Studying the Topology and Dynamics of Elasto-inertial Channel Flow Turbulence Using the Invariants of the Velocity Gradient Tensor and Dynamic Mode Decomposition

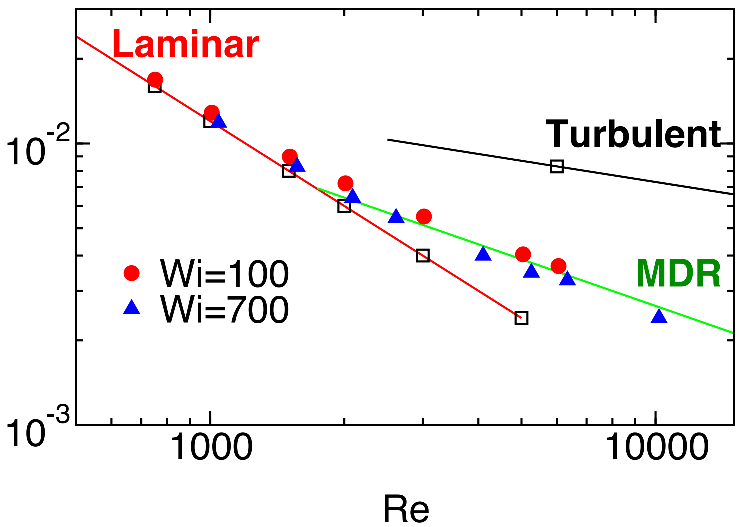
J. Soria [1,5], Y. Dubief [2], V. Terrapon [3] & I. Moreno-Bermejo [4]

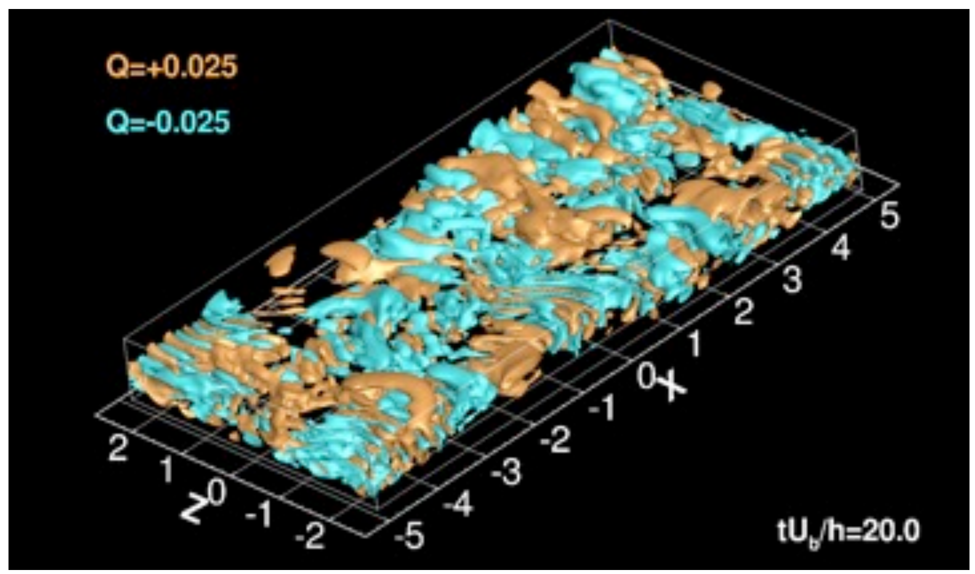
[1]Laboratory for Turbulence Research in Aerospace and Combustion,
Dept. of Mechanical and Aerospace Engineering, Monash University, Melbourne, Australia
[2]School of Engineering, University of Vermont, Burlington, VT 05405, USA
[3]Aerospace & Mechanical Engineering Department, University of Liège, Belgium
[4]Center for Turbulence Research, Stanford University, CA, USA
[5]Dept. of Aeronautical Engineering, King Abdulaziz University, Jeddah, Kingdom of Saudi Arabia

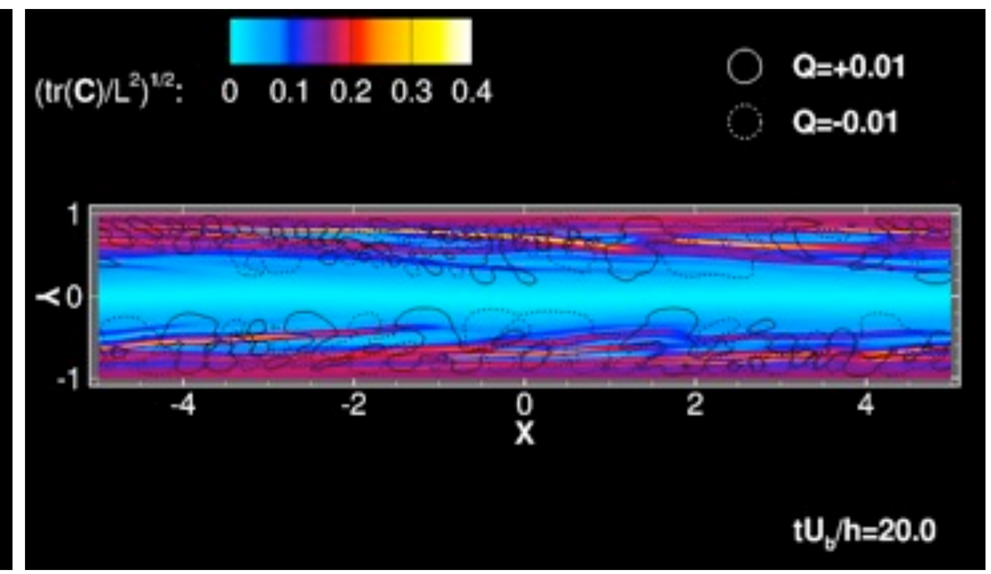
Vermont Advanced Computing Center
National Institutes of Health.
Marie Curie FP7 Career Integration Grant
Australian Research Council
Part of this research was conducted during the 2012 CTR Summer Program at Stanford

American Physical Society
65th Annual Fall DFD Meeting
November 18 - 20, 2012
San Diego, California

Introduction







$$\partial_t \mathbf{C} + (\mathbf{u} \cdot \boldsymbol{\nabla})\, \mathbf{C}$$

Formation of sheets of C

$$\nabla^2 p = 2Q + \frac{1-\beta}{Re} \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \cdot \mathbf{T})$$

Excitation of extensional sheet flow and elliptical pressure redistribution of energy

$$\mathbf{C} \cdot (\boldsymbol{\nabla}\mathbf{u}) + (\boldsymbol{\nabla}\mathbf{u})^t \cdot \mathbf{C} - \mathbf{T}$$

