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Mechanics and Characteristics of Transition to Turbulence in Elasto-Inertial Turbulence

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Context and objectives

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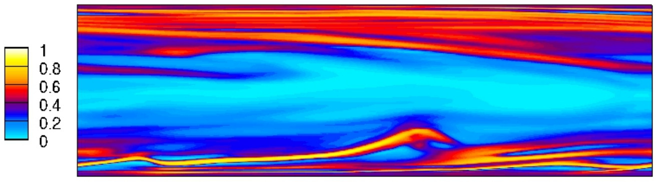
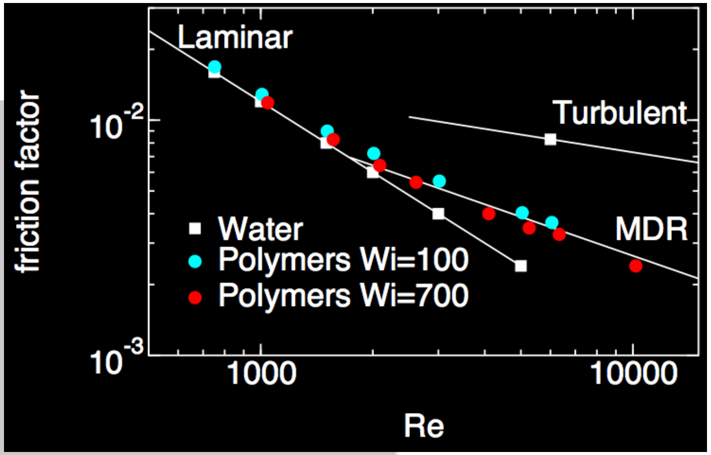
Polymer extension



Channel flow



$$\partial_t \mathbf{C} + (\mathbf{u}\cdot\nabla)\mathbf{C}$$

- Formation of sheets

$$\nabla^2 p = 2Q_a - \frac{1-\beta}{Re}\nabla\cdot(\nabla\cdot\mathbf{T})$$

- Excitation of extensional sheet flow
- Elliptical pressure redistribution of energy

$$\mathbf{C}(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T\mathbf{C} - \mathbf{T}$$

- Increase of extensional viscosity

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Context and objectives

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Context and objectives

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- How do long-range interactions through pressure contribute to transition?
- What locations in the flow contribute most to instability excitation?
- What is the relative contribution of both terms in the Poisson equation?

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Equations

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Navier-Stokes for dilute polymer solutions

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla} p + \frac{\beta}{Re}\nabla^2\mathbf{u} + \boxed{\frac{1-\beta}{Re}\boldsymbol{\nabla}\cdot\mathbf{T}}$$

Polymer stress – FENE-P model

$$\mathbf{T} = \frac{1}{Wi}\left(\frac{\mathbf{C}}{1-\mathrm{tr}(\mathbf{C})/L^2} - \mathbf{I}\right)$$

Polymer conformation tensor

$$\partial_t \mathbf{C} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{C} = \mathbf{C}(\boldsymbol{\nabla}\,\mathbf{u}) + (\boldsymbol{\nabla}\,\mathbf{u})^{\mathrm{T}}\mathbf{C} - \mathbf{T}$$

Equations

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Simulated by-pass transition

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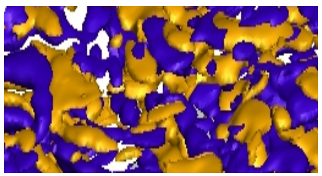
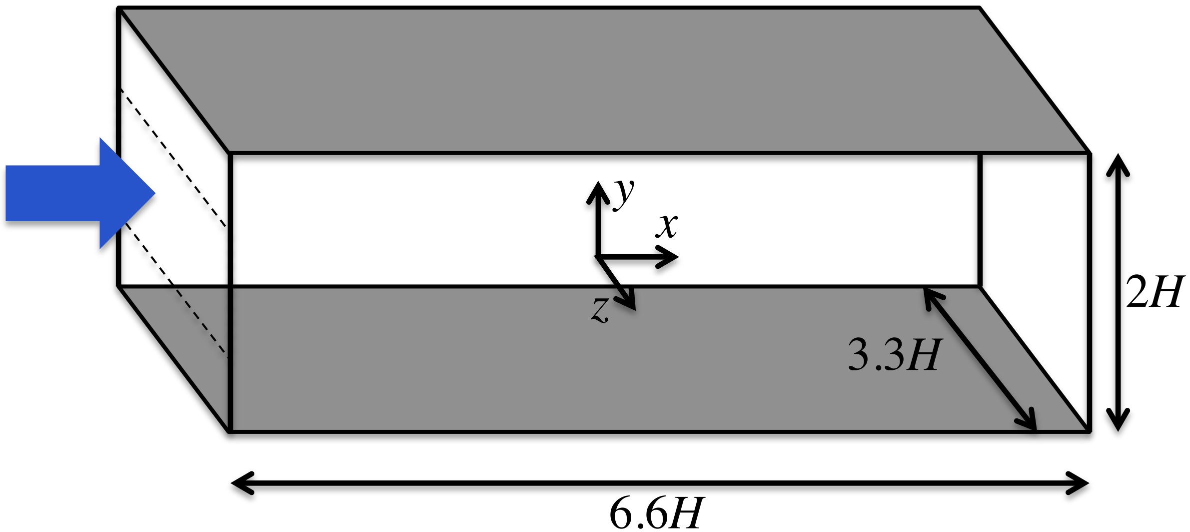


Periodic channel flow



Parameters

- $Wi = 400$
- $L = 200$
- $Re = U_b H / \nu = 1000$
- $\beta = 0.9$

1. **Plug flow with slip**
2. **Injection of homogeneous isotropic turbulence in channel center**
3. **No-slip at the wall**

