The University of Vermont

A New State of Turbulence: Elasto-Inertial Turbulence

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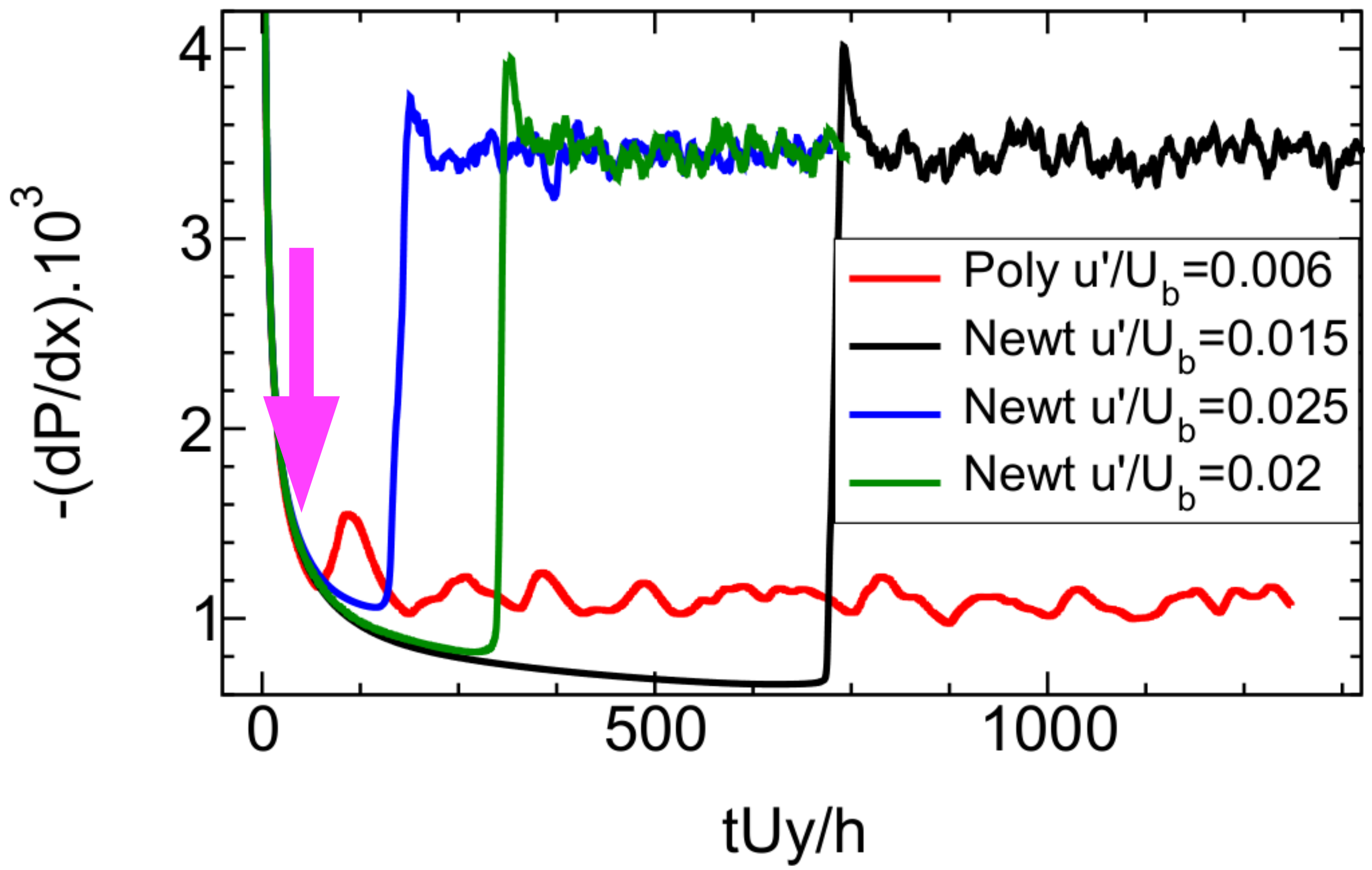
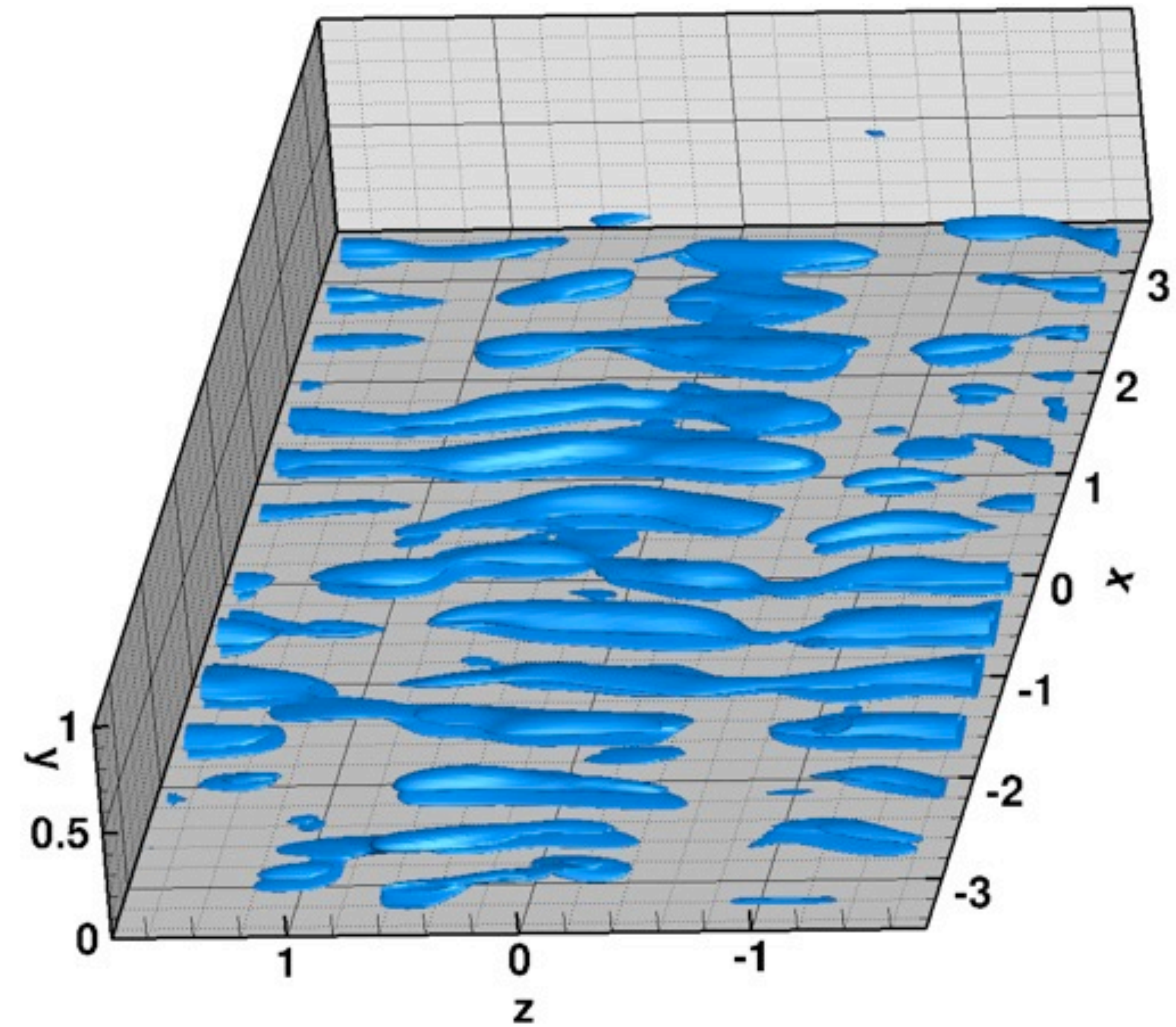
Study performed during the
2012 Center for Turbulence Research Summer Program

