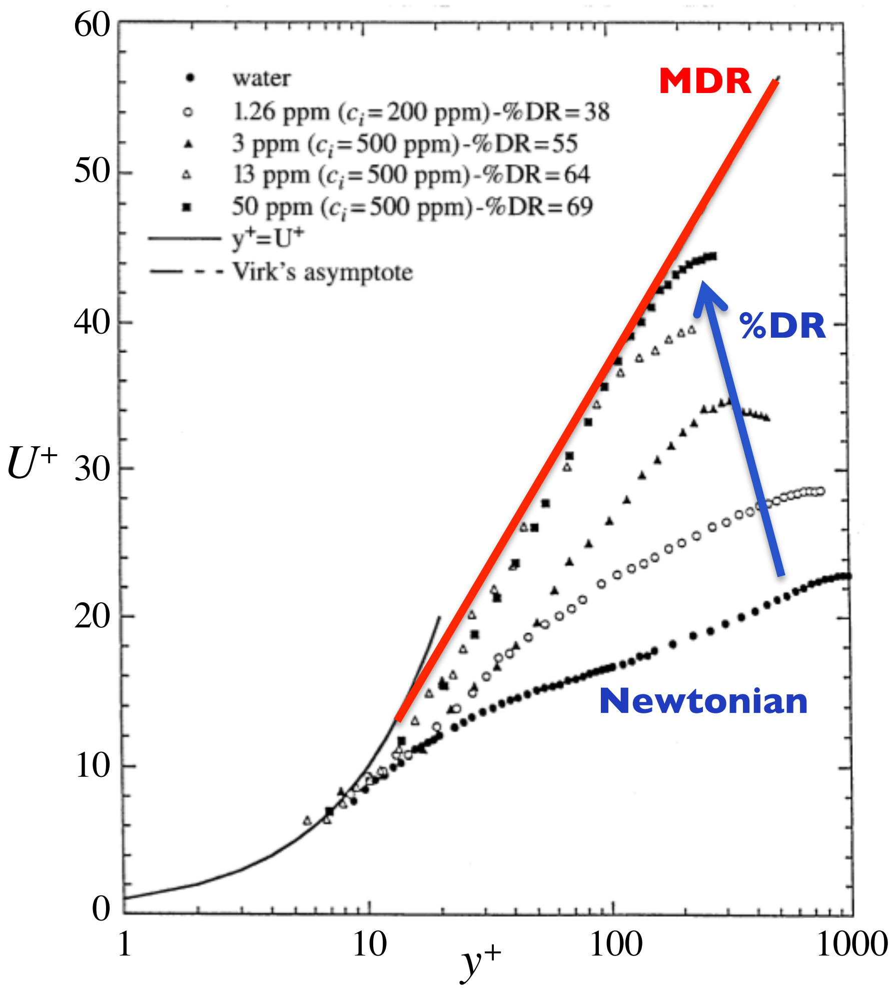
Mean velocity profile



Modified log law

- Newtonian $U^+ = 0.4 \ln(y^+) + 5.5$
- Parallel shift at low drag reduction (LDR)
- Change of slope at high drag reduction (HDR)
- From buffer layer to edge of boundary layer

MDR asymptote

- Universal
- Not laminar
- Virk's log-law $U^+ = 11.7 \ln(y^+) - 17$

Experimental measurements (Warholic *et al.*, 1999)

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Changes in velocity fluctuations

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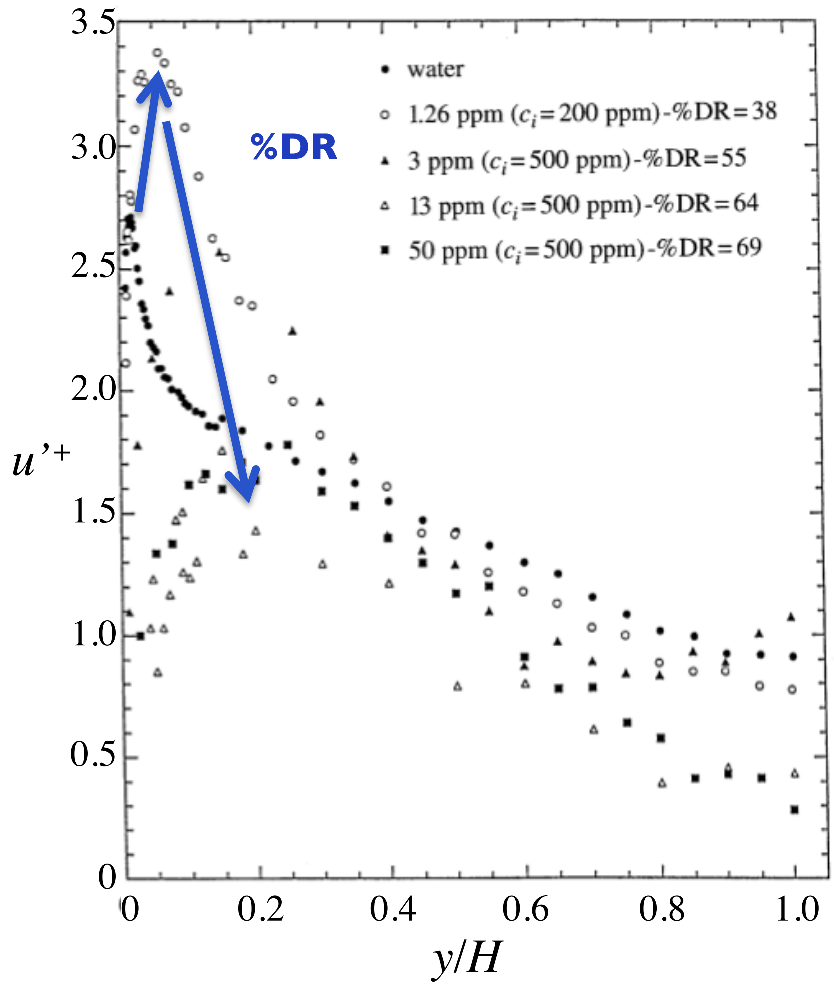
Streamwise velocity fluctuations



Experimental measurements (Warholic *et al.*, 1999)

- First, increase in peak value
- Damping of peak value at higher DR
- Maximum value further away from the wall
- Thicker buffer layer

- Decrease of maximum streamwise velocity fluctuations not always observed

Changes in velocity fluctuations

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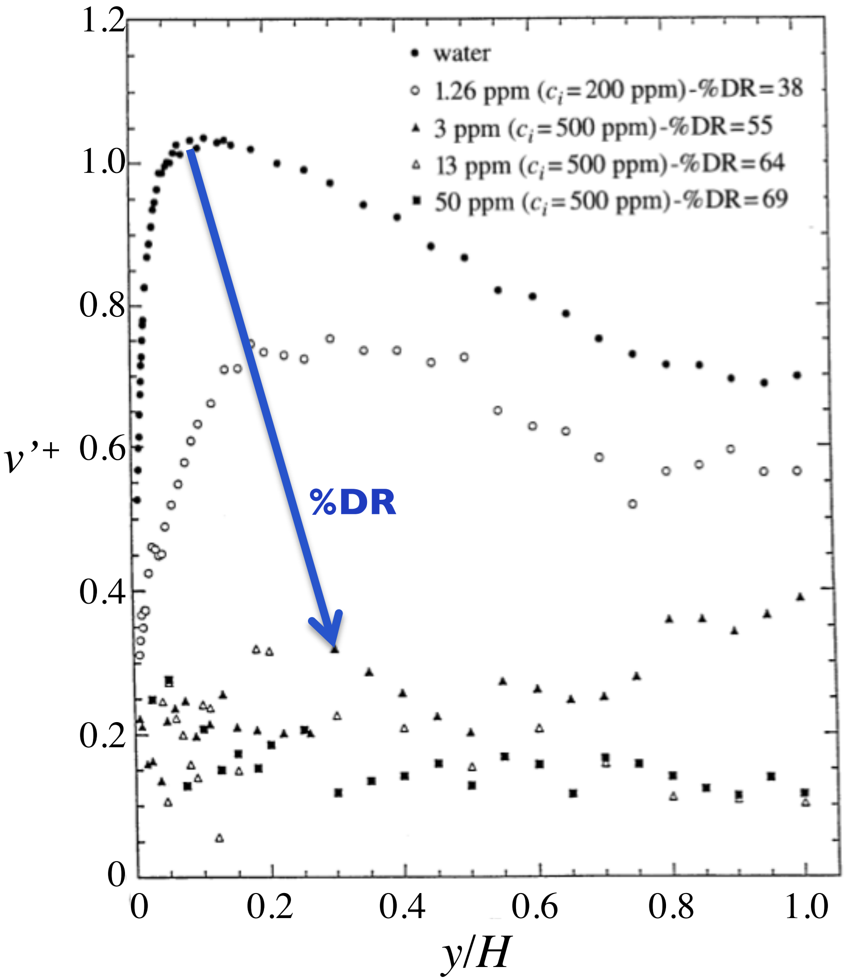
Wall-normal velocity fluctuations

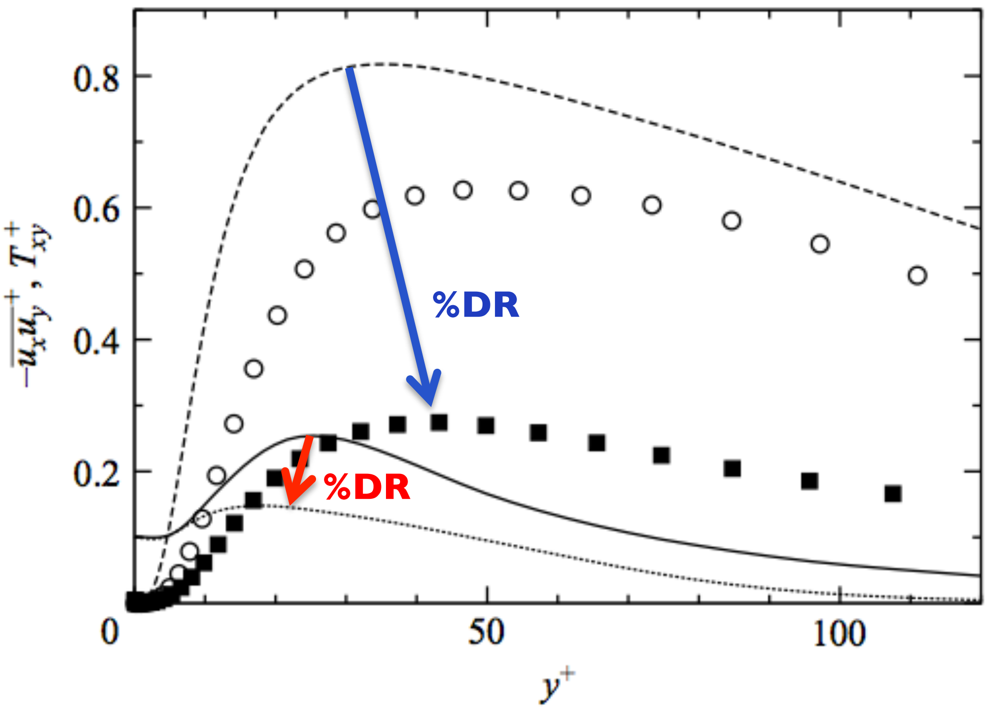


- Strong damping with increasing DR
- Maximum value further away from the wall

Experimental measurements (Warholic *et al.*, 1999)

Changes in Reynolds stress

Reynolds stress and polymer stress



Simulations (Dubief *et al.*, 2004)

- Strong damping of Reynolds stress
- Maximum value further away from wall
- Increasing contribution of polymer stress

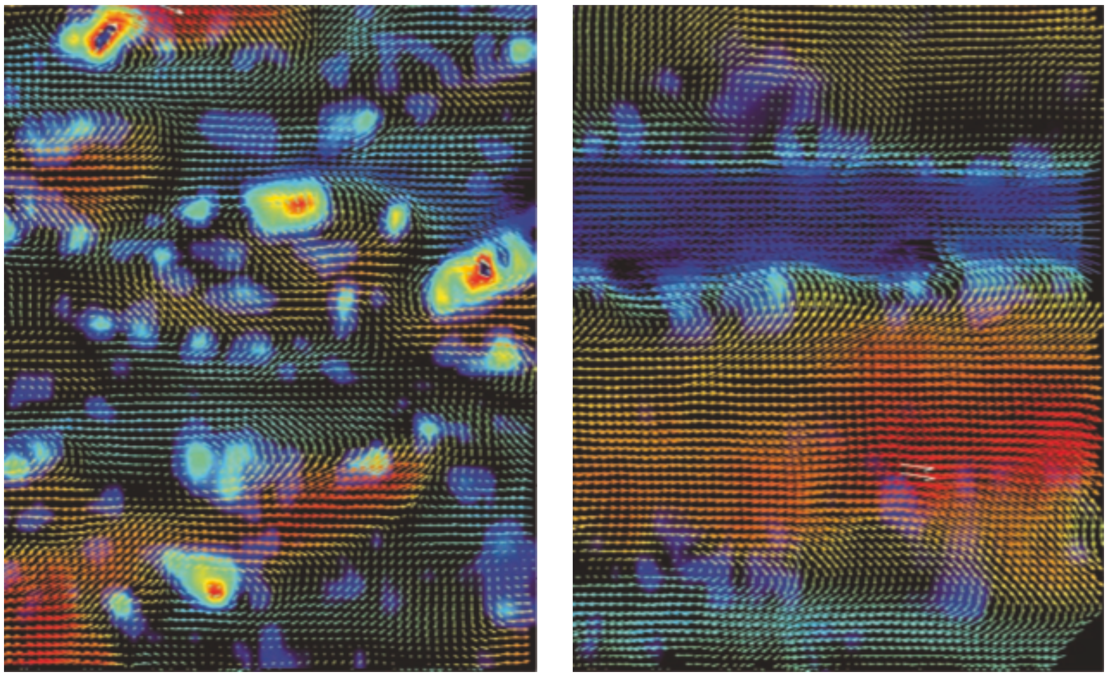
$$-\overline{u'v'}^+ + \beta \frac{\mathrm{d}U^+}{\mathrm{d}y^+} + (1-\beta)T_{xy}^+ = 1 - \frac{y^+}{h^+}$$

Reynolds stress · **Viscous shear stress** · **Polymer stress ("stress deficit")**

- Does Reynolds stress vanishes at MDR or is it finite?

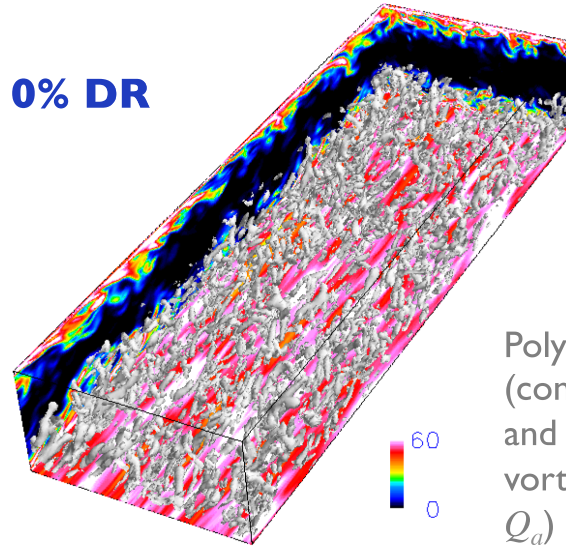
Changes in coherent structures

0% DR **50% DR**

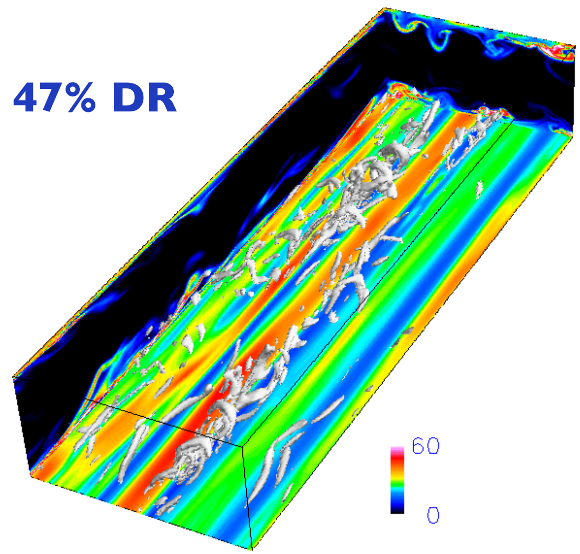


PIV measurements in viscoelastic boundary layer (White *et al.*, 2004)

0% DR



47% DR



Polymer stretch (contours on bottom and side walls) and vortices (isosurfaces of Q_a) from DNS

- Strong damping of quasi-streamwise vortices
- Coarsening and stabilization of streaks

Mechanism

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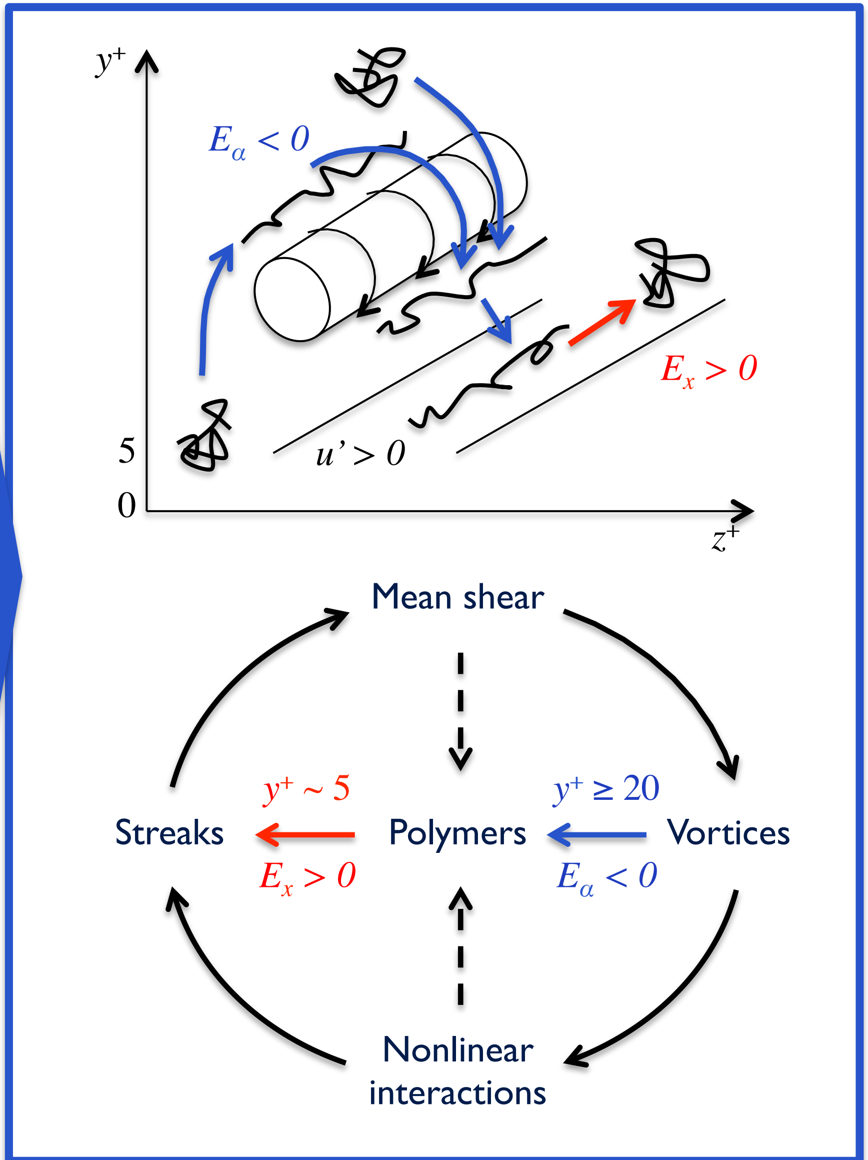
Polymer – turbulence interactions