Transition is promoted by viscoelasticity

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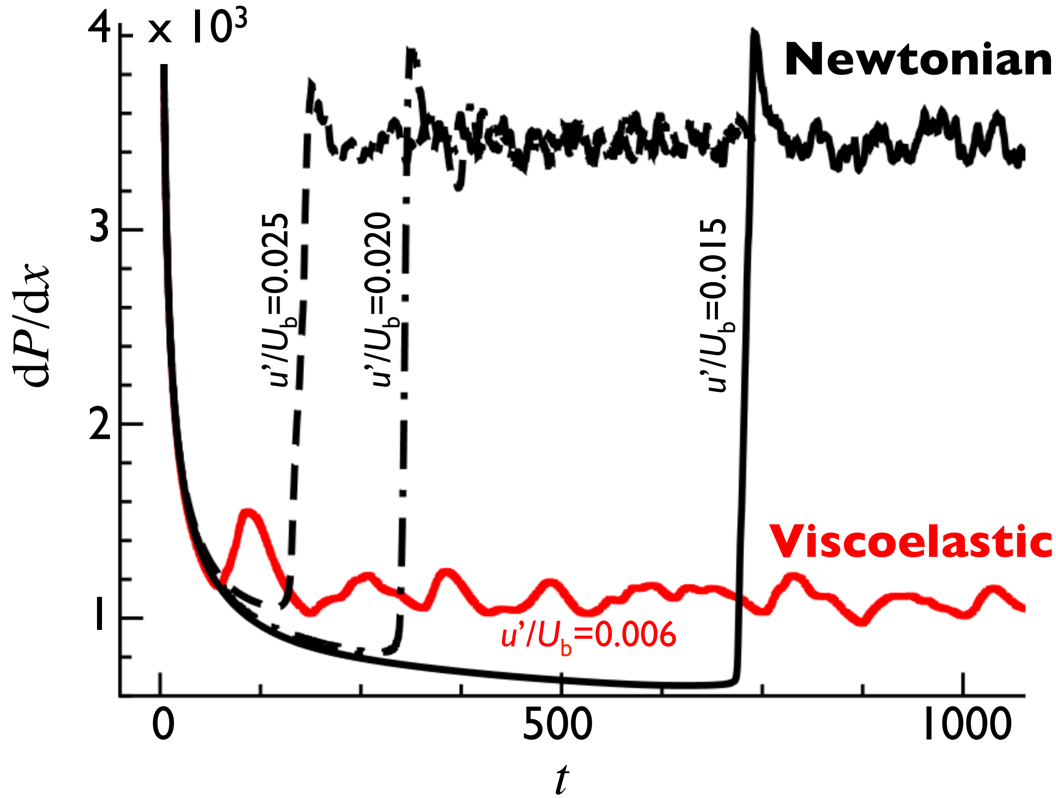
Evolution of wall friction



- Viscoelastic flow transitions with **weaker** perturbations
- Same perturbation level would lead to **laminar** Newtonian flow
- Viscoelastic flow transitions to **MDR** and stays at MDR

2 states of the evolution considered

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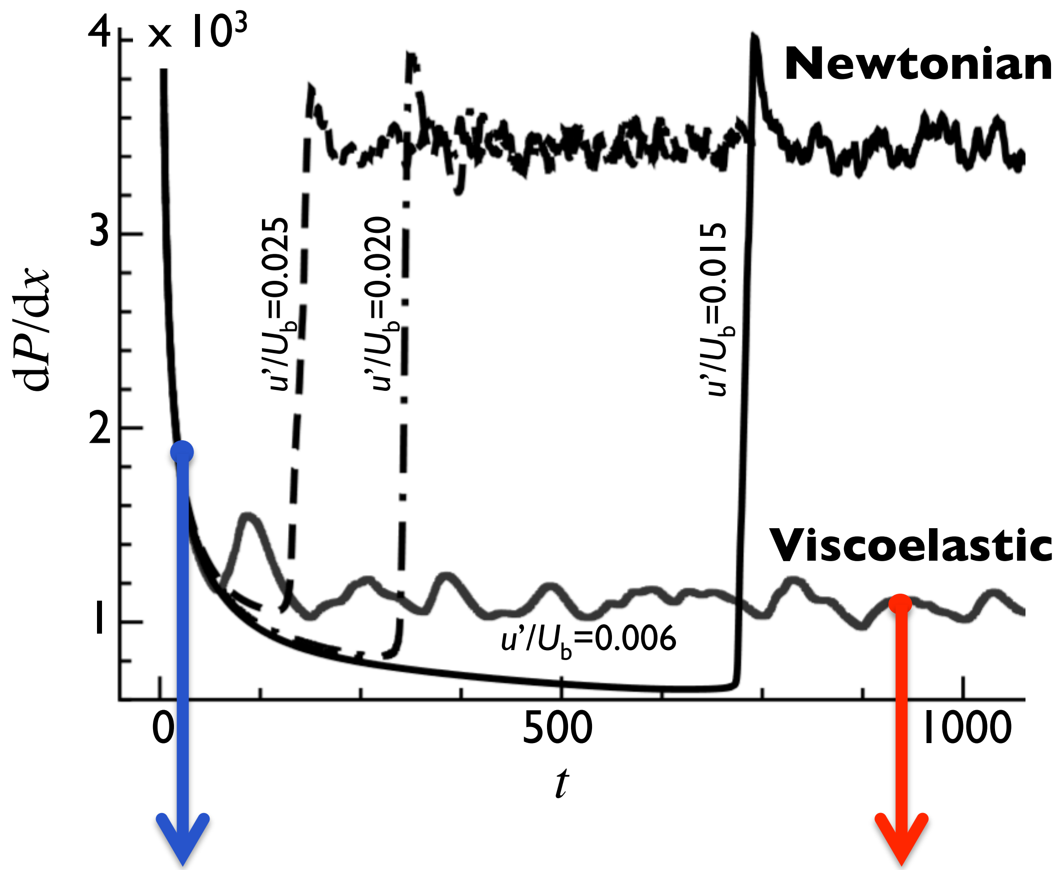
Evolution of wall friction



