

**INVESTIGATING THE VARIABILITY IN DAILY TRAFFIC
COUNTS USING ARIMAX AND SARIMA(X) MODELS:
ASSESSING THE IMPACT OF HOLIDAYS
ON TWO DIVERGENT SITE LOCATIONS**

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ABSTRACT

In this paper, daily traffic counts are explained and forecasted by different modeling philosophies, namely the ARIMAX and SARIMA(X) modeling approaches. Special emphasis is put on the investigation of the seasonality in the daily traffic data and on the identification and comparison of holiday effects at different site locations. To get prior insight in the cyclic patterns present in the daily traffic counts, spectral analysis provides the required framework to highlight periodicities in the data. Data originating from single inductive loop detectors, collected in 2003, 2004 and 2005, are used for the analyses. Four traffic count locations are investigated in this study, an upstream and downstream traffic count location on a highway that is excessively used by commuters and an upstream and downstream traffic count location on a highway that is typified by leisure traffic. The different modeling techniques pointed out that weekly cycles appear to determine the variation in daily traffic counts. The comparison between seasonal effects and holiday effects at different site locations revealed that both the ARIMAX and SARIMAX modeling approach are valid frameworks for the identification and quantification of possible influencing effects. The technique yielded the insight that holiday effects play a noticeable role on highways that are excessively used by commuters, while holiday effects have a more ambiguous effect on highways typified for their leisure traffic. Modeling of daily traffic counts on secondary roads, and simultaneous modeling of both the underlying reasons of travel and revealed traffic patterns, certainly are challenges for further research.

1 BACKGROUND

Reliable predictions of travel behavior, traffic performance, and traffic safety are essential requirements for governments to lead an efficient policy. Policy tools like advanced traveler information systems (ATIS), advanced traffic management systems (ATMS) and control strategies such as ramp metering depend on the quality of forecasts of traffic volumes (1). Therefore, a deeper understanding in the events that affect the traffic performance will improve the quality of the predictions and consequently policy measures can be based upon more accurate data.

A good overview of the different techniques that exist to investigate the variability in daily traffic counts is provided by Han and Song (2) and Van Arem et al. (3). A first category that can be distinguished are time series models, which can be further divided into Box and Jenkins techniques, smoothing techniques, Kalman filtering theory and spectral analysis. Early applications of Box and Jenkins techniques in the field of traffic forecasting were implemented by Ahmed and Cook (4) and Nihan and Holmesland (5). More advanced techniques are more recently applied, including ARIMA(X) (Auto-Regressive Integrated Moving Average Models with Intervention X-variables (6)), Seasonal ARIMA models (7) and Kohenen enhanced ARIMA (KARIMA) models (8), and multivariate approaches such as the multivariate state space approach (9) and vector autoregressive and dynamic space time models (10). The first application of Kalman filtering theory within the field of traffic flow forecasting can be attributed to Okatuni and Stephanedes (11). Xie et al. (12) also used the Kalman filter to forecast traffic volumes, but now with discrete wavelet decomposition.

Neural network models are the second category of techniques that can be identified. Smith and Demetsky (13) were among the first that applied the technique in the domain of traffic flow forecasting. A performance evaluation of neural networks was made by Yun et al. (14). Time-delay neural networks (15), dynamic neural networks using a resource allocating network (16), and Bayesian combined neural networks (17) appear to be valuable neural network modeling examples.

Other techniques that are used for predicting traffic volumes include non-parametric models (18), cluster-based methods (19), principal component analysis (20), pattern recognition (21), fuzzy set theory (22) and support vector machines (23).

Cools et al. (24) used ARIMA modeling and Box-Tiao modeling (two Box and Jenkins techniques) to show that events such as special holidays (e.g. Christmas Day, Easter Sunday) and school holidays (e.g. spring half-term) have a significant influence on daily traffic performance. The authors highlighted that the underlying reason is the fact that these events influence mobility in different ways. First, these events can affect the demand for activities and the supply of activity opportunities. Second, these events can have an influence on the distribution of passenger and goods trips to vehicles and transport services. Finally, these events can have an effect on the infrastructure (e.g. available parking facilities) and their associated management systems.

One of the limitations of the latter study was that the analysis was done on a very specific location. Moreover, seasonality was not explicitly taken into account. To open the pathway of generalizing the results, the analyses presented in this paper are performed on two diverse site locations, where both the upstream and downstream traffic counts are investigated. Furthermore, this paper explicitly focuses on seasonality and recurrences in daily traffic data.

The main objectives of this study are the unraveling of the variability in daily traffic counts, the identification and comparison of holiday effects at different site locations, the prediction of future traffic volumes, and the validation of the suggested modeling framework. The cyclicity in the daily traffic data will be explored using spectral analysis. To quantify holiday effects and predict future traffic counts, autoregressive integrated moving average models with explanatory variables (ARIMAX) and seasonal autoregressive integrated moving average models with explanatory variables (SARIMAX) will be the main statistical model approaches envisaged. Note that the combination of a regression model with ARIMA or SARIMA errors raises the opportunity to build a model with desirable statistical properties, and thus to minimize the risk of erroneous model interpretation (25).

