ASSESSMENT OF THE EFFECT OF MICRO-SIMULATION ERROR ON KEY TRAVEL INDICES: EVIDENCE FROM THE ACTIVITY-BASED MODEL FEATHERS

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ABSTRACT

Current transportation models often do not explicitly address the degree of uncertainty in travel forecasts. Of particular interest in activity-based travel demand models is the model uncertainty that is caused by the statistical distributions of random components, i.e. micro-simulation error. Therefore, the main objective of this paper is to assess the impact of micro-simulation error on two key travel indices, namely the average daily number of trips per person and the average daily distance traveled per person. The effect of micro-simulation error will be investigated by running the activity-based modeling framework FEATHERS 200 times using the same 10% fraction of the population. Results show that micro-simulation errors are limited especially when disaggregation is limited to two levels. Notwithstanding, results indicate that for more elaborate analyses a 10% fraction might not be sufficient. The size of micro-simulation error increases along with complexity. Moreover, more commonly used transport modes such as using the car as driver have a lower error rate. Further research should investigate the impact of the population fraction on the micro-simulation error rates. Besides, one could also investigate other aspects (e.g. the number of activities) involved in the activity-scheduling process.

1 BACKGROUND

Rising concerns over increasingly intolerable externalities have generated particular interest in how transport planning policies might at least moderate the well-known negative effects of transport and support the principles of sustainable development. This has lead to the development of models which are both used to make long term predictions and to account for an explicit management of travel demand (e.g. road pricing), which objective is to alter travel behavior without necessarily embarking on large-scale infrastructure expansion.

The relevance of using travel demand models for this purpose is reflected by the multitude of European Research projects in different countries (e.g. Four Futures of Europe (*1*), Mobility 2030: Meeting the Challenges to Sustainability (*2*), TransVisions (*3*)). In these projects, projections for the future are often done by means of rather straightforward assumptions: all or most future trends are translated into changes in either travel time and transport costs and these variables are subsequently used to estimate mode outcomes (e.g. The TREMOVE model (*4*)). Therefore, the need of adopting more behaviorally sound models, that do not assess the anticipated changes in behavior by means of a simple transformation, is high. Activity-based models are implemented within a micro-simulation environment and they implement the more behaviorally sound environment by means of individual decision rules about activities and about the way they are dispersed in space and time. As such, they also provide a theoretically and conceptually more sound framework for forecasting travel behavior in comparison with more traditional methods (*5*, *6*).

Current models often do not explicitly address the degree of uncertainty in travel forecasts. Since transportation models are used to predict the likely impacts of various policy measures such as congestion charging and new transport infrastructure, it is imminent for decision-making to have an estimate not only of the most likely outcome, but also to know the possible range and variability of future transport predictions and their corresponding probabilities (*7*). After all, estimates of for instance the financial viability of infrastructure projects are highly dependent on the accuracy of travel demand forecasts (*8*).

Consider the following example that illustrates the significance of accuracy in forecasts to the effective allocation of funds: Bangkok’s Skytrain (costing about 2 billion US dollars) was hugely over-dimensioned because the passenger forecast was 2.5 times higher than the actual ridership. As a result, station platforms are too long for the shortened trains that now operate the system, a large number of trains are superfluous, terminals are to large, etc (*9*). Moreover, in a more elaborate study of 210 projects in 14 nations with a combined cost of $59 billion, in 9 out of 10 transit projects, transit ridership was overestimated by an average of 106%, and half of all roadway projects had prediction errors of more than 20% (*10*). These errors can be attributed to many causes, from cynical interpretations of models to achieve political aims of leveraging federal investment in desired projects, to uncertainty in model assumptions and errors in model specifications (*11*).

In essence, uncertainty in model results can be divided into two components; input uncertainty and model uncertainty (*12*). Input uncertainty expresses the fact that future values of the exogenous variables are unknown. Model uncertainty is caused by two elements (*7*), namely by specification errors (omitted variables, inappropriate assumptions on functional form and statistical distributions for random components), and errors due to the use of parameter estimates instead of the true values (the model is estimated on a sample of the population only). Of particular interest in activity-based travel demand models is the model uncertainty that is caused by micro-simulation, namely the fact that the results are stochastic, meaning that the forecast changes each time the seeds to the random number generators used in the simulation change (*13*). Therefore, the main objective of this paper is to assess the impact of micro-simulation error on two key travel indices, namely the average daily number of trips per person and the average daily distance traveled per person The effect of micro-simulation error will be investigated by means of the activity-based modeling framework FEATHERS (*14*). In Section 2, the set-up of the experiment will be described in more detail. Afterwards, factors that contribute to the error rates will be highlighted in Section 3 and discussed in Section 4. Finally, the main conclusions are recapitalized in Section 5.