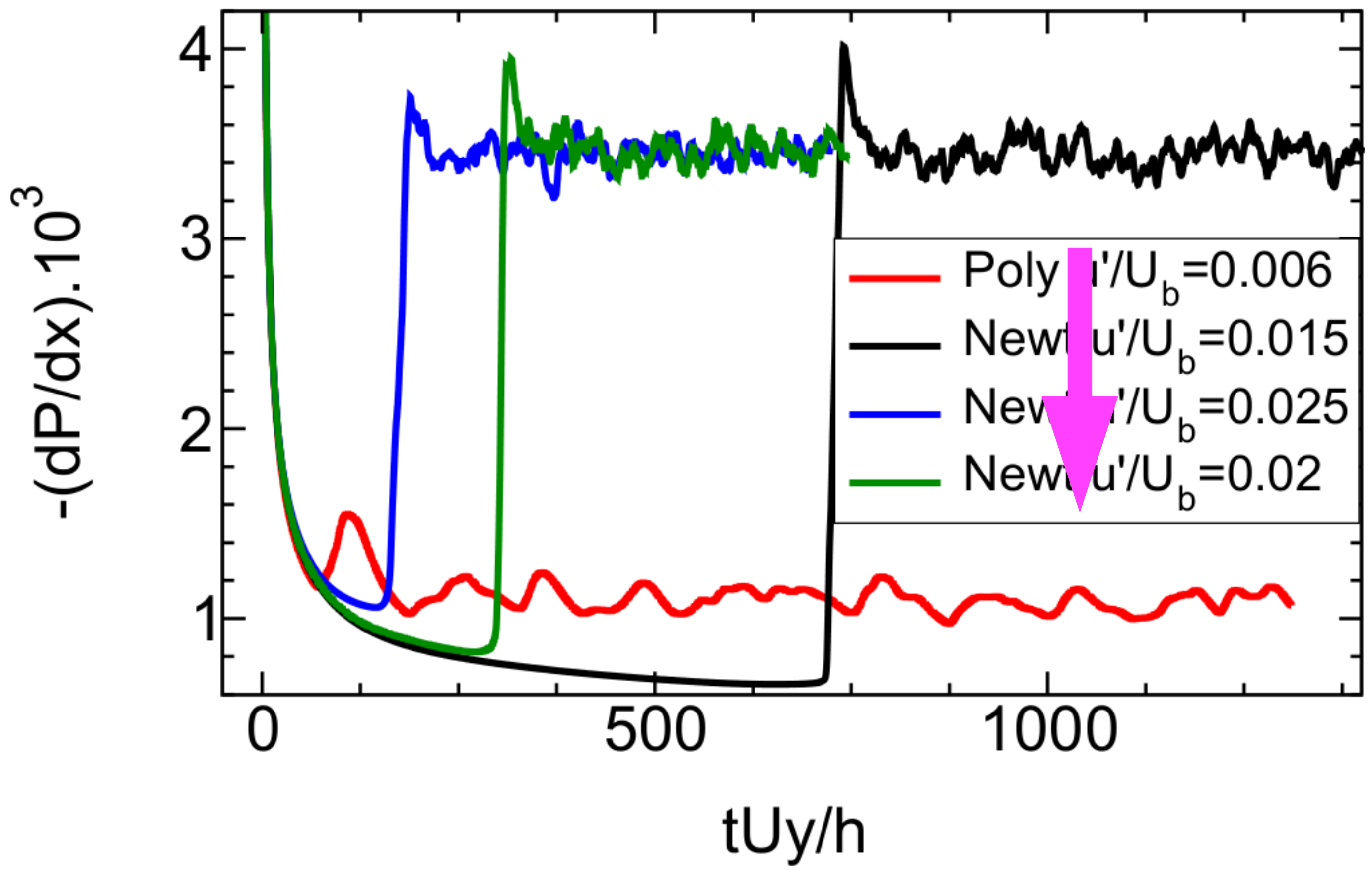
First simulations of EIT via early turbulence

