Applications of Security-Constrained Optimal Power Flows

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Abstract - **This paper deals with the Security-Constrained Optimal Power Flow (SCOPF) problem. We first revisit both preventive and corrective variants of the SCOPF problem. Then we present the nonlinear Interior-Point Method (IPM) which we use for the solution of the SCOPF problem. Next, we provide numerical results, on a 60-bus test system, for three main SCOPF applications, namely: minimum overall cost of generation, minimum cost of removing congestion and maximum power transfer computation. We finally discuss some critical issues related to the SCOPF problem.**

Keywords: **optimal power flow, security-constrained optimal power flow, interior-point method**.

1 INTRODUCTION

The Optimal Power Flow (OPF) problem was introduced in the 60's [1, 2]. The main limitation of OPF formulation is that it focuses on the optimization of a single system configuration at the time while the System Operator (SO) needs to know: (i) how robust the system is with respect to various credible contingencies and (ii) how (much costs) to meet also operating constraints for these contingencies, beside the pre-contingency ones. While the item (i) can be tackled by a mere security analysis performed at the OPF solution, the item (ii) requires the development of new tools. That led to the formulation of the Security-Constrained Optimal Power Flow (SCOPF) problem [3] as a natural extension of the OPF which takes into account pre-contingency (base case) constraints and also (steady-state) post-contingency constraints together. Nowadays, modern SCOPF software handle also (voltage or transient) stability constraints, most often expressed as surrogate power flow limits for some interfaces [4, 5].

The SCOPF problem is formulated in its general form as nonlinear, non-convex, large-scale, static optimization problems with both continuous and discrete variables. It aims at optimizing some objective by acting on available control means while satisfying some equality constraints (e.g., network power flow equations) and inequality constraints (e.g., physical and operational limits of equipments). These constraints relate to both base case and some plausible (mainly "N-1") post-contingency states.

As regards the SCOPF problem formulation two approaches can be distinguished: "preventive" [3] and "corrective" [6]. The difference between these variants is that, in the preventive SCOPF, the re-scheduling of control variables in post-contingency states is not allowed, except of those with automatic response to contingencies (e.g., the active power of generators participating at primary or secondary frequency control, transformers controlling some voltages, etc).

Nowadays (SC)OPF computations are essential in power systems planning, operational planning and real-time operation, both in an integrated and a deregulated electricity industry [7, 8]. This paper presents some key SCOPF applications in such environments: minimum overall generation cost [3, 7], minimum cost of removing congestion [6, 9] and maximum power transfer computation [10].

Among the above mentioned SCOPF objectives, the determination of the security-constrained (economic) generation dispatch is no doubt the most important (SC)OPF application in both deregulated and vertically integrated environments. In particular, OPF or SCOPF are used, admittedly mostly still based on the DC approximation, in some deregulated environments to compute *locational marginal prices* (or *nodal prices*) for electricity [8].

In a deregulated environment the SO is responsible to remove (generally at the least cost) congestion in both day-ahead market and real-time. A system is said to be "congested" when some predefined operating and/or security constraints (e.g., thermal, voltage magnitude, voltage stability, angle stability, etc.) are violated in the current or in a foreseen operating state. Security constraints refer to "N" and some plausible "N-K" system configurations. Congestion management consists in controlling the system such that all security constraints are satisfied. Clearly, power systems were confronted to congestions also in a vertically integrated environment. In such an environment the congestion management most often consists in modifying the economic dispatch at the least cost such that no security constraint is violated.

Finally, the computation of maximum power transfer (loadability margin) between two ("source"-"sink") areas of a power system is extremely important, especially in a deregulated environment where one has to allocate in a fair manner transmission capacity among various simultaneous power transactions. In a deregulated environment such loadability margin is called Total Transfer Capability (TTC) and is, in turn, the major component for the determination of Available Transfer Capability (ATC) between areas.

We solve the SCOPF problem by the Interior-Point Method (IPM) [11]. The latter is a very appealing method mainly due to its speed of convergence and ease of handling inequality constraints.

The remaining of the paper is organized as follows.

Section 2 introduces successively the preventive and the corrective approaches to the SCOPF problem. Section 3 presents the basics of the IPM. Section 4 provides numerical results for three main SCOPF applications. Some critical issues relative to the SCOPF problem are discussed in Section 5. Finally, some conclusions and future works are presented in Section 6.