Increase of extensional viscosity in sheets

MONASH University

Flow Topology

- Chong *et al.* (1990) generalised the idea of critical point theory by attaching the origin of a non-rotating, translating coordinate system to every fluid particle
- in this reference frame the flow at the origin is a critical point
- topological character of the flow pattern of the fluid particle is governed by $A_{ij} = \frac{\partial u_i}{\partial x_j}$ (VGT)
- the topological character is Galilean Invariant
- VGT has characteristic equation

$$\lambda_i^3 + P_A \lambda_i^2 + Q_A \lambda_i + R_A = 0$$

- λ_i are the eigenvalues of A_{ij}, P_A, Q_A and R_A are the 1st, 2nd and 3rd tensor invariants

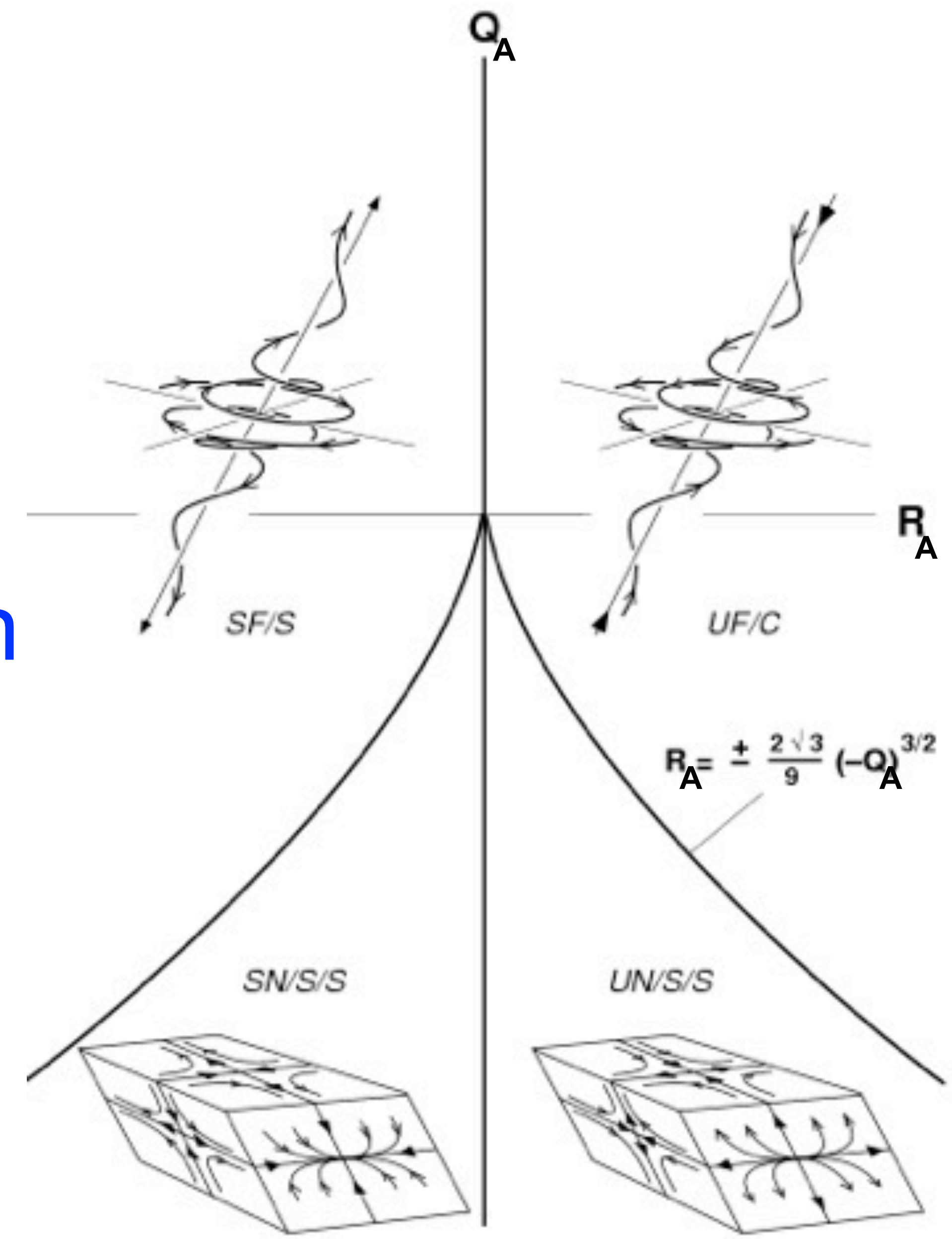
Flow Topology (Chong *et al.* 1990, Soria *et al.* 1994)

- incompressible flows, invariants of VGT A_{ij}:

$$P_A = -A_{ii} = 0,$$

$$Q_A = -\frac{1}{2} A_{ij} A_{ji}$$

$$R_A = -\frac{1}{3} A_{ij} A_{jk} A_{ki}$$



- local topology dependents only on Q_A and R_A

- D_A is the discriminant of A_{ij}:

$$D_A = \frac{27}{4} R_A^2 + Q_A^3$$

Flow Topology (Chong *et al.* 1990, Soria *et al.* 1994)

- A_{ij} can be split: $A_{ij} = S_{ij} + W_{ij}$
  - S_{ij} - rate-of-strain tensor (symmetric ∴ real eigenvalues)
    - 3 corresponding invariants (~~P_S~~, Q_S, R_S)
    - $\alpha_1, \alpha_2, \alpha_3$ are eigenvalues = principal strain rates s.t. $\alpha_1 \leq \alpha_2 \leq \alpha_3$
  - W_{ij} - rate-of-rotation tensor (skew-symmetric ∴ complex eigenvalues)
    - 3 corresponding invariants (~~P_W~~, Q_W, ~~R_W~~)
  - $P_S = P_W = R_W = 0$
  - Q_S is **negative definite** $Q_S = -\frac{1}{2}S_{ij}S_{ji}$ → $\epsilon = -4\nu Q_S$?
  - Q_W is **positive definite** $Q_W = -\frac{1}{2}W_{ij}W_{ji}$ → $Q_W = \frac{\omega^2}{4}$, where $\omega^2 = \omega_i\omega_i$
  - ***sgn(R_S) = sgn(α_2)*** $R_S = -\frac{1}{3}S_{ij}S_{jk}S_{ki}$
  - Truesdell (1954) introduced kinematic vorticity number $\mathscr{K} = \left(\frac{Q_W}{-Q_S}\right)^{1/2}$
    - local measure of rotational strength to rate of irrotational stretching of fluid element: κ = ∞ (solid body rotation), κ = 0 (irrotational stretching)