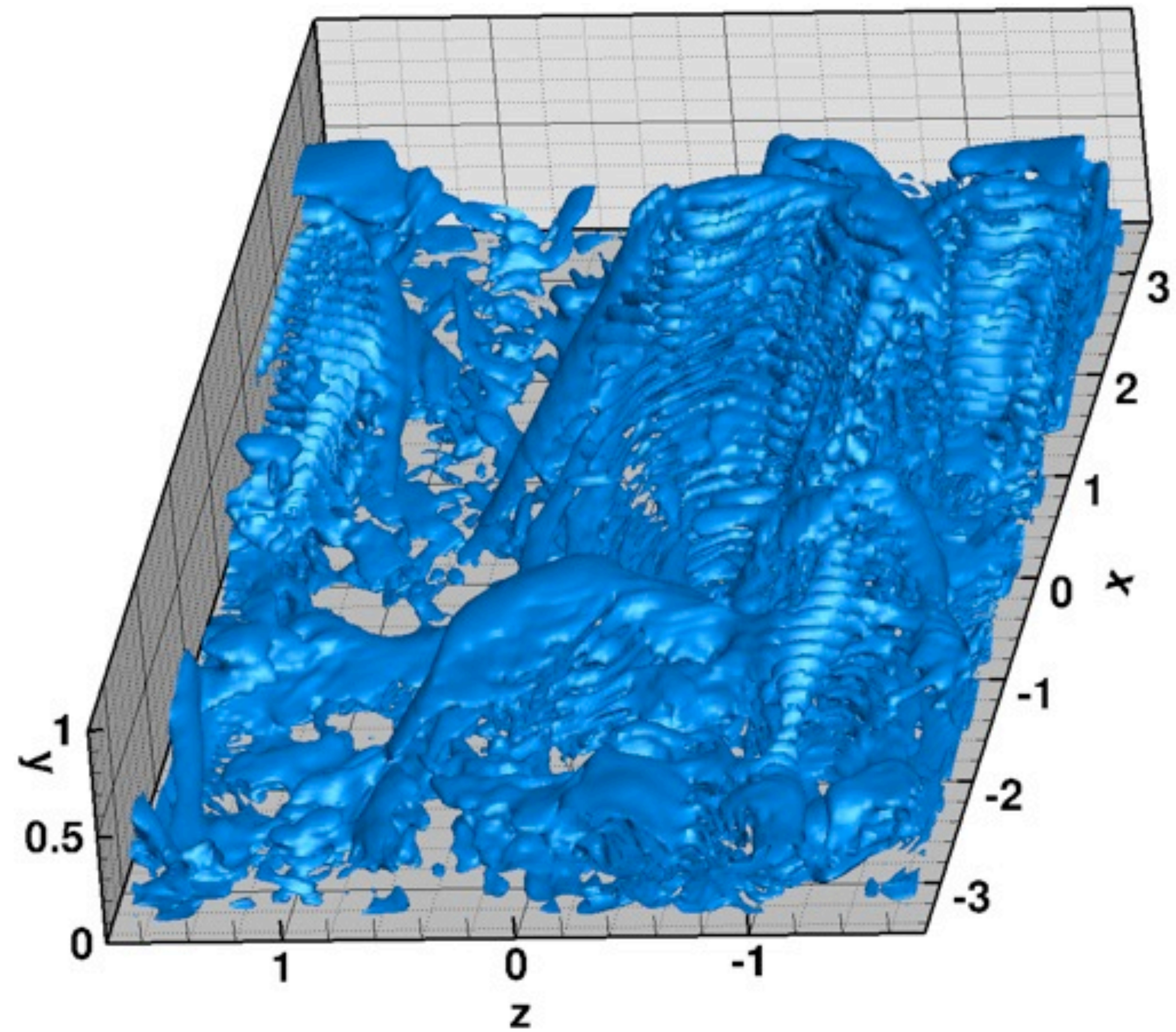




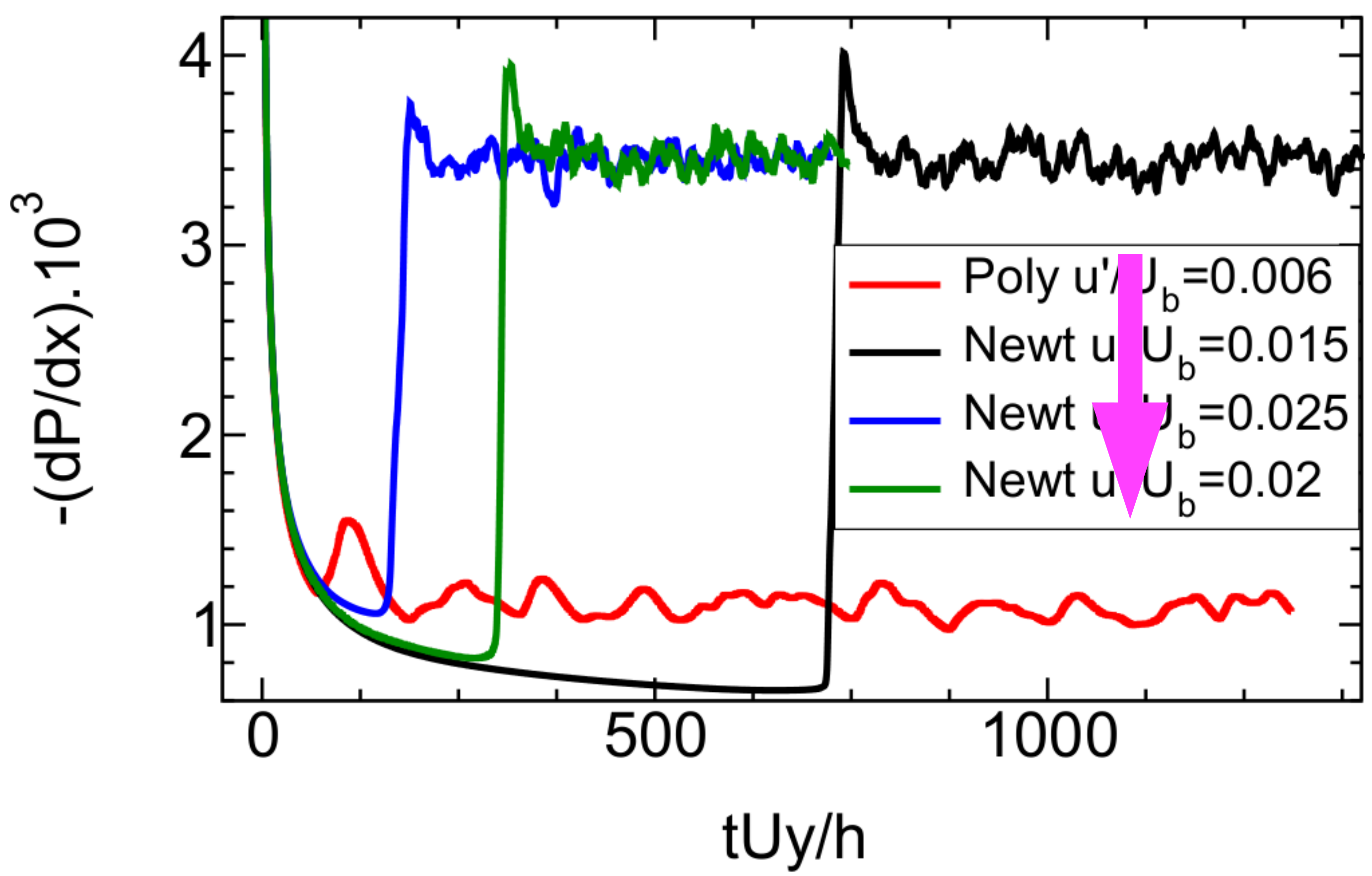
Dubief, White, Shaqfeh, Terrapon, CTR summer