- Energy transfer from flow to polymers
  - Kinetic to elastic energy
  - In up- and downwashes
  - Around near-wall vortices
  - Stretching caused by bi-axial extensional flow
- Energy transfer from polymers to flow
  - Into high-speed streaks
  - Very close to the wall

Dubief *et al.* (2004), Terrapon *et al.* (2004)



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  - **MDR**
  - Early turbulence
  - Transition
- Conclusions and future work

Transition in Newtonian BL

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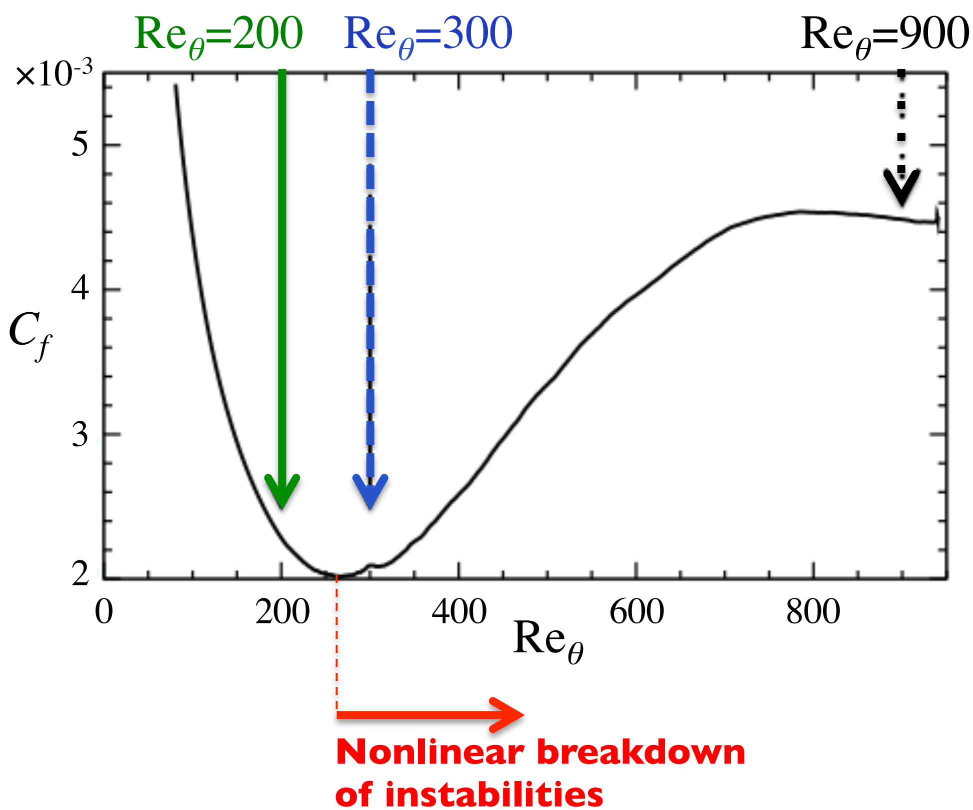
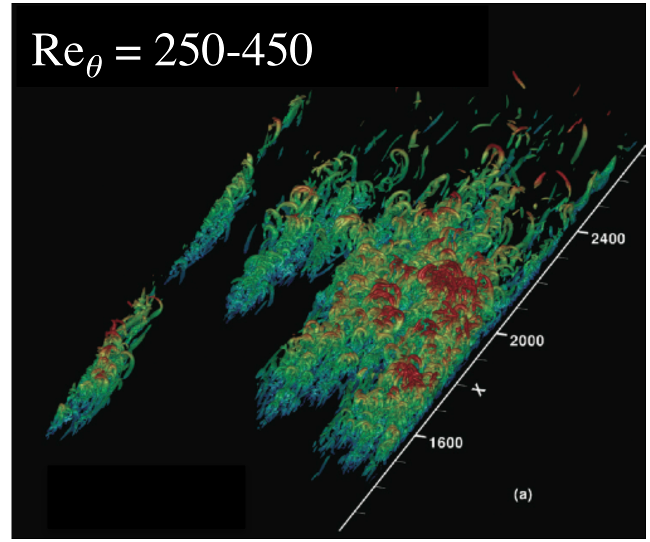
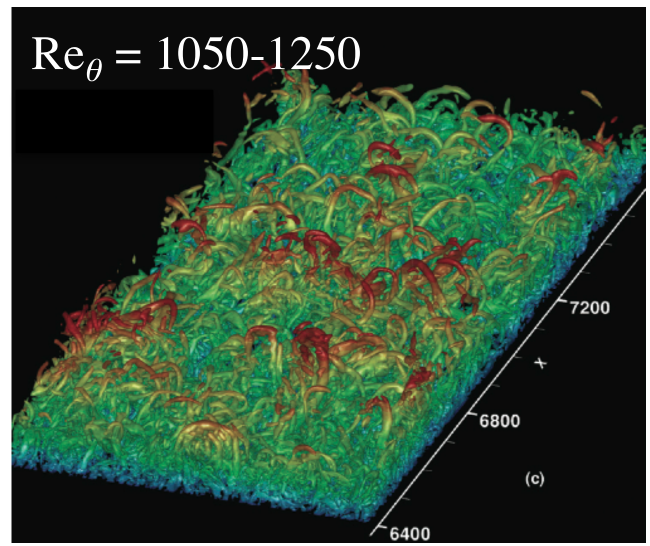
Skin friction coefficient (ZPGBL)







DNS of a ZPGBL (Wu & Moin, 2009)

Transition in Newtonian BL

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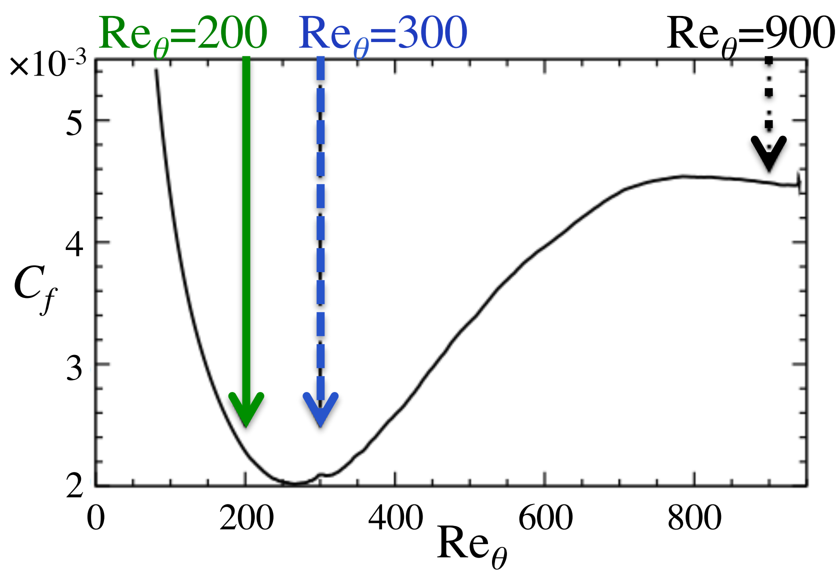
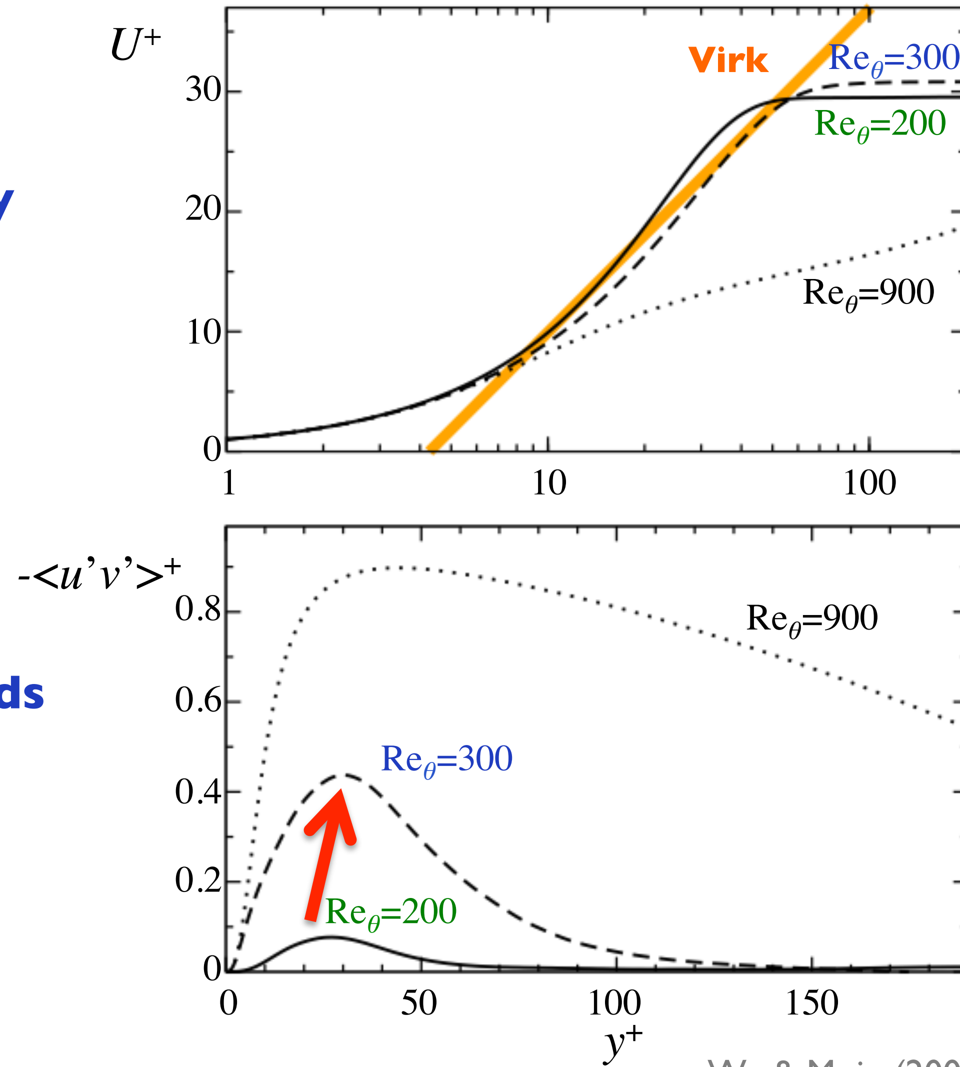
Skin friction coefficient (ZPGBL)



Virk's asymptote very similar to velocity profiles around nonlinear breakdown of instabilities

Mean velocity

Reynolds stress



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MDR and Newtonian transitional flow

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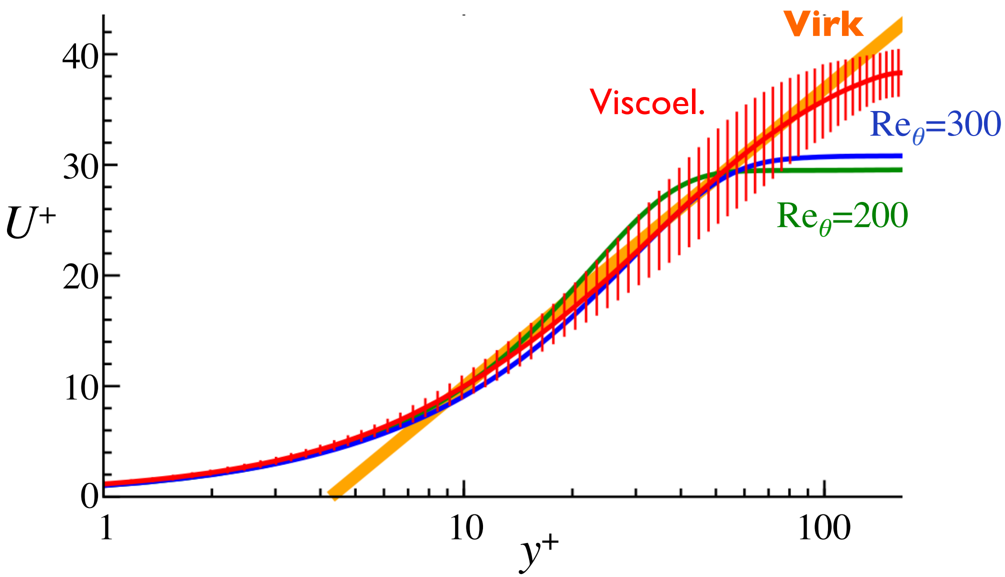
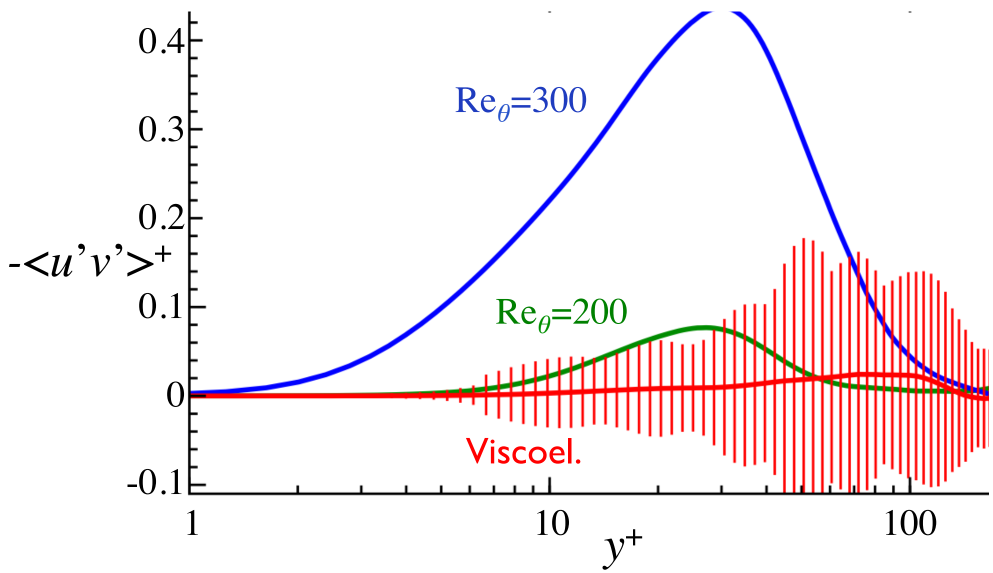
Mean velocity



Reynolds stress



- Velocity profile at MDR (Virk) similar to Newtonian transitional flow
- Time fluctuations at MDR span both pre- and post-breakdown velocity profiles

- Average MDR Reynolds stress negligible
- Time fluctuations at MDR larger than pre-breakdown Newtonian case
- Correlation with $\mathrm{d}P/\mathrm{d}x$

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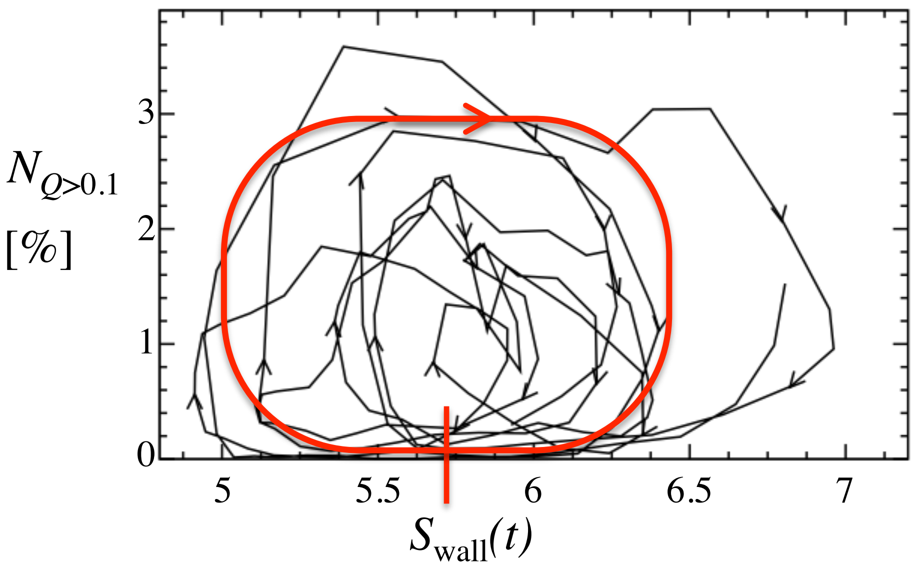
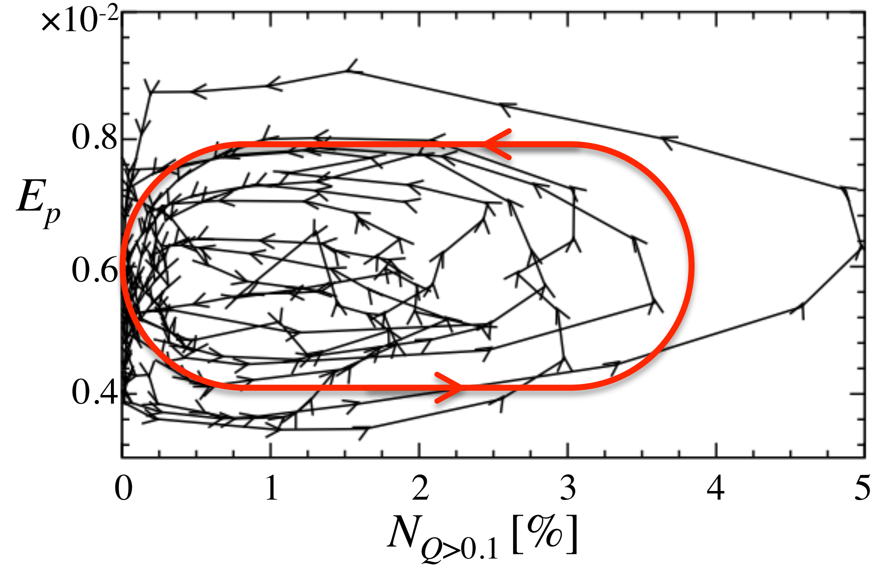
MDR oscillating behavior

Vortical activity vs. wall-shear



- Correlation between increasing vortical activity and increasing shear stress
- When vortices are damped, shear stress decreases

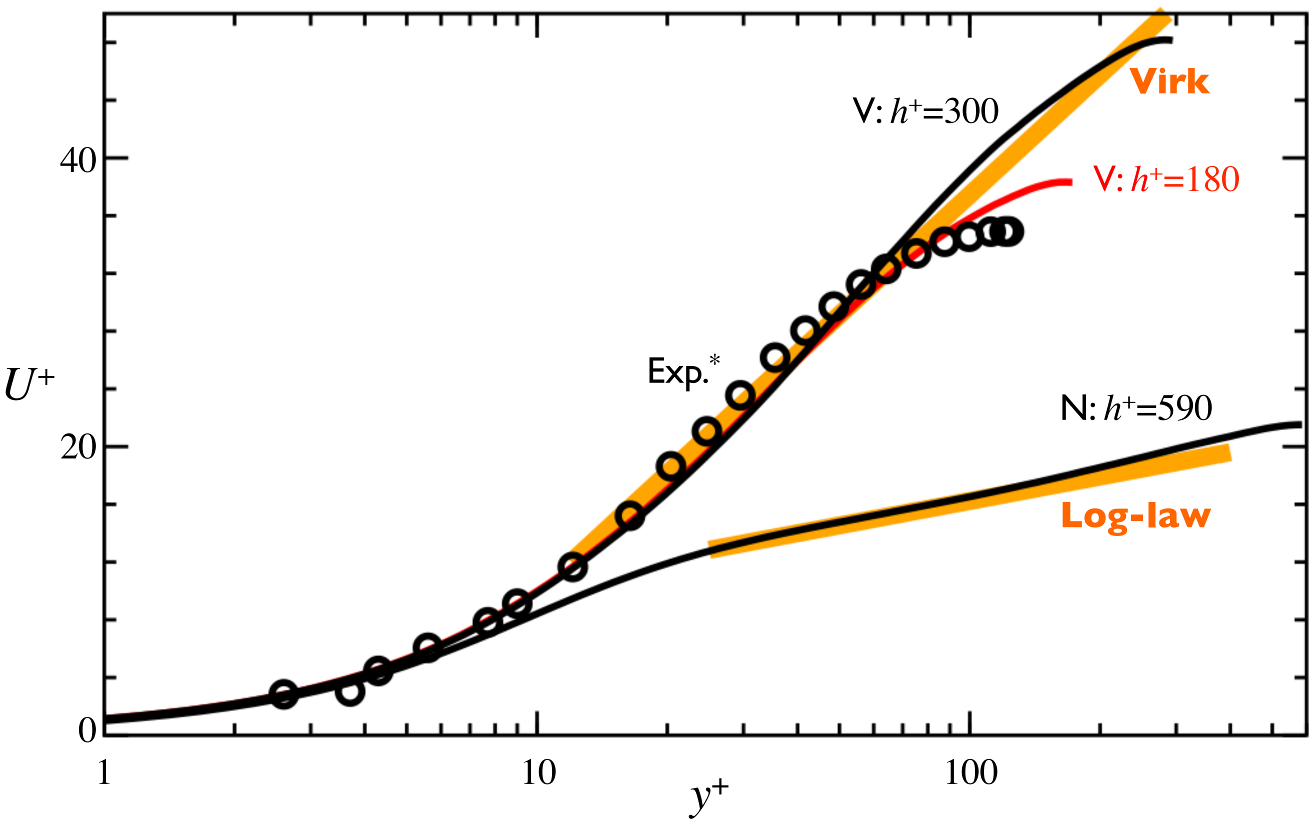
Elastic energy vs. vortical activity



- Elastic energy increases when vortical activity grows
- Elastic energy peaks during decaying portion of vortical activity
- Rapid drop when vortical activity vanishes
- Phase lag depends on elasticity

Is MDR logarithmic?