2 EXPERIMENT

As outlined in the introduction, the impact of micro-simulation on key travel indices will be investigated using the FEATHERS activity-based modeling framework. In essence, FEATHERS is a rule- and agent-based micro-simulation model developed for Flanders, the Dutch speaking and northern part of Belgium (*14*). The core activity scheduler of the model is based on the scheduling model that is present in the ALBATROSS model (*15-16*) which was developed for the Netherlands.

For the experiment, a 10% fraction of the population (corresponding to 616,160 persons) will be simulated. Although Castiglione et al. (*17*) used the full population of households to systematically analyze the impact of (micro-)simulation error for the San Francisco model, Walker (*13*) indicated that this is not always necessary, and computation times could be saved using only a fraction of the whole population. Moreover, in most applications it suffices to synthesize only a fraction of the total population. Arentze and Timmermans (*18*) indicate that a fraction of 10% would suffice to for instance reveal the mobility effects on a national level of even a small increase in fuel price.

To estimate the error due to (micro-)simulation, the FEATHERS model will be run 200 times. For all these 200 runs the same 10% fraction of the population will be taken to ensure that the variability in the model outputs is due to the model uncertainty and not due to the selection of a different sample of households. For each of these runs, the most prevalent travel indices in Flemish policy practice (see e.g. the reports on the Flemish ‘national’ household travel surveys 2008 (*19*) and 2009 (*20*)) will be computed, namely the average daily number of trips per person and the average daily distance traveled per person. These travel indices are computed for the entire sample, as well as for particular target segments. A subdivision of these travel indices will be made based on one particular travel facet, namely mode choice, as well as for the socio-demographic variables age and gender. In addition, the cross-tabulations of the travel facet and socio-demographic variables are also computed. This way, one of the most important gains of micro-simulation, namely the preservation of the entire richness of the population throughout the modeling process, is explicitly tested.

Based on the 200 runs the averages and standard deviations of the number of trips and distances are calculated. In this study, the micro-simulation error rate is defined as the standard deviation divided by the mean, which yields the relative standard deviation when compared to its mean. A value of 1.27% of this error rate is considered to be an acceptable barrier as in this case, as the corresponding 95% confidence bounds define a range of 5% deviation ($1.27\% \times z_{0.025} \times 2 = 1.27\% \times 1.96 \times 2$), given the fact that the key indices (number of trips and total trip distance) are normally distributed. The latter hypothesis will be tested using the Shapiro-Wilk test (*21*), the Cramer-von Mises test (*22*) and the Anderson-Darling test (*23*).

In addition to the descriptive segmentation of the micro-simulation error rate, a linear regression model will be estimated to explain the variances in the micro-simulation errors. The dependent variable in this model will be micro-simulation error rate for a particular setting (e.g. the overall mean, the mean for males, etc). The explanatory variables that are going to be used for the analysis are the complexity, which is defined as the number of cross-tabulations of the error rate (e.g. when the combined effect of transport mode and age is considered the complexity is 20 (4 transport modes × 5 age categories), transport mode (all modes (reference category), car as driver, car as passenger, slow modes, and public transport), gender (all gender (reference category), males, females), and age (all age classes (reference category), 18-34 years, 35-54 years, 55-64 years, 65-74 years, and 75+ years). Note that the different categories of the transport mode and socio-demographic uniquely match the categories defined within the FEATHERS framework (*14*).

3 RESULTS

Before elaborating on the segmentation and analysis of the micro-simulation error, it is important that, as assumed, the mean number of trips and distance travelled tabulated for different segments in each run are normally distributed according to the different normality tests. Consequently, the 1.27% error bound indicating an acceptable amount of micro-simulation error is defensible.