program 2010, Annual Research Briefs 2010.

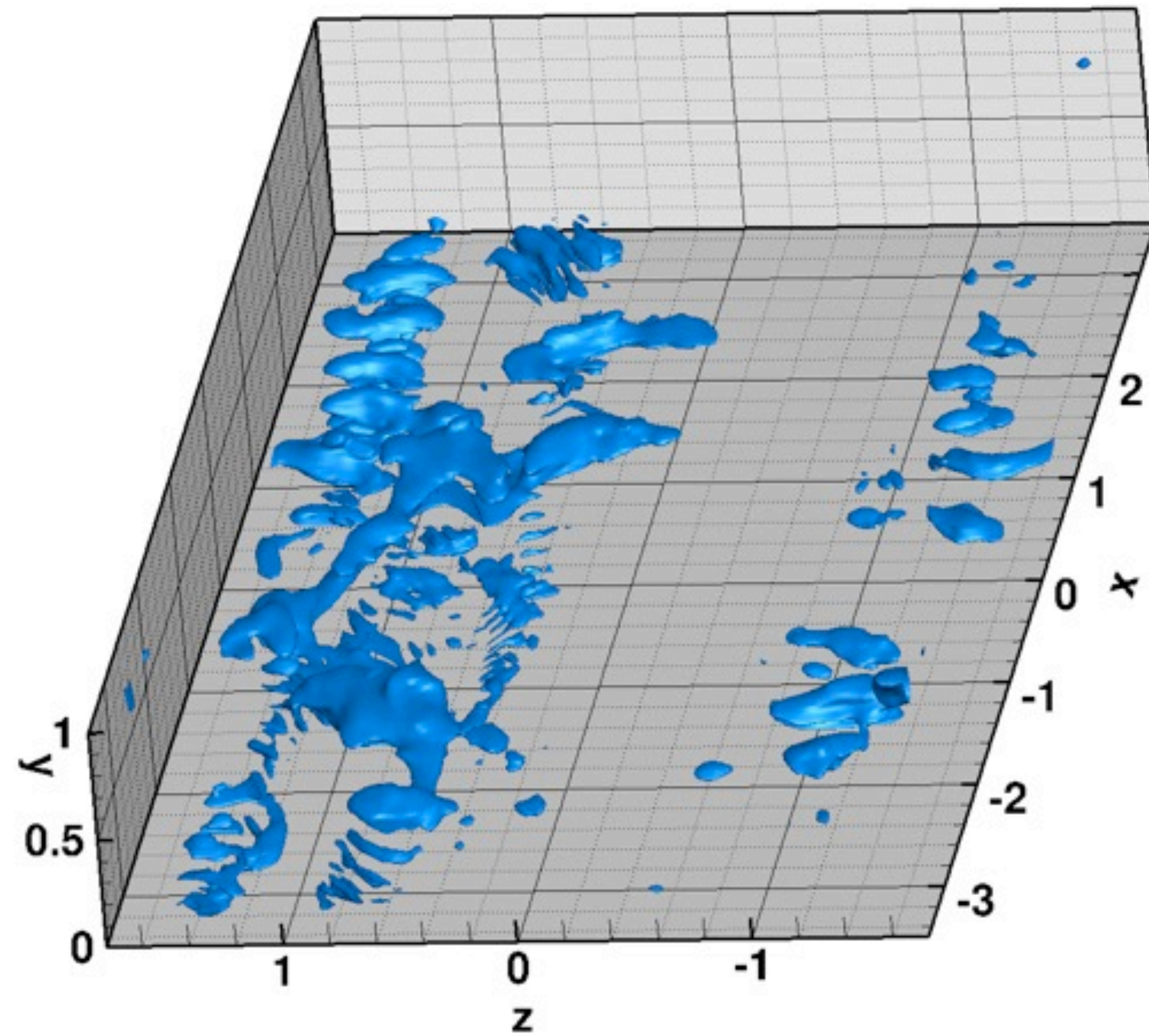
First simulations of EIT via early turbulence