2 DATA

The variability in daily traffic will be investigated by analyzing the impact of cyclic patterns, day-of-week effects and holidays on daily highway traffic counts. While traditional ‘short term’ traffic forecasting investigates predominantly 15 and 30-minute traffic flows, in this paper daily traffic flows are used to investigate cyclicity. The reason for this choice is to filter out possible shifts in traffic volume due to changes in time of day (caused by accidents, ...). In this section, first, the dependent variable (daily traffic count) is further explored for all four traffic count locations. Then, the different explanatory variables, often called interventions in time series terminology, are described.

2.1 Daily Traffic

The aggregated daily traffic counts originate from minute data coming from single inductive loop detectors, collected in 2003, 2004 and 2005 by the Vlaams Verkeerscentrum (Flemish Traffic Control Center). Four traffic count locations are investigated in this study, displayed in Figure 1. The first two are located on the E314 Highway, a highway that is one of the entranceways of Brussels, and thus excessively used by commuters. The detectors in Gasthuisberg (Leuven, Belgium) are used to analyze the upstream traffic counts on this highway. The detectors in Herent (Leuven, Belgium) are used to analyze the downstream traffic counts. The second two traffic count locations are located on the E40 Highway, a highway that is one of the accesses to the Belgian seashore, and thus typified by leisure traffic. Both the upstream and downstream traffic counts are analyzed by data coming from detectors in Zandvoorde (Belgium). To refine the attractiveness of the Belgian seashore, it is noteworthy to mention that Belgium has a moderate maritime climate.

Minutely, the loop detectors generate four statistics: the number of cars driven by, the number of trucks driven by, the occupancy of the detector and the time-mean speed of all vehicles (26). Adding up the number of cars and trucks for all lanes in a specific direction, being two lanes for all four traffic count locations under study, yields a total traffic count for each minute. Although single loop detectors can distinguish between cars and trucks, it was decided to use the aggregate of both car and truck traffic to analyze the impact of holidays, as the distinction between cars and trucks is made by means of an algorithm which has an inferior performance during congested periods.

2.2 Holiday Effect

To assess the effect of holidays on traffic counts, it is necessary to identify which holidays occur in Belgium. The following holiday occasions were considered: Christmas vacation, spring half-term, Easter vacation, Labor Day, Ascension Day, Whit Monday, vacation of the construction industry (three weeks, starting the second Monday of July), Our Blessed Lady Ascension, fall break (including All Saints' Day and All Soul's Day), and finally Remembrance Day. Note that the national holiday, occurring on July 21, is included in the vacation of the construction industry. To evaluate the effect of all these holidays, the adjacent weekends, were considered to be a holiday too. For holidays occurring on a Tuesday or on a Thursday, respectively the Monday and weekend before, and the Friday and weekend after, were also defined as a holiday, because often people have a day-off at those days, and thus have a leave of several days, which might be used to go on a long weekend or on a short holiday. To model the effect of the above described holidays a dummy variable was created; "normal" days were coded zero, and holidays were coded one.

2.3 Day Effects

Next to the holiday effects, also day-of-week effects are envisaged in this study. Six dummy variables are created in order to model the day-of-week effect. Note that in general it is necessary to create $k - 1$ dummy variables to analyze the effect of a categorical variable with k classes (27). It was chosen to represent the first six days of the week (Monday until Saturday) by respectively one dummy each, equal to one for the day the dummy represents, and zero elsewhere. Representing the first six days by six dummies entails that the remaining day, the Sunday, is treated as a reference day, implying that for all traffic counts that were collected on a Sunday, the corresponding six dummies are coded zero.

3 METHODOLOGY

In this study, special emphasis is put on the investigation of cyclicity in the daily traffic data and on the identification and comparison of holiday effects at different site locations. To get prior insight in the cyclic patterns present in the daily traffic counts, spectral analysis provides the required framework to highlight periodicities in the data.

For forecasting daily traffic counts, two modeling philosophies are explored. The basic principle of the first philosophy is the fact that consecutive traffic counts are correlated, and that therefore present and future values can be (partially) explained by past values. In this paper SARIMA-models are fitted as this type of models is extremely suitable in taking into account seasonality in the data.

The second philosophy is the regression philosophy. The basic premise of this approach is the idea that the dependent variable, in this study the daily traffic counts, could be explained by other variables. Notwithstanding, the linear regression model only yields interpretable parameter estimates when different underlying assumptions are satisfied. Since correlation between error terms is present, two accommodations to the classical linear regression model, namely the ARIMAX-model, sometimes referred to as the Box-Tiao-model, and the SARIMAX-model, are investigated. The latter models are capable of taking into account dependencies between error terms.