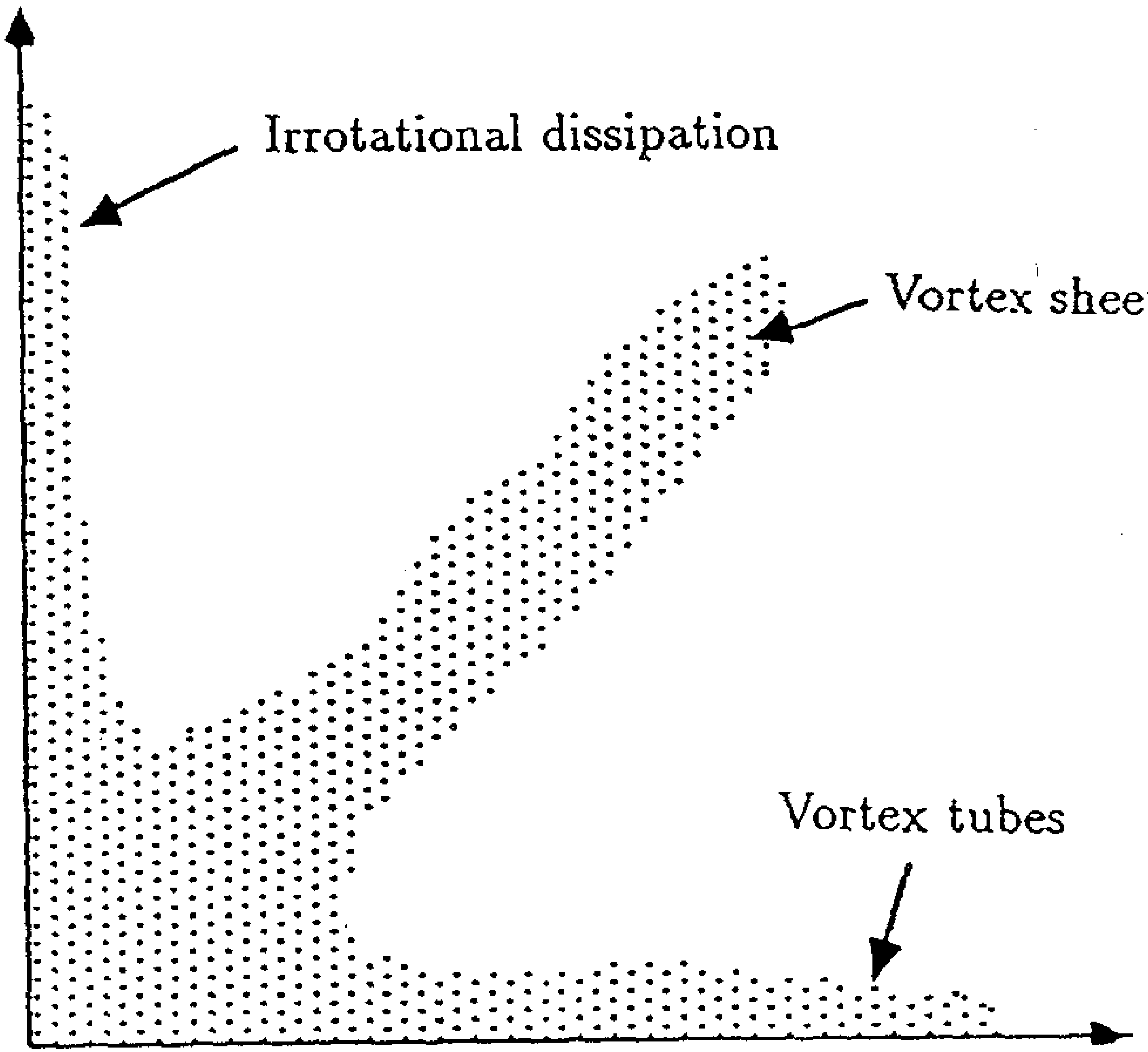
$$Q_A = Q_S + Q_W$$

MONASH University

LTRAC

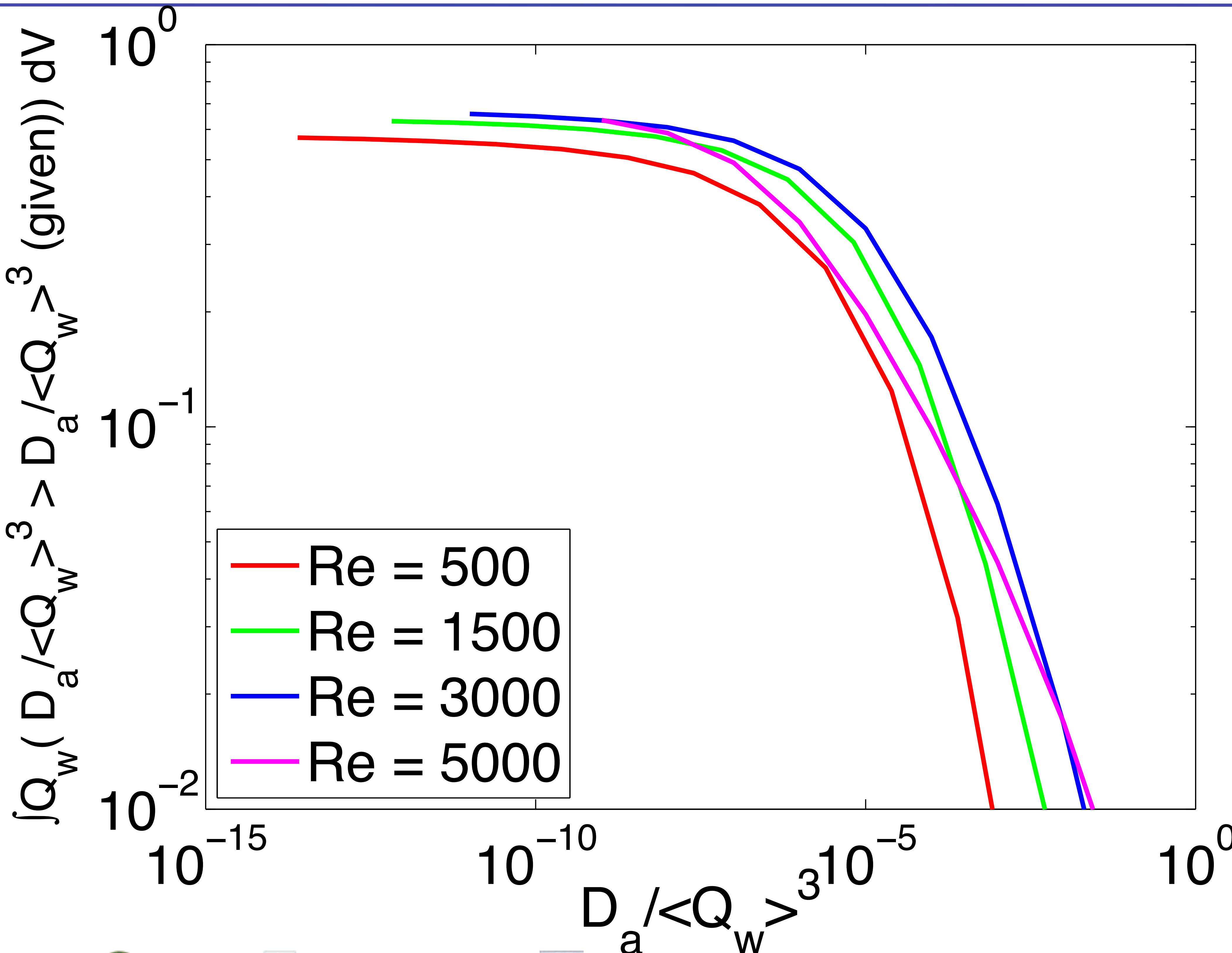
JPDF of Q_W - $-Q_S$ and its relationship to turbulence structure (Perry & Chong (1994))