First simulations of EIT via early turbulence





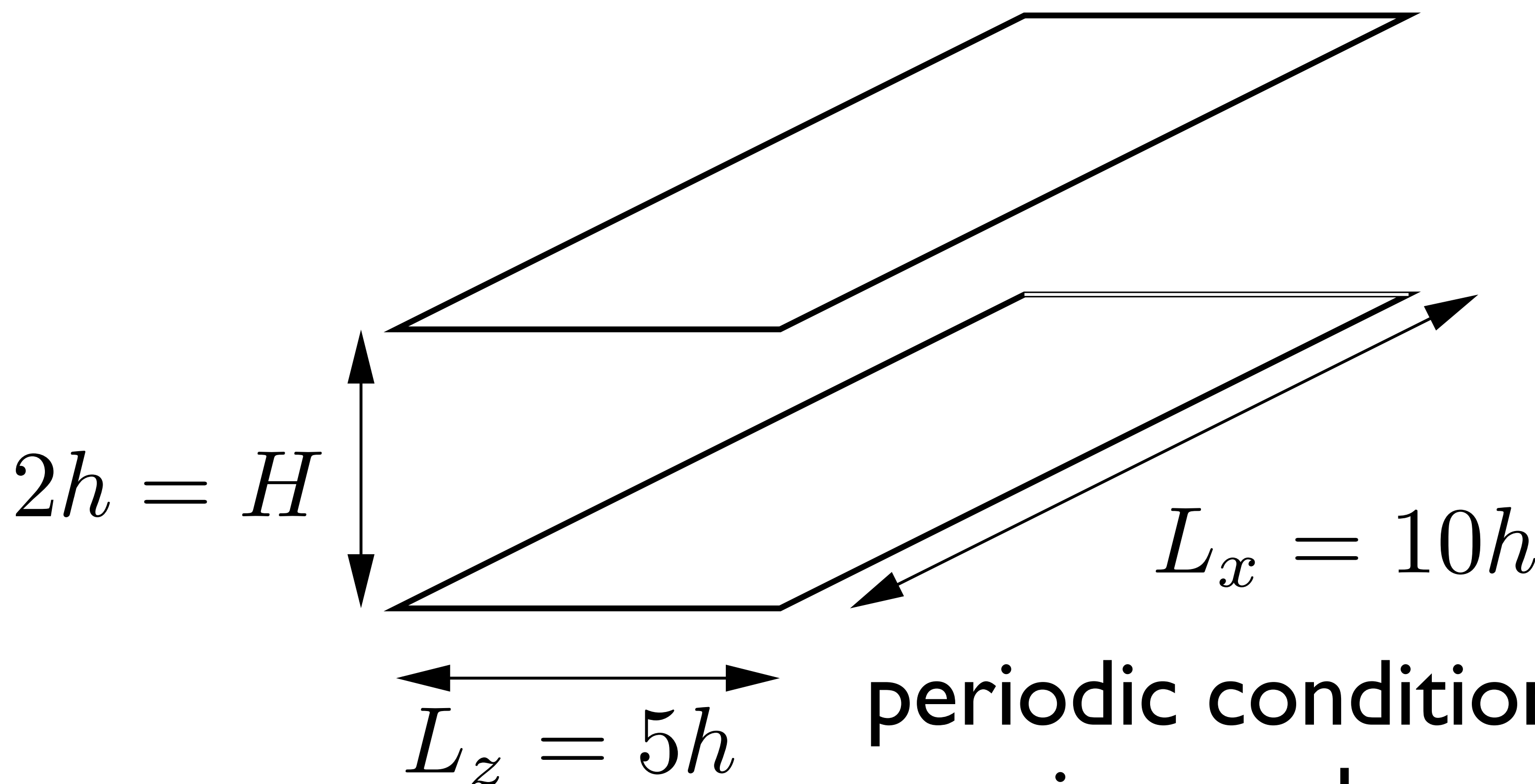
See Vincent Terrapon's talk for more details

Investigative methods

Direct numerical simulation of periodic channel flow with a weak initial wall perturbation

$$Re = \frac{U_b H}{\nu}$$

$$N_x \times N_y \times N_z = 256 \times 151 \times 256$$



periodic conditions in x and z

Initial wall perturbation designed to trigger turbulence at *Re*=6000 in water

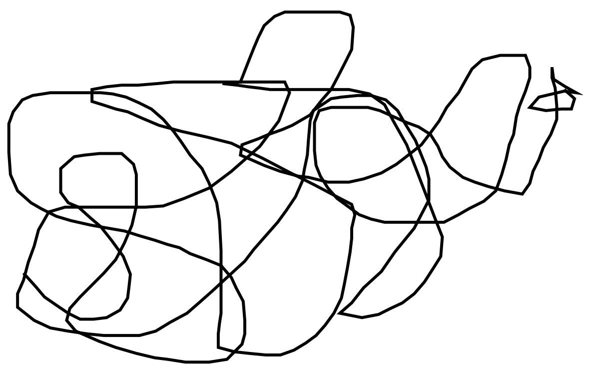
$$v_{(x,y=\pm h,z,t)} = \mathcal{H}(t)\left[A\sin\left(\frac{8\pi}{L_x}x\right)\sin\left(\frac{8\pi}{L_z}z\right) + \epsilon(t)\right]$$

$$\mathcal{H}(t) = \begin{cases} 1 & \text{for } \frac{tU_b}{h} < 1 \\ 0 & \text{for } \frac{tU_b}{h} \geq 1 \end{cases}$$