Before nonlinear breakdown of instabilities

After nonlinear breakdown of instabilities

Spatial statistics

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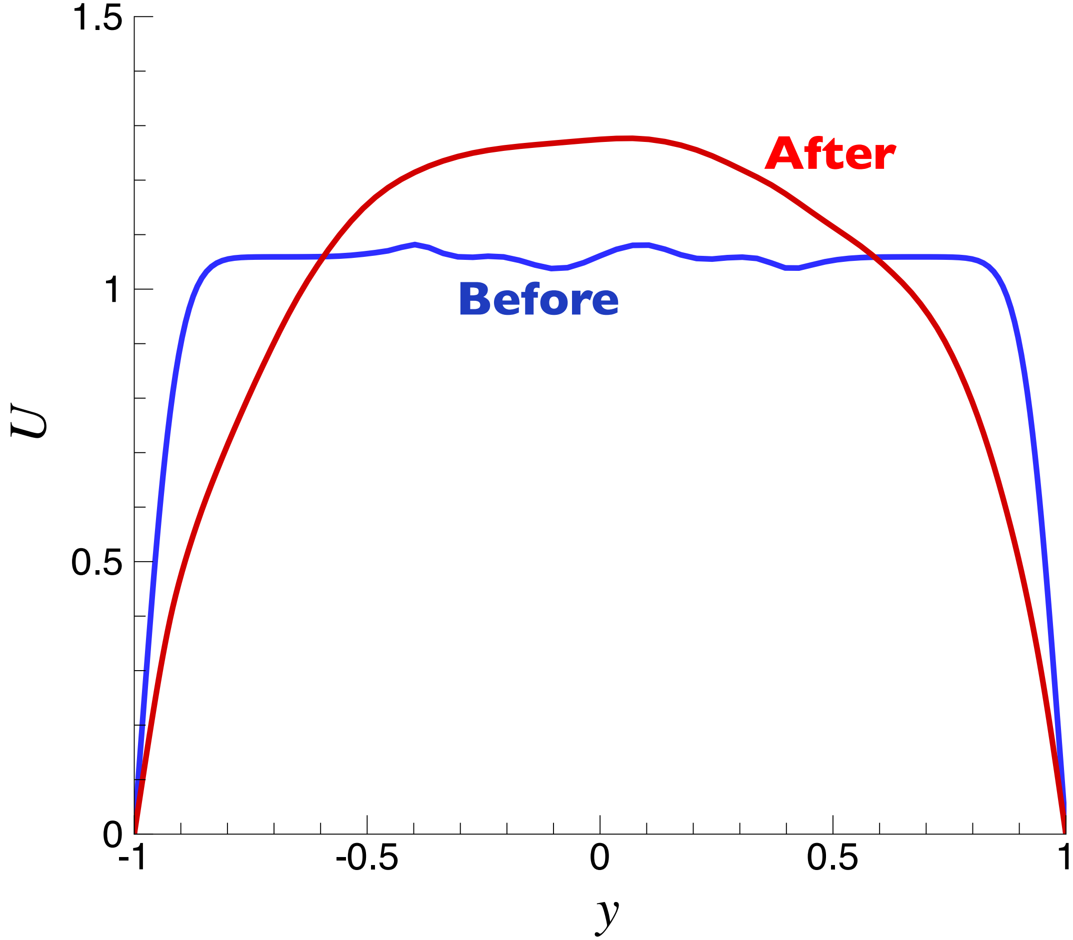
Mean velocity profile



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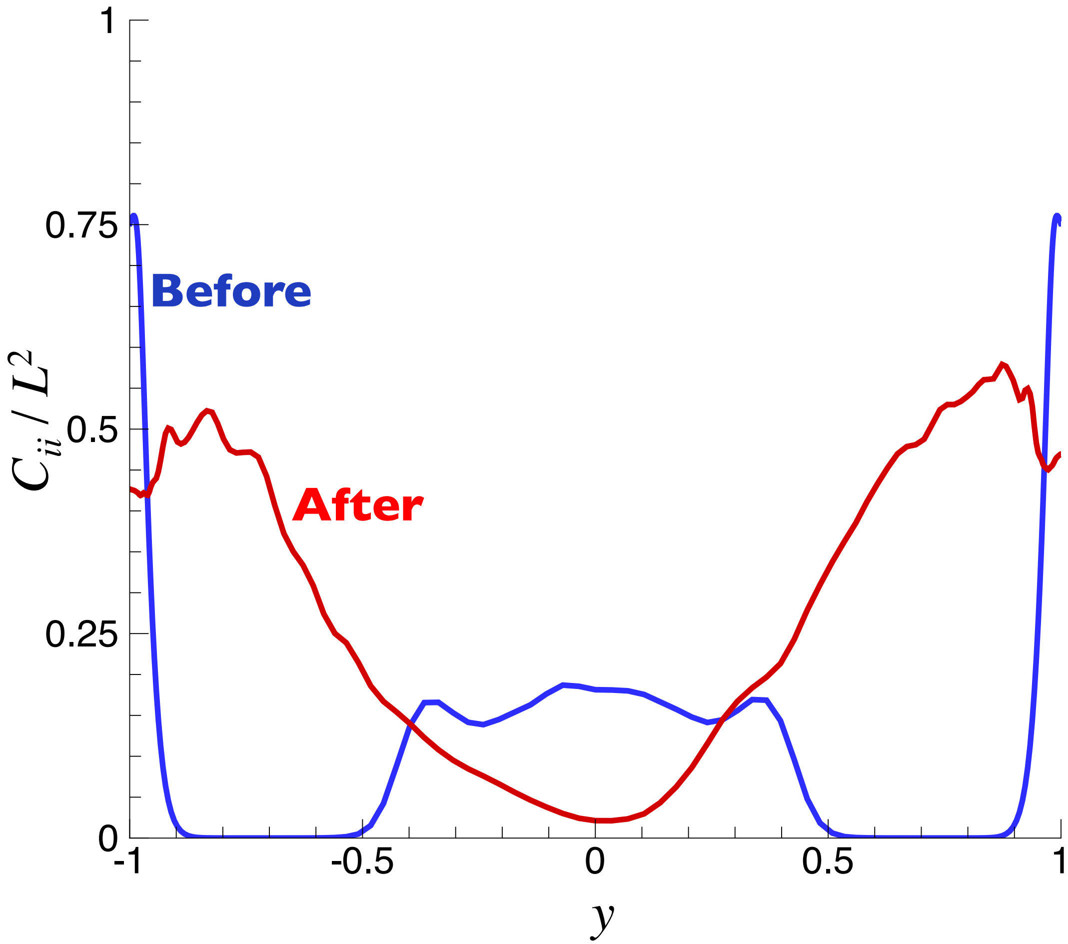
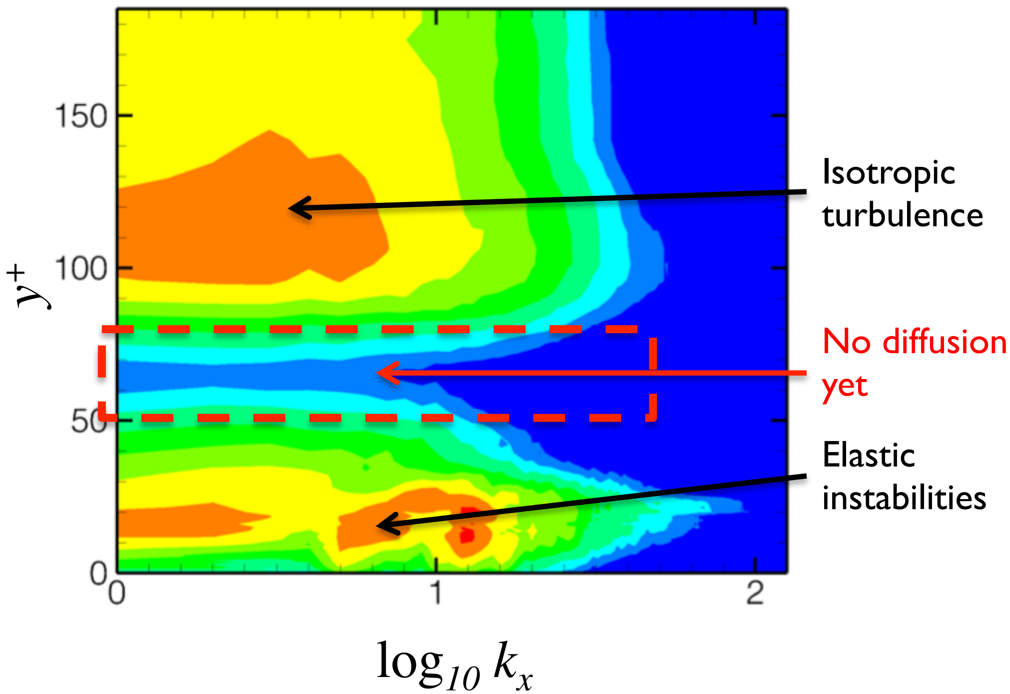
Spatial statistics