$-Q_s = \frac{1}{2} S_{ij} S_{ij} \sim$ Dissipation (for "Newtonian Fluid")

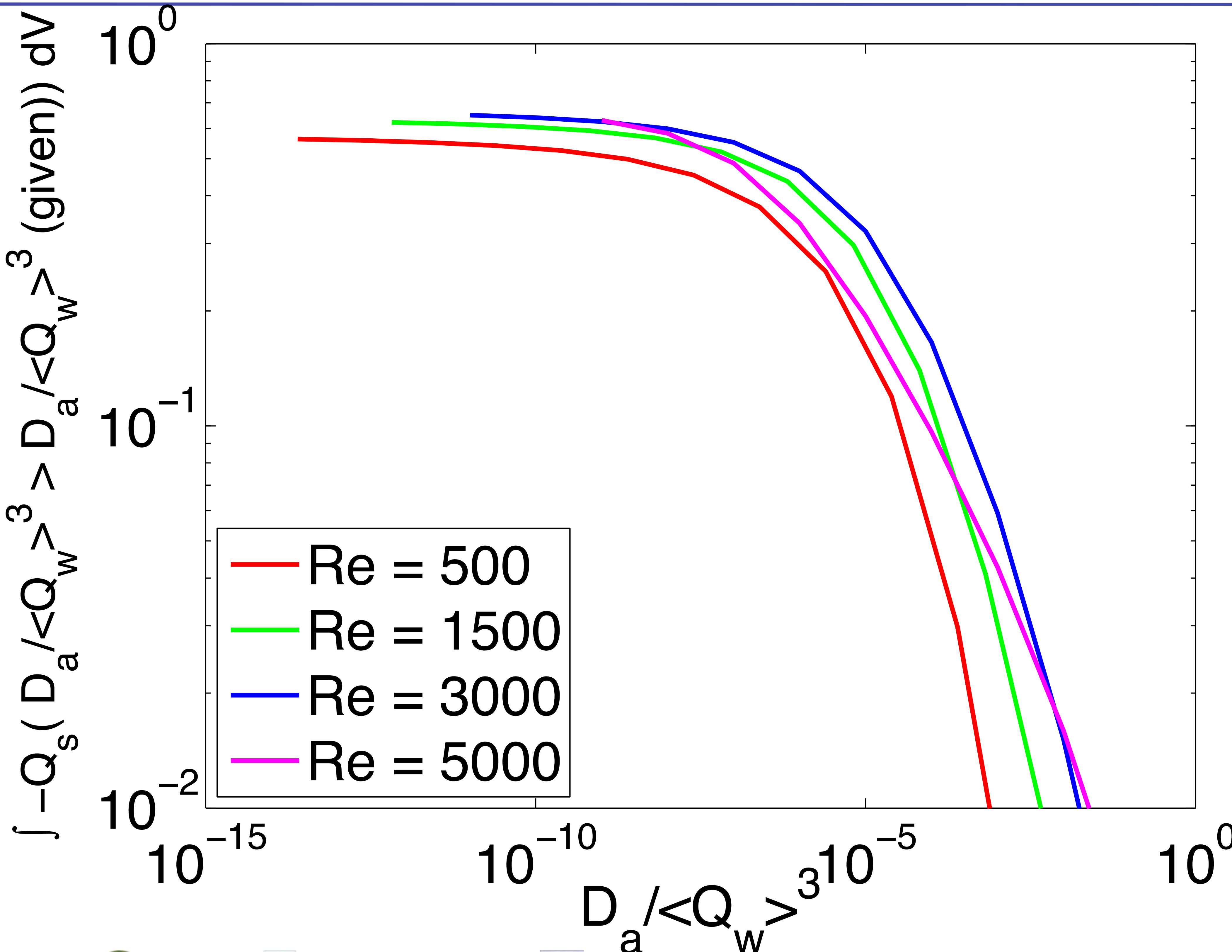


$Q_w = \frac{1}{2} W_{ij} W_{ij} \sim$ Enstropy density

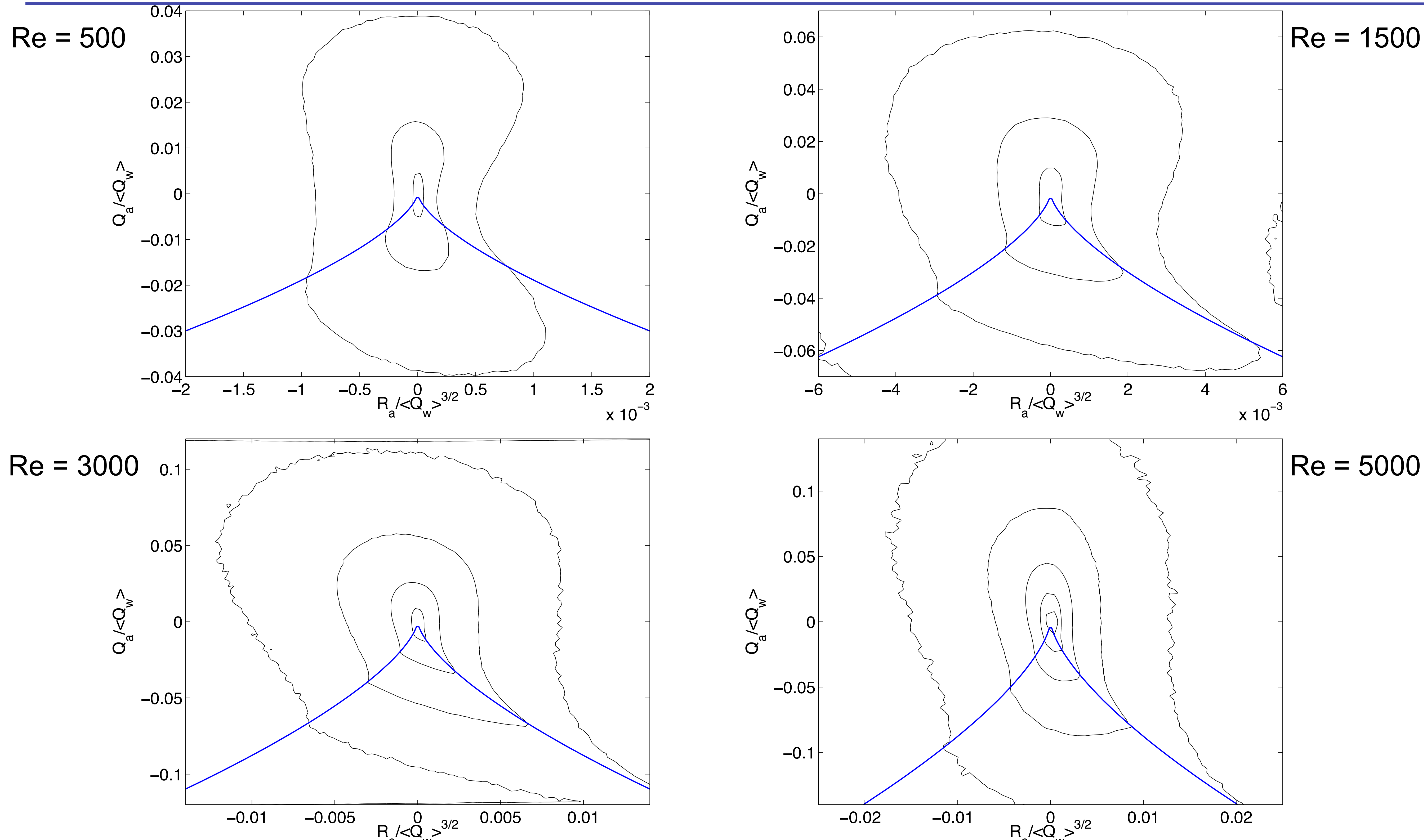
Results:
Conditional volume integrals when $D_A > |D_A \text{ (given)}|$
Enstrophy due to focal regions