2 STATEMENT OF THE SCOPF PROBLEM

2.1 The Preventive Approach

The benchmark of the Preventive Security-Constrained Optimal Power Flow (PSCOPF) problem can be compactly formulated as follows:

$$\min_{\mathbf{x}_0,\ldots,\mathbf{x}_c,\mathbf{u}_0} f(\mathbf{x}_0, \mathbf{u}_0) \tag{1}$$

$$\text{s.t.} \quad \mathbf{g}_k(\mathbf{x}_k, \mathbf{u}_0) = \mathbf{0} \quad k = 0, \ldots, c \tag{2}$$

$$\mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_0) \geq \mathbf{0} \quad k = 0, \ldots, c \tag{3}$$

where f is a (real-valued) function representing the objective to optimize, $\mathbf{g}_k$ (resp. $\mathbf{h}_k$) is the set of equality (resp. inequality) constraints for the k-th system configuration ($k = 0$ corresponds to the base case, while $k = 1, \ldots, c$ corresponds to the k-th post-contingency state, c being the number of contingencies considered), $\mathbf{x}_k$ is the vector of state variables (i.e., real and imaginary part of voltage at all buses[1]) for the k-th system topology and $\mathbf{u}_0$ is the vector of base case control variables (e.g., active and reactive generator powers, controllable transformer ratio, shunt element reactance, load apparent power, etc.).

Equations (2) mainly refer to base case and post-contingency network power flow equations. Inequality constraints (3) concern physical limits of equipments (e.g., bounds on active and reactive generator powers, controllable transformer ratio, shunt element reactance, etc.), (steady-state) security operational limits (e.g., mainly limits on branch currents and voltage magnitudes) and possibly transient and/or voltage stability limits [4, 5].

Incidentally, the OPF problem may be stated as (1-3) where equality and inequality constraints (2) and (3) are written for the base case only ($k = 0$). Thus, the size of the PSCOPF problem is $(c + 1)$ times greater than that of the OPF.

Observe that in PSCOPF one acts only on the base case control means $\mathbf{u}_0$ while trying to satisfy both base case and post-contingency (equality and inequality) constraints. Indeed, in the preventive SCOPF the re-scheduling of control variables in post-contingency states is not allowed, except of those with automatic response to contingencies, e.g., the active power of generators participating at primary or secondary frequency control, transformers controlling some voltages, etc. These control variable changes between base case and post-contingency have not be shown explicitly in the PSCOPF model (1-3) to lighten the presentation.

Since the SO actions and/or some corrective schemes are not modeled in post-contingency states, PSCOPF may lead to (very) high operating costs and can hence be seen as a bit conservative [6, 9]. In addition, especially under stressed operating conditions, there could be no feasible solution to the PSCOPF problem or the latter could be very constrained, and consequently more difficult to obtain [5]. On the other hand, post-contingency limits (e.g., on branch currents and voltage magnitudes) can be somewhat relaxed with respect to the base case ones, hence expanding the feasible region. Anyway, the PSCOPF indicates the price to pay for ensuring system security with respect to a set of plausible contingencies.

[1] we have used a rectangular model to express (complex) bus voltages

2.2 The Corrective Approach

The corrective approach to the SCOPF problem relies on the fact that some post-contingency constraint violation (e.g., thermal, bus voltage magnitude, etc.) can be endured up to (at least) several minutes without damaging the corresponding equipment, which lets enough time for some (automatic or not) corrective actions to be implemented.

The benchmark of the Corrective Security-Constrained Optimal Power Flow (CSCOPF) problem can be compactly stated as follows [9]:

$$\min_{\mathbf{x}_0,\ldots,\mathbf{x}_c,\mathbf{u}_0,\ldots,\mathbf{u}_c} f(\mathbf{x}_0, \mathbf{u}_0) \tag{4}$$

$$\text{s.t.} \quad \mathbf{g}_k(\mathbf{x}_k, \mathbf{u}_k) = \mathbf{0} \quad k = 0, \ldots, c \tag{5}$$

$$\mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_k) \geq \mathbf{0} \quad k = 0, \ldots, c \tag{6}$$

$$|\mathbf{u}_k - \mathbf{u}_0| \leq \Delta\mathbf{u}_k^{max} \quad k = 1, \ldots, c \tag{7}$$

The main difference with respect to the preventive SCOPF approach (1-3) stems from the allowing of post-contingency control variables rescheduling in order to remove contingent constraints violation. However, in order to prevent unrealistic variations of control variables under the effect of a contingency, "coupling" constraints between the base case and post-contingency values of control variables are included (7). $\Delta\mathbf{u}_k^{max}$ represents the vector of maximal allowed variation of control variables between the base case and k-th post-contingency state. This maximal bound is determined by both the time allowed for correction and control variables rate of changes in response to a contingency.