The remainder of this section provides a brief recapitulation of underlying mathematical theory of the proposed time series models. For an introduction on time series techniques, the reader is referred to Shumway and Stoffer (28). In Yaffe and McGee (29), and Brocklebank and Dickey (30) a comprehensive overview of how to fit time series models using the statistical software SAS, is given.

3.1 Spectral Analysis

Spectral analysis is a statistical approach to detect regular cyclical patterns or periodicities. In spectral analysis the data are transformed with a fine Fourier transformation and decomposed into waves of different frequencies (31). The Fourier transform decomposition of the series x_t is:

$$x_t = \frac{a_0}{2} + \sum_{k=1}^m \left[a_k \cos w_k t + b_k \sin w_k t \right]$$

where t is the time subscript, x_t are the data, n is the number of observations in the series, m is the number of frequencies in the Fourier decomposition ($m = \frac{n}{2}$ if n is even; $m = \frac{n-1}{2}$ if n is odd), a_0 is the mean term ($a_0 = 2\bar{x}$), a_k are the cosine coefficients, b_k are the sine coefficients, and w_k are the Fourier frequencies ($w_k = \frac{2\pi k}{n}$).

Functions of the Fourier coefficients a_k and b_k can be plotted against frequency or against wave length to form periodograms, estimates of a theoretical quantity called a spectrum. The amplitude periodograms, also referred to as the periodogram ordinates, can then be smoothed to form spectral density estimates. The weight function used for the smoothing process, $W(\cdot)$, is often called the spectral window. The following simple triangular weighting scheme will be used to produce a weighted moving average estimate for the spectral density of the series: $\frac{1}{64\pi}, \frac{2}{64\pi}, \frac{3}{64\pi}, \frac{4}{64\pi}, \frac{3}{64\pi}, \frac{2}{64\pi}, \frac{1}{64\pi}$.

3.2 SARIMA Modeling

SARIMA modeling is a time series technique that accommodates ARIMA modeling to take into account seasonality in the data. It is an approach that tries to predict current and future values of a variable by using a weighted average of its own past values. If the series Y_t is modeled as a SARIMA $(p,d,q) \times (P,D,Q)_s$ process, then the model is given by:

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D Y_t = \theta(B)\Theta(B^s)e_t,$$

where s is the length of the periodicity (seasonality); $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$ is the non-seasonal autoregressive (AR) operator of order p and $\phi_1, \phi_2, \dots, \phi_p$ the corresponding non-seasonal AR parameters; $\Phi(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \dots - \Phi_P B^{Ps}$ the seasonal AR operator of order P and $\Phi_1, \Phi_2, \dots, \Phi_P$ the equivalent seasonal AR parameters; $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$ the non-seasonal moving average (MA) operator of order q and $\theta_1, \theta_2, \dots, \theta_q$ the associated non-seasonal MA parameters; $\Theta(B^s) = 1 - \Theta_1 B^s - \Theta_2 B^{2s} - \dots - \Theta_Q B^{Qs}$ the seasonal MA operator of order Q

and $\Theta_1, \Theta_2, \dots, \Theta_Q$ the corresponding seasonal MA parameters; $1-B^d$ the non-seasonal differencing operator of order d to produce non-seasonal stationarity of the d -th differenced data (usually $d = 0, 1$, or 2); and finally $1-B^{sD}$ the seasonal differencing operator of order D to produce seasonal stationarity of the D -th differenced data (usually $D = 0, 1$, or 2). In the above model equation B^i is used as a backshift operator on Y_t , and is defined as $B^i Y_t = Y_{t-i}$.

Note that a SARIMA-model is only valid, when the series satisfies the requirement of weak stationarity. This requirement is fulfilled when the mean value function is constant and does not depend on time, and when the variance around the mean remains constant over time (28). A transformation, like taking the logarithm or the square root of the series, often proves to be a good remedial measure to achieve constancy of the series's variance (27). To achieve stationarity in terms of the mean, it sometimes is required to difference the original series. Successive changes in the series are then modeled instead of the original series. Therefore in its most general form, as represented above, the SARIMA model includes a seasonal and non-seasonal differencing operator.

3.3 ARIMAX and SARIMAX Modeling

In contrast with purely modeling a series Y_t as a combination of its past values, the regression approach tries to explain the series Y_t with other covariates. Attention is needed when the classical linear regression approach is applied to time series, as the assumption of independence of the error terms is often violated because of autocorrelation (the error terms being correlated among themselves). The transgression of this assumption increases the risk for erroneous model interpretation, because the true variance of the parameter estimates may be seriously underestimated (27).