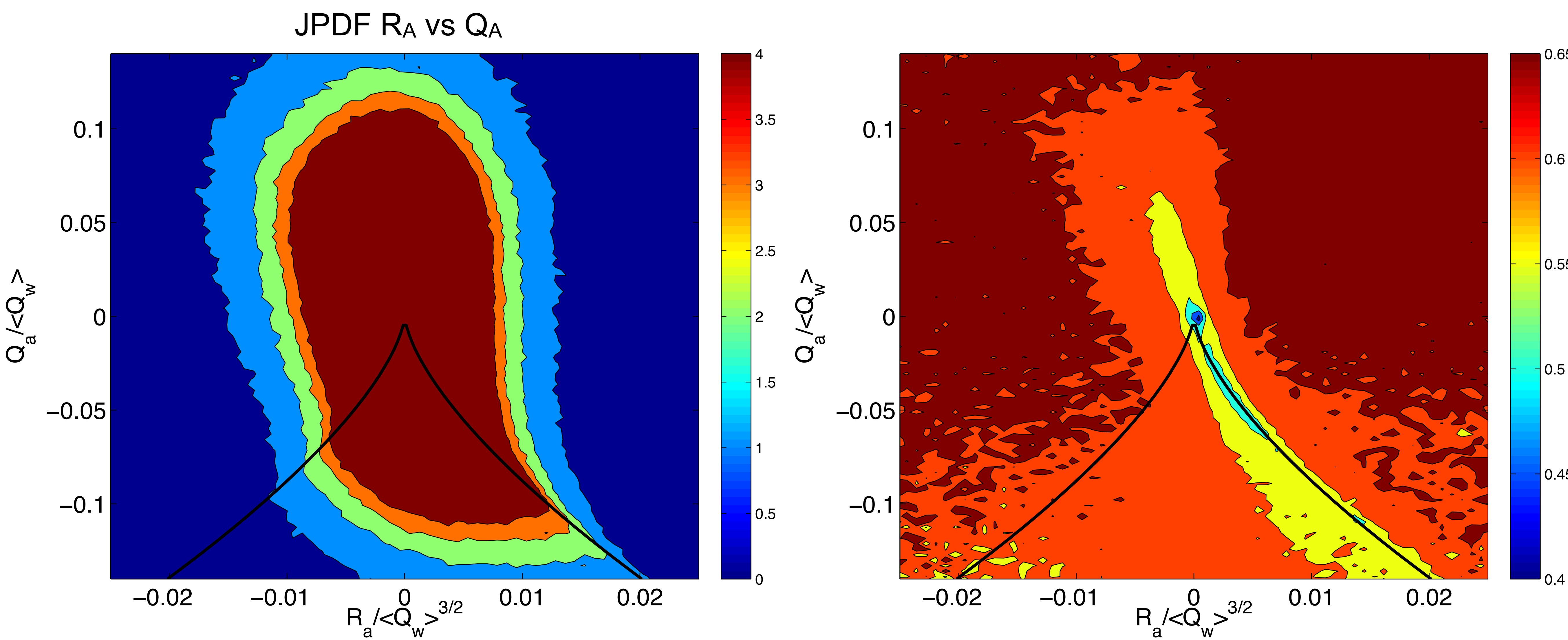
Results:
Conditional volume integrals when $D_A > |D_A \text{ (given)}|$
"Dissipation" of mechanical energy due to focal regions

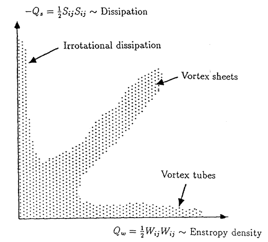


Results:
JPDF R_A - Q_A



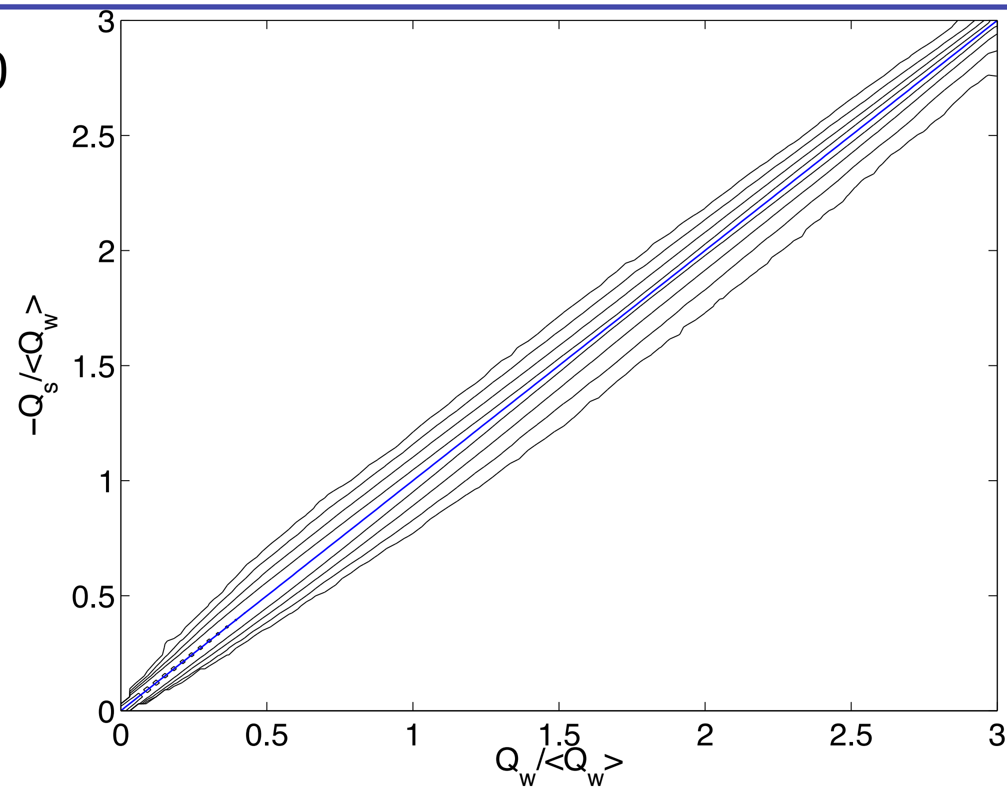
Results:
Expected value of polymer stretch conditioned on (R_A, Q_A) for Re = 5000