Note that, the PSCOPF can be seen as a particular case of a CSCOPF, obtained for $\Delta\mathbf{u}_k^{max} = \mathbf{0},\ \forall k = 1, \ldots, c$. Observe also that the value of the objective of a CSCOPF is lower (resp. upper) bounded by the value of the objective of a OPF (resp. PSCOPF).

3 INTERIOR-POINT METHOD

3.1 Obtaining the Optimality Conditions

The SCOPF formulations (1-3) and (4-7) can be compactly written as a general nonlinear programming problem:

$$\min_{\mathbf{y}} f(\mathbf{y}) \tag{8}$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{y}) = \mathbf{0} \tag{9}$$

$$\mathbf{h}(\mathbf{y}) \geq \mathbf{0} \tag{10}$$

where $f(\mathbf{y})$, $\mathbf{g}(\mathbf{y})$ and $\mathbf{h}(\mathbf{y})$ are assumed to be twice continuously differentiable, $\mathbf{y}$ is an m-dimensional vector that encompasses both control variables ($\mathbf{u}_k$) and state variables ($\mathbf{x}_k$), $\mathbf{g}$ is a p-dimensional vector of functions and $\mathbf{h}$ is a q-dimensional vector of functions.

The IPM encompasses four steps to obtain optimality conditions. One first adds slack variables to inequality constraints, transforming them into equality constraints and non-negativity conditions on slacks:

$$\begin{aligned} &\min_{\mathbf{y}} f(\mathbf{y}) \\ \text{s.t.} \quad &\mathbf{g}(\mathbf{y}) = \mathbf{0} \\ &\mathbf{h}(\mathbf{y}) - \mathbf{s} = \mathbf{0} \\ &\mathbf{s} \geq \mathbf{0} \end{aligned}$$

where the vectors $\mathbf{y}$ and $\mathbf{s} = [s_1, \ldots, s_q]^T$ are called *primal variables*.

The inequality constraints are then eliminated by adding them to the objective function as logarithmic barrier terms, resulting in the following equality constrained optimization problem:

$$\begin{aligned} &\min_{\mathbf{y}} f(\mathbf{y}) - \mu \sum_{j=1}^{q} \ln s_j \\ \text{subject to:} \quad &\mathbf{g}(\mathbf{y}) = \mathbf{0} \\ &\mathbf{h}(\mathbf{y}) - \mathbf{s} = \mathbf{0} \end{aligned}$$

where μ is a positive scalar called *barrier parameter* which is gradually decreased to zero as iterations progress. Let us remark that at the heart of IPM is the main theorem from [11], which proves that as μ tends to zero, the solution $\mathbf{y}(\mu)$ converges to a local optimum of the problem (8-10).

Next, one transforms the equality constrained optimization problem into an unconstrained one, by defining the Lagrangian:

$$L_\mu(\mathbf{z}) = f(\mathbf{y}) - \mu \sum_{j=1}^{q} \ln s_j - \boldsymbol{\lambda}^T \mathbf{g}(\mathbf{y}) - \boldsymbol{\pi}^T [\mathbf{h}(\mathbf{y}) - \mathbf{s}]$$

where the vectors of Lagrange multipliers $\boldsymbol{\lambda}$ and $\boldsymbol{\pi}$ are called *dual variables* and $\mathbf{z} = [\mathbf{s} \;\; \boldsymbol{\pi} \;\; \boldsymbol{\lambda} \;\; \mathbf{y}]^T$.