Mean velocity profile

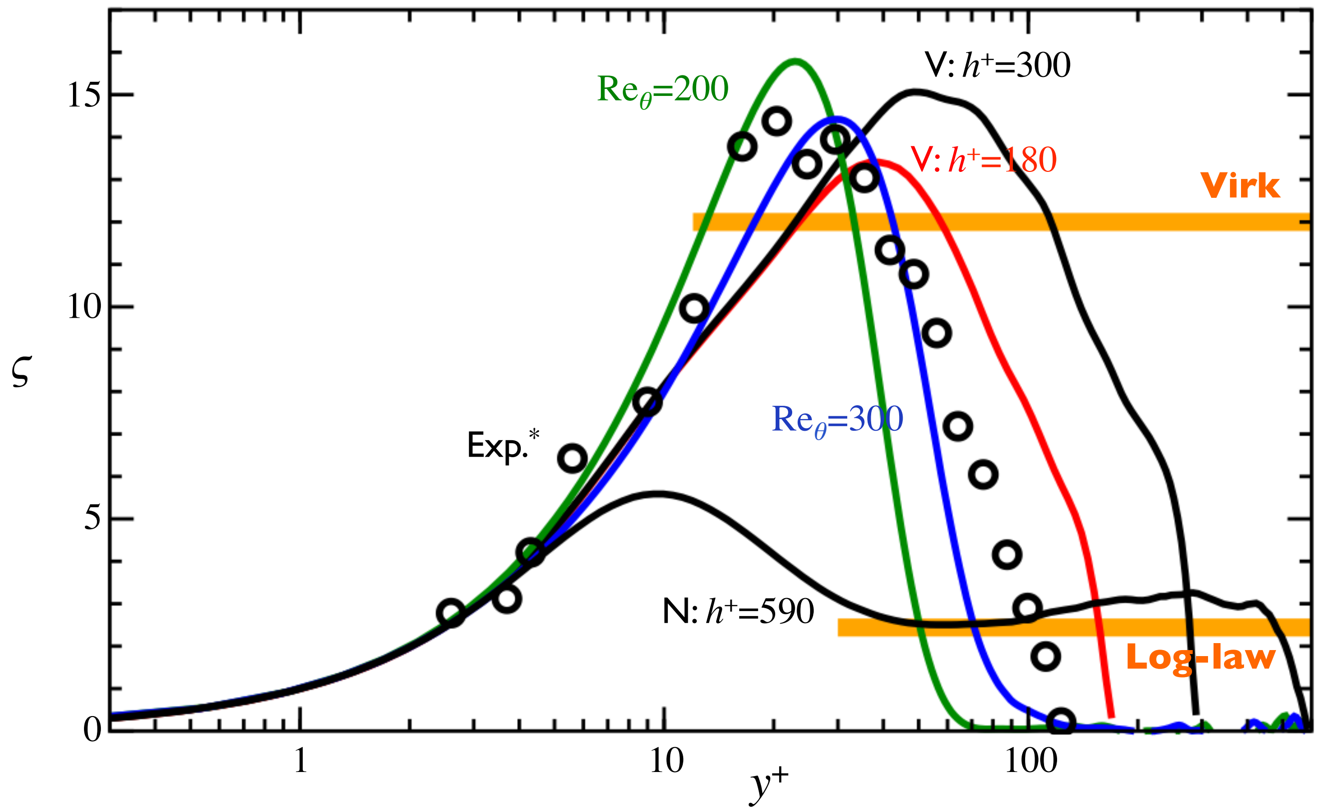


- MDR experiment, BL transitional flow, and MDR simulations straddle Virk's asymptote
- But is it a logarithmic scaling?

* Escudier *et al.*(2009)

Is MDR logarithmic?

Indicator function $\zeta = y^+(dU^+/dy^+)$



- If logarithmic behavior, indicator function should be constant
- No apparent logarithmic behavior
  - MDR simulations
  - MDR experiment
  - BL transitional flow
- Possibly due to low Re but other experiments at higher Re show similar velocity profile
- Need more data at higher Re

White *et al.*(2012)

* Escudier *et al.*(2009)

Our understanding of MDR

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- MDR is a **transitional state** corresponding to the onset of the **nonlinear breakdown of instabilities**
- MDR flow **oscillates** between pre- and post-breakdown state
  - Polymers disrupt the near-wall autonomous cycle of wall turbulence
  - Vortices are damped out (or sufficiently weakened as to "turn-off" their ability to stretch polymers)
  - Stretching mechanism for polymers is limited to wall-shear and spanwise shear layers between streaks (both modest source of polymer stretching compared to biaxial-extensional flow)
  - Polymers recoil, allowing instabilities to grow and new vortices to form
- Oscillation "amplitude" depends on **polymer elasticity**
- There is **no apparent logarithmic scaling** of velocity at MDR or in transitional flows
- There are indications of **elastic instabilities** at small scales that contribute to the regeneration of vortices following the low vortical activity

White *et al.* (2012), Xi & Graham (2010), L'vov *et al.* (2004)

Outline

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- Context
- Models and numerical implementation
- Polymer drag reduction
- Elasto-inertial turbulence
  - MDR
  - **Early turbulence**
  - Transition
- Conclusions and future work

Transitional viscoelastic flows

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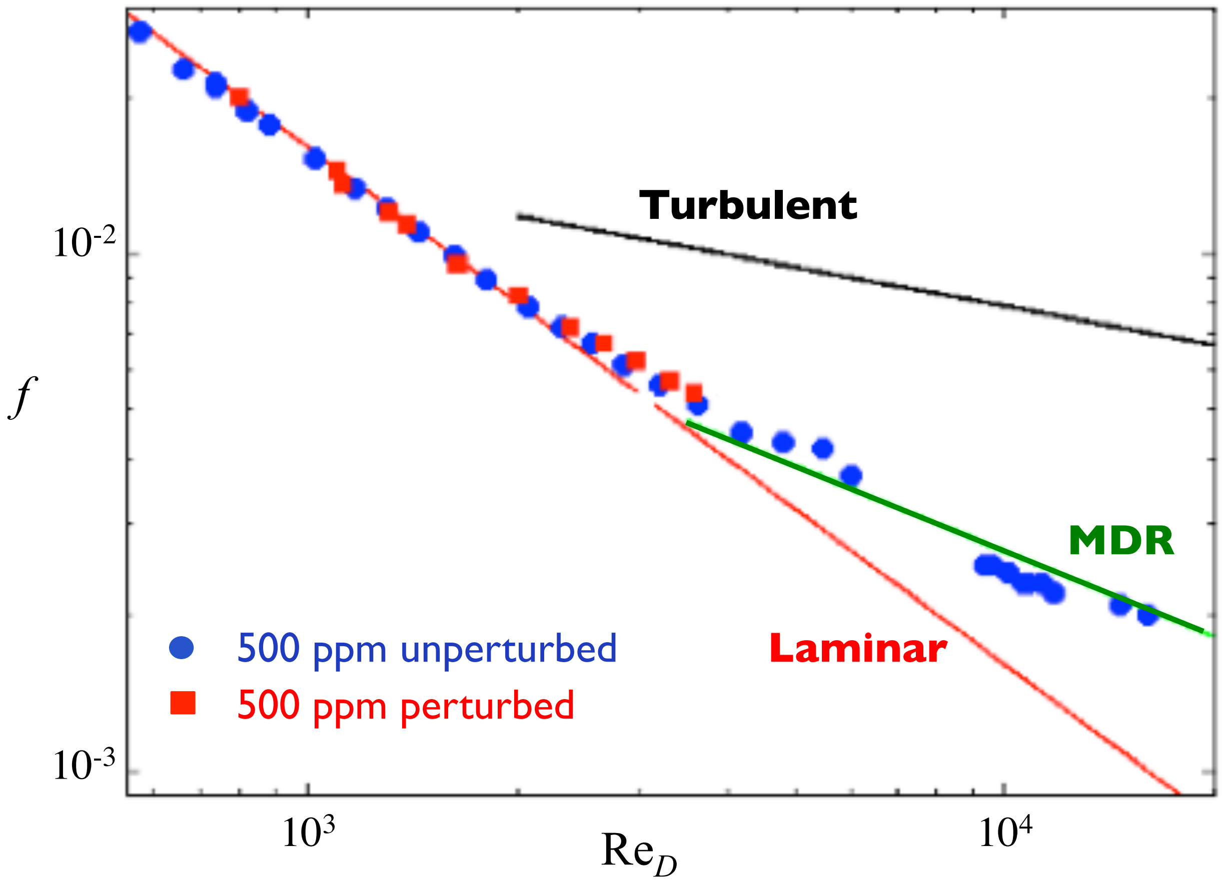
Pipe flow experiment with PAAm solution

Friction factor



Samanta *et al.* (2012, submitted)

- Departure from laminar state at Re~800
- Smooth transition from laminar to MDR state
- Flow dynamics controlled by elastic and inertial instabilities

Transitional viscoelastic flows

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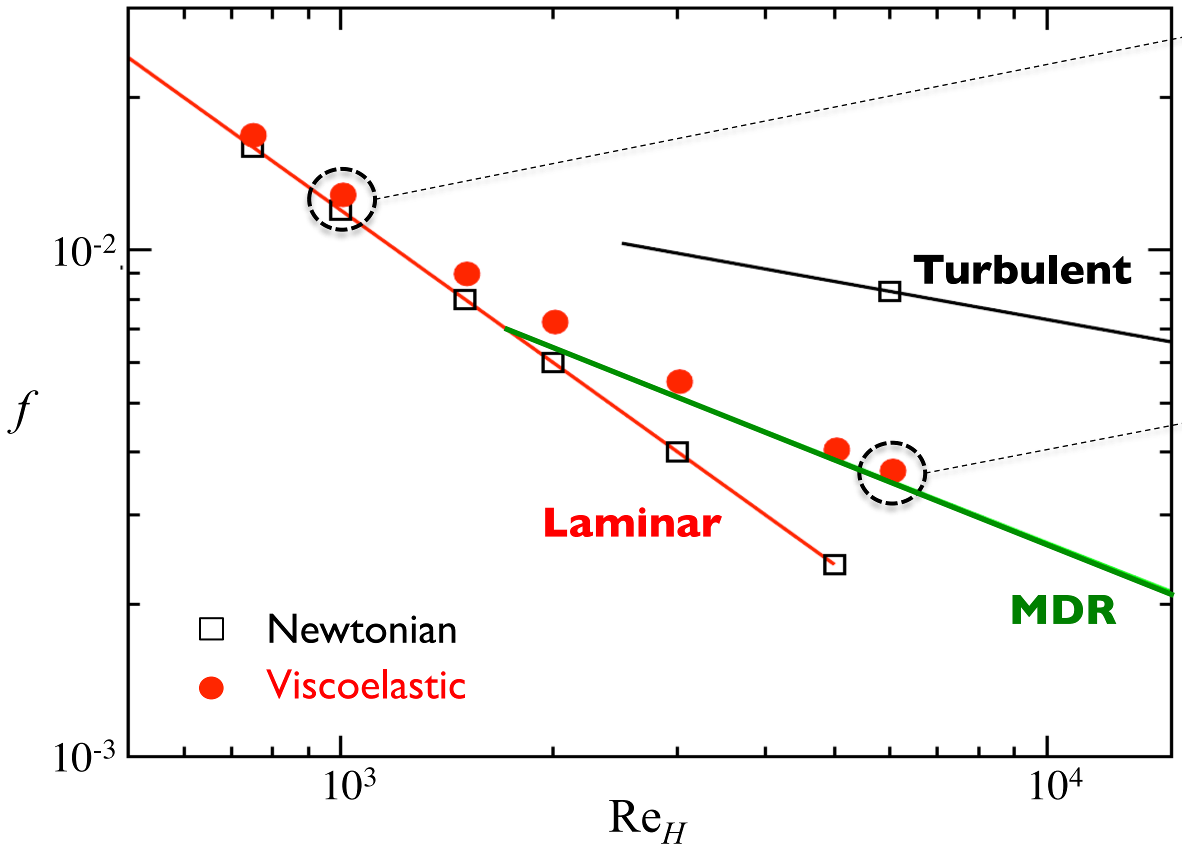
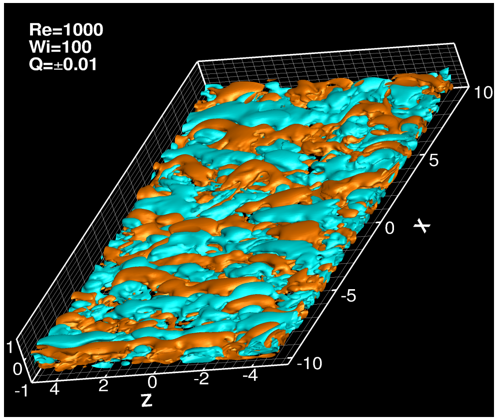
Channel flow simulations

Friction factor

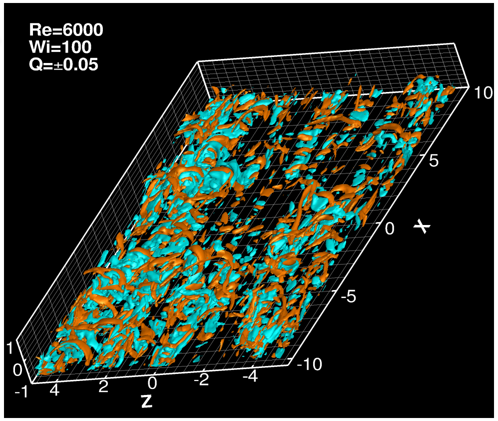




Re = 1000

- Not laminar
- Elastic contributions

Isosurface of Q_a invariant



Re = 6000

- Inertial & elastic contributions
- Turbulent?
- New state?

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Qualitative flow behavior

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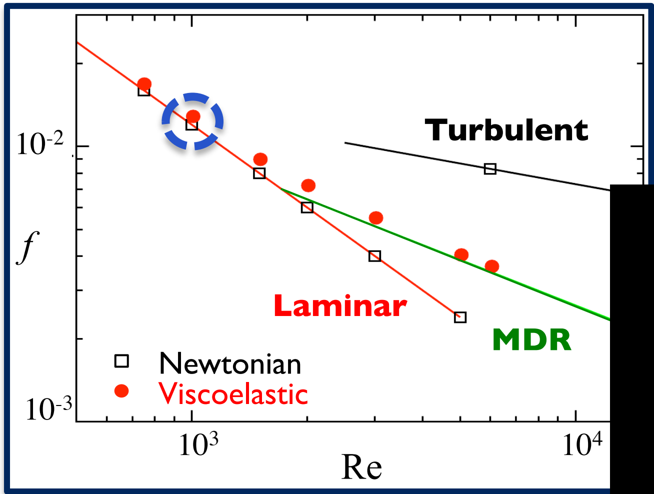
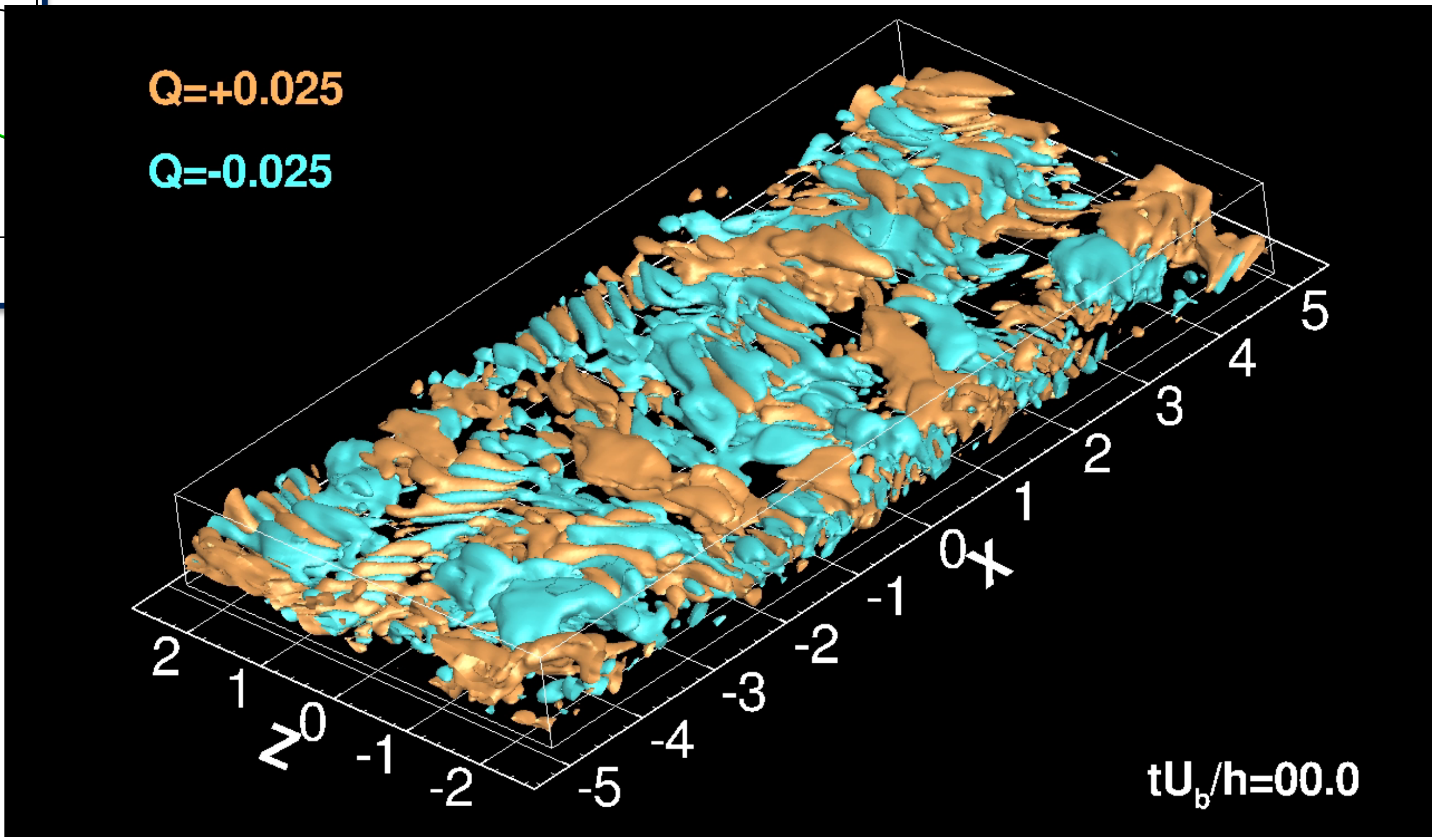
Re = 1000
Wi = 8