ARIMAX and SARIMAX models provide the required modeling frameworks to rectify the problem of autocorrelation by describing the errors terms of the linear regression model by respectively an $ARIMA(p, d, q)$ and $SARIMA(p, d, q) \times (P, D, Q)_s$ process. Formally the ARIMAX and SARIMAX models can be represented by the following equations:

$$Y_t = \beta_0 + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \dots + \beta_k X_{k,t} + \frac{1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q}{1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p} \varepsilon_t,$$

$$Y_t = \beta_0 + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \dots + \beta_k X_{k,t} + \frac{1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q}{1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p} \frac{1 - \Theta_1 B^s - \Theta_2 B^{2s} - \dots - \Theta_Q B^{Qs}}{1 - \Phi_1 B^s - \Phi_2 B^{2s} - \dots - \Phi_P B^{Ps}} \varepsilon_t,$$

the first being the formal representation of the ARIMAX model, the latter of the SARIMAX model, where Y_t is the t -th observation of the dependent variable, $X_{1,t}, X_{2,t}, \dots, X_{k,t}$ the corresponding observations of the explanatory variables, $\beta_0, \beta_1, \beta_2, \dots, \beta_k$ the parameters of the regression part, and where $\phi_1, \phi_2, \dots, \phi_p$, $\Phi_1, \Phi_2, \dots, \Phi_P$, $\theta_1, \theta_2, \dots, \theta_q$ and $\Theta_1, \Theta_2, \dots, \Theta_Q$ are the weights for the non-seasonal and seasonal autoregressive terms and moving average terms. The remaining error terms ε_t are assumed to be white noise. Note that for clarity of the formulae the differencing operators were left out of the equations.

The parameters of the ARIMAX and SARIMAX models are estimated using Maximum Likelihood. Studies, comparing least squares methods with maximum likelihood methods for

this family of models, show that maximum likelihood estimation gives more accurate results (30). The likelihood function is maximized via nonlinear least squares using Marquardt's method (32). When differencing of the error terms is required to obtain stationarity, all dependent and independent variables should be differenced (25).

3.4 Model Evaluation

In order to compare the different types of models that are considered, and to make comparisons between the models for upstream and downstream traffic intensity on the one hand, and between the models for different highways on the other hand, objective criteria are needed to determine the models' performance (33). To determine the appropriateness of the models and to substantiate the validity of the proposed modeling framework, the following criteria were considered: the Akaike Information Criterion (AIC), the Mean Square Error (MSE) and the Mean Absolute Percentage Error (MAPE). Only the latter criterion can be applied for comparing models of different traffic count locations. By constructing the models on a training data set containing the first 75% of the observations, the remaining 25% of the observations make up a validation or test data set. This test data set can then be used to assess the forecasting performance of the models, by calculating the MSE and MAPE for the predictions for this test data. The choice of these percentages is arbitrary, but common practice in validation studies (see e.g. Wets et al. (34)).

The following three definitions determine the three criteria that are considered. The Akaike Information Criterion (AIC) is defined as $-2 \times \log \text{likelihood} + 2 \times \text{number of parameters}$ in the model. The Mean Square Error (MSE) equals the Sum of all Squared Errors (SSE) divided by its degrees of freedom, which are calculated by subtracting the number of parameters in the model from the number of observations. The Mean Absolute Percentage Error (MAPE) is defined as the average of the absolute values of the proportion of error at a given point of time. Models with lower values for these criteria are considered to be the more appropriate ones (35).

To evaluate the predictive strength of the proposed models, and more precisely to test whether the differences in MAPE for the different models are significant, the following statistical testing procedures are used: the Friedman test and the Wilcoxon signed-rank test (18). The predictions (test data) of the different forecasting methods are ranked, and based on these ranked predictions the nonparametric repeated measures tests are performed. The Friedman test evaluates the null hypothesis that three or more related samples are from the same population, and thus is used to assess whether the MAPEs for the different approaches are equal or not. The Wilcoxon signed-rank test evaluates the null hypothesis that two related samples have the same distribution. This test is adopted to test whether pairwise differences in MAPE are significant or not.

4 RESULTS

In this Section, the results are presented, the parameter estimates of the models are interpreted, and the different models are compared with each other. First, the periodicities in the data are highlighted. Then, the results of the three model approaches are provided, and their performances are carefully assessed. Finally, the models for upstream traffic count data and downstream traffic count data are compared for the two highways, and then differences in variability between the two highways are discussed.

4.1 Spectral Analysis

Prior insight in the cyclic patterns present in the daily traffic counts can be obtained by looking at the results of the spectral analysis presented in Figure 2. This figure displays the spectral density estimates against the periods. From this figure it is clear that for three of the four traffic count locations the spectral density reaches a local maximum in period 3.5 and a global maximum in period 7. This global maximum can be interpreted as a weekly recurring pattern in the traffic data. For the remaining traffic count location (E40, Downstream) only a local maximum in period 7 is attained. Note that the other maxima (in periods 2.33 and 3.5) also contribute in explaining weekly cyclicity, as repetition of these patterns also yields a weekly pattern. In addition to the weekly periodicity, differences between the two highways can be highlighted: the weekly structure accounts for almost all variability on the E413 highway (typified by commuting traffic), while weekly patterns only partially explain the variability on the E40 highway (characterized by leisure traffic).