$\epsilon(t)$: random noise

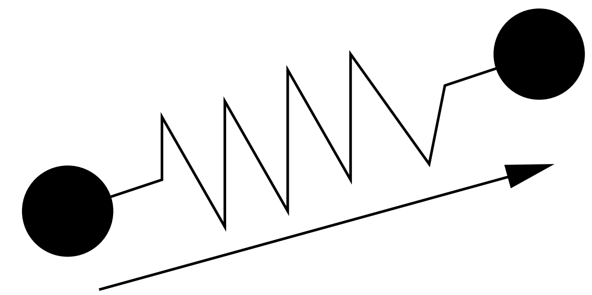
Viscoelastic flow model

FENE-P model for Newtonian flow



polymer molecule

reduced model



$\mathbf{q}$: end-to-end vector

continuum model

$$\mathbf{C} = C_{ij} = \mathbf{q} \otimes \mathbf{q} = q_i q_j$$

Polymer solution parameter

$$Wi = \frac{\text{polymer solution relaxation time scale}}{\text{flow time scale}}$$

L: maximum polymer extension

$$\beta = 0.9 = \frac{\text{solvent viscosity}}{\text{zero-shear polymer solution viscosity}}$$

Viscoelastic flow model (FENE-P)

- Momentum transport equation

$$\partial_t \mathbf{u} + \boldsymbol{\nabla} \mathbf{u} \otimes \mathbf{u} = -\boldsymbol{\nabla} p + \frac{\beta}{Re} \nabla^2 \mathbf{u} + \frac{1-\beta}{Re} \boldsymbol{\nabla} \cdot \mathbf{T}$$

- Polymer stress tensor

$$\mathbf{T} = T_{ij} = \frac{1}{Wi} \left(\frac{C_{ij}}{1 - C_{kk}/L^2} - \delta_{ij} \right)$$

- Conformation stress tensor

$$\partial_t C + (\mathbf{u} \cdot \boldsymbol{\nabla}) \, \mathbf{C} = \mathbf{C} \cdot (\boldsymbol{\nabla} \mathbf{u}) + (\boldsymbol{\nabla} \mathbf{u})^t \cdot \mathbf{u} - \mathbf{T}$$

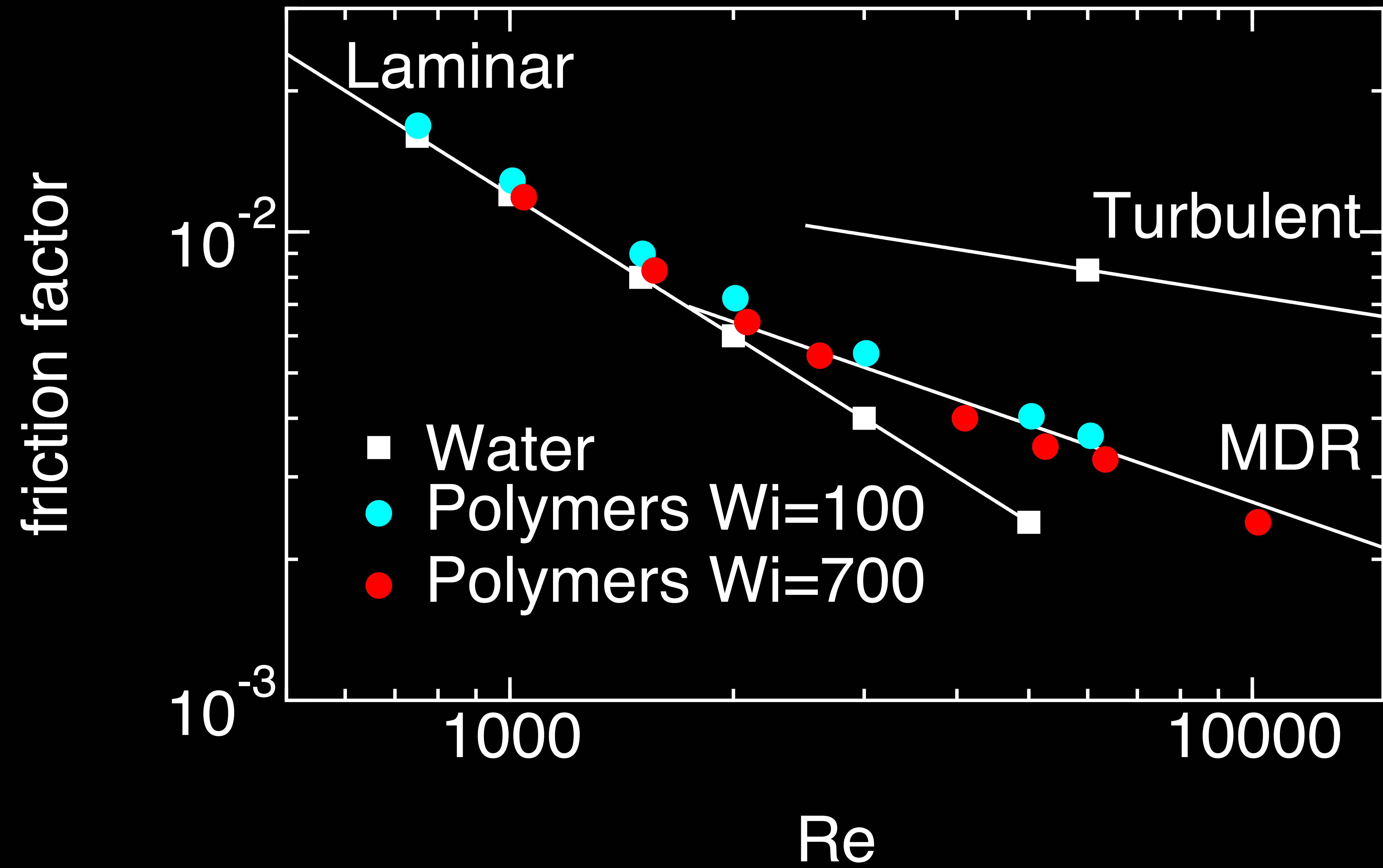
advection polymer stretching-internal forces