Mean polymer extension



Excitation of instabilities

Power spectrum of the elastic energy before nonlinear breakdown of instabilities



- Instabilities in near-wall regions not caused by diffusion of turbulence from channel center
- What triggers the instability?

Poisson equation for pressure

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Extended Poisson equation

$$\nabla^2 p = \underbrace{2Q_a + \frac{1-\beta}{Re}\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\mathbf{T})}_{f(\mathbf{x})}$$

Poisson equation for pressure

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Extended Poisson equation

$$\nabla^2 p = 2Q_a + \underbrace{\frac{1-\beta}{Re}\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\mathbf{T})}_{f(\mathbf{x})}$$

+ B.C.

Pressure kernel – Green function G

$$p(\boldsymbol{\xi}) = \int_V G(\boldsymbol{\xi},\mathbf{x})f(\mathbf{x})\mathrm{d}\mathbf{x} \equiv \int_V F(\boldsymbol{\xi},\mathbf{x})\mathrm{d}\mathbf{x}$$

"Influence" function $F(\mathbf{x},\xi)$ represents the contribution of point $\mathbf{x}$ to the pressure at point ξ

Poisson equation for pressure

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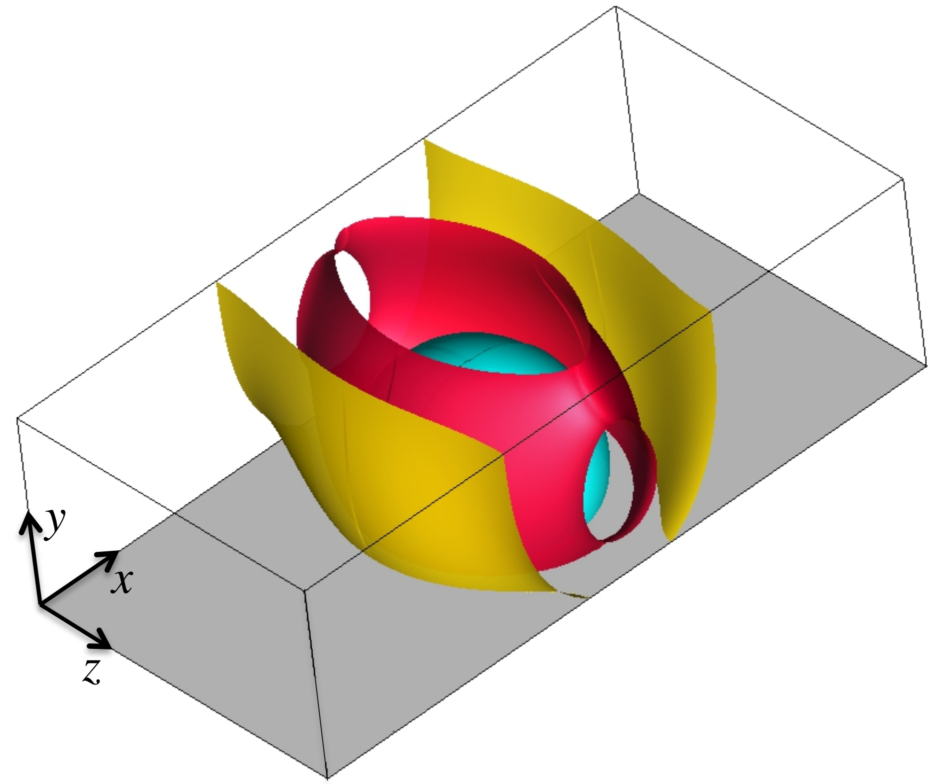
"Influence" function $F(\mathbf{x}, \xi)$ represents the contribution of point $\mathbf{x}$ to the pressure at point ξ

Green function $G(0, -0.9H, 0; x, y, z)$



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Influence function

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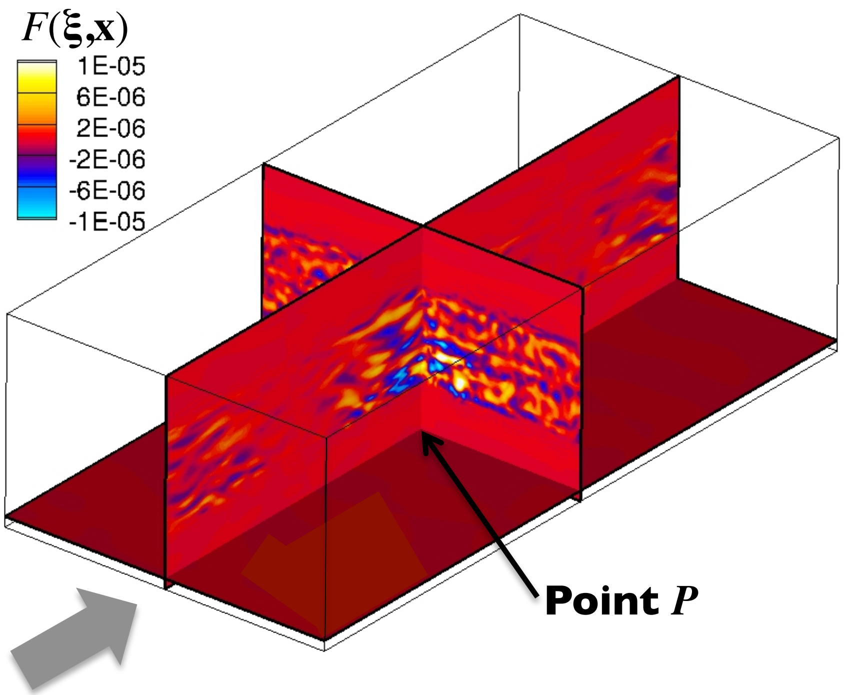
Before nonlinear breakdown of instabilities