Second invariant of the velocity gradient tensor: $Q_a = ½ (\Omega^2\text{-}S^2)$

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Qualitative flow behavior

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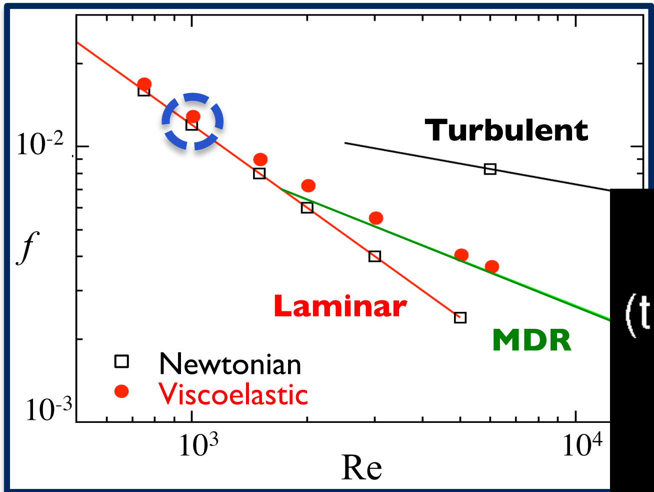
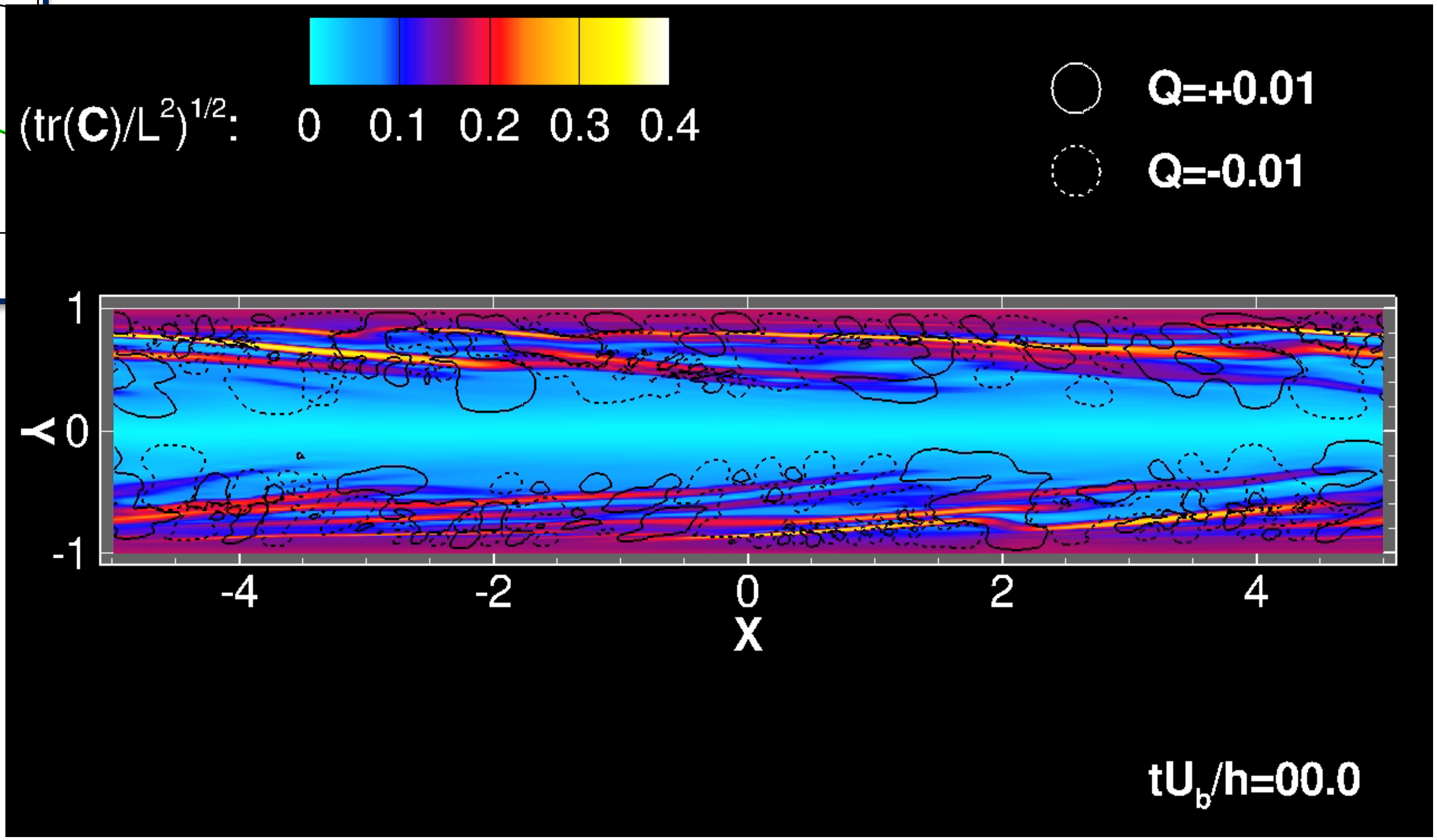
Re = 1000
Wi = 8





Polymer extension $(C_{ii} / L^2)^{1/2}$

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Qualitative flow behavior

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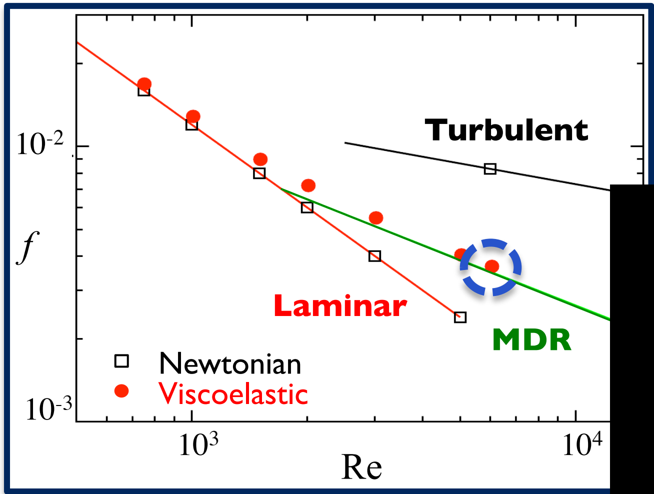
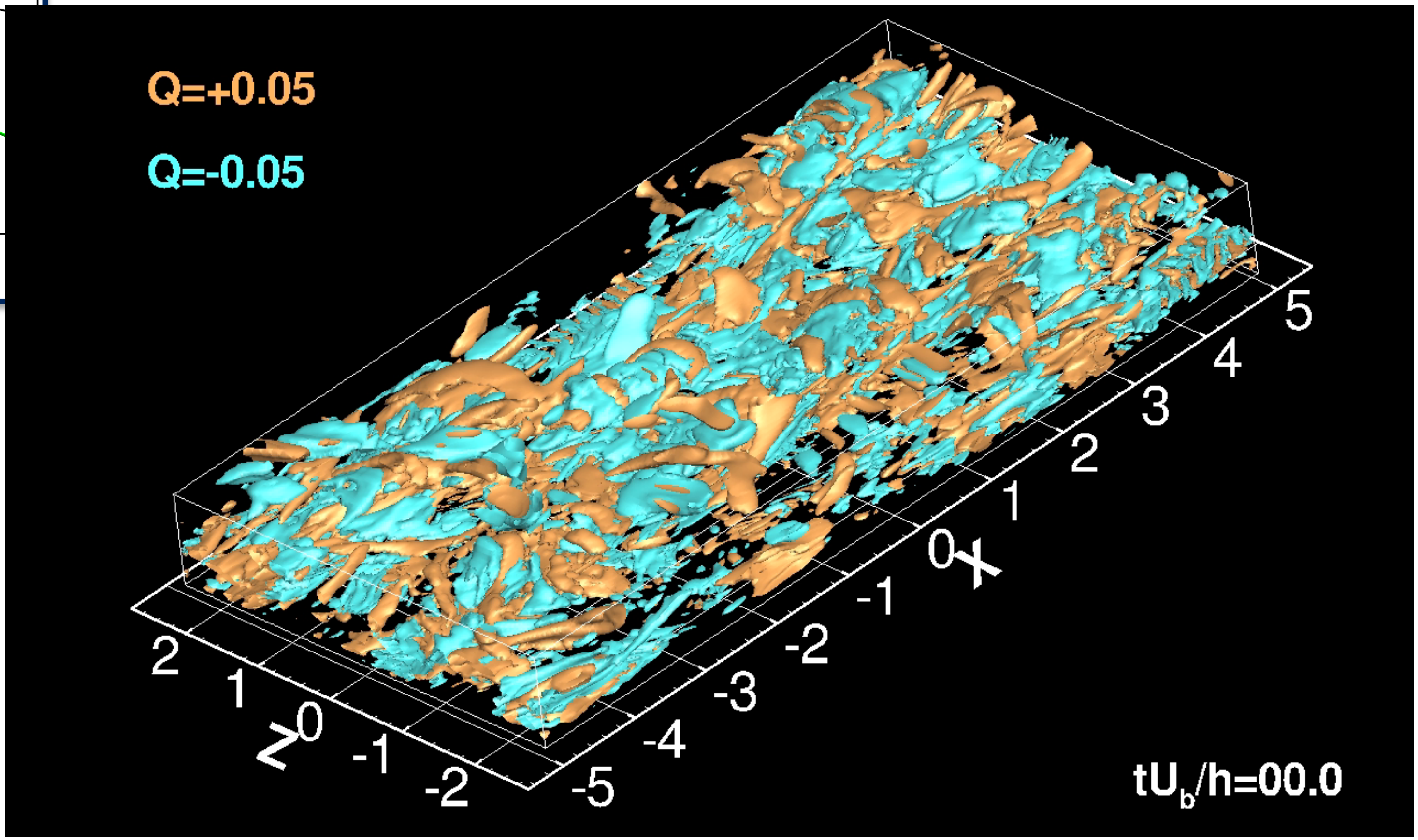
Re = 6000
Wi = 8





Second invariant of the velocity gradient tensor: $Q_a = \frac{1}{2}\,(\Omega^2 - S^2)$

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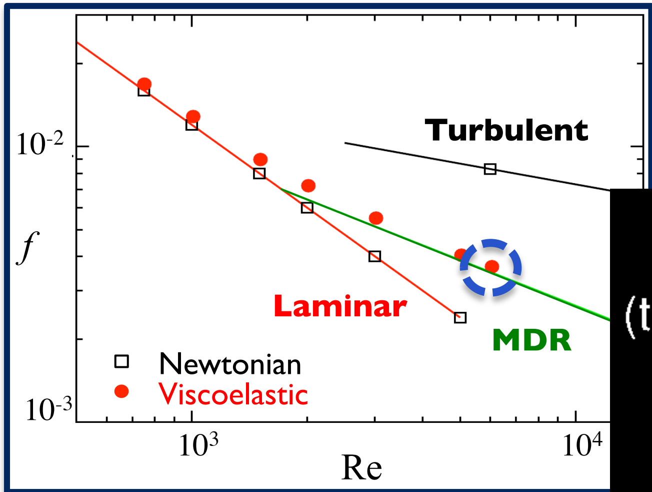
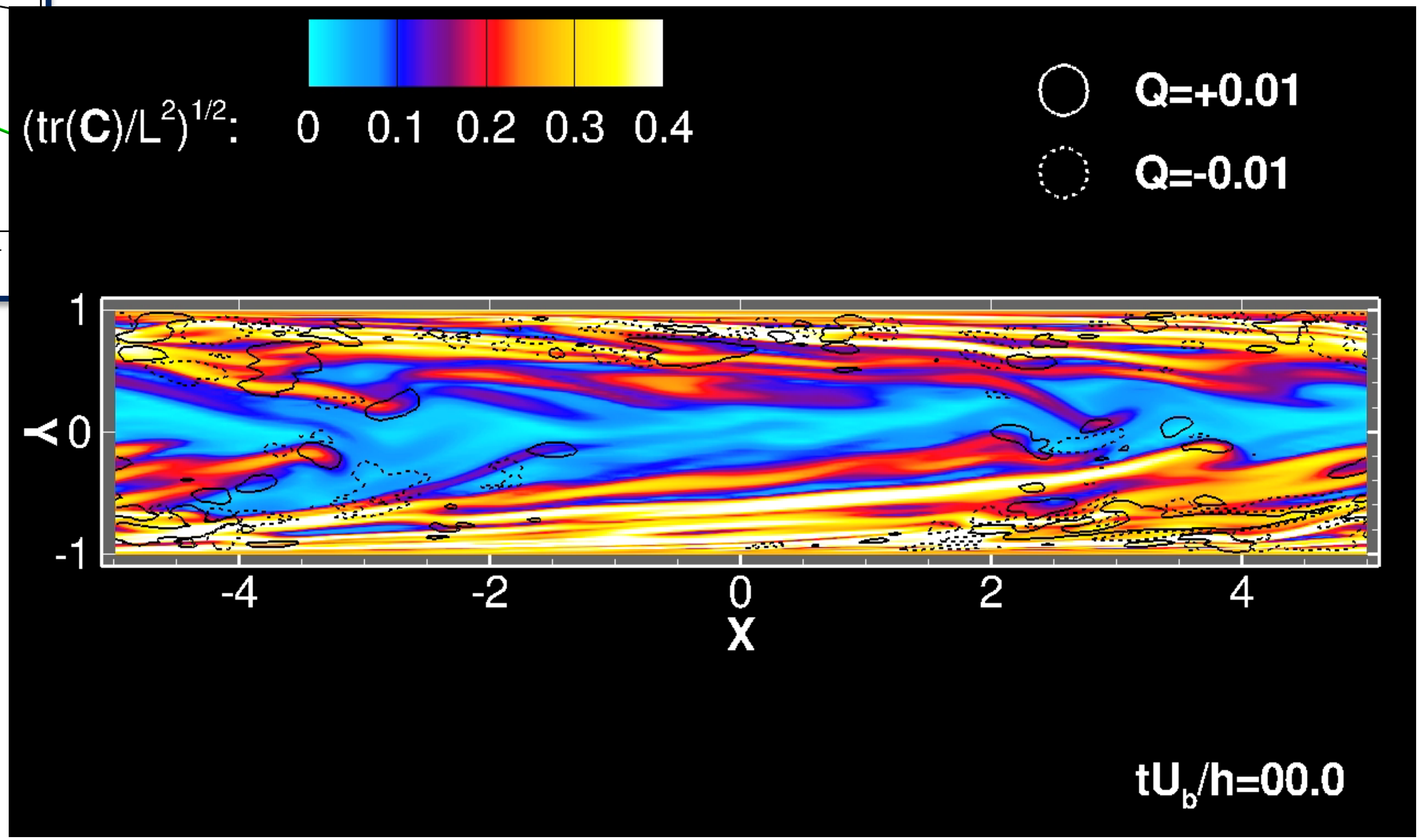
Qualitative flow behavior

Re = 6000
Wi = 8





Polymer extension $(C_{ii} / L^2)^{1/2}$

Flow topology

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Local flow pattern at fluid particle

Velocity gradient tensor:

$$A_{ij} = (\nabla \mathbf{u})_{ij}$$

Invariants:

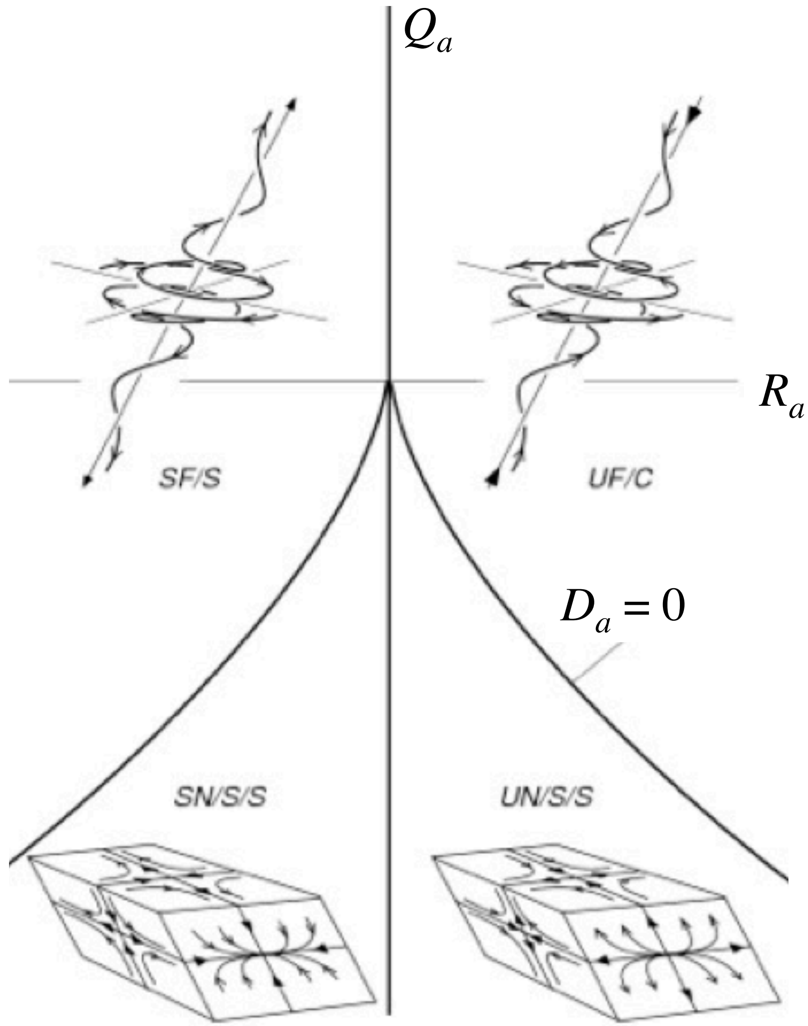
$$P_a = -A_{ii} = 0$$

$$Q_a = -\frac{1}{2} A_{ij} A_{ji}$$

$$R_a = -\frac{1}{3} A_{ij} A_{jk} A_{ki}$$

Discriminant:

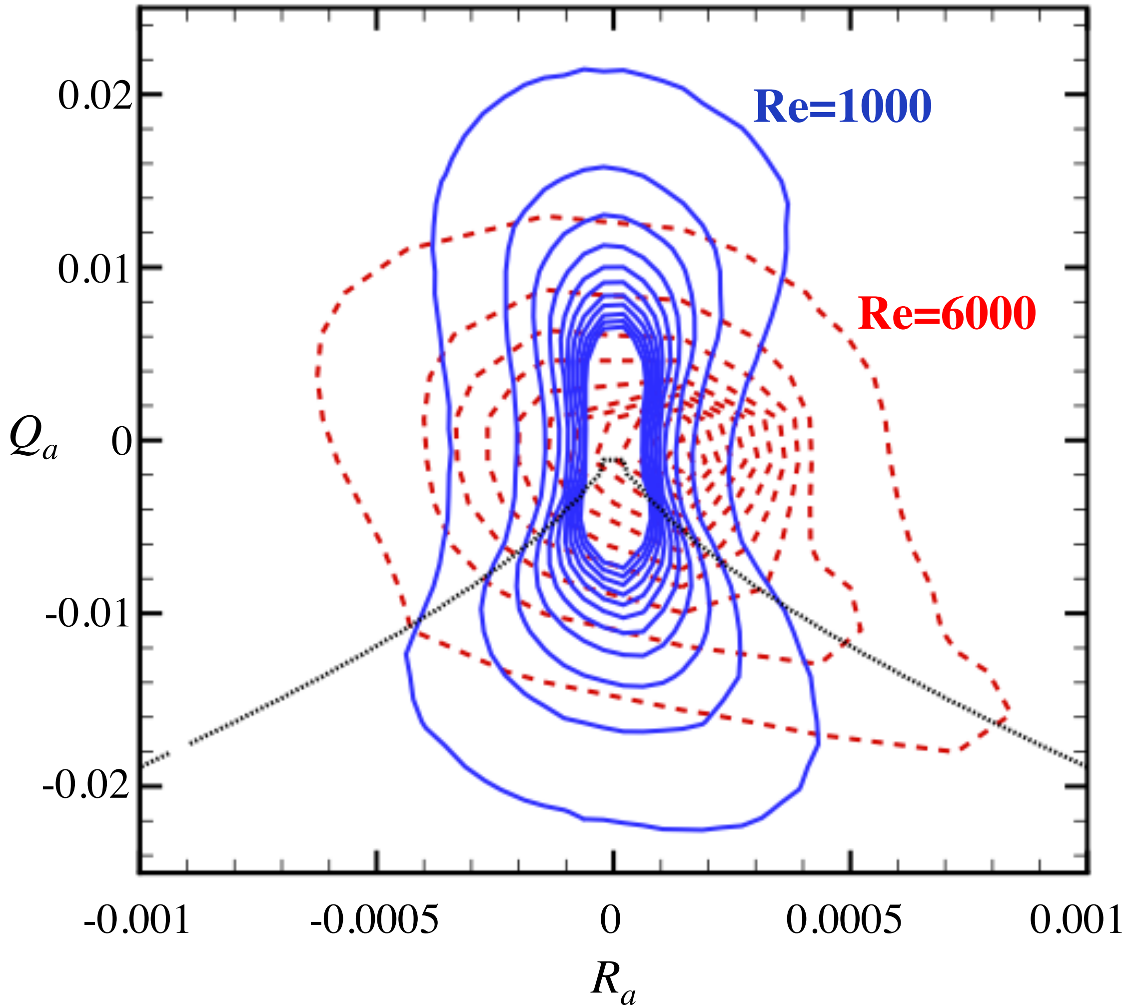
$$D_a = \frac{27}{4} R_a^2 + Q_a^3$$



Chong *et al.* (1990), Soria *et al.* (1994)

Flow topology

EIT flow (JPDF)



- At low Re, symmetric distribution around 2D flow (R_a=0)
- At higher Re, "teardrop" shape similar to Newtonian turbulence

Flow topology

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Velocity gradient tensor:

$$A_{ij} = (\nabla \mathbf{u})_{ij} = S_{ij} + \Omega_{ij}$$

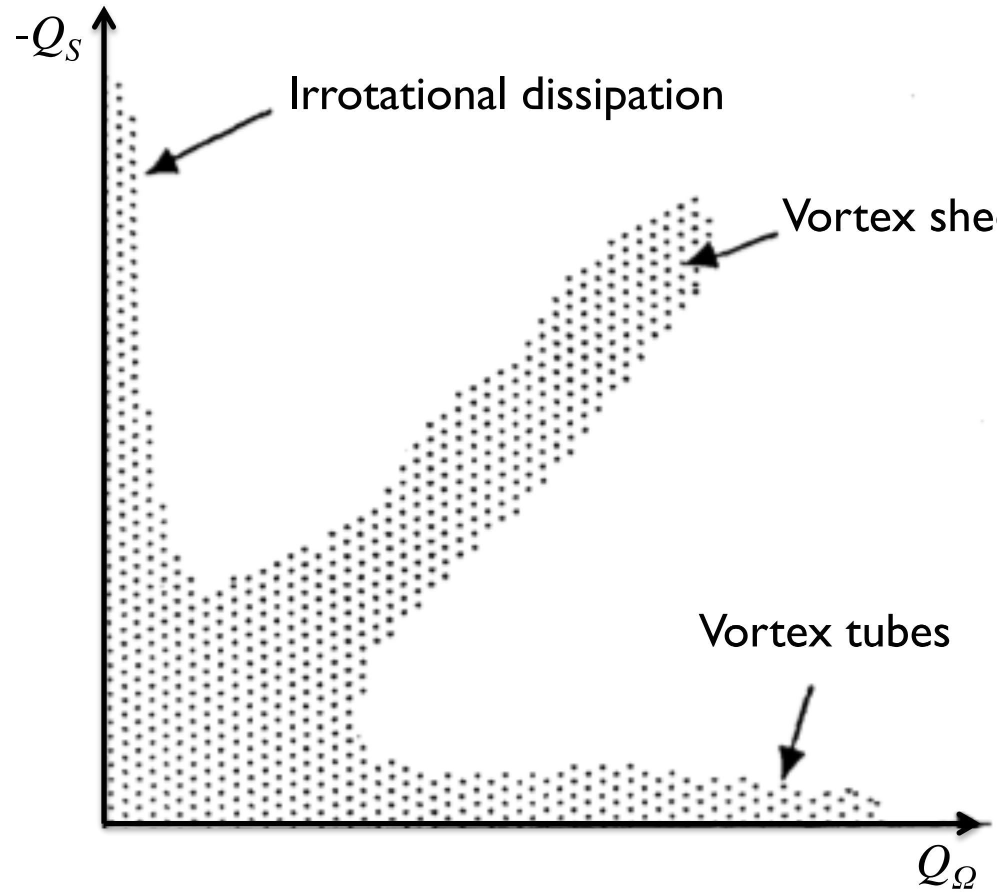
2nd invariant

$$Q_a = Q_S + Q_\Omega$$

$$Q_S = -\frac{1}{2} S_{ij} S_{ji}$$

$$Q_\Omega = -\frac{1}{2} \Omega_{ij} \Omega_{ji}$$

Newtonian turbulent flow (JPDF)



Flow topology

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Velocity gradient tensor:

$$A_{ij} = (\nabla \mathbf{u})_{ij} = S_{ij} + \Omega_{ij}$$

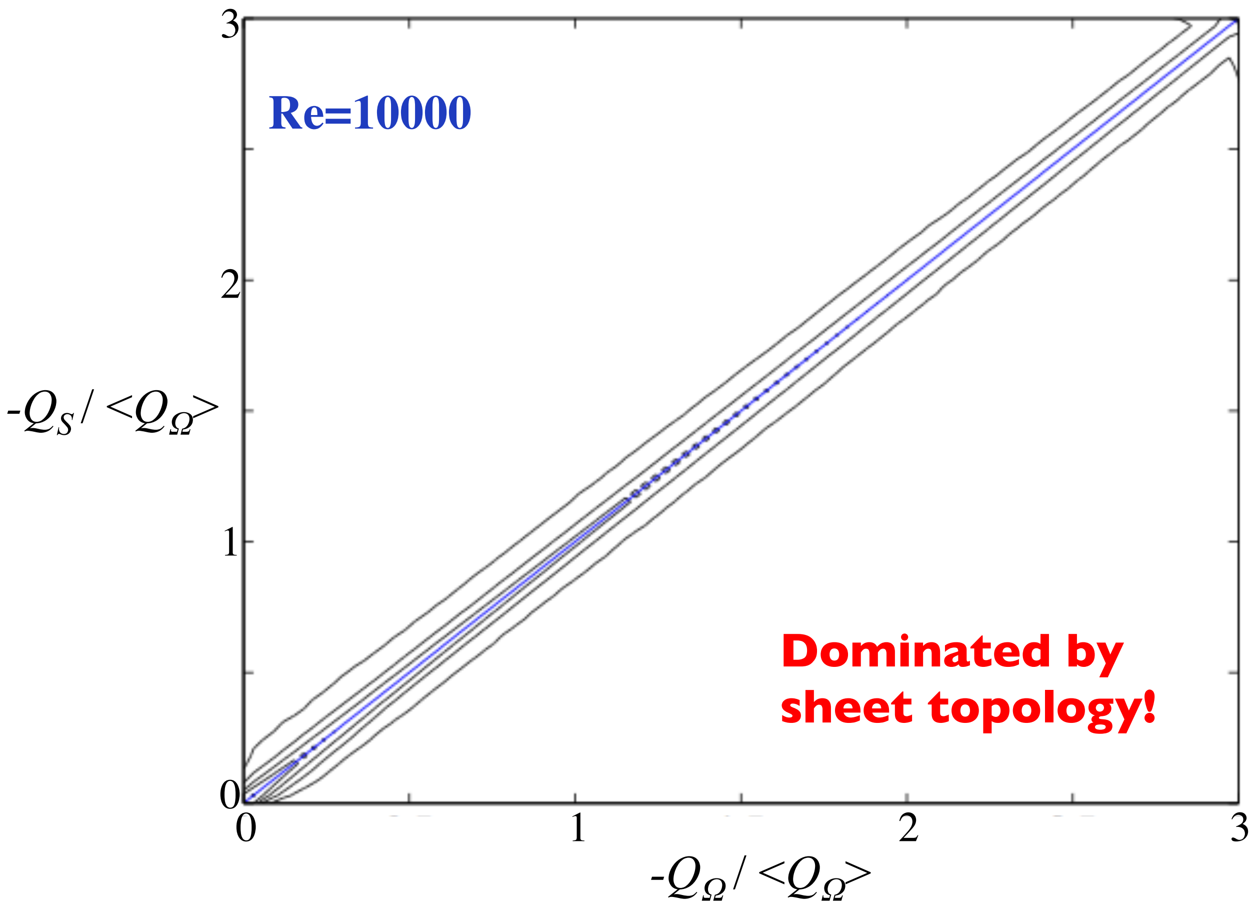
2nd invariant

$$Q_a = Q_S + Q_\Omega$$

$$Q_S = -\frac{1}{2} S_{ij} S_{ji}$$

$$Q_\Omega = -\frac{1}{2} \Omega_{ij} \Omega_{ji}$$

EIT flow (JPDF)



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Observations

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- Flow is perfectly **laminar** in the absence of polymers
- Polymer addition creates a **self-sustained** chaotic flow consisting of **trains** of cylindrical weakly rotational regions (positive Q_a) and weakly extensional regions (negative Q_a)
- There is a **hierarchy** of cylindrical structures, the smallest one being of the order of the **Kolmogorov** scale
- The polymer extension field is organized in **sheets**
- Polymers cause the flow to evolve from pure shear flow to mix **extensional-shear** flow
- The cylindrical Q_a structures are attached to sheets of large polymer extension
- Elasto-inertial turbulence results from the combination of the **hyperbolic** transport equation of $\mathbf{C}$ and the **elliptic** equation of p
- Pressure **redistributes** energy with trains of cylindrical structures to attenuate the anisotropy caused by sheets of extensional viscosity

Proposed mechanism

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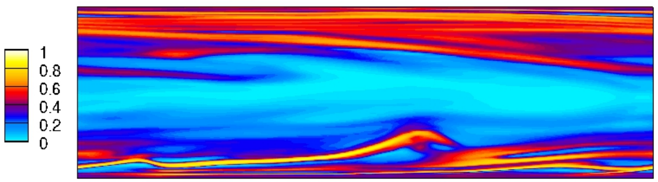
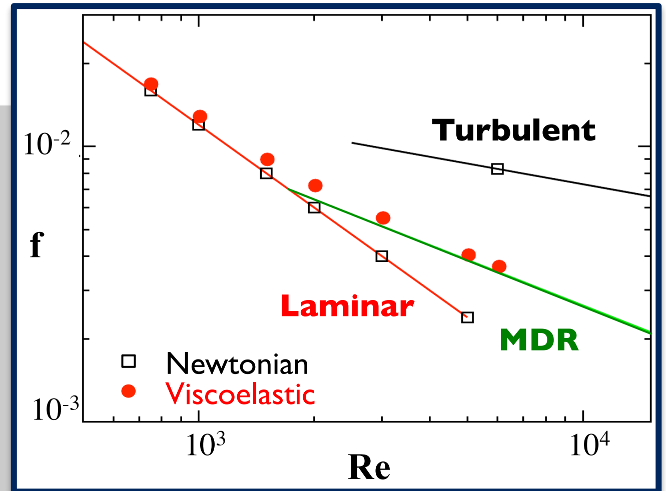
Polymer extension



Channel flow



$$\partial_t \mathbf{C} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{C}$$

- Formation of sheets

$$\nabla^2 p = 2Q_a - \frac{1-\beta}{Re}\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\mathbf{T})$$

- Excitation of extensional sheet flow
- Elliptical pressure redistribution of energy

$$\mathbf{C}(\boldsymbol{\nabla}\,\mathbf{u}) + (\boldsymbol{\nabla}\,\mathbf{u})^{\mathrm{T}}\mathbf{C} - \mathbf{T}$$

- Increase of extensional viscosity (anisotropic)

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Outline

- Context
- Models and numerical implementation
- Polymer drag reduction
- Elasto-inertial turbulence
  - MDR
  - Early turbulence
  - **Transition**
- Conclusions and future work

By-pass transition

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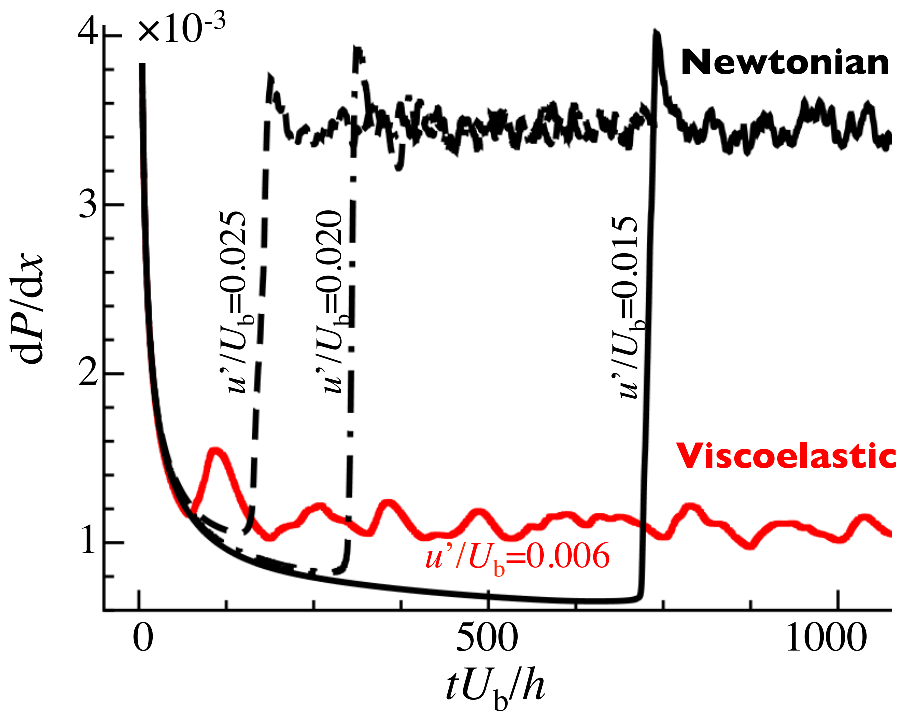
Re = 10000	Wi = 40
$L = 200$	$\beta = 0.9$

Time evolution of drag



- Viscoelastic flow transitions with **weaker** perturbations
- Same perturbation level would lead to **laminar** Newtonian flow
- Viscoelastic flow transitions to **MDR** and stays at MDR

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By-pass transition

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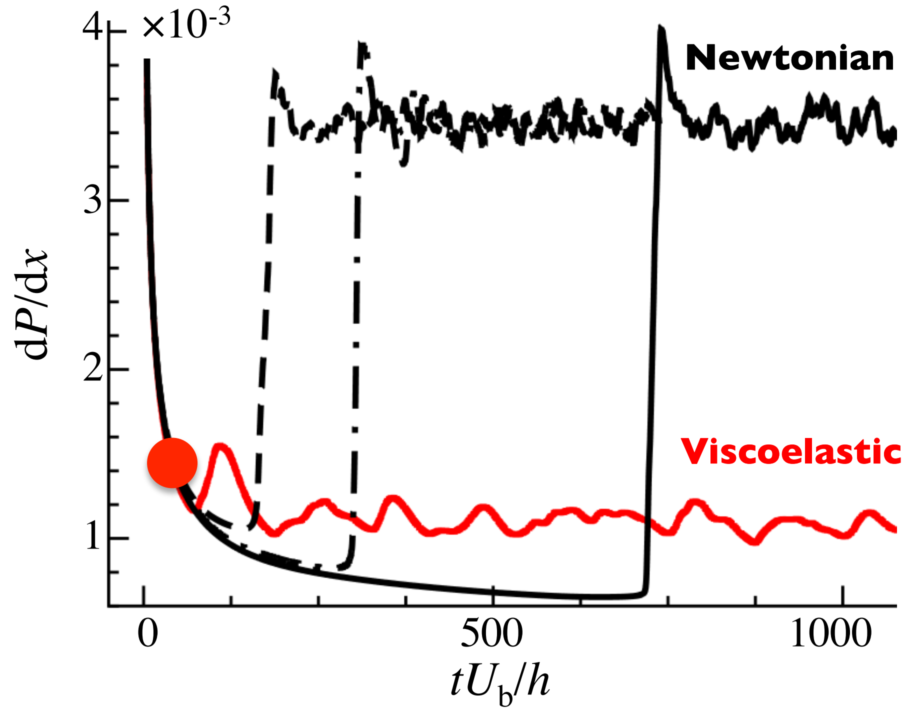
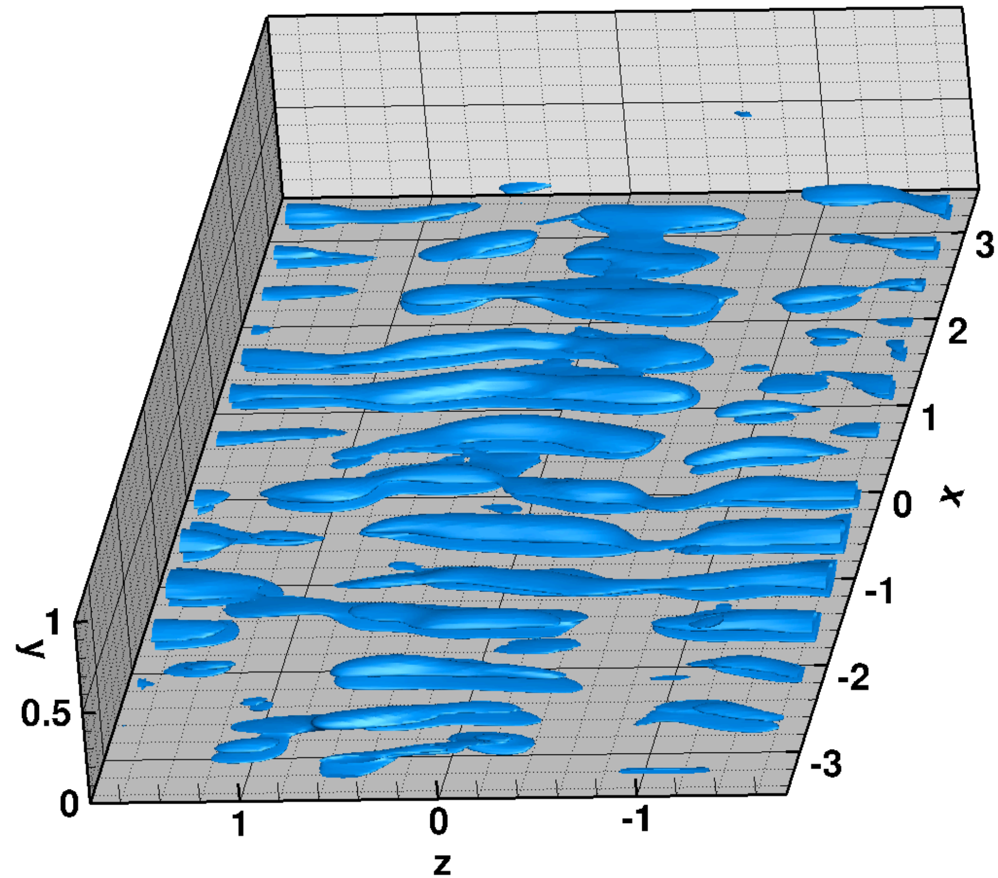
Time evolution of drag