3.1 Identification of Influencing Factors

In order to identify potential influencing factors, a subdivision of the micro-simulation error for both travel indices (number of trips, total trip distance) is made based on mode choice, socio-demographic variables and cross-effects.

3.1.1 Mode Choice

Investigation of Figure 1 reveals that the overall error is higher for the daily distance traveled per person than for the daily number of trips. This can be accounted for by the fact that the rule-based activity-scheduler within the FEATHERS framework schedules the type of activities in an earlier step than the activity locations, the latter step dependent upon the earlier steps in the activity-scheduler. For a detailed description of the sequential steps in the activity-scheduler the reader is referred to Arentze et al. (*24*).

Besides, one could note that the micro-simulation error rate is smaller for more commonly used transport modes, both for the number of trips and total distance travelled. The ordering of the magnitude of the micro-simulation error rate appears to be reciprocal to the share the transport mode has in respectively the number of trips and distance traveled. Nonetheless, all error relates are acceptable and only marginally change the estimates of the key indices.

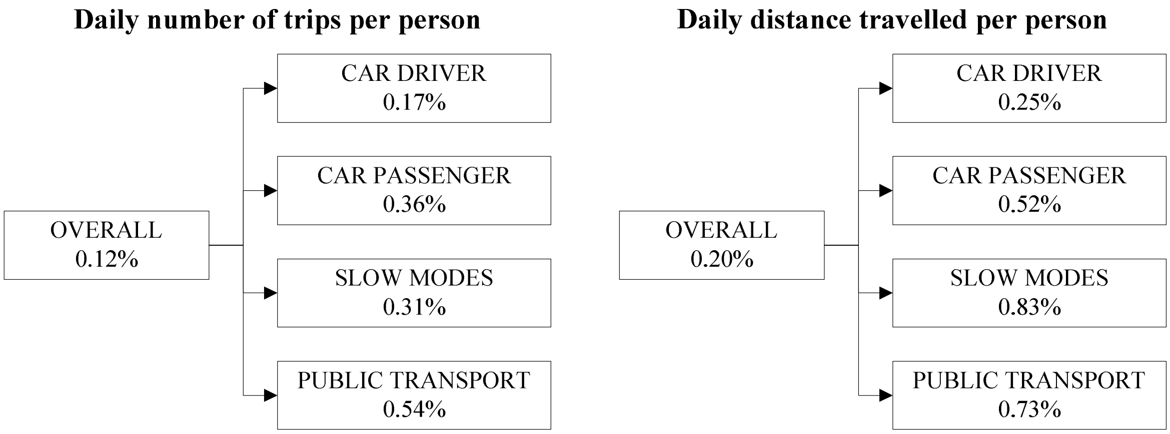

FIGURE 1 Micro-simulation error for different transport modes

3.1.2 Socio-Demographical Variables

Next to the travel facet mode choice, it is also interesting to look at the potential impact of socio-demographical variables. Figures 2 and 3 provide insight in the effect of the variables age and gender on the micro-simulation error rate of respectively the number of trips and distance traveled. Gender appears to have only a limited effect on the error rates, whereas the error rates differ apparently more between different age categories. At least two explanations can be formulated to explain the dissimilarities in error rates between different age groups. The first explanation is the fact that the first two age classes (18-34 years and 35-54 years) involve a larger population than age classes (55-64 years and 65-74 years), which on their turn involve a larger population than the oldest age category. A second reason is the fact that this age class involves way more people that do not travel at all. Consequently, the choice facet of engaging in out-of-home activities or not does have a larger effect of this group, potentially increasing the micro-simulation error rate. Overall, the error due to micro-simulation is acceptable, even for the cross-tabulation of both age and gender, with exception of the combined age and gender effect for the oldest age category with respect to the daily distance traveled. The latter categories (males 75+ and fames 75+) marginally exceed the 1.27 error bound.