4.2 SARIMA Modeling

For all four traffic count locations it was required to develop the corresponding SARIMA models on differenced data in order to obtain stationarity. A thorough investigation of the autocorrelation function and the partial autocorrelation function of the residuals was required, to evaluate which Autoregressive (AR) and moving average (MA) factors were required for the model building process. The following SARIMA models were obtained using the AIC as selection criterion:

E314, Upstream (Gasthuisberg): SARIMA (1,1,1) \times (1,0,1)₇

E314, Downstream (Herent): SARIMA (2,0,1) \times (0,1,1)₇

E40, Upstream (Zandvoorde): SARIMA (1,1,1) \times (1,1,2)₇

E40, Downstream (Zandvoorde): SARIMA (0,1,2) \times (1,1,1)₇

The estimates for these final obtained SARIMA models for the four traffic count locations can be formally represented by the following equations:

$$\text{E314, Upstream (Gasthuisberg): } 1 - B \quad Y_t = \frac{1 - 0.812B \quad 1 - 0.999B^7}{1 - 0.349B \quad 1 - B^7} \varepsilon_t$$

$$\text{E314, Downstream (Herent): } 1 - B^7 \quad Y_t = \frac{1 - 0.775B \quad 1 - 0.994B^7}{1 - 1.256B + 0.304B^2} \varepsilon_t$$

$$\text{E40, Upstream (Zandvoorde): } 1 - B \quad 1 - B^7 \quad Y_t = \frac{1 - 0.926B \quad 1 - 1.593B^7 + 0.598B^{14}}{1 - 0.485B \quad 1 - 0.700B^7} \varepsilon_t$$

$$\text{E40, Downstream (Zandvoorde): } 1 - B \quad 1 - B^7 \quad Y_t = \frac{1 - 0.457B - 0.324B^2 \quad 1 - 0.978B^7}{1 - 0.077B^7} \varepsilon_t$$

The above described models all contain seasonal and non-seasonal moving average factors, and addition seasonal and/or non-seasonal autoregressive factors are included if required. Notice that if these models would be worked out completely, also other autoregressive

and moving factors play a role. Investigation of the SARIMA models draws immediate attention to the seasonality in the data: a seven-day cyclicity seems to predetermine daily traffic counts. This can be seen from the fact that a seasonal difference operator (taking the 7th order difference) is included in three of the four models and that the seasonal autoregressive and moving average (SARMA) factors for the first model (E314, Upstream) are very close or equal to one. Moreover, other SARMA factors play an important role, indicating that traffic counts can be explained by weekly cyclic patterns.

4.3 ARIMAX and SARIMAX Modeling

Like for the SARIMA modeling approach also for the ARIMAX and SARIMAX modeling approaches it was necessary to develop a model on differenced data to achieve (weak) stationarity; and the intercept was dropped from the equations to attain realistic interpretations. Recall that when differencing is applied, the intercept is interpreted as a deterministic trend, and this interpretation is not always realistic (36). The final error terms obtained in the four models were accepted to be ‘white noise’ according to the Ljung-Box Q^* -statistics (37). Parameter estimates for the ARIMAX and SARIMAX models are shown in Table 1. The standard errors (S.E.), and values of the significance tests are provided as well. The estimates of the (S)ARIMA parameters are not shown, as they serve as a remedial measure for autocorrelation, and because focus lies on the interpretation of the regression part of the models. No day-of-week effects were included in the SARIMAX models, as seasonal differencing of the day-of-week variables would yield variables having a zero variance, and thus the model estimation would become infeasible. For the traffic count site location counting upstream traffic on the E314 highway (Gasthuisberg), it would have been feasible to include day-of-week effects, as no seasonal difference operator was used in this model. Nonetheless, when day-of-week effects were included, the seasonal AR and MA parameters were not significant and the model unfolded into the ARIMAX model.

From Table 1 one can see that the day-of-week effects are significant for all four traffic count locations: all six individual day-of-week dummy variables were significant on three of the four locations, while on the remaining traffic count location (E40 Downstream) half of the day-of-week dummy variables turned out to be significant. Note that the spectral analysis also pinpointed this contrast between the latter location and the rest. This could be partially explained by the fact that in general on Sundays the lowest traffic counts are observed compared to other days, while on this particular site location, on Sundays the intensity rates peak due to traffic generated from people returning home from their leisure trip at the seashore area. Furthermore, the analysis revealed that the holiday effects are only significant for the traffic count locations on the E314 highway. For the traffic counts locations on the E40 highway the holiday effects were not significant. Therefore, for these leisure locations the parameter estimates for both the ARIMAX model including holiday effects and the ARIMAX model without holiday effects are presented in Table 1. For the SARIMAX models only the models including the holiday effects are displayed, as the models without holiday effects are obviously the SARIMA models described in the previous section.