Source of contribution to pressure at point P from relative "unorganized" free-stream turbulence in channel center

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Influence function

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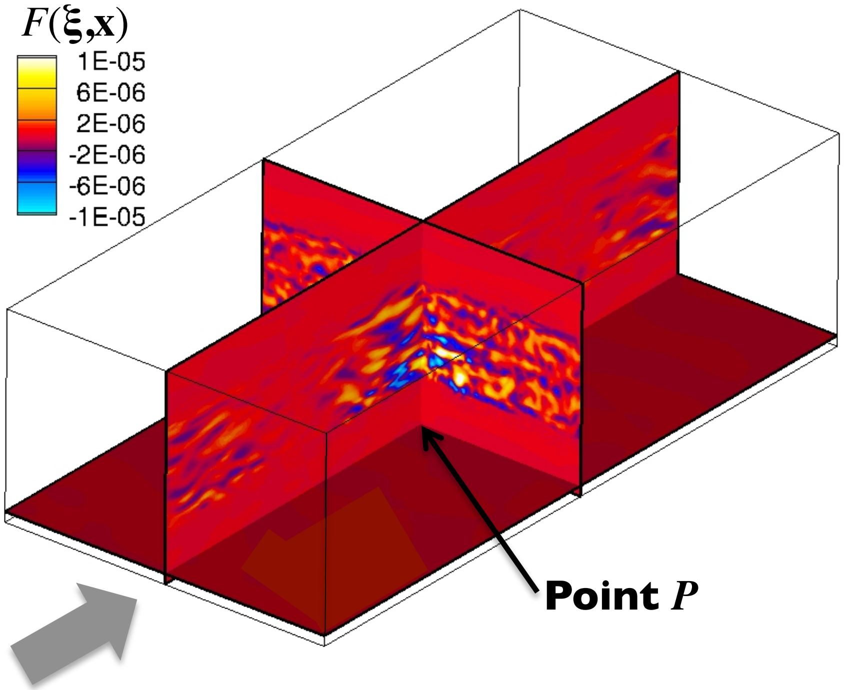
Before nonlinear breakdown of instabilities



Source of contribution to pressure at point P from relative "unorganized" free-stream turbulence in channel center

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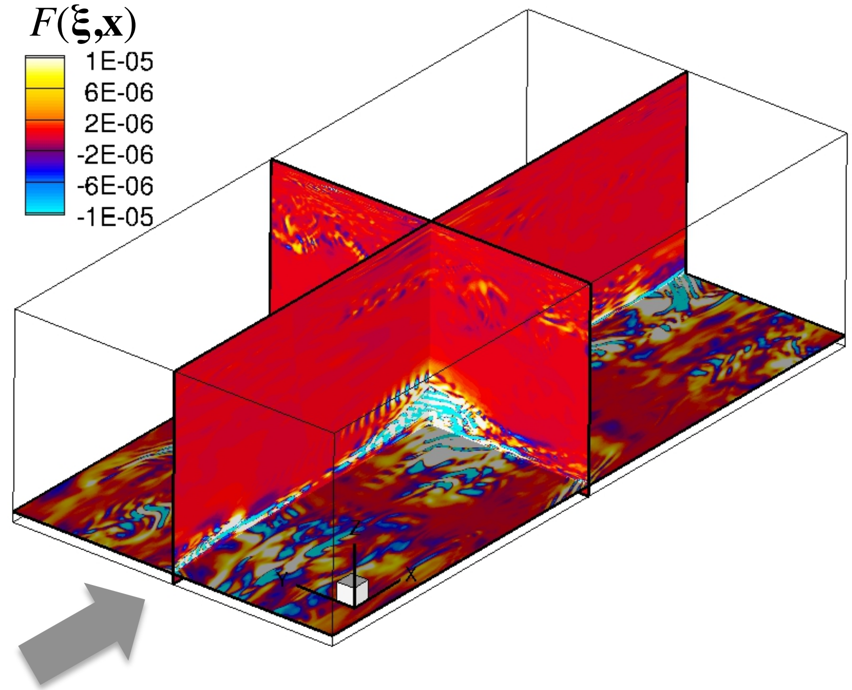

Source of contribution to pressure at point P from elastically induced more "organized" structures in near-wall region

Plane-averaged pressure fluctuations

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$$\nabla^2 p = 2Q_a - \frac{1-\beta}{Re}\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\mathbf{T})$$

$$p(\boldsymbol{\xi}) = \int_V G(\boldsymbol{\xi},\mathbf{x})f(\mathbf{x})\mathrm{d}\mathbf{x}$$

$$\overline{p^2}(\eta) = \int_{-H}^{H} h(y;\eta)\,\mathrm{d}y$$

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Plane-averaged pressure fluctuations

$$\nabla^2 p = 2Q_a - \frac{1-\beta}{Re}\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\mathbf{T}) \quad \blacktriangleright \quad p(\boldsymbol{\xi}) = \int_V G(\boldsymbol{\xi},\mathbf{x})f(\mathbf{x})\mathrm{d}\mathbf{x} \quad \blacktriangleright \quad \overline{p^2}(\eta) = \int_{-H}^{H} h(y;\eta)\,\mathrm{d}y$$