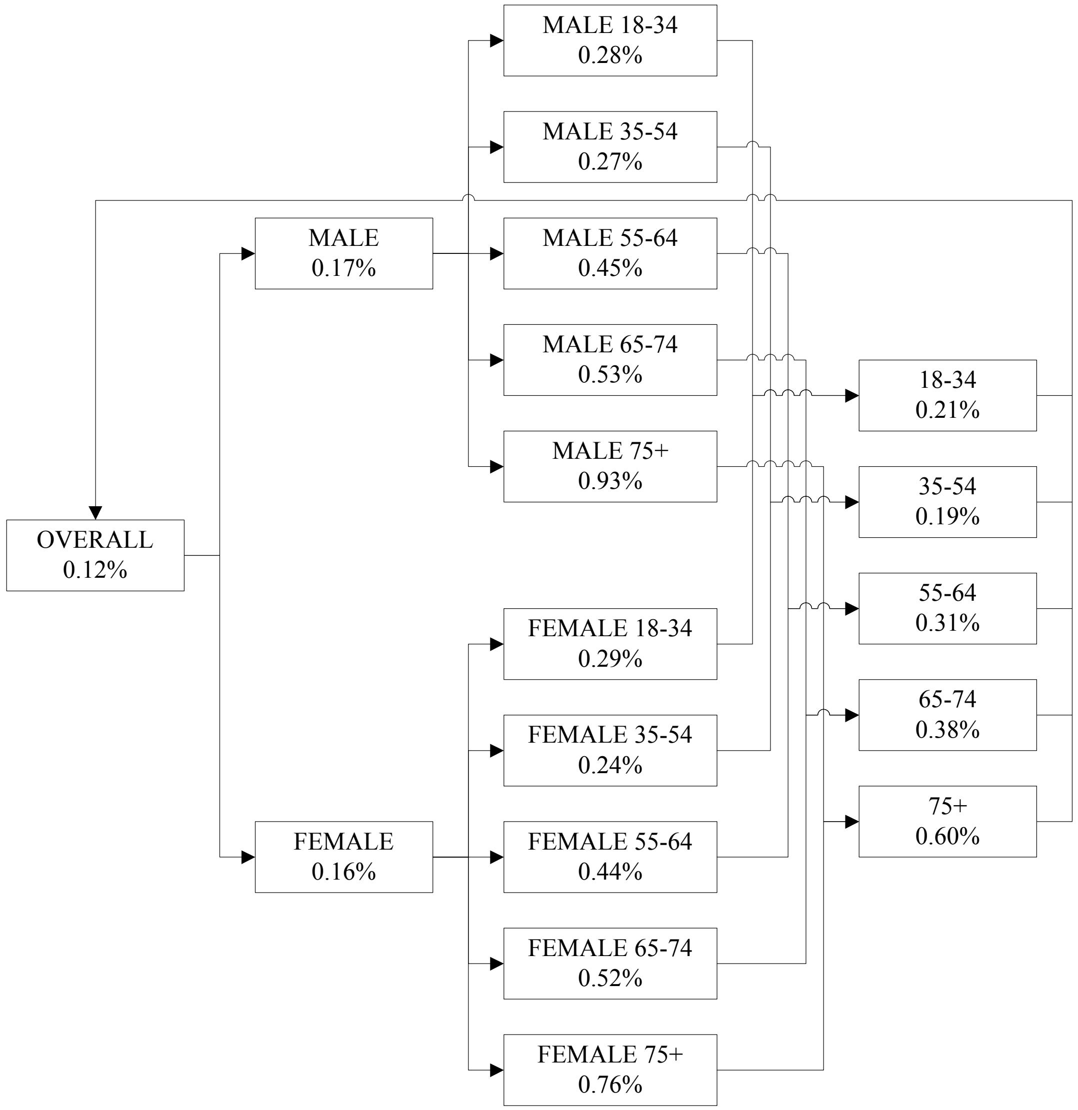

FIGURE 2 Micro-simulation error for different socio-demographic categories based on the daily number of trips per person.