Finally, the *perturbed* KKT first order necessary optimality conditions of the resulting problem are obtained by setting to zero the derivatives of the Lagrangian with respect to all unknowns [11]:

$$\begin{bmatrix} \nabla_{\mathbf{s}} L_\mu(\mathbf{z}) \\ \nabla_{\pi} L_\mu(\mathbf{z}) \\ \nabla_{\lambda} L_\mu(\mathbf{z}) \\ \nabla_{\mathbf{y}} L_\mu(\mathbf{z}) \end{bmatrix} = \begin{bmatrix} -\mu\mathbf{e} + \mathbf{S}\,\boldsymbol{\pi} \\ -\mathbf{h}(\mathbf{y}) + \mathbf{s} \\ -\mathbf{g}(\mathbf{y}) \\ \nabla f(\mathbf{y}) - \mathbf{J}_{\mathbf{g}}(\mathbf{y})^T \boldsymbol{\lambda} - \mathbf{J}_{\mathbf{h}}(\mathbf{y})^T \boldsymbol{\pi} \end{bmatrix} = \mathbf{0}$$

where $\mathbf{S}$ is a diagonal matrix of slack variables, $\mathbf{e} = [1, \ldots, 1]^T$, $\nabla f(\mathbf{y})$ is the gradient of f, $\mathbf{J}_{\mathbf{g}}(\mathbf{y})$ is the Jacobian of $\mathbf{g}(\mathbf{y})$ and $\mathbf{J}_{\mathbf{h}}(\mathbf{y})$ is the Jacobian of $\mathbf{h}(\mathbf{y})$.

3.2 The Primal Dual Interior Point Algorithm

We briefly outline the primal dual interior point (PDIP) algorithm to solve the perturbed KKT optimality conditions:

1. Set iteration count $i = 0$. Chose $\mu^0 > 0$. Initialize $\mathbf{z}^0$, taking care that slack variables and their corresponding dual variables are strictly positive $(\mathbf{s}^0, \boldsymbol{\pi}^0) > \mathbf{0}$.

2. Solve the linearized KKT conditions for the Newton direction $\Delta\mathbf{z}^i$:

$$\mathbf{H}(\mathbf{z}^i) \begin{bmatrix} \Delta\mathbf{s}^i \\ \Delta\boldsymbol{\pi}^i \\ \Delta\boldsymbol{\lambda}^i \\ \Delta\mathbf{y}^i \end{bmatrix} = \begin{bmatrix} \mu^i\mathbf{e} - \mathbf{S}^i\boldsymbol{\pi}^i \\ \mathbf{h}(\mathbf{y}^i) - \mathbf{s}^i \\ \mathbf{g}(\mathbf{y}^i) \\ -\nabla f(\mathbf{x}^i) + \mathbf{J}_{\mathbf{g}}(\mathbf{y}^i)^T \boldsymbol{\lambda}^i + \mathbf{J}_{\mathbf{h}}(\mathbf{y}^i)^T \boldsymbol{\pi}^i \end{bmatrix}$$

   where $\mathbf{H}(\mathbf{z}^i)$ is the second derivative Hessian matrix $(\partial^2 L_\mu(\mathbf{z}^i)/\partial\mathbf{z}^2)$.

3. Determine the maximum step length $\alpha^i \in (0, 1]$ along the Newton direction $\Delta\mathbf{z}^i$ such that $(\mathbf{s}^{i+1}, \boldsymbol{\pi}^{i+1}) > \mathbf{0}$:

$$\alpha^i = \min\left\{1, \gamma \min_{\Delta s_j^i < 0} \frac{-s_j^i}{\Delta s_j^i}, \gamma \min_{\Delta \pi_j^i < 0} \frac{-\pi_j^i}{\Delta \pi_j^i}\right\} \quad (11)$$

   where $\gamma \in (0, 1)$ is a safety factor aiming to ensure strict positiveness of slack variables and their corresponding dual variables. A typical value of the safety factor is $\gamma = 0.99995$. Update solution:

$$\begin{aligned} \mathbf{s}^{i+1} &= \mathbf{s}^i + \alpha^i \Delta\mathbf{s}^i & \boldsymbol{\pi}^{i+1} &= \boldsymbol{\pi}^i + \alpha^i \Delta\boldsymbol{\pi}^i \\ \mathbf{y}^{i+1} &= \mathbf{y}^i + \alpha^i \Delta\mathbf{y}^i & \boldsymbol{\lambda}^{i+1} &= \boldsymbol{\lambda}^i + \alpha^i \Delta\boldsymbol{\lambda}^i \end{aligned}$$