2[nd] invariant Q_a of the velocity gradient tensor

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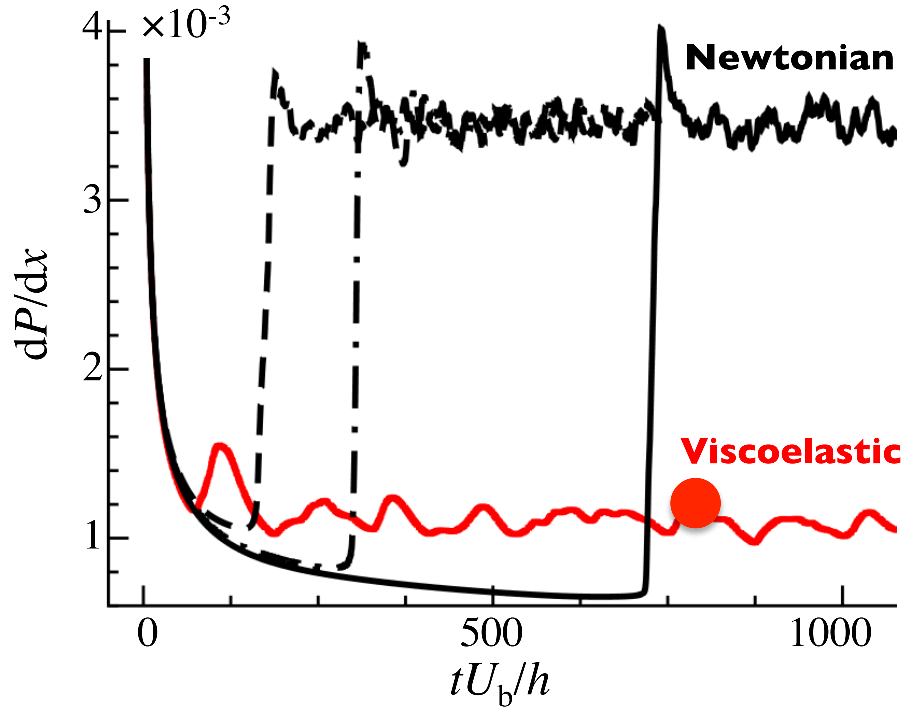
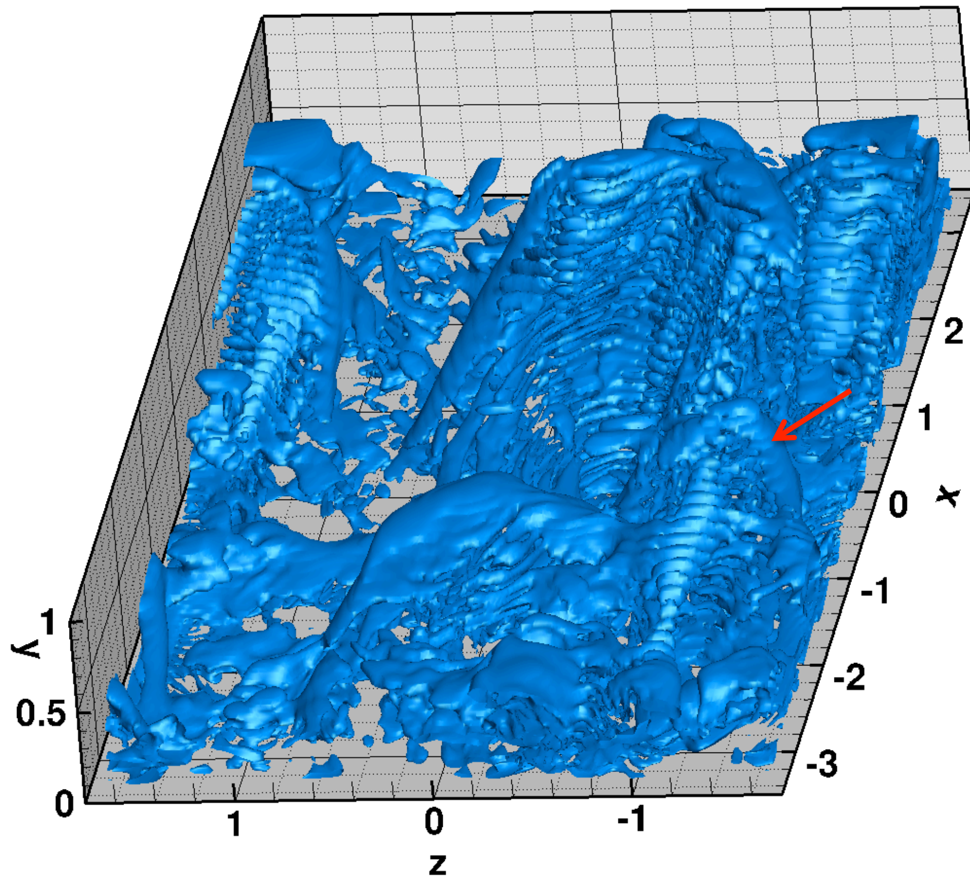
By-pass transition

Time evolution of drag

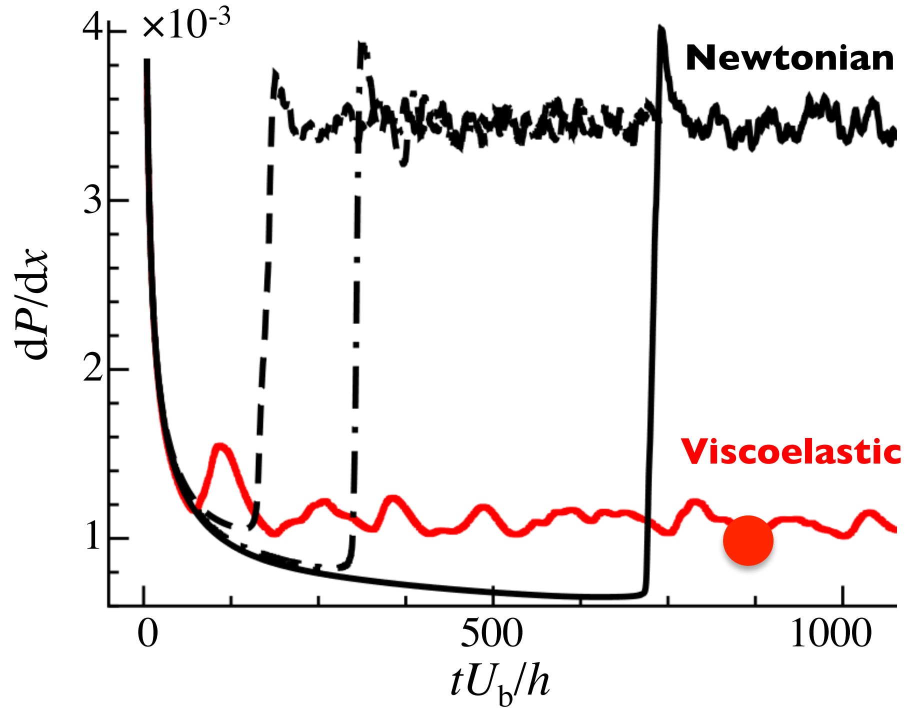


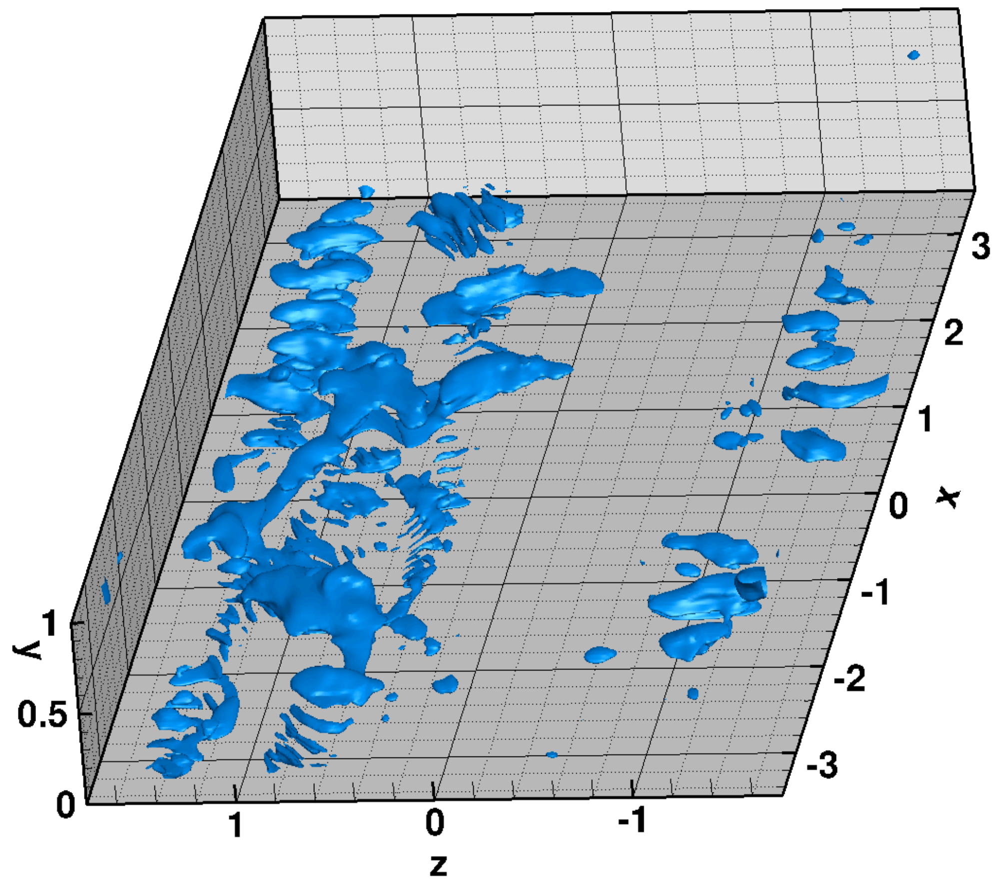


2nd invariant Q_a of the velocity gradient tensor

By-pass transition

Time evolution of drag

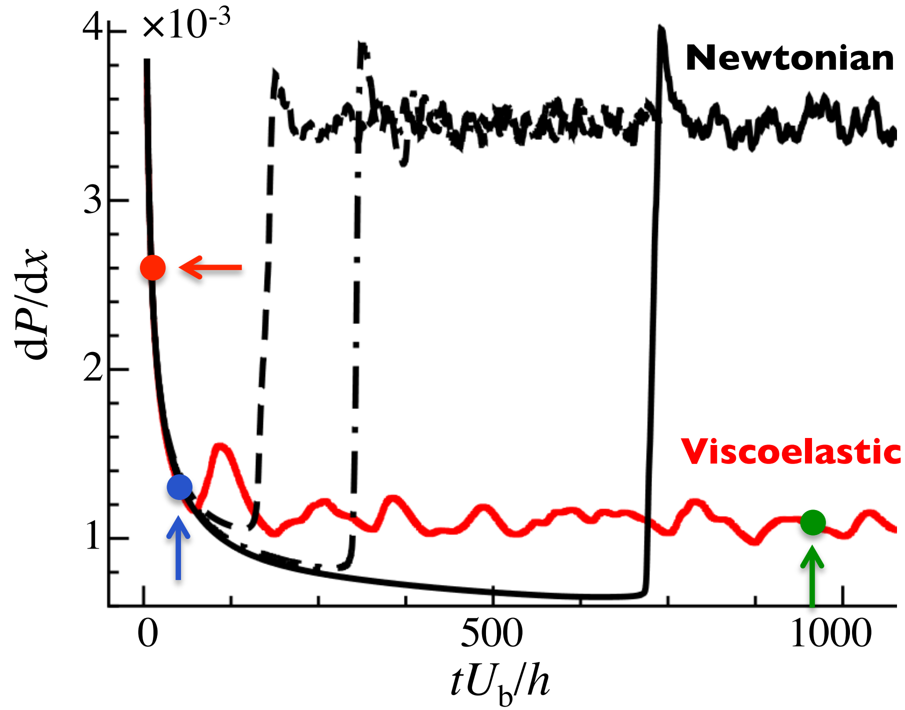




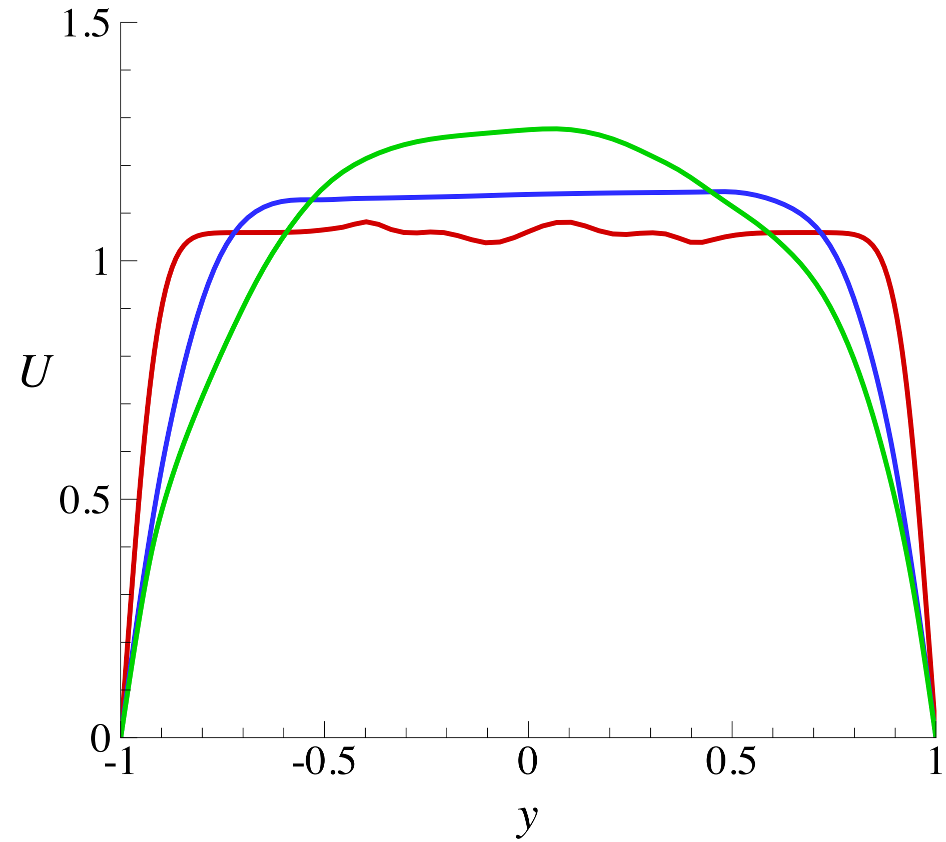
2nd invariant Q_a of the velocity gradient tensor

Spatial statistics

Time evolution of drag



Mean velocity profile



Spatial statistics

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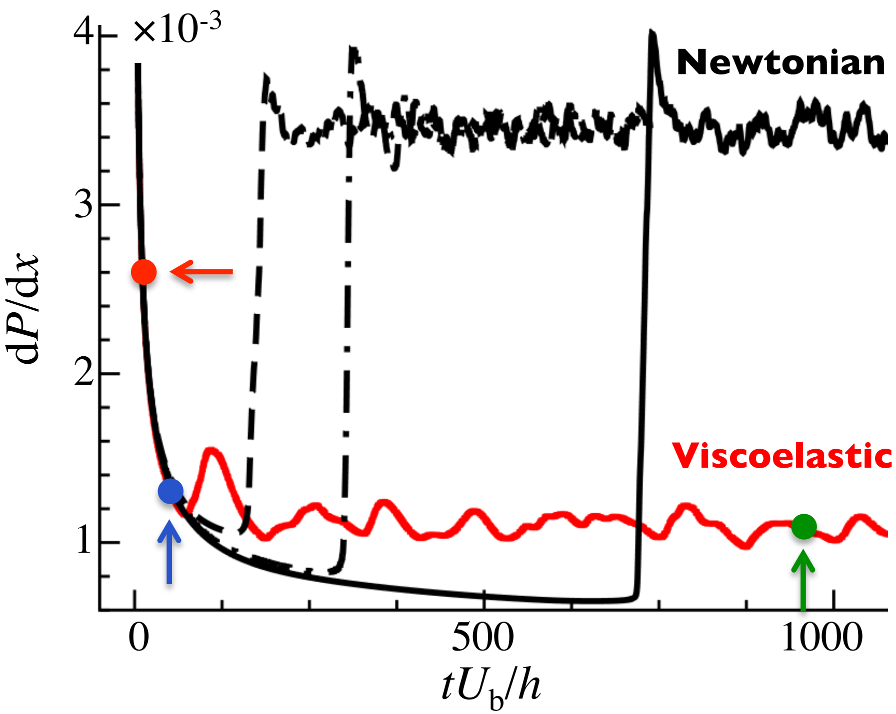
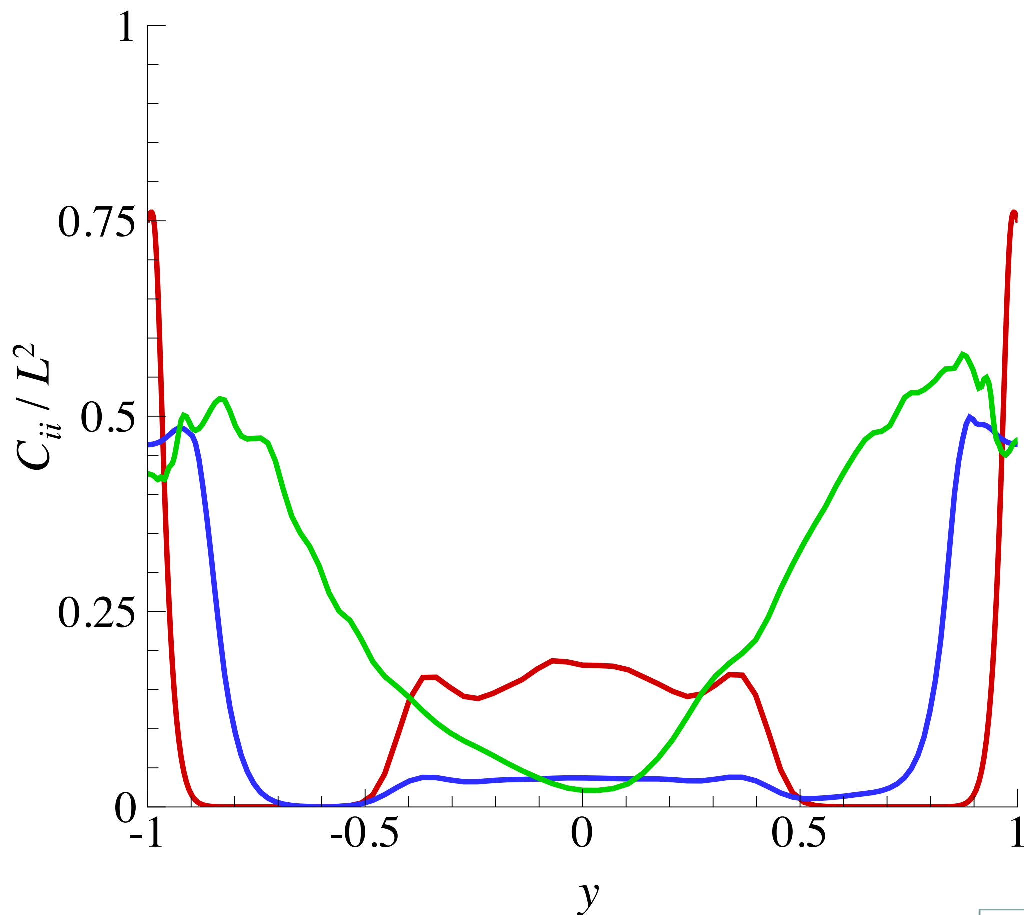
Time evolution of drag



Mean polymer extension



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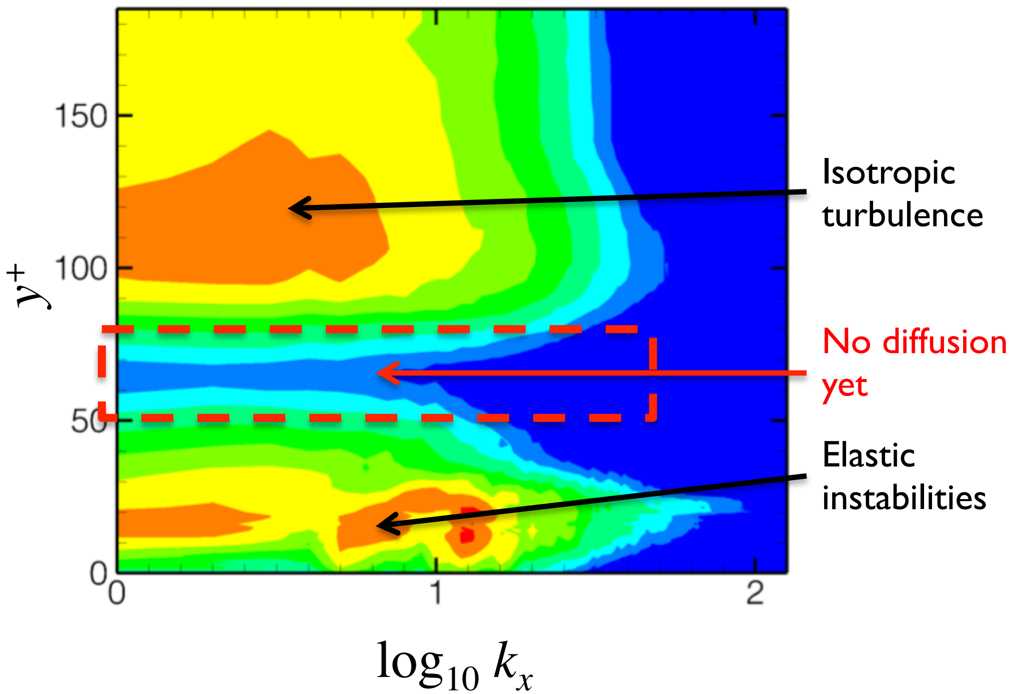
Excitation of instabilities

Power spectrum of the elastic energy before nonlinear breakdown of instabilities



- Instabilities in near-wall regions not caused by diffusion of turbulence from channel center
- What triggers the instability?