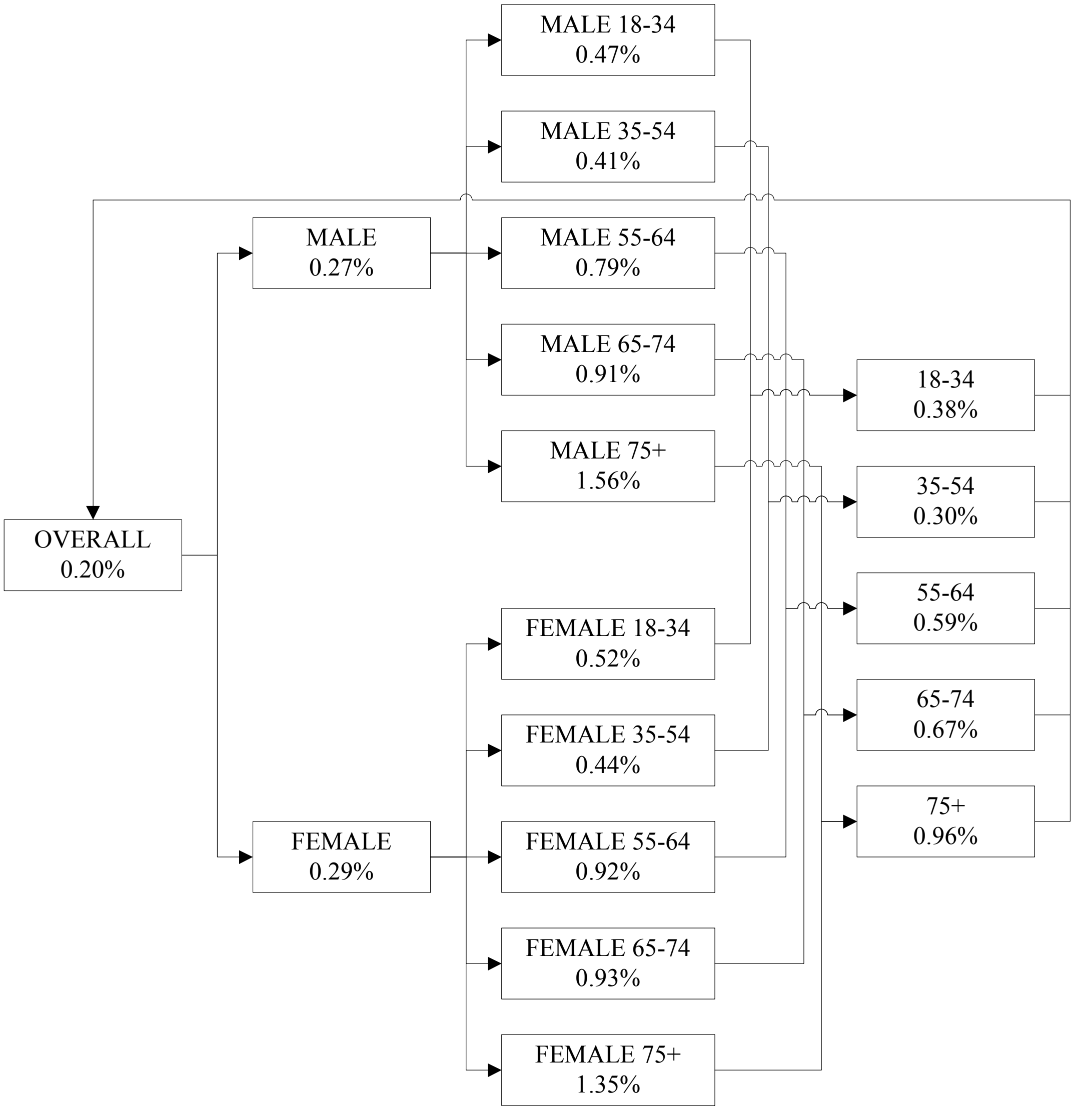

FIGURE 3 Micro-simulation error for different socio-demographic categories based on the daily distance travelled per person.

3.1.3 Combined Effects

A final segmentation could be made based on the combined effect of socio-demographic variables and mode choice. Figures 4 and 5 display the cross-tabulations of gender and mode choice, but obviously also other cross-tabulations (i.e. the two-way cross-tabulation age and mode choice, and three-way cross-tabulation age, gender and mode choice) can be made. From both figures it becomes apparent that error-rates increase as complexity increases. Three-way tabulations (Table 1) reveal that some micro-simulation error rates, especially those on trip distance often exceed the 1.27% boundary. This suggests that for detailed disaggregate analyses of travel behavior, running the full population sample rather than a fraction might be necessary.