4. Check convergence. A (locally) optimal solution is found and the optimization process terminates when: primal feasibility, scaled dual feasibility, scaled complementarity gap and objective function variation from an iteration to the next fall below some tolerances [12]:

$$\max\left\{\max_j \{-h_j(\mathbf{y}^i)\}, ||\mathbf{g}(\mathbf{y}^i)||_\infty\right\} \leq \epsilon_1$$

$$\frac{||\nabla f(\mathbf{y}^i) - \mathbf{J}_{\mathbf{g}}(\mathbf{y}^i)^T \boldsymbol{\lambda} - \mathbf{J}_{\mathbf{h}}(\mathbf{y}^i)^T \boldsymbol{\pi}^i||_\infty}{1 + ||\mathbf{y}^i||_2 + ||\boldsymbol{\lambda}^i||_2 + ||\boldsymbol{\pi}^i||_2} \leq \epsilon_1$$

$$\frac{\rho^i}{1 + ||\mathbf{y}^i||_2} \leq \epsilon_2$$

$$\frac{|f(\mathbf{y}^i) - f(\mathbf{y}^{i-1})|}{1 + |f(\mathbf{y}^i)|} \leq \epsilon_2$$

   where $\rho^i = (\mathbf{s}^i)^T \boldsymbol{\pi}^i$ is called *complementarity gap*.

5. If convergence was not achieved, update the barrier parameter:

$$\mu^{i+1} = \sigma^i \frac{\rho^i}{q}$$

   where usually $\sigma^i = 0.2$. Set $i = i + 1$ and go to step 2.

4 NUMERICAL RESULTS

In this section we present numerical results, on a modified variant of the "Nordic32" system [13], for three (SC)OPF applications. A summary of this test system is given in Table 1, where: n, g, d, b, l, t, o and s denote the number of: buses, generators, loads, branches, lines, all transformers, transformers with controllable ratio and shunt elements, respectively.

Table 1: Nordic32 test system summary

n	g	d	b	l	t	o	s
60	23	22	81	57	31	4	12

The (SC)OPF has been coded in C++ and runs under Cygwin or Linux environments. All tests have been performed on a PC Pentium IV of 2.8-GHz and 512-Mb RAM.

The convergence tolerances, appearing at the step 4 of the PDIP algorithm, are set to $\epsilon_1 = 10^{-4}$ and $\epsilon_2 = 10^{-6}$.

For all SCOPF problems that follow, we consider a set of 12 contingencies, typically the loss of significant transmission branches that do not create islands. We also consider the same limit for every type of inequality constraint (e.g., relative to: branch current, voltage magnitude, etc.) in both pre- and post-contingency states.

4.1 Minimizing Overall Generation Cost

We first focus on minimizing the overall generation cost by three different approaches: OPF (no contingency constraints), PSCOPF (1-3) and CSCOPF (4-7).

We also consider two types of optimization problems: problem A and problem B. Problem A corresponds to a classical security-constrained economic dispatch. The control variables used are: generator active and reactive powers. The equality constraints are the bus active and reactive power flow equations. The inequality constraints include bounds on generator active and reactive powers and limits on branch currents. Problem B corresponds to a "full" (SC)OPF. One acts thus on the following control variables: generator active and reactive powers, controllable transformer ratio and shunt reactance. The equality constraints are again the bus active and reactive power flow equations. The inequality constraints include bounds on all above mentioned control variables as well as limits on bus voltage magnitudes and branch currents. The bus voltage magnitudes are allowed to vary between 0.95 pu and 1.05 pu. Finally, for both problems, when using the CSCOPF approach the coupling constraints of type (7) are included. In the latter constraints, the maximal allowed variation of control variables between the base case and every k-th post-contingency state, $\Delta \mathbf{u}_k^{max}$ has been chosen as a fraction p of the physical range of variation of control variables ($\mathbf{u}_{max} - \mathbf{u}_{min}$). We have used the value $p = 0.1$ for every generator active power, and $p = 0.5$ for every shunt reactance or controllable transformer ratio.

Table 2 yields the value of the objective (the overall generation cost) for problems A and B and for the three optimization approaches. As expected, for both problems the value of the objective increases when adding contingency constraints to the basic OPF, as one can observe when comparing the columns labelled OPF and PSCOPF as well as OPF and CSCOPF. One can also observe that the value of the objective is lower for CSCOPF than for PSCOPF (see the last two columns of the table), since post-contingency controls are allowed to vary.