Poisson equation for pressure

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Extended Poisson equation

$$\nabla^2 p = \underbrace{2Q_a + \frac{1-\beta}{Re}\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\mathbf{T})}_{f(\mathbf{x})}$$

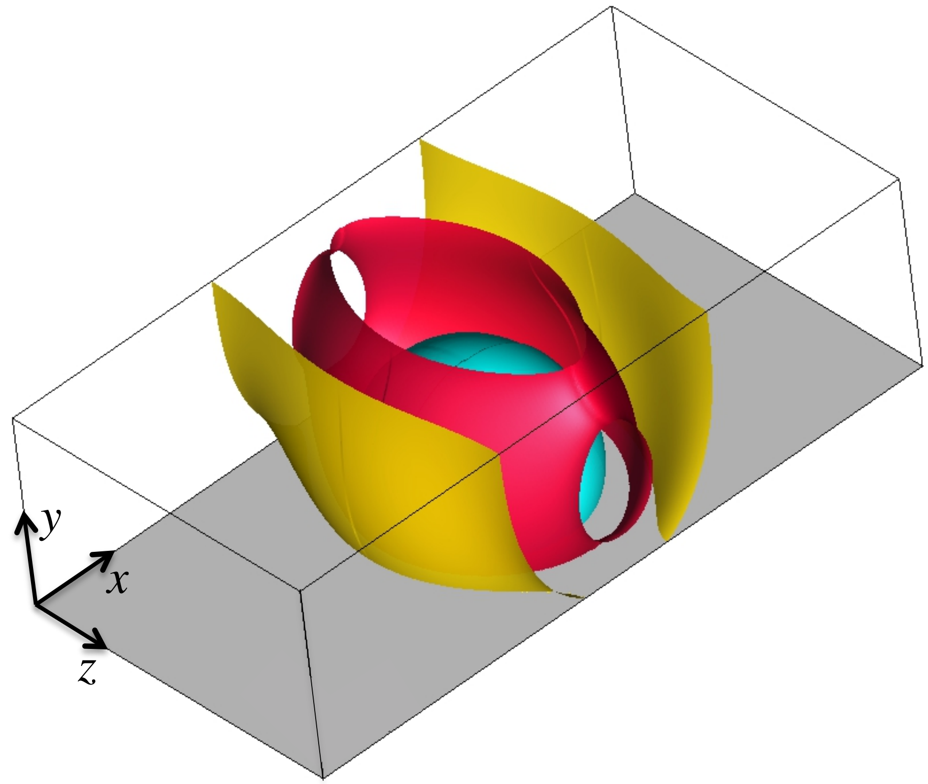
+ B.C.

Pressure kernel – Green function G

$$p(\boldsymbol{\xi}) = \int_V G(\boldsymbol{\xi},\mathbf{x})f(\mathbf{x})\mathrm{d}\mathbf{x} \equiv \int_V F(\boldsymbol{\xi},\mathbf{x})\mathrm{d}\mathbf{x}$$

"Influence" function $F(\mathbf{x}, \xi)$ represents the contribution of point $\mathbf{x}$ to the pressure at point ξ

Green function $G(0, -0.9H, 0; x, y, z)$



Influence function

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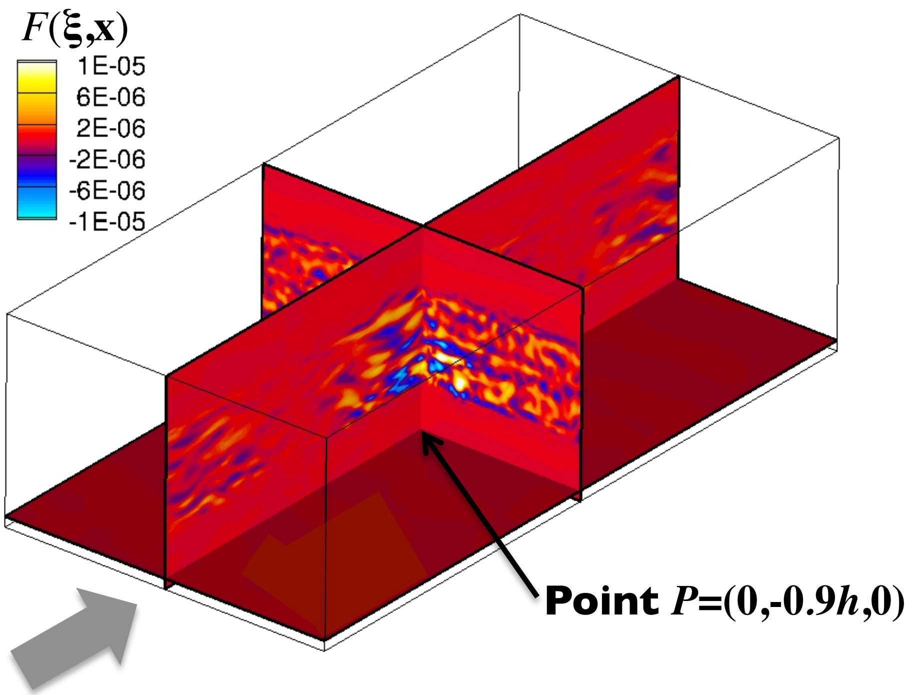
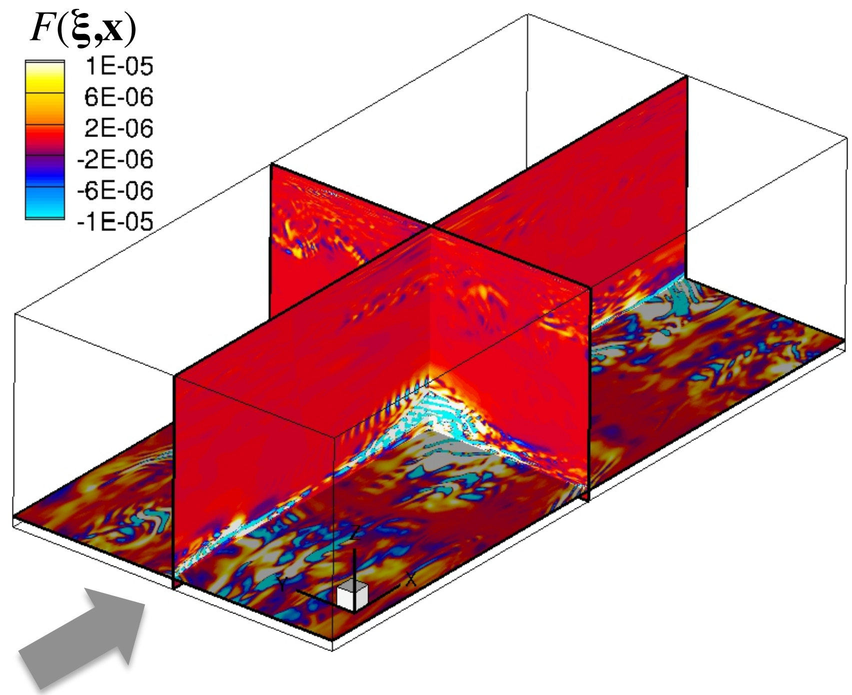
Before nonlinear breakdown of instabilities



Source of contribution to pressure at point P from relative "unorganized" free-stream turbulence in channel center

After nonlinear breakdown of instabilities



Source of contribution to pressure at point P from elastically induced more "organized" structures in near-wall region

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Plane-averaged pressure fluctuations

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$$\nabla^2 p = 2Q_a - \frac{1-\beta}{Re}\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\mathbf{T})$$

$$p(\boldsymbol{\xi}) = \int_V G(\boldsymbol{\xi},\mathbf{x}) f(\mathbf{x})\mathrm{d}\mathbf{x}$$

$$\overline{p^2}(\eta) = \int_{-h}^{h} h(y;\eta)\,\mathrm{d}y$$

- $h(y;\eta)$ represents the influence of plane y on the pressure fluctuations averaged over plane η
- Three contributions
  - Q_a alone
  - $\mathbf{T}$ alone
  - the interaction of Q_a and $\mathbf{T}$

Plane-averaged pressure fluctuations

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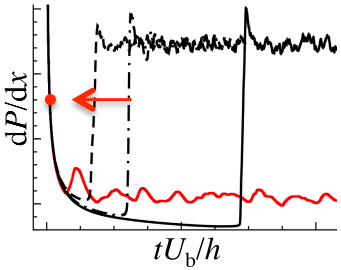
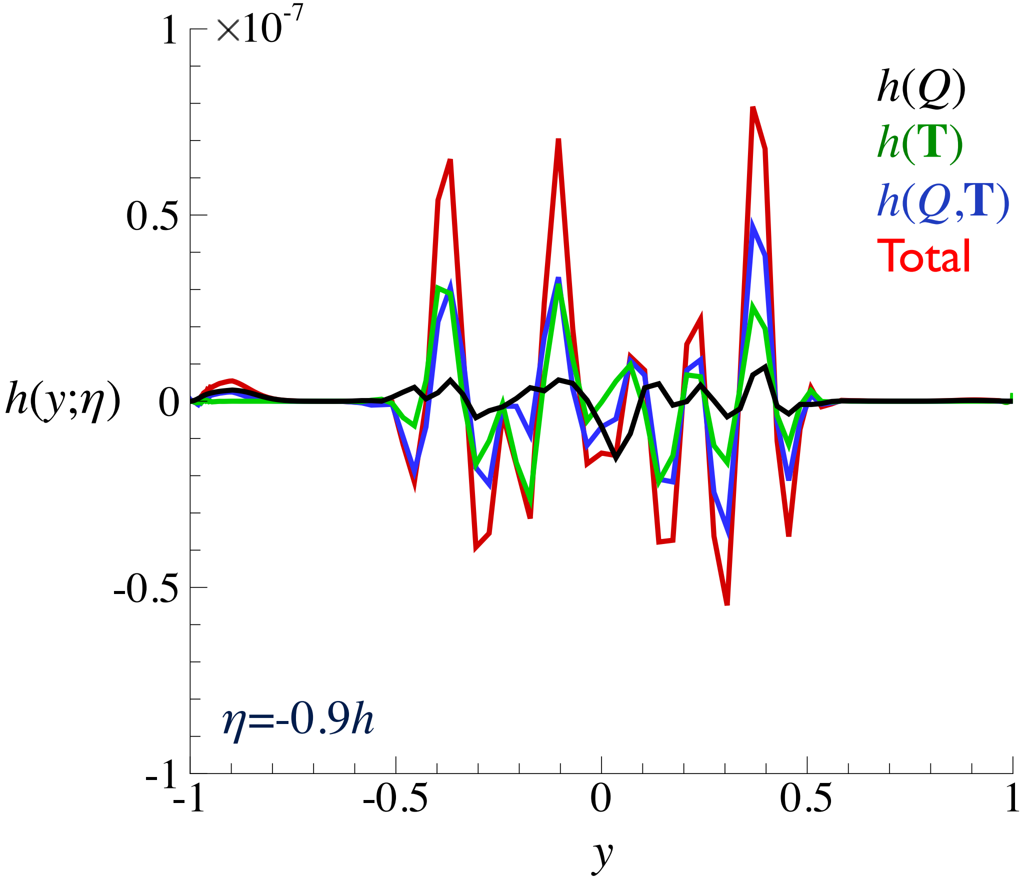


$$\overline{p^2}(\eta) = \int_{-h}^{h} h(y;\eta)\,\mathrm{d}y$$



Before nonlinear breakdown of instabilities

- Major contribution from
  - free-stream turbulence at center of the channel
  - polymer stress

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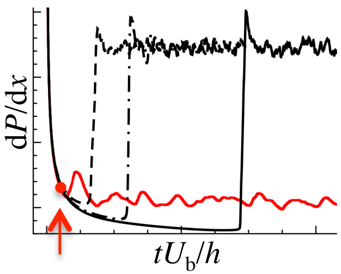
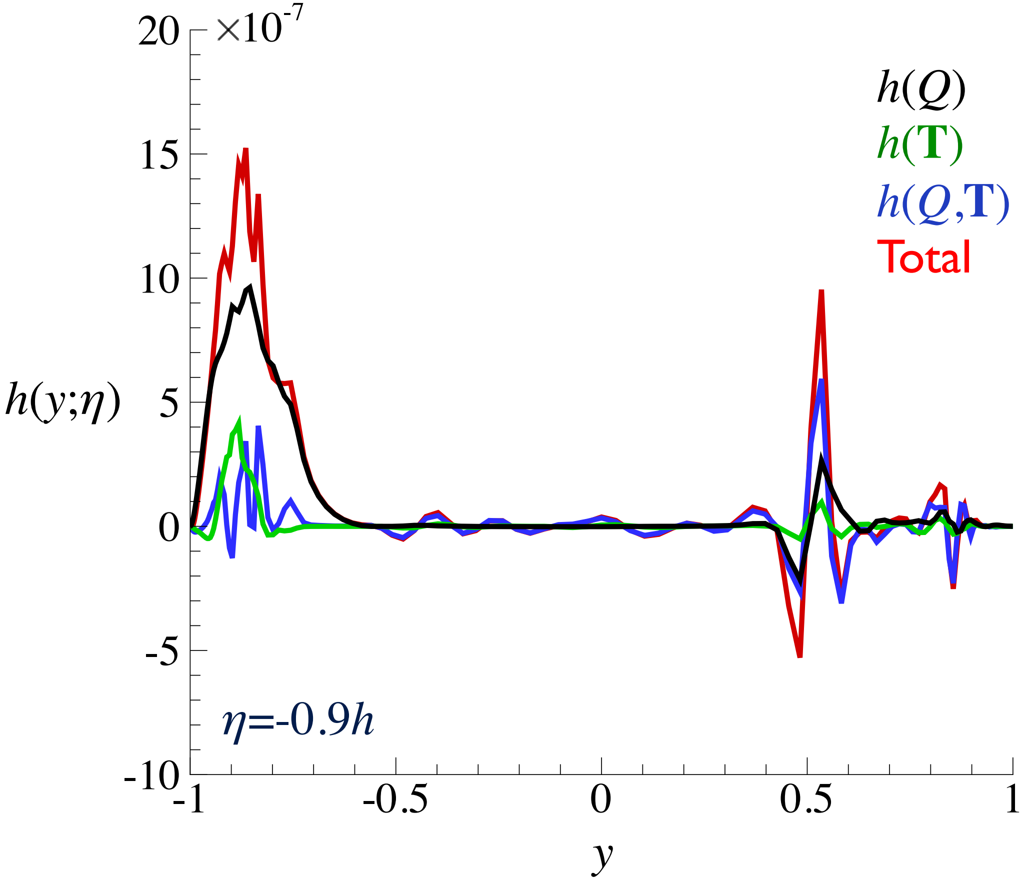
Plane-averaged pressure fluctuations



$$\overline{p^2}(\eta) = \int_{-h}^{h} h(y;\eta)\,\mathrm{d}y$$



After nonlinear breakdown of instabilities

- Major contribution from
  - near-wall region
  - Q
- Contribution from free-stream turbulence negligible

Plane-averaged pressure fluctuations

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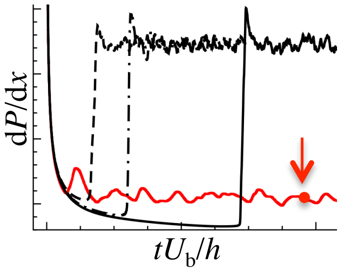
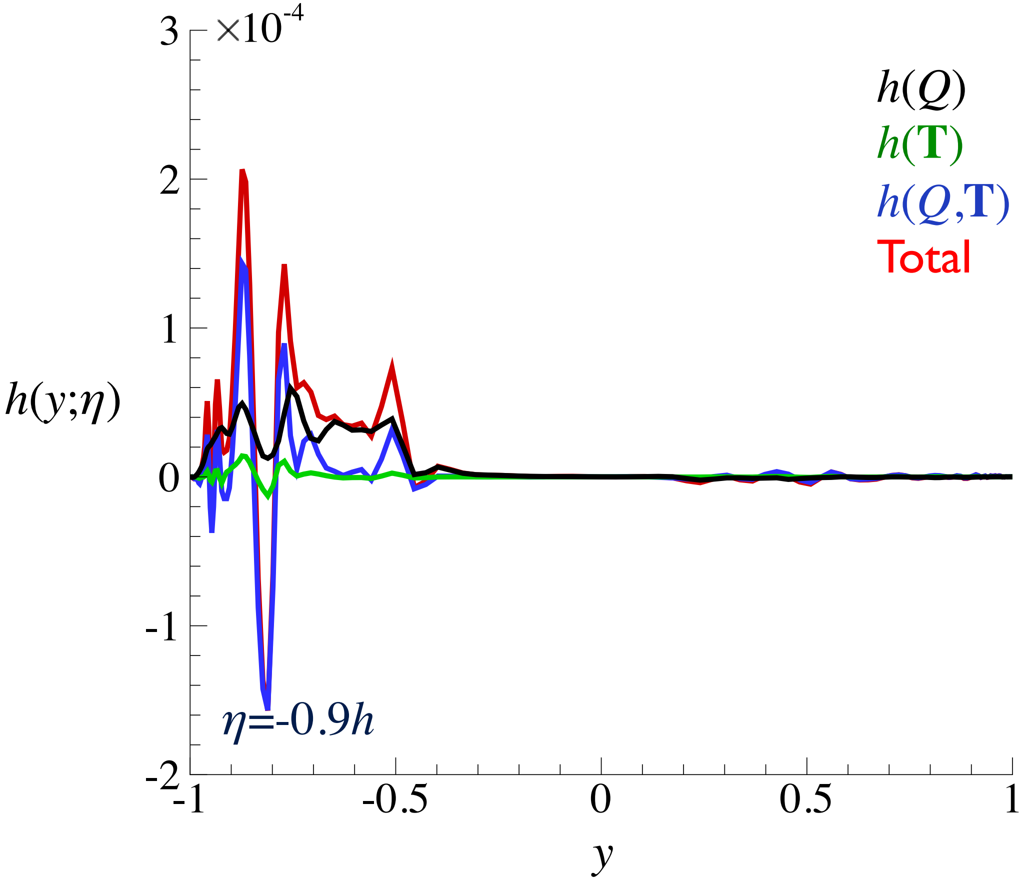