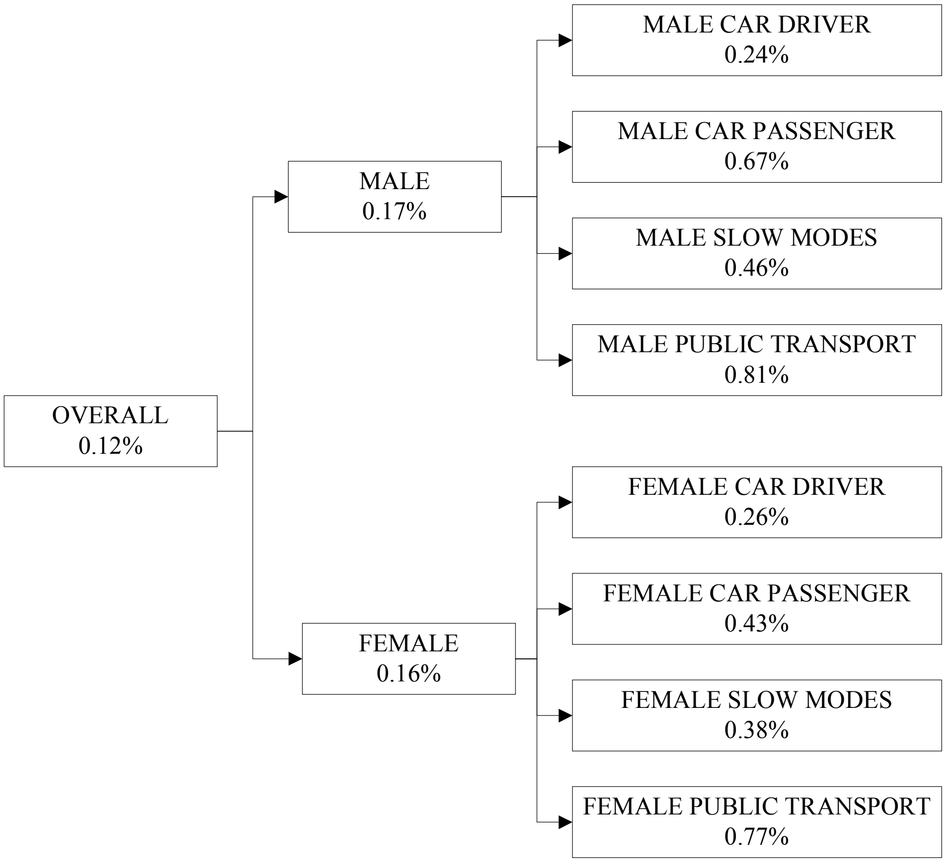

FIGURE 4 Micro-simulation error for combined gender and mode choice categories based on the daily number of trips per person.