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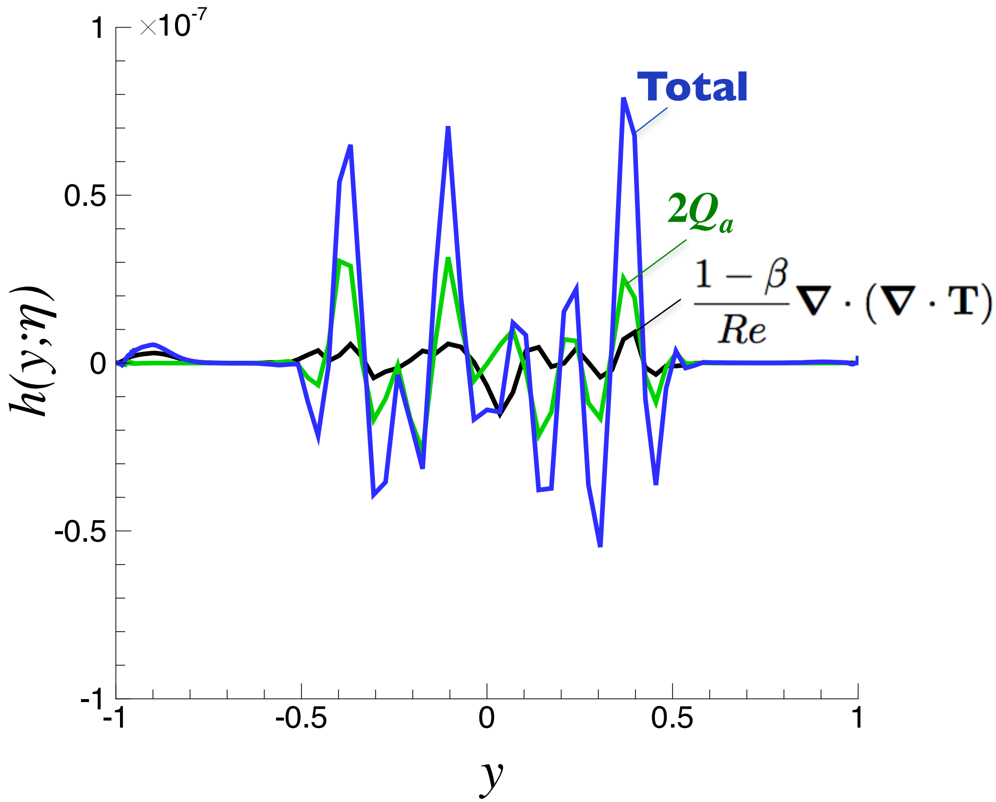

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Plane-averaged pressure fluctuations

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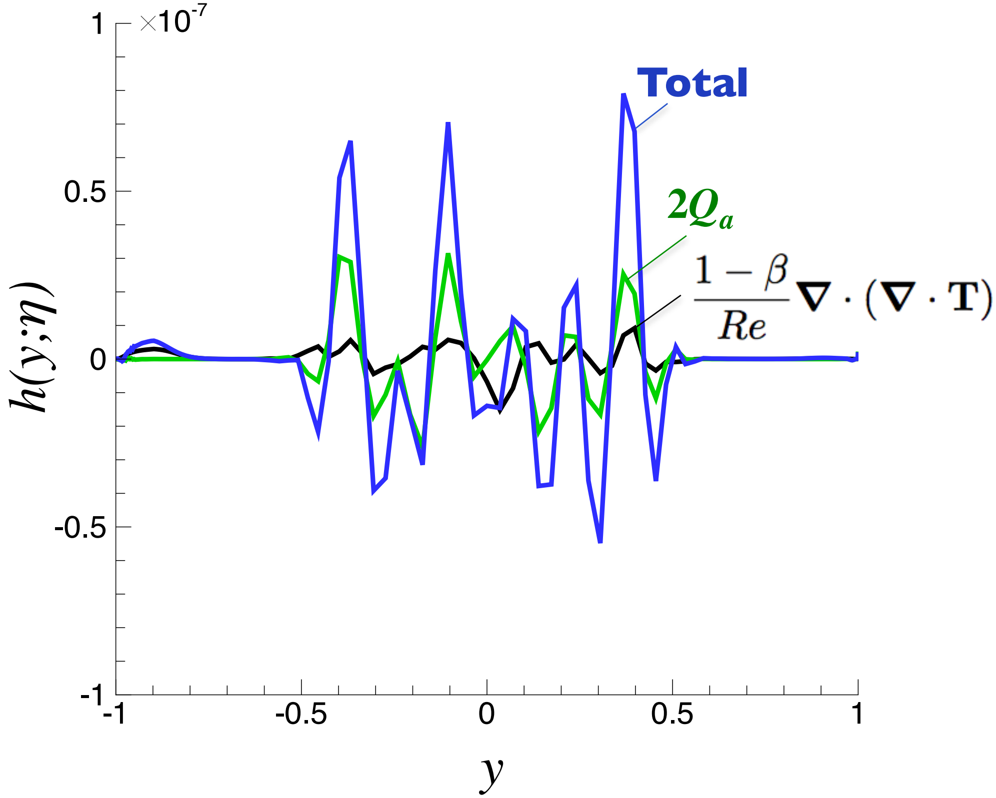
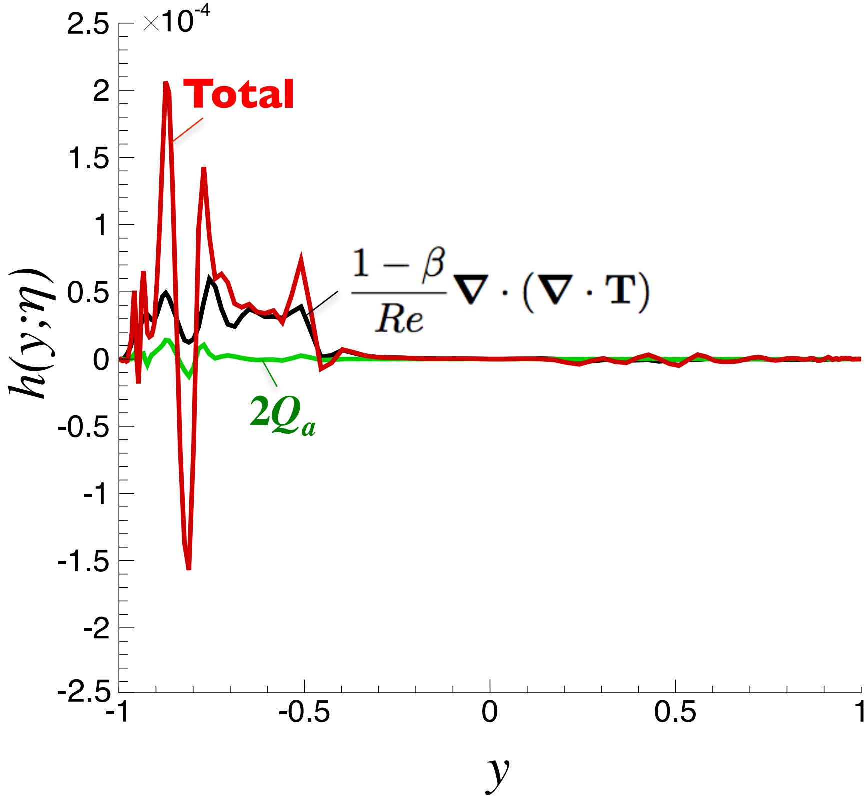