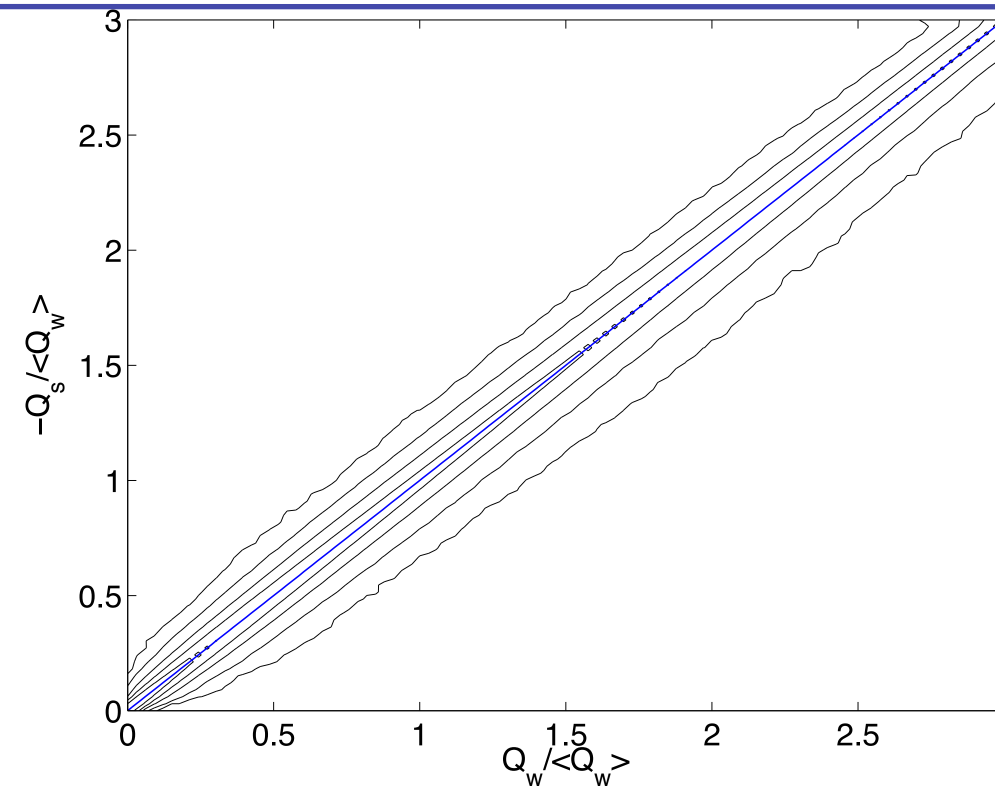


Results: JPDF Q_W - $-Q_S$

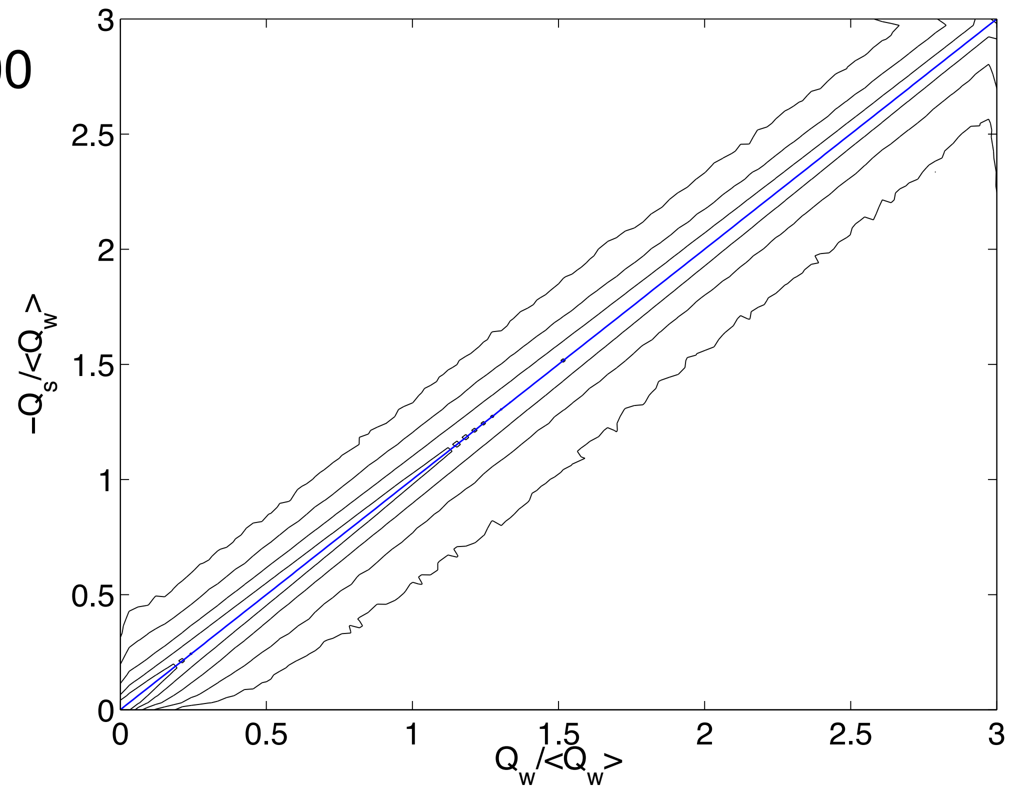
Re = 500



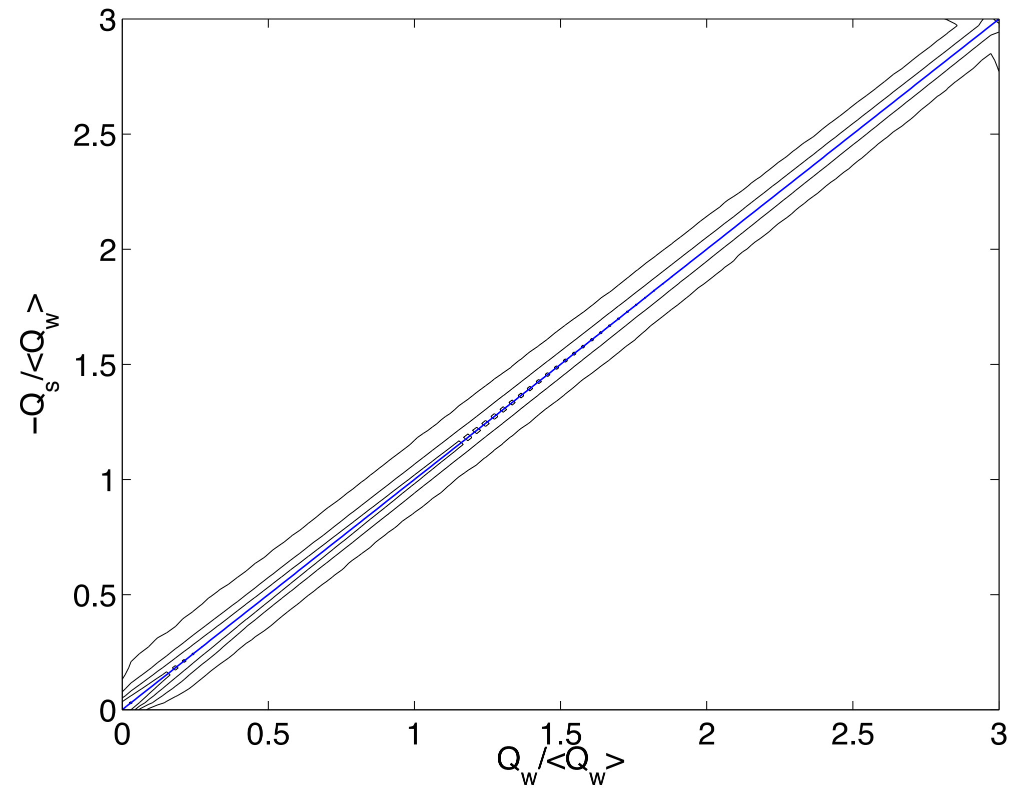
Re = 1500



Re = 3000



Re = 5000



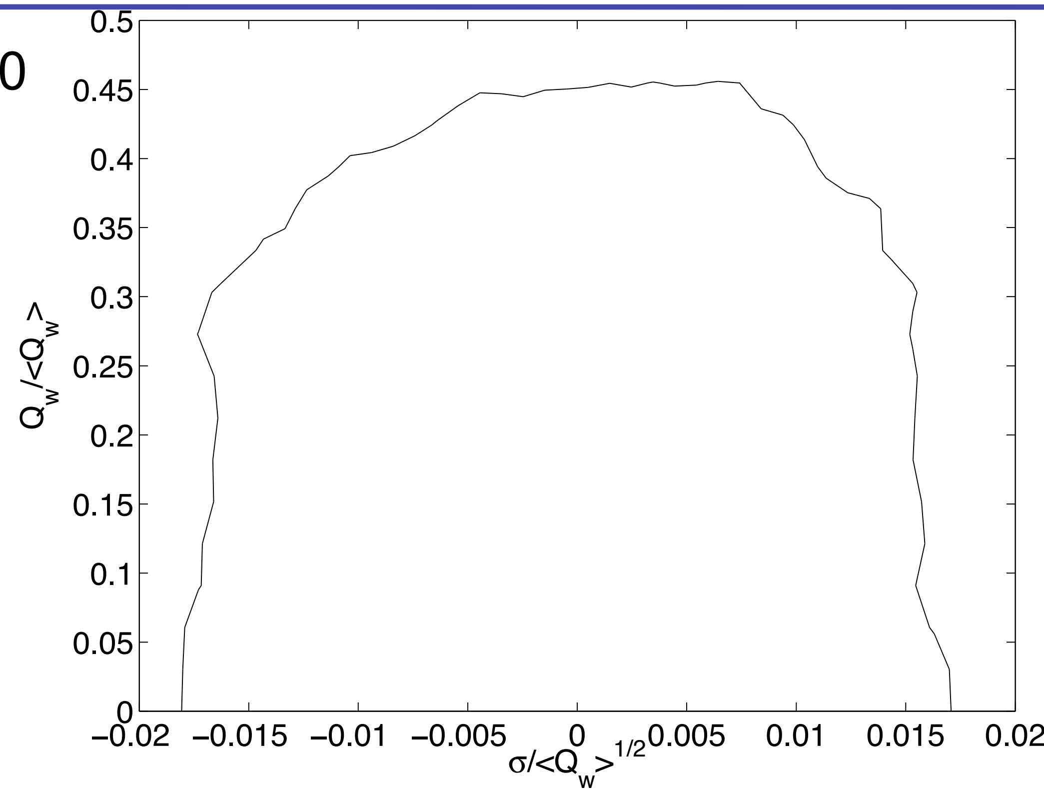
MONASH University

Université de Liège

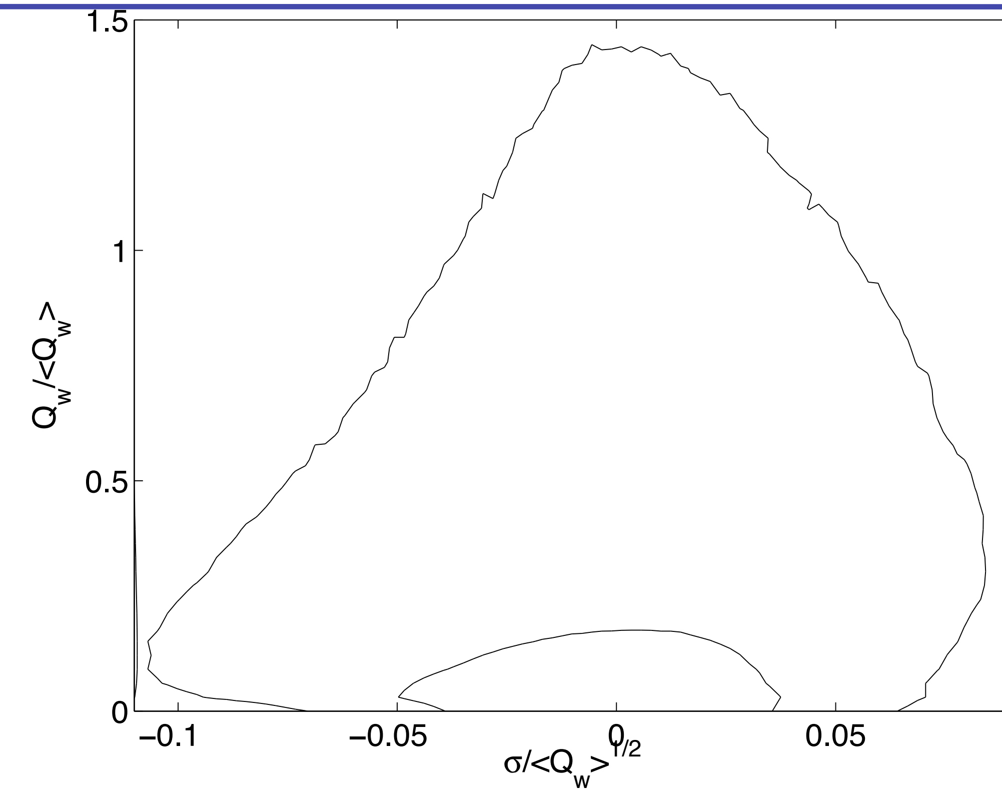
LTRAC

Results:
JPDF Σ - Q_W

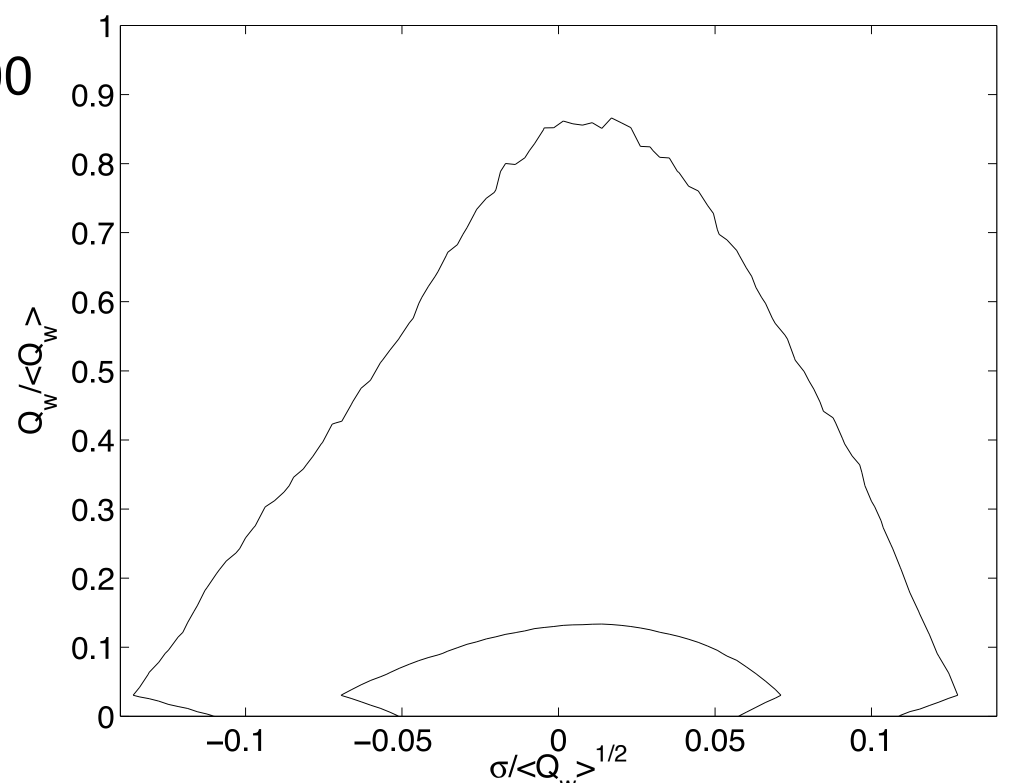
Re = 500



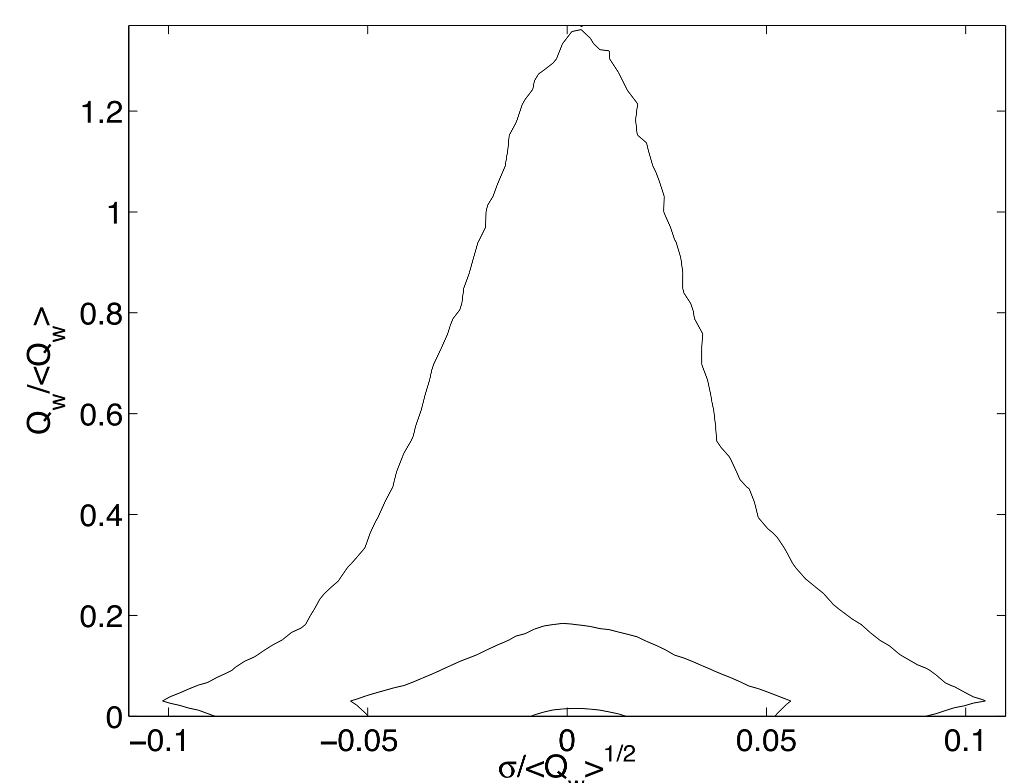
Re = 1500



Re = 3000



Re = 5000



MONASH University

LTRAC

Summary

- in the transition from laminar regime (Re = 500) focal regions occupy ~57% of the volume containing ~56% of the enstrophy and "dissipate" ~57% of the mechanical energy
- while in the EIT regime (Re = 5000) they occupy ~64% of the volume containing ~63% of the enstrophy and "dissipate" ~64% of the mechanical energy
- during the transition form laminar to the EIT regime, the JPDF of R_A v. Q_A evolves from a somewhat symmetric shape around the 2-D flow axis (R_A = 0) to the more tear-drop shape but which is different to that found in Newtonian turbulent flows
- throughout the transition form laminar to the EIT regime the dominant structure of the flow is sheet like as evidenced by the JPDF of Q_W v. $-Q_S$
- polymer stretch in the EIT regime exhibits minima which are UFC topology and lie along the null discriminant which represents axisymmetric contraction topology