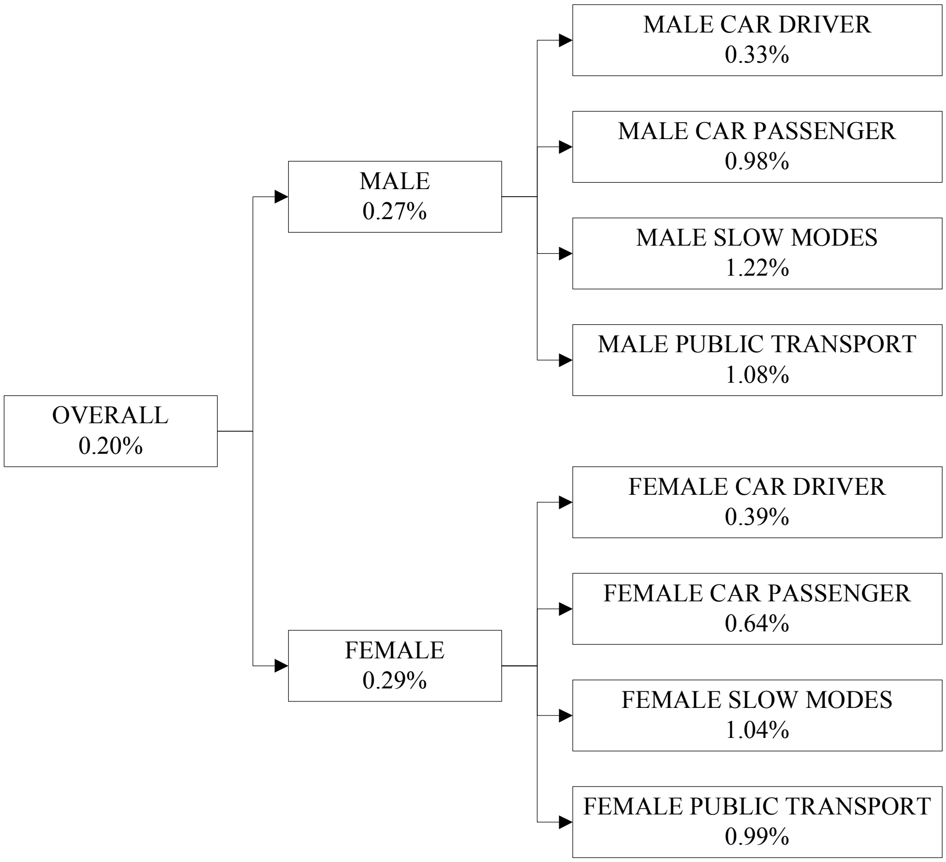

FIGURE 5 Micro-simulation error for combined gender and mode choice categories based on the daily distance travelled per person.

TABLE 1 Micro-Simulation Error for Combined Socio-Demographic and Mode Choice Categories

Gender	Age	Car Driver	Slow Modes	Public Transport	Car Passenger
Micro-simulation error based on the daily number of trips per person					
Male	18-34	0.46%	0.72%	1.18%	0.97%
Male	35-54	0.34%	0.73%	*1.38%*	1.17%
Male	55-64	0.65%	1.18%	*2.71%*	*2.10%*
Male	65-74	0.70%	1.24%	*3.08%*	*2.05%*
Male	75+	1.20%	*1.77%*	*4.18%*	*3.49%*
Female	18-34	0.51%	0.69%	1.08%	0.70%
Female	35-54	0.37%	0.70%	*1.42%*	0.78%
Female	55-64	0.72%	1.04%	*2.77%*	1.15%
Female	65-74	0.80%	1.15%	*2.97%*	1.16%
Female	75+	1.07%	*1.62%*	*3.89%*	*1.72%*
Micro-simulation error based on the daily distance travelled per person					
Male	18-34	0.61%	*1.95%*	*1.63%*	*1.46%*
Male	35-54	0.47%	*1.82%*	*1.68%*	*1.55%*
Male	55-64	1.00%	*3.43%*	*3.22%*	*2.71%*
Male	65-74	1.10%	*3.29%*	*3.59%*	*2.89%*
Male	75+	*1.78%*	*4.73%*	*5.31%*	*4.52%*
Female	18-34	0.77%	*1.87%*	*1.37%*	1.06%
Female	35-54	0.59%	*1.87%*	*1.77%*	1.08%
Female	55-64	1.16%	*3.40%*	*3.45%*	*1.65%*
Female	65-74	1.23%	*3.37%*	*4.04%*	*1.76%*
Female	75+	*1.81%*	*4.37%*	*4.92%*	*2.40%*

Values in italic pinpoint micro-simulation error rates that exceed 1.27%.

3.2 Model Results

Next to the subdivision of the micro-simulation error rate according to transport mode, age and gender, a linear regression model is estimated to explain the variances in the micro-simulation error rates. Gender had no significant impact on the micro-simulation error rates, neither on the error rates of the number of trips, nor on the error rates of the distance traveled, and was therefore not remained in the final models. The parameter estimates of these models are displayed in Table 2. Both models predict more than 80% of the variability in the error rates.

From Table 2 one could note that, as expected, the micro-simulation error increases along with the complexity (the level of disaggregation, defined as the number of cross-tabulations of the error-rate). In addition, error rates for the most common transport mode (car driver) are significantly lower than for the other transport modes (the estimate of car driver is significantly lower both in the model predicting the micro-simulation error in terms of the number of trips and in the model predicting the micro-simulation error in terms of the distance traveled). Finally, age has an increasing effect of the error rates, the older the age category considered, the higher the error rate. This supports the thesis the variability in older age-classes is higher due to the higher share of non-travelers and more essential more non-workers.