Polymer effects

- Polymer solution viscosity decreases in shear flow for large concentration
- Polymer solution viscosity (dramatically) increases in extensional flows due to the increase in polymer extension

Realism of our DNS

- The hyperbolicity of the conformation tensor transport equation is respected as best as numerically possible (Dubief et al., 2005)
- Polymer parameters are such that the shear thinning effect is small but the extensional viscosity is large (increasing with *Wi*), as expected for low polymer concentrations
- The following movies are representative of the dominant dynamics of the respective flows over a very long simulation time
- Our simulations reproduce the evolution of the friction factor as a function of the Reynolds numbers observed in pipe flow experiments (Samanta, Dubief, Holzner, Shäfer, Morozov, Wagner, Hof, submitted)



Polymers create their own turbulence at subcritical Reynolds numbers

Equations of Elasto-Inertial Turbulence

$$\partial_t \mathbf{C} + (\mathbf{u} \cdot \boldsymbol{\nabla}) \mathbf{C} = \mathbf{C} \cdot (\boldsymbol{\nabla} \mathbf{u}) + (\boldsymbol{\nabla} \mathbf{u})^t \cdot \mathbf{C} - \mathbf{T}$$

$$\boldsymbol{\nabla} \cdot \left[\partial_t \mathbf{u} + \boldsymbol{\nabla} \mathbf{u} \otimes \mathbf{u} = -\boldsymbol{\nabla} p + \frac{\beta}{Re} \nabla^2 \mathbf{u} + \frac{1-\beta}{Re} \boldsymbol{\nabla} \cdot \mathbf{T} \right]$$

$$\Downarrow$$

$$\nabla^2 p = 2Q + \frac{1-\beta}{Re} \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \cdot \mathbf{T})$$

$$Re = \frac{U_b H}{\nu} = 1000,\ Wi = \lambda_p \dot{\gamma} = 100$$

Isosurfaces of *Q* the second invariant of velocity gradient tensor

$$Q = \frac{1}{2}\left(\mathbf{\Omega}^2 - \mathbf{S}^2\right) = \frac{1}{8}\left[\left(\nabla\mathbf{u} - \nabla\mathbf{u}^t\right)^2 - \left(\nabla\mathbf{u} + \nabla\mathbf{u}^t\right)^2\right]$$

- Flow is perfectly laminar in the absence of polymers

- Polymer addition creates a self-sustained chaotic flow consisting of trains of cylindrical weakly rotational regions (positive *Q*) and weakly extensional regions (negative *Q*)

- There is a hierarchy of cylindrical structures, the smallest one being of the order of the Kolmogorov scale

$$Re = \frac{U_b H}{\nu} = 1000,\ Wi = \lambda_p \dot{\gamma} = 100$$

Contours of polymer extension and *Q*

$$\sqrt{\frac{\mathrm{Trace}}{L^2}}$$

The polymer extension field is organized in sheets

Polymers cause the flow to evolve from pure shear flow to mix extensional-shear flow

The cylindrical *Q* structures are attached to sheets of large polymer extension

$$Re = \frac{U_b H}{\nu} = 6000,\ Wi = \lambda_p \dot{\gamma} = 700$$

Isosurfaces of *Q* the second invariant of velocity gradient tensor

$$Q = \frac{1}{2}\left(\boldsymbol{\Omega}^2 - \mathbf{S}^2\right) = \frac{1}{8}\left[\left(\boldsymbol{\nabla}\mathbf{u} - \boldsymbol{\nabla}\mathbf{u}^t\right)^2 - \left(\boldsymbol{\nabla}\mathbf{u} + \boldsymbol{\nabla}\mathbf{u}^t\right)^2\right]$$

- The flow is at the maximum drag reduction state
- Vortices are highly intermittent
- Long periods of elasto-inertial turbulence

Equations of Elasto-Inertial Turbulence

$$\partial_t \mathbf{C} + (\mathbf{u} \cdot \boldsymbol{\nabla}) \mathbf{C} = \mathbf{C} \cdot (\boldsymbol{\nabla} \mathbf{u}) + (\boldsymbol{\nabla} \mathbf{u})^t \cdot \mathbf{C} - \mathbf{T}$$