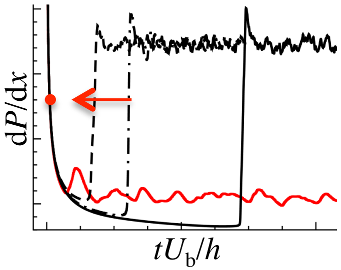
$$\overline{p^2}(\eta) = \int_{-h}^{h} h(y;\eta)\,\mathrm{d}y$$



Much after nonlinear breakdown of instabilities

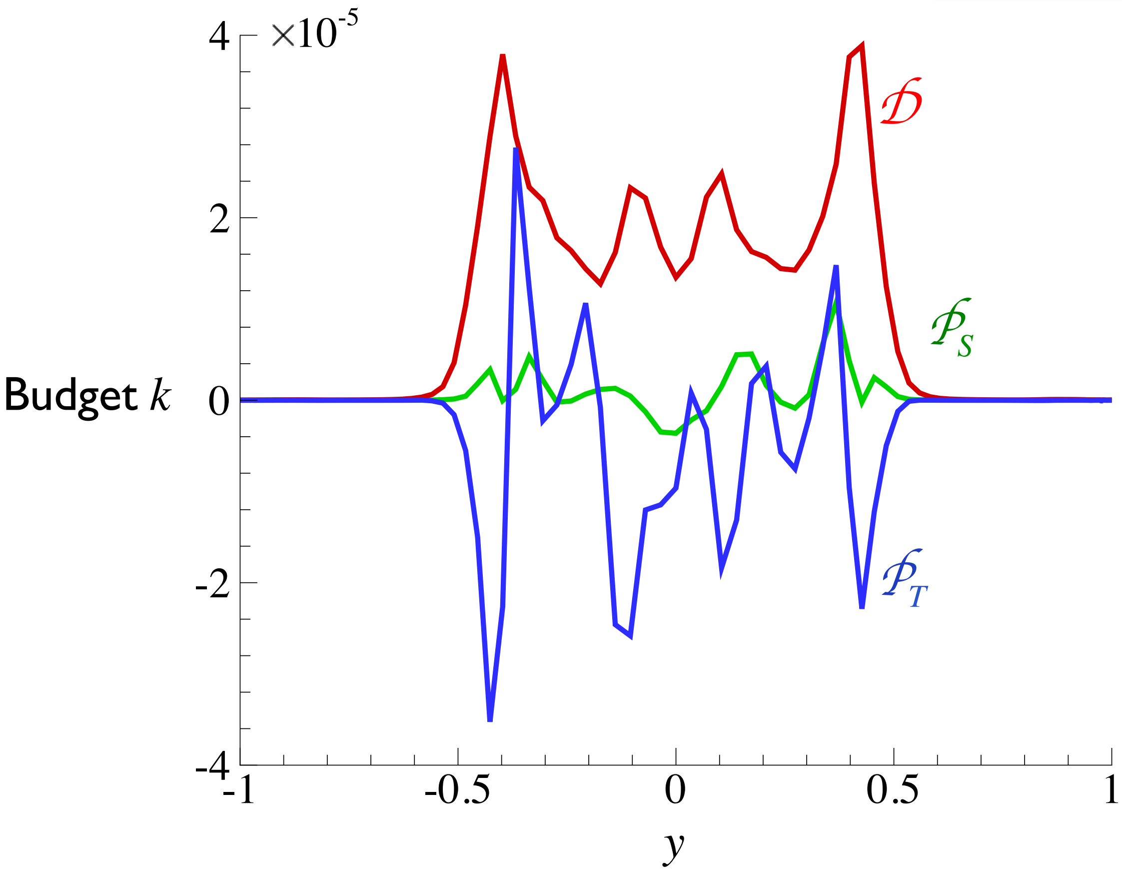
- Major contribution from
  - near-wall region
  - the interaction between Q and $\mathbf{T}$
- Free-stream turbulence fully decayed
- Activity mostly at lower wall at this specific instant

Budget of turbulent kinetic energy



$$\frac{\partial k}{\partial t} = \mathcal{T}_k + \mathcal{P}_S - \mathcal{P}_T - \mathcal{D}$$

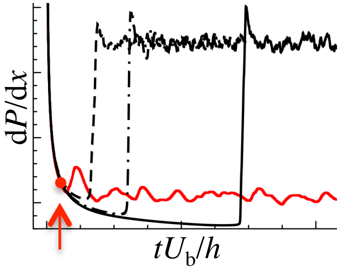
Transport
Production from mean shear
Dissipation
Production of elastic energy



Before nonlinear breakdown of instabilities

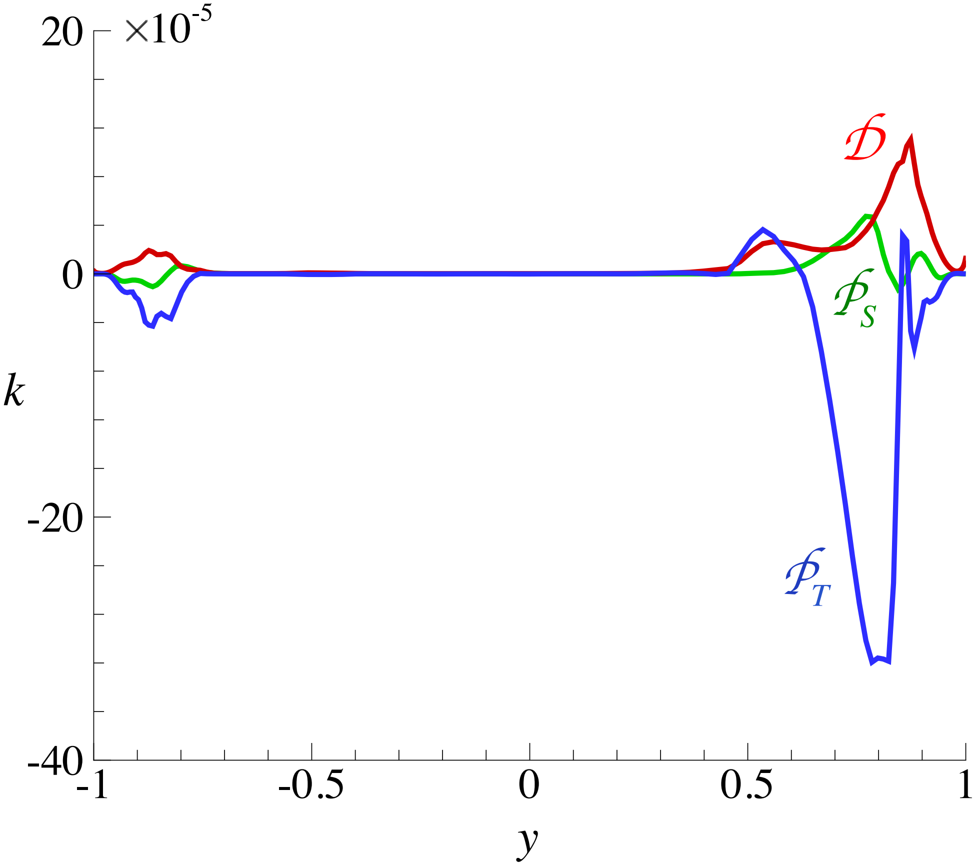
- Dissipation dominates
- Larger production of k from polymers
- Free-stream turbulence decays

Budget of turbulent kinetic energy



$$\frac{\partial k}{\partial t} = \mathcal{T}_k + \mathcal{P}_S - \mathcal{P}_T - \mathcal{D}$$

Transport
Production from mean shear
Dissipation
Production of elastic energy



After nonlinear breakdown of instabilities

- Production of k from polymers dominates
- Larger values at upper wall

Budget of turbulent kinetic energy

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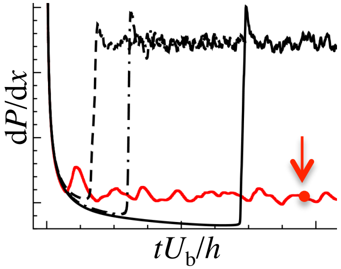
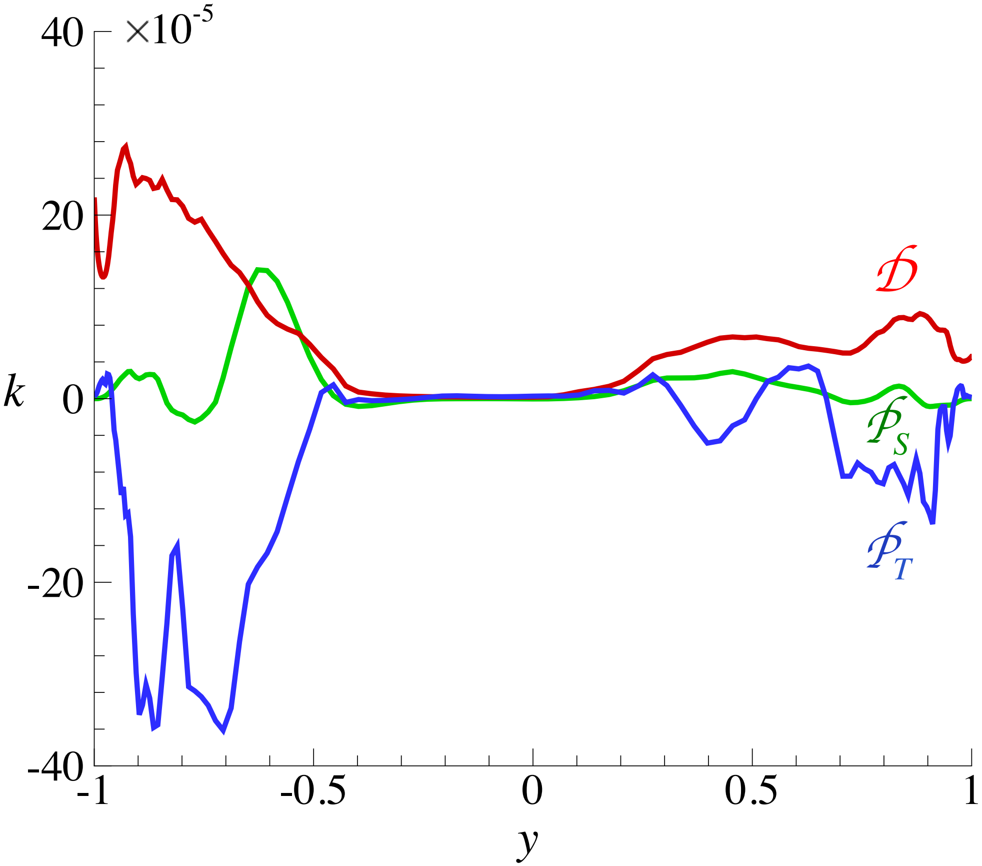


$$\frac{\partial k}{\partial t} = \mathcal{T}_k + \mathcal{P}_S - \mathcal{P}_T - \mathcal{D}$$

Transport
Production from mean shear
Dissipation
Production of elastic energy



Much after nonlinear breakdown of instabilities

- Production of k from polymers dominates
- Larger values at lower wall

Outline

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- Context
- Models and numerical implementation
- Polymer drag reduction
- Elasto-inertial turbulence
- **Conclusions and future work**

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Conclusion

Viscoelasticity leads to new phenomena

- Common to many fluids (e.g., blood)
- Viscoelasticity can dramatically reduce drag at higher Reynolds number
- Viscoelasticity can promote departure from laminar state at lower Reynolds number
- Elasto-Inertial Turbulence (EIT) identified as new regime explaining this seemingly contradictory behavior

Drag reduction

- Up to 80% drag reduction can be achieved with dilute polymer or surfactant solutions
- Maximum drag reduction achievable bounded by MDR asymptote
- Polymers stretched in bi-axial flow regions around near-wall vortices
- Polymers damp quasi-streamwise vortices in near-wall region
- Polymers can store kinetic energy from the flow as elastic energy
- Polymers release elastic energy to the flow in high-speed streaks

Conclusion

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Maximum drag reduction

- MDR is transitional state corresponding to onset of nonlinear breakdown of instabilities
- MDR flow oscillates between pre- and post-breakdown state
- No apparent logarithmic scaling of velocity at MDR or in transitional flows
- Elastic instabilities at small scales contribute to regeneration of vortices

Elasto-inertial turbulence

- EIT identified as new regime describing both MDR and early turbulence
- Hyperbolic transport equation leads to creation of thin sheets of high polymer extension and large extensional viscosity
- Self-sustained chaotic flow consisting of trains of cylindrical weakly rotational and extensional regions
- Pressure redistributes energy with trains of cylindrical structures to attenuate the anisotropy caused by sheets of extensional viscosity
- Transfer of elastic energy from polymers into turbulent kinetic energy of the flow

Conclusion

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Transition

- Long-range excitation of elastic instabilities through pressure
- Feeding of energy to elastic instabilities from mean flow
- Transfer of elastic energy from the polymers into turbulent kinetic energy of the flow
- Elastic instabilities self-sustained

Numerics

- Accurate numerical approach is required
- Small-scales must be captured to simulate EIT
- Use of global artificial dissipation prevents formation of elastic instabilities

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Conclusion

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Overall

- Viscoelastic turbulent flows are a very exciting area
- EIT explains seemingly contradictory phenomena in viscoelastic turbulence
- EIT provides support to de Gennes' theory
- Understanding EIT is relevant for biofluids, but also to better understand and control Newtonian turbulence

Future work

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- Further characterize EIT
- Understand the exact mechanisms during transition process
- Develop new control strategies for turbulent flows
- Identify potential role of EIT in biofluid flows

Future work – DMD analysis

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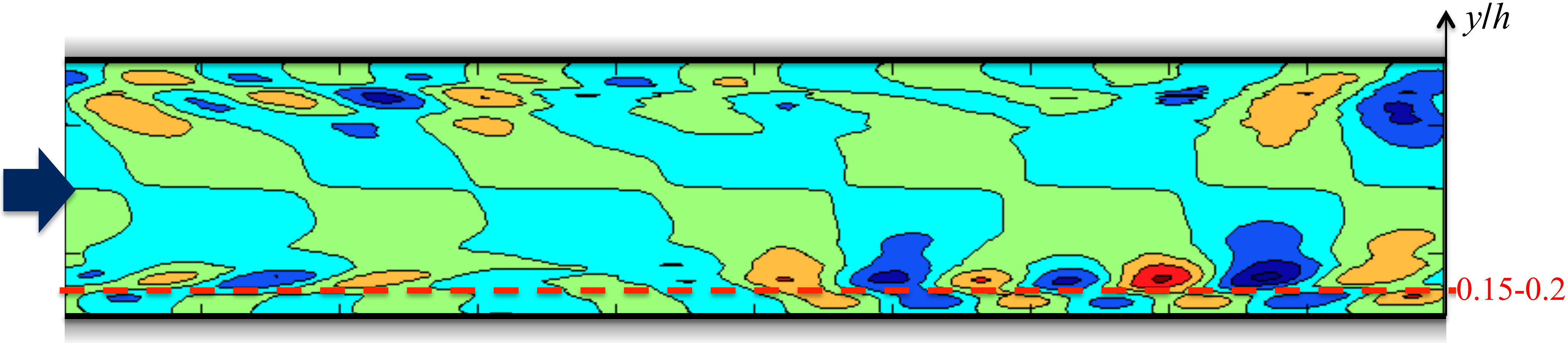
Most amplified mode from DMD analysis

Q_a invariant (streamwise – wall-normal plane) at Re=1000

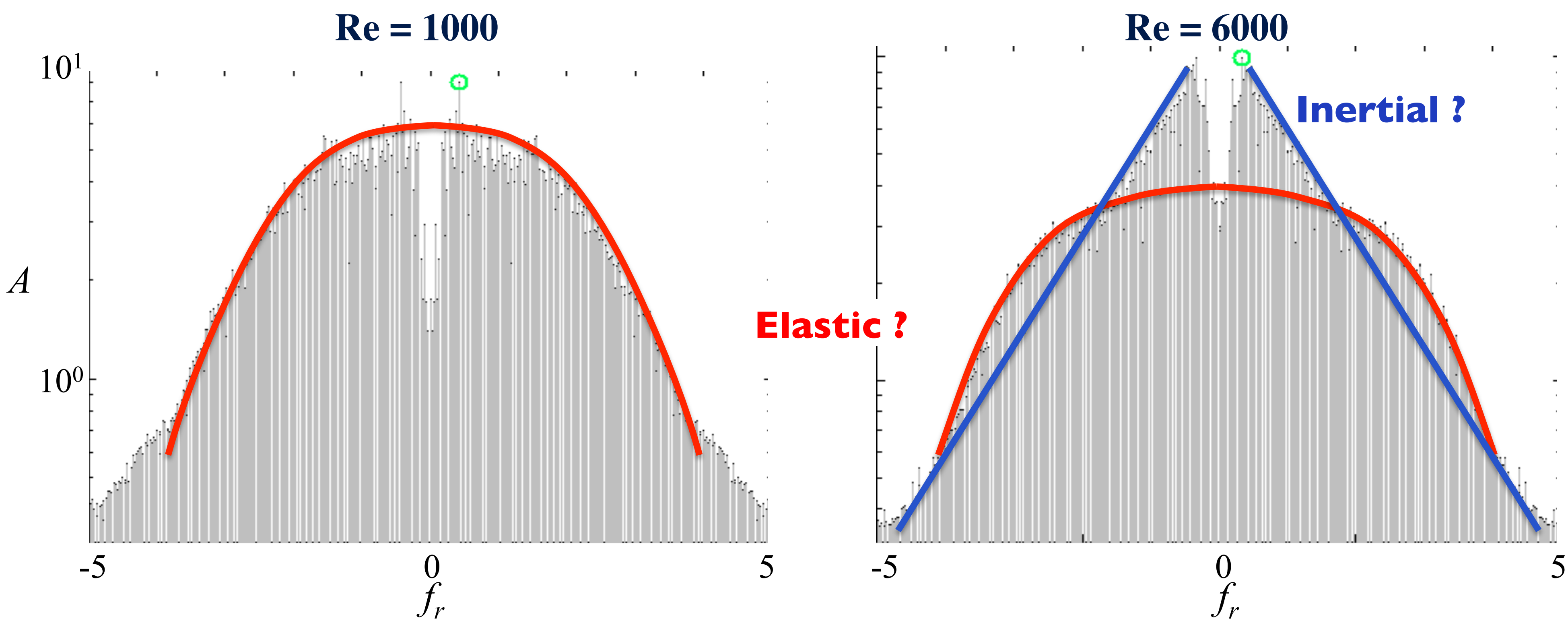


- Mostly two-dimensional structures
- Located in near-wall region
- Alternating pressure minima and maxima
- “Discontinuity” close to wall corresponding to location of maximum of mean polymer extension (critical layer?)

Future work – DMD analysis

Mode amplitude as a function of frequency from DMD analysis

Q_a invariant (streamwise – wall-normal plane)



- Shape change with increasing Re
- At larger Re, two apparent contributions
- Hypothesis: elastic and inertial (to be verified)