After nonlinear breakdown



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Conclusions

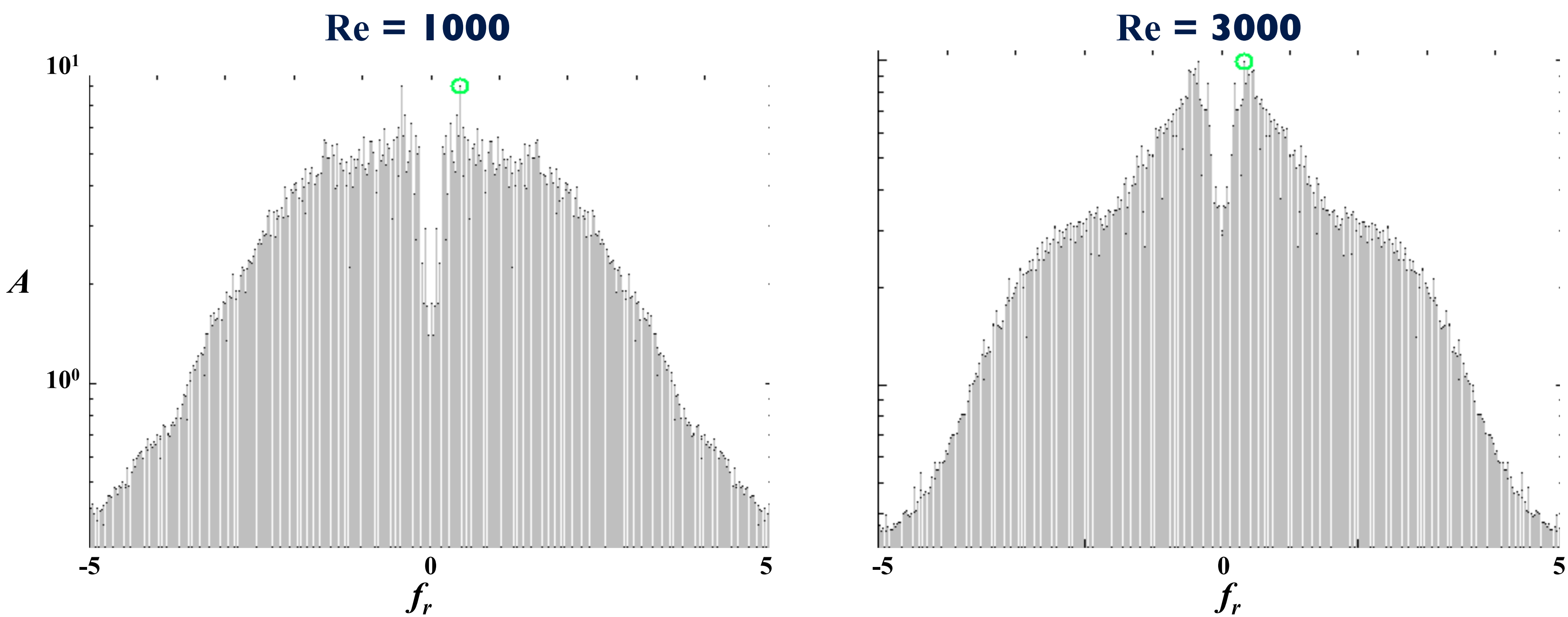
Current understanding of viscoelastic transition

- Long-range excitation of elastic instabilities through pressure
- Feeding of energy to elastic instabilities from mean flow
- Elastic instabilities then self-sustained
- Polymeric contribution in Poisson equation dominates kinematic contribution

Future work – DMD analysis

Mode amplitude as a function of frequency from DMD analysis

Q invariant (streamwise – wall-normal plane)



- Shape change with increasing Re
- At larger Re, two apparent contributions
- Hypothesis: elastic and inertial (to be verified)

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Future work – DMD analysis

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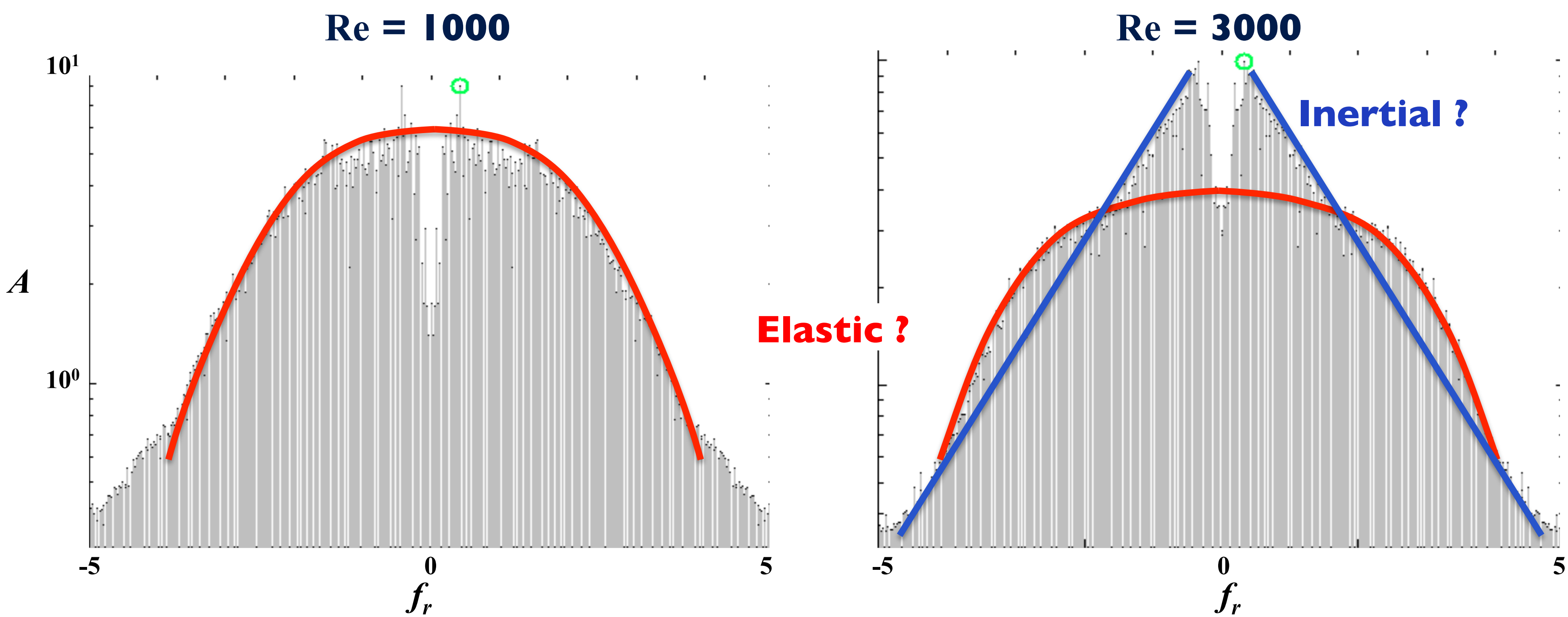