4.4 Model Comparison

When the performance of the different modeling philosophies is assessed, it is clear from Table 2 that all three model approaches perform reasonably well in explaining the variability of

daily traffic counts: the three criteria that are based on the training data (AIC, MSE and MAPE) favor different modeling approaches suggesting that the three model approaches tested are valid approaches for investigating daily traffic counts. Concerning forecasting of daily traffic counts the ARIMAX models outperformed the SARIMA and SARIMAX models on three locations based on the two criteria that are based on the test data (MSE and MAPE) and for the fourth location (E314 Downstream) only small differences in performance are observed. This suggests that when the focus is put on forecasting, the use of the ARIMAX model approach should be preferred.

Besides, forecasting on the E314 yields more reliable results than on the E40 highway: MAPEs of 7.12% and 8.24% are observed for respectively upstream and downstream traffic for the E314 highway, while on the E40 highway only percentages of 10.07% and 9.16% were attained for the best models. This finding matches perfectly with the results from the spectral analysis, namely that the weekly structure accounts for almost all variability on the E314 highway (typified by commuting traffic), while weekly patterns only partially explain the variability on the E40 highway (characterized by leisure traffic). The superior forecasting on the E314 highway is also evidenced by Figure 3: the predictions of the daily traffic counts on the E314 highway (upper plots) are much closer to the actual values, than the ones on the E40 highway (lower plots).

When the statistical testing procedures are used to assess the predictive value of the models, the Friedman tests all indicate that significant differences in MAPE exist. For three of the four locations the corresponding p-values are below 0.01. For the models for downstream traffic on the E40 highway, the differences are only borderline significant (p-value equals 0.046). Note that for this location the MAPEs based on the test data set are indeed much closer to each other, when compared to the other locations. When the differences are tested in pairwise comparison, accounting for multiple testing, significance differences can be found between most of the MAPEs, except for the models predicting downstream traffic on the E40 highway.

The sundry techniques all highlighted a weekly cyclic behavior on all four locations. Yet, holiday effects turned out to have only a significant impact on the upstream and downstream traffic of the E314 highway. For the traffic count locations on the E40 highway no significant holiday effects were retrieved. Nevertheless, further elaboration on this insignificance of the holiday-effect is worthwhile, since the daily travel time expenditure on commuting is clearly lower on holidays than on regular days (38). Thus, one can conclude that for the E40 traffic count locations, the decrease in the number of vehicles due to fewer commuting traffic on holidays is compensated by an increase in the number of vehicles due to leisure traffic, which is shown by the non-significant effect of holidays on the sum of all traffic, as considered here. The simultaneous analysis of travel goals and traveling itself (traffic counts) seems therefore an interesting avenue for further research.

The comparison of upstream and downstream traffic count locations yields quite diverse results for the locations on the E314 highway and the locations on the E40 highway (Table 1). For the first highway, upstream and downstream traffic seem to yield very comparable findings; significant lower traffic counts during the weekend and during holidays and maximum levels of traffic intensity on Wednesdays and Fridays. Controversially, the upstream and downstream traffic locations of the E40 result in quite divergent outcomes; upstream traffic seems to top on Fridays and is least intense on Sundays, while downstream traffic reaches the maximum on Sundays. This discrepancy between upstream and downstream can be (partially) explained by

the fact that people make a weekend trip to the seashore, starting their leisure trip on Friday evening, and making their trip back home on Sunday.

5 CONCLUSIONS AND FURTHER RESEARCH

In this study, three modeling approaches, namely SARIMA, ARIMAX and SARIMAX were considered to predict daily traffic counts. These different modeling techniques, as well as the spectral analysis, pointed out the significance of the day-of-week effects: weekly cycles seem to determine the variation of daily traffic flows. The comparison of day-of-week effects or seasonal effects and holiday effects at different site locations revealed that all three modeling approaches perform reasonably well in explaining the variability of daily traffic counts, favoring the ARIMAX model, when the focus is on forecasting daily traffic counts. Results revealed that the ARIMAX and SARIMAX modeling approaches are valid frameworks for identification and quantification of possible influencing effects. Nonetheless, the explicit incorporation of day-of-week effects in ARIMAX yields additional insight for policy decision makers. The technique yielded the insight that holiday effects play a noticeable role on highways that are excessively used by commuters, while holiday effects have a more ambiguous effect on highways typified for their leisure traffic.

The results discussed in this paper generalize the findings of Cools et al. (24) so that policy makers can fine-tune current policy measures based on these results. Thus, the performance of policy tools like advanced traveler information systems (ATIS) and advanced traffic management systems (ATMS) can be improved. An example are online calendars pinpointing the days with high expected traffic volumes. Travelers can use the information provided reschedule and/or adapt their planned travel trips. A second example is to focus policy actions such as carpooling initiatives on the most traffic intense days. For the upstream traffic for the E40 highway for instance, focus should be put on stimulating alternatives for the Friday-traffic. The examples illustrate that the findings of this study contribute in achieving an important goal, namely the policy keystone ‘more acceptable and reliable travel times’.