TABLE 2 Parameter Estimates of Linear Regression Model Predicting the Micro-Simulation Error Rate

	Trips		**Distance traveled**	
Parameter	**Estimate**	**Std. Err.**	**Estimate**	**Std. Err.**
Intercept	0.000440	0.001337	0.001210	0.001600
Complexity	0.000232	0.000046	0.000365	0.000055
Transport mode (p-value < 0.001)				
- Car driver	-0.003352	0.001583	-0.006116	0.001894
- Car passenger	0.003162	0.001583	0.002115	0.001894
- Public transport	0.010940	0.001583	0.010385	0.001894
- Slow modes	-0.000017	0.001583	0.009752	0.001894
- All modes	0.000000	n.a.	0.000000	n.a.
Age (p-value < 0.001)				
- 18-34 years	-0.002981	0.001703	-0.004248	0.002037
- 35-54 years	-0.002473	0.001703	-0.004195	0.002037
- 55-64 years	0.002603	0.001703	0.004632	0.002037
- 65-74 years	0.003620	0.001703	0.006029	0.002037
- 75+ years	0.009366	0.001703	0.014573	0.002037
- All ages	0.000000	n.a.	0.000000	n.a.
Fit statistic	$R^2 = 81.7\%$		$R^2 = 87.0\%$	

n.a.: not available (reference category)

4 DISCUSSION

To conduct the experiment the FEATHERS activity-based model was run 200 times. This number should be enough to carefully assess the effect of micro-simulation. Although literature that systemically investigates micro-simulation error of activity-based travel demand models is limited, the largest number of runs retrieved was 100 (*17*). Therefore it was concluded that running the FEATHERS activity-based model 200 times was certainly sufficient to draw valid conclusions concerning micro-simulation error.

The results support the thesis formulated by Arentze and Timmermans (*18*) that a fraction of 10% suffices for revealing the mobility effects on a national level of policy measures such as increases in travel costs (e.g. due to increasing fuel prices or road pricing), at least for activity-based models with a similar activity scheduler as the FEATHERS and ALBATROSS models.

It is important to stress that the sample 10% fraction was used in each of the 200 runs. Moreover, all required attributes for the population were either available from the census (the vast majority of socio-demographic and travel-related), or simulated before hand. Therefore, the resulting micro-simulation errors reported in this paper are purely due to the variation caused by the activity-scheduler in the FEATHERS framework.

Another point that needs attention is the fact that the impact of model uncertainty, i.e. micro-simulation error is investigated on two specific travel indices (the average daily number of trips per person and the average daily distance traveled per person). The applied methodology can be applied to other more complex travel indicators without complicating the simulation or analysis of results.

5 CONCLUSIONS

In this paper, model uncertainty caused by the statistical distributions of random components, i.e. micro-simulation error was investigated by means of 200 runs of the FEATHERS activity-based model using a 10% fraction. Results showed that micro-simulation errors are limited especially when disaggregation is limited to two levels. Notwithstanding, results indicated that for more elaborate analyses a 10% fraction might not be sufficient. A well-considered definition of the various categories of explanatory variables used in the activity-based model is therefore needed to ensure that more elaborate analysis still yield acceptable results for various cross-tabulations. The size of micro-simulation error increases along with complexity, as could be expected. Moreover, more commonly used transport modes such as using the car as driver have not surprisingly a lower error rate.

Further research should investigate the impact of the population fraction on the micro-simulation error rates. Although computationally burdensome, expanding the experiment for different sampling rates might provide a solid base for selecting the required fraction. After all, these would allow the analysis to balance computational complexity and micro-simulation error. The results presented in this paper contribute significantly as they provide support for the thesis of Arentze and Timmermans (*18*) that the model results using a 10% fraction will be certainly stable enough for strategic decision makings. Concerning more detailed analysis, especially for car drivers, accurate model results can be obtained. As reducing car use is often of key interest, the FEATHERS model using this 10% fraction will yield satisfactory results. Besides repeating the experiment for different fraction, one could also investigate other aspects (e.g. the number of activities) involved in the activity-scheduling process.

In addition to looking at micro-simulation errors, further research should investigate the input uncertainty present in the model. To pin-point the effect of input uncertainty, this paper already provided the insight to account for the micro-simulation effects that might obfuscate the effects of input uncertainty.

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