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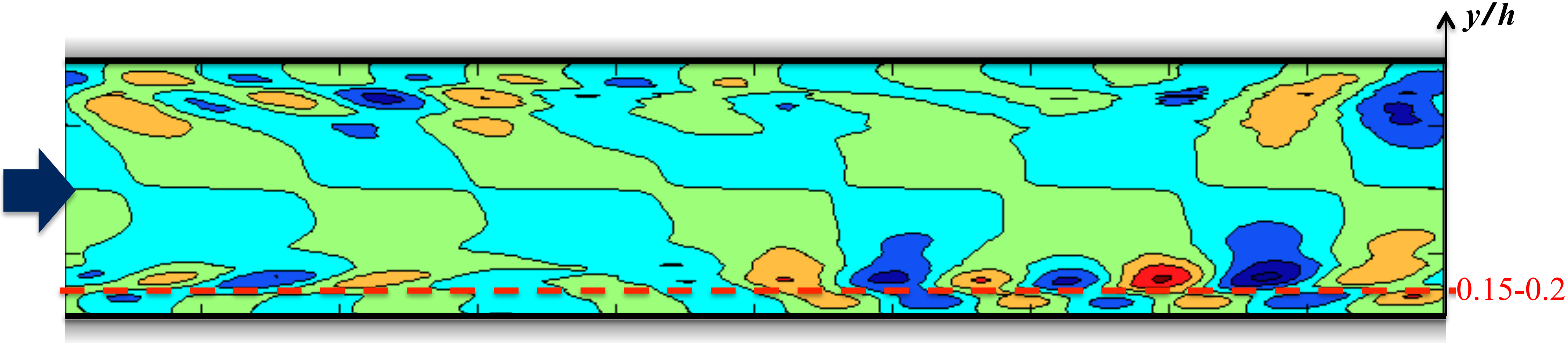
Future work – DMD analysis

Most amplified mode from DMD analysis
Q invariant (streamwise – wall-normal plane) at Re=1000



- Mostly two-dimensional structures
- Located in near-wall region
- Alternating pressure minima and maxima
- "Discontinuity" close to wall corresponding to location of extremum of mean polymer extension (critical layer?)

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Vortices and extensional flow

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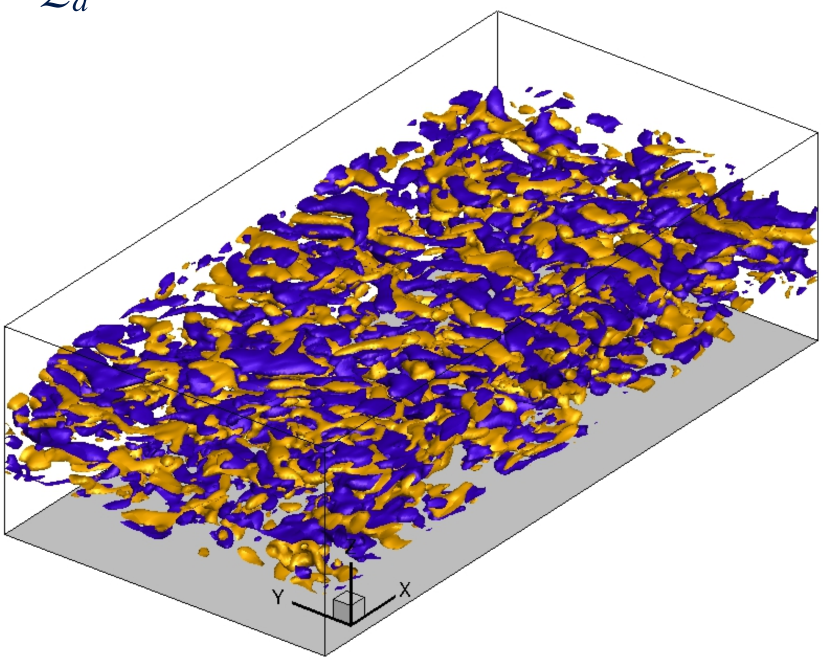
Before nonlinear breakdown of instabilities

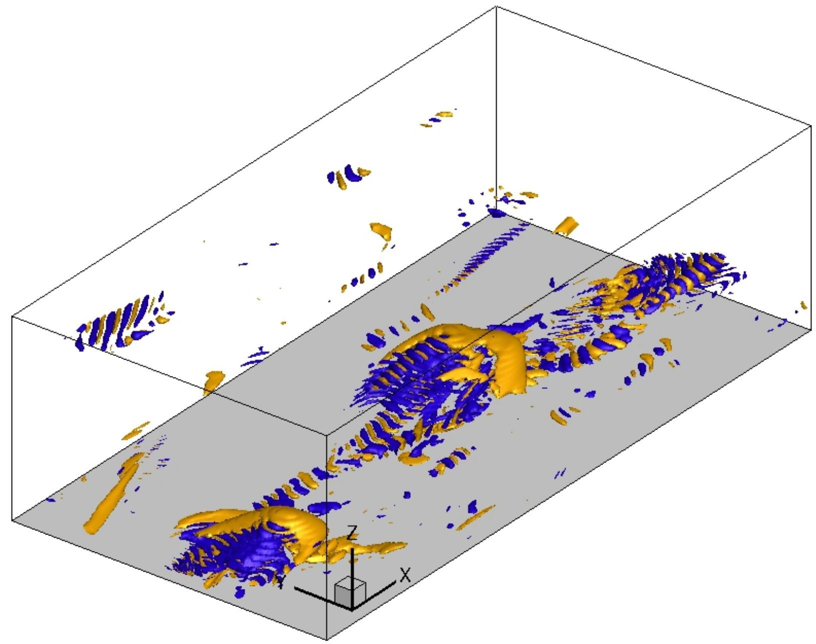
- $Q_a = 0.02$
- $Q_a = -0.02$



Weak homogeneous isotropic turbulence

After nonlinear breakdown of instabilities

- $Q_a = 0.2$
- $Q_a = -0.2$



Few hairpin-like vortices followed by train of alternating "rotational" and "straining" flow at very small scales

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