Further generalization of the results is possible, when even more traffic patterns of other parts of the road network are analyzed. Modeling of daily traffic counts on secondary roads, and simultaneous modeling of different traffic counts locations is certainly an important pathway for further research. A key challenge will be the simultaneous modeling of both the underlying reasons of travel, and revealed traffic patterns.

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TABLE 1 Parameter Estimates for the ARIMAX and SARIMAX Models

Parameter	Estimate	S.E.	t-value	p-value	Estimate	S.E.	t-value	p-value
E314 Upstream (Gasthuisberg)					E314 Downstream (Herent)			
<i>Model</i>	<i>ARIMAX (1,1,1)</i>				<i>ARIMAX (1,1,1)</i>			
Holiday	-4197	295	-14.2	<0.001	-3863	351	-11.0	<0.001
Monday	9203	256	36.0	<0.001	9011	308	29.3	<0.001
Tuesday	10832	293	37.0	<0.001	10548	362	29.2	<0.001
Wednesday	11522	303	38.1	<0.001	11022	378	29.2	<0.001
Thursday	11311	301	37.6	<0.001	10863	376	28.9	<0.001
Friday	11983	291	41.2	<0.001	11028	359	30.7	<0.001
Saturday	1390	252	5.5	<0.001	1428	303	4.7	<0.001
<i>Model</i>	<i>SARIMAX (1,1,1) × (1,0,1)₇</i>				<i>SARIMAX (2,0,1) × (0,1,1)₇</i>			
Holiday	-4219	290	-14.6	<0.001	-3839	351	-10.9	<0.001
Parameter	Estimate	S.E.	t-value	p-value	Estimate	S.E.	t-value	p-value
E40 Upstream (Zandvoorde)					E40 Upstream (Zandvoorde)			
<i>Model</i>	<i>ARIMAX (1,1,2) with holiday effect</i>				<i>ARIMAX (3,1,1) with holiday effect</i>			
Holiday	-196	164	-1.2	0.234	95	143	0.7	0.504
Monday	933	126	7.4	<0.001	-193	112	-1.7	0.086
Tuesday	1121	158	7.1	<0.001	-398	135	-3.0	0.003
Wednesday	1370	164	8.3	<0.001	-473	131	-3.6	<0.001
Thursday	1651	163	10.1	<0.001	-222	131	-1.7	0.089
Friday	3119	156	20.0	<0.001	-107	134	-0.8	0.424
Saturday	738	124	6.0	<0.001	-2122	110	-19.3	<0.001
<i>Model</i>	<i>ARIMAX (1,1,2) no holiday effect</i>				<i>ARIMAX (3,1,1) no holiday effect</i>			
Monday	963	124	7.8	<0.001	-207	110	-1.9	0.059
Tuesday	1155	155	7.5	<0.001	-416	133	-3.1	0.002
Wednesday	1406	162	8.7	<0.001	-491	129	-3.8	<0.001
Thursday	1680	162	10.4	<0.001	-237	129	-1.8	0.066
Friday	3147	154	20.4	<0.001	-121	132	-0.9	0.362
Saturday	737	124	6.0	<0.001	-2122	110	-19.3	<0.001
<i>Model</i>	<i>SARIMAX (1,1,1) × (1,1,2)₇</i>				<i>SARIMAX (0,1,2) × (1,1,1)₇</i>			
Holiday	-178	161	-1.1	0.269	50	135	0.4	0.711

TABLE 2 Criteria for Model Comparisons

	Training data			Test data	
	AIC	MSE	MAPE	MSE	MAPE
<i>E314 Gasthuisberg (Upstream)</i>					
SARIMA (1,1,1) \times (1,0,1) ₇	15275.0	6,654,557	5.37%	17,877,684	8.85%
ARIMA(X) (1,1,1)	*15074.8	*5,451,893	*4.90%	*10,380,923	*7.12%
SARIMA(X) (1,1,1) \times (1,0,1) ₇	15116.6	5,515,467	5.06%	18,557,140	10.18%
<i>E314 Herent (Downstream)</i>					
SARIMA (2,0,1) \times (0,1,1) ₇	15387.0	8,905,383	6.05%	16,779,060	9.13%
ARIMA(X) (1,1,1)	15383.8	*7,939,285	*5.82%	14,453,314	8.46%
SARIMA(X) (2,0,1) \times (0,1,1) ₇	*15295.4	7,994,919	5.84%	*13,501,012	*8.24%
<i>E40 Zandvoorde (Upstream)</i>					
SARIMA (1,1,1) \times (1,1,2) ₇	*13883.0	1,440,592	6.67%	3,883,197	11.92%
ARIMA(X) (1,1,2) with holiday	13978.7	*1,433,216	*6.59%	*3,246,433	10.11%
ARIMA(X) (1,1,2) no holiday	13978.2	1,433,935	6.60%	3,296,933	*10.07%
SARIMA(X) (1,1,1) \times (1,1,2) ₇	13883.8	1,440,650	6.67%	3,948,399	12.14%
<i>E40 Zandvoorde (Downstream)</i>					
SARIMA (0,1,2) \times (1,1,1) ₇	*13661.1	1,101,579	6.06%	3,025,826	9.67%
ARIMA(X) (3,1,1) with holiday	13775.2	1,090,283	5.91%	3,052,143	9.24%
ARIMA(X) (3,1,1) no holiday	13753.7	*1,089,540	*5.89%	*3,003,765	*9.16%
SARIMA(X) (0,1,2) \times (1,1,1) ₇	13662.9	1,102,956	6.07%	3,017,484	9.59%