[2] CPU time concerns the optimization process only.

Table 2: Value of the objective (in $/h)

problem	OPF	PSCOPF	CSCOPF
problem A	9560	10126	9619
problem B	9557	10153	9615

Table 3 provides the number of iterations to convergence and the CPU times[2] (in seconds) for different problems and optimization approaches. Note that, a great advantage of using the IPM is that the number of iterations to convergence is generally little influenced by the size of the problem.

Table 3: Number of iterations to convergence and CPU times

problem	OPF		PSCOPF		CSCOPF	
	iters	time	iters	time	iters	time
problem A	23	0.39	39	7.4	29	3.5
problem B	31	0.58	56	13.2	51	16.3

Tables 4 and 5 show the number and the type of binding constraints at optimum for problems A and B, respectively. In these tables columns labelled with P_g, Q_g, I, V, r, x, and cpl refer to constraints relative to generator active power, generator reactive power, branch current, bus voltage magnitude, controllable transformer ratio, shunt reactance and coupling constraints, respectively. The high number of active power generator constraints binding at optimum is due to the fact that we have considered a linear cost curve for all generators participating in the optimization.

Table 4: Number and type of active constraints for problem A

approach	P_g	Q_g	I	cpl	total
OPF	13	0	4	-	17
PSCOPF	12	1	4	-	17
CSCOPF	34	0	3	23	60

Table 5: Number and type of active constraints for problem B

approach	P_g	Q_g	I	V	r	x	cpl	total
OPF	15	0	3	20	0	2	-	40
PSCOPF	13	1	5	190	0	7	-	216
CSCOPF	28	0	4	40	0	7	9	88

Table 6 yields the number of binding contingencies at the optimum, i.e., those contingencies that lead to active post-contingency constraints different than the base case ones. Obviously, when performing the SCOPF computation subject only to the binding contingencies one obtains the same optimum as when considering the full contingency set.

Table 6: Number of binding contingencies at the optimum

problem	PSCOPF	CSCOPF
problem A	3	2
problem B	5	1

Note finally that, the dual variable (Lagrange multiplier) at the optimal solution associated with each active power flow equation (representing the variation of the overall generation cost for an increment of the active load at that bus) is called locational marginal price (or nodal price) at that bus and could be used to price electricity in some liberalised electricity markets [8].

4.2 Minimizing Congestion Cost

We now consider the problem of minimizing the congestion cost by the same approaches: OPF, PSCOPF and CSCOPF. The objective function is expressed as: $\min \sum_{i=1}^{g} (c_i^+ \Delta Pg_i^+ - c_i^- \Delta Pg_i^-)$, where, for the i-th generator, c_i^+ (resp. c_i^-) is its incremental (resp. decremental) price [8] and $\Delta Pg_i^+ \geq 0$ (resp. $\Delta Pg_i^- \geq 0$) is its active power increase (resp. decrease) with respect to the base case.

In order to assess the robustness of these optimization approaches to remove severe (thermal) congestions we choose a very "congested" base case, with 7 branches being overloaded up to 147 %. Obviously, when simulating (by an AC power flow software) the contingency impact at the base case, at the corresponding post-contingency state most of these branches are (even higher) overloaded, while new branches becoming also overloaded.

As in the previous example, we also consider two types of optimization problems: problem C and problem D. The control variables allowed in problem C are: generator active and reactive powers. The equality constraints are the bus active and reactive power flow equations. The inequality constraints include bounds on generator active and reactive powers and limits on branch currents. For problem D one acts on the following control variables: generator active and reactive powers, controllable transformer ratio and shunt reactance. The equality constraints are again the bus power balances. The inequality constraints include bounds on all above mentioned control variables as well as limits on branch currents and bus voltage magnitudes. The latter are allowed to vary between 0.95 pu and 1.05 pu. Note that, for problem D, at the base case as well as at all post-contingency states some voltages are out of the allowable variation range, which creates voltage congestion.

Tables 7 and 8 provide the value of the objective (the congestion cost) and the total amount of generation rescheduled to remove the congestion, respectively. The effort of removing the thermal/and or voltage congestion is shared for problem C (resp. D) among 6 (resp. 7) generators for the OPF, 7 generators for the PSCOPF and 6 generators for the CSCOPF. Note that, if a congestion can not be removed by rescheduling generation only, load curtailment can also be taken into account.