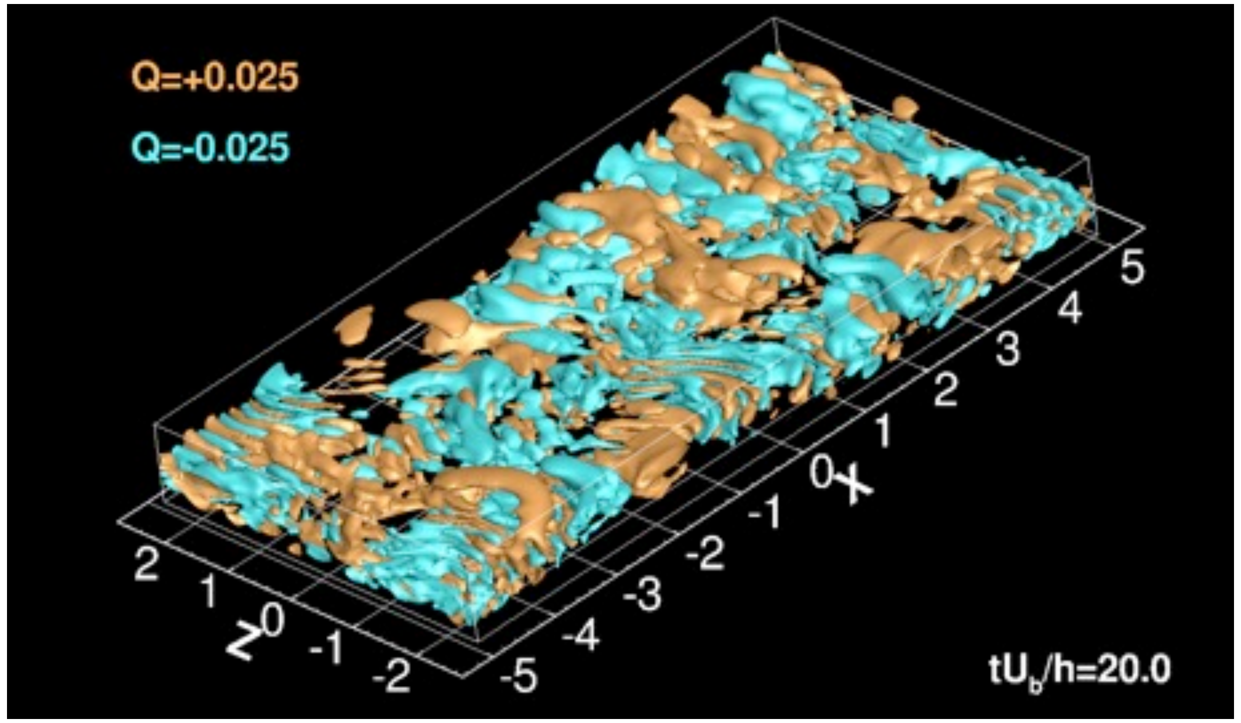
$$\boldsymbol{\nabla} \cdot \left[\partial_t \mathbf{u} + \boldsymbol{\nabla} \mathbf{u} \otimes \mathbf{u} = -\boldsymbol{\nabla} p + \frac{\beta}{Re} \nabla^2 \mathbf{u} + \frac{1-\beta}{Re} \boldsymbol{\nabla} \cdot \mathbf{T} \right]$$

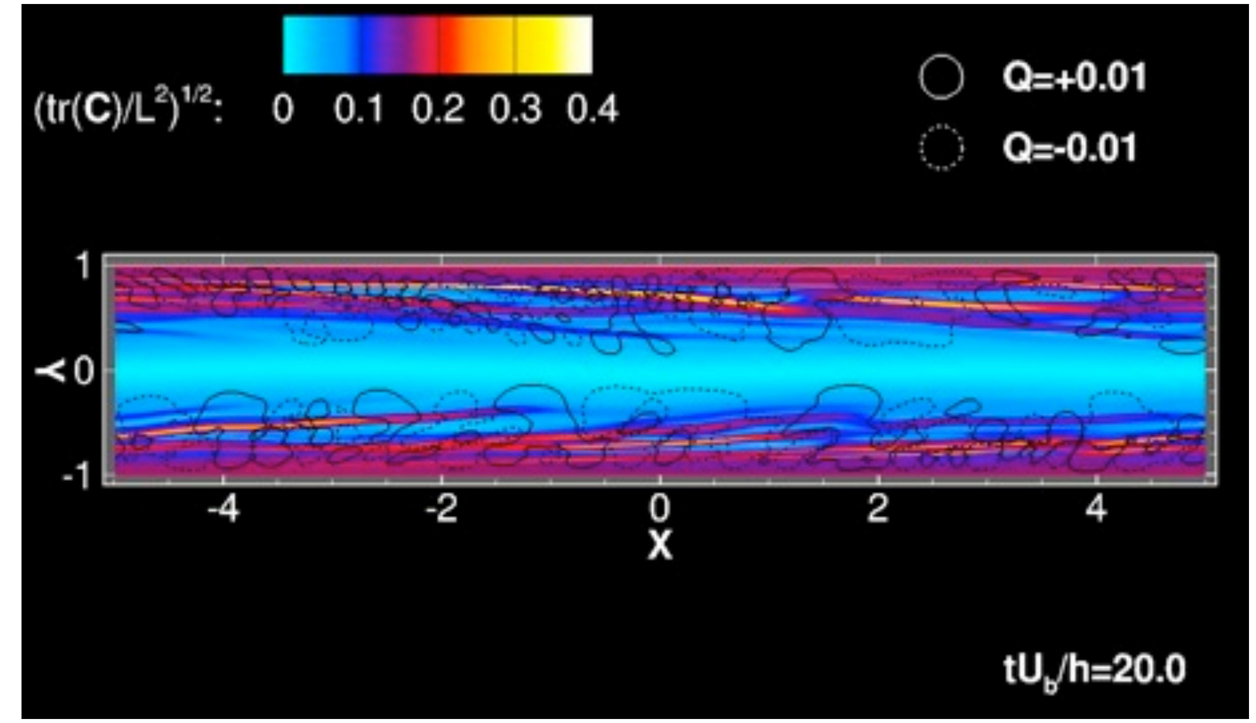
$$\Downarrow$$

$$\nabla^2 p = 2Q + \frac{1-\beta}{Re} \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \cdot \mathbf{T})$$

- Elasto-inertial turbulence results from the combination of the hyperbolic transport equation of C and the elliptic equation of p
- Pressure redistributes energy with trains of cylindrical structures to attenuate the anisotropy caused by sheets of extensional viscosity

Mechanism of Elasto-Inertial Turbulence





$$\partial_t \mathbf{C} + (\mathbf{u} \cdot \boldsymbol{\nabla})\, \mathbf{C}$$

Formation of sheets of C

$$\nabla^2 p = 2Q + \frac{1-\beta}{Re} \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \cdot \mathbf{T})$$

Excitation of extensional sheet flow and elliptical pressure redistribution of energy

$$\mathbf{C} \cdot (\boldsymbol{\nabla} \mathbf{u}) + (\boldsymbol{\nabla} \mathbf{u})^t \cdot \mathbf{C} - \mathbf{T}$$

Increase of extensional viscosity in sheets

Stick around

- Julio Soria will tell you everything you need to know about the flow topology of EIT
- Vincent Terrapon will show the long range interactions that trigger EIT in a bypass transition flow
- and all the other talks

Acknowledgments

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