Table 7: Value of the objective (in $/h)

problem	Value of the objective (in $/h)		
	OPF	PSCOPF	CSCOPF
problem C	599	884	643
problem D	620	895	659

Table 8: The total amount of generation rescheduling (MW)

problem	MW rescheduled		
	OPF	PSCOPF	CSCOPF
problem C	1210	1638	1304
problem D	1250	1674	1334

Table 9 provides the number of iterations to convergence and the CPU times for different problems and optimization approaches.

Table 9: Number of iterations to convergence and CPU times

problem	OPF		PSCOPF		CSCOPF	
	iters	time	iters	time	iters	time
problem C	21	0.34	28	4.5	36	5.4
problem D	23	0.44	53	12.1	55	15.4

Tables 10 and 11 show the number and the type of binding constraints at optimum for problems C and D, the meaning of the columns label being the same as in the previous Section. Note that, the high number of active power generation active constraints at the optimum is owing to the fact that, for this objective function, each generator power shift with respect to its base case value is split into two positive variables, ΔPg_i^+ and ΔPg_i^-.

Table 10: Number and type of active constraints for problem C

approach	P_g	Q_g	I	*cpl*	total
OPF	40	0	5	-	45
PSCOPF	63	1	25	-	89
CSCOPF	41	0	11	16	68

Table 11: Number and type of active constraints for problem D

approach	P_g	Q_g	I	V	r	x	*cpl*	total
OPF	40	1	5	23	0	1	-	70
PSCOPF	63	2	5	268	0	4	-	342
CSCOPF	41	1	9	48	0	6	18	123

All three optimizers succeed to remove the thermal and/or voltage congestion, despite a significant number of branch currents and voltage magnitudes binding constraints at the optimum, in both pre- and post-contingency states. This proves once more the efficiency of the IPM to solve optimization problems with a large number of constraints active at the optimum.

Finally, Table 12 yields the number of binding contingencies at the optimum.

Table 12: Number of binding contingencies at the optimum

problem	PSCOPF	CSCOPF
problem A	4	1
problem B	5	1

4.3 Computing Maximum Power Transfer

We finally tackle the problem of determining the maximum power transfer (the TTC) between two ("source"-"sink") areas of a power system, by means of two approaches: OPF and PSCOPF. We assume that all loads in the sink area are increased proportionally to their base case consumptions, and that increase is covered (for simplicity) by a single "slack" generator in the source area. The control variables considered are: the value of the power transfer and slack generator active output. The equality constraints concern buses active and reactive power balance. The inequality constraints include limits on: generator reactive power, branch current and bus voltage magnitudes. The latter are allowed to vary between 0.90 pu and 1.10 pu. More details about loadability limit computation are available in [14].

Note that, a power transfer (loadability) limit computed with the (SC)OPF static models (1-3) or (4-7) can correspond to a voltage stability limit, a thermal limit, a voltage magnitude limit or any combination of those.

The loadability margin is 807 MW (resp. 173 MW) when using the OPF (resp. the PSCOPF). In both approaches one branch attaining its maximum current, in the base case (resp. in a post-contingency state) when using the OPF (resp. the PSCOPF), prevents us to obtain a larger power transfer. These margins indicate, obviously, that base power transfer can be considerably limited when adding supplementary contingency constraints. Now, if one takes off branch current constraints from the above computation model, the new loadability margin increases to 821 MW (resp. 616 MW) when using the OPF (resp. the PSCOPF). These margins are now limited by one voltage reaching its minimal bound (0.90 pu), in the base case (resp. in a post-contingency state).

5 CRITICAL ISSUES IN SCOPF

The major drawback of the (brute force) approach to the SCOPF benchmark adopted in this paper is the high dimensionality of the problem, especially for large power systems and/or when many contingent cases have to be considered. A first limitation stems from the memory capacity of current computers. Secondly, although in real life applications most postulated contingencies do not constrain the optimum, including them into the SCOPF increases the complexity of the computations, due to shrinking the feasible region, and can hence lead to algorithmic/numerical problems. This is especially true under stressed operating conditions.

As regards the PSCOPF, a widely used approach to mitigate these drawbacks combines three modules: a PSCOPF which considers a subset of potentially active contingencies, a (steady-state) security analysis and a contingency filtering technique [3, 7, 16]. This approach requires to iterate between these modules until all post-contingency constraints are satisfied. The PSCOPF can be further simplified by adding to the base case constraints only relevant post-contingency inequalities, linearized around the base case optimized operating point, while dropping all post-contingency equality constraints (which are checked at the optimal solution) [4, 5, 6, 7]. This approach requires iterating between the solution of the PSCOPF and the linearization of post-contingency inequality constraints until some convergence criteria are met.

Contingency filtering is an essential step in the sequential PSCOPF solution. Its goal is to efficiently identify an as small as possible subset, of the initial contingency set, containing all the binding contingencies at the benchmark PSCOPF solution. Most such contingency filtering schemes rank various contingencies according to a severity index, which accounts for post-contingency constraints violation [6]. Another interesting alternative consists in identifying a set of "umbrella" contingencies to be finally included in the PSCOPF computation [15]. Such umbrella contingencies are identified by solving relaxed PSCOPF formulations, which take into account only the contingency of concern and base case constraints. Are considered as umbrella, those contingencies having the highest Lagrange multipliers (according to some norm L_i, e.g., $i = 1, 2, \dots, \infty$) corresponding to the post-contingency bus active power balance.

Another technique to further reduce the size of a PSCOPF and to speed-up computations consists in decomposing and distributing the problem among several processors, each one solving (asynchronously) only a limited subset of post-contingency states [16, 17].

The contingency filtering for the CSCOPF is a delicate task due to the difficulty to implement time-varying actions into a classical power flow software. The CSCOPF is usually delt with by Benders decomposition [4, 5, 9, 10]. In this approach the original CSCOPF problem is decomposed into a master problem and several slave problems, each corresponding to a harmful contingency case. Thus, a slave problem encompasses mainly the post-contingency constraints relative to a contingency and provides a linear constraint (Benders cut) to the master problem. The latter contains base case constraints and Benders cuts stemming from slave problems. At each iteration the slave problems fed the master problem with improved Benders cuts until convergence is reached. Clearly, the simultaneous solution of slave problems is possible; it can significantly speed-up computations. Although good results have been reported with this approach, note, however, that Benders decomposition requires the convexity of the feasible region which can not be guaranteed in SCOPF.

Since in a the deregulated context, the "N-1" criterion is felt as an obstacle to competition, the PSCOPF variant may be deemed too conservative. On the other hand, relying too much on corrective actions, the CSCOPF variant increases considerably the risk of blackouts. Whereas the cost of preventive actions is easy to calculate, getting a reliable estimate of the corrective actions cost is a challenging problem, especially when post-contingency load curtailment is allowed. The future is most probably in a careful tradeoff between both preventive and corrective SCOPF variants, with the objective to minimize the overall cost of pre- and post-contingency control actions [18].

The SCOPF problems considered in this work are formulated deterministically. One of their main limitation is that they do not take into account the likelihood of the various contingencies, but rather treats them all as equiprobable, which may lead to (very) high operation costs for (very) low probable events. It may thus be more advantageous to formulate the SCOPF problem stochastically to better reflect the occurrence probability of each contingency, as proposed in [15, 18].

6 CONCLUSION

This paper has presented our recent developments in the field of SCOPF. We have implemented both SCOPF variants: the preventive one and the corrective one. They have been successfully tested, on a 60-bus system, for three key SCOPF applications: minimum overall generation cost, minimum congestion cost and maximum power transfer computation. We have also discussed some critical issues related to the SCOPF problem.

The IPM succeeds to solve all SCOPF problems considered in this work. Expectedly, in IPM, the number of iterations to convergence is little influenced by the size of the problem, but rather by its difficulty, often revealed by a high number of constraints that are active at the optimum. As it is known that the IPM is highly sensitive to the choice of the starting point, some future work has to be devoted to the choice of a more robust initial point. Admittedly, this topic is even more stringent in SCOPF than in OPF applications, due to the effect of various contingencies on equality and inequality constraints.

A future natural extension of this work is the sequential solution of the PSCOPF problem, with a particular focus on the finding of robust and efficient heuristic techniques to quickly identify binding contingencies at the benchmark PSCOPF solution.

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