* best model for a specific traffic count location according to the evaluation criterion

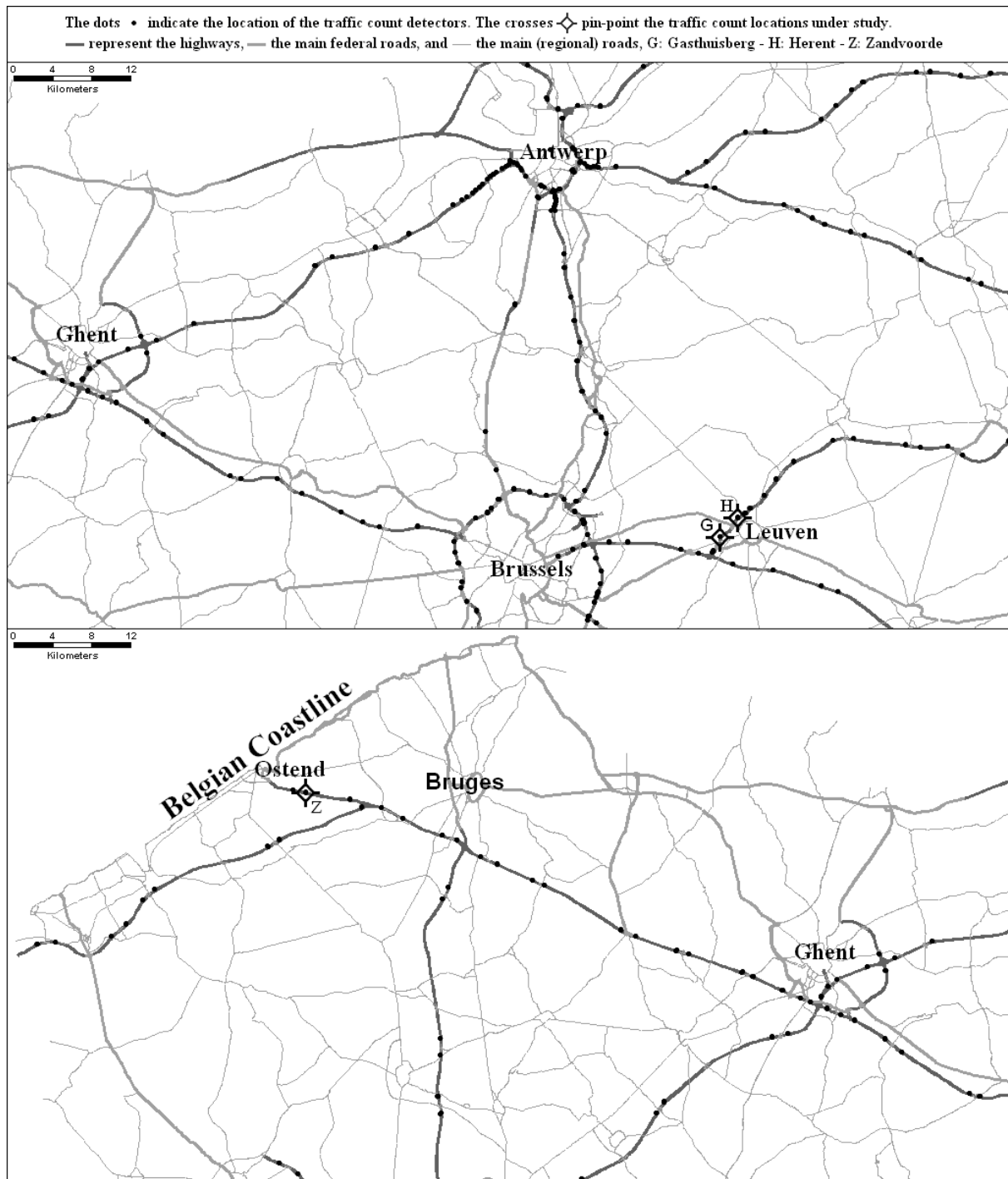


FIGURE 1 Geographical representation of the traffic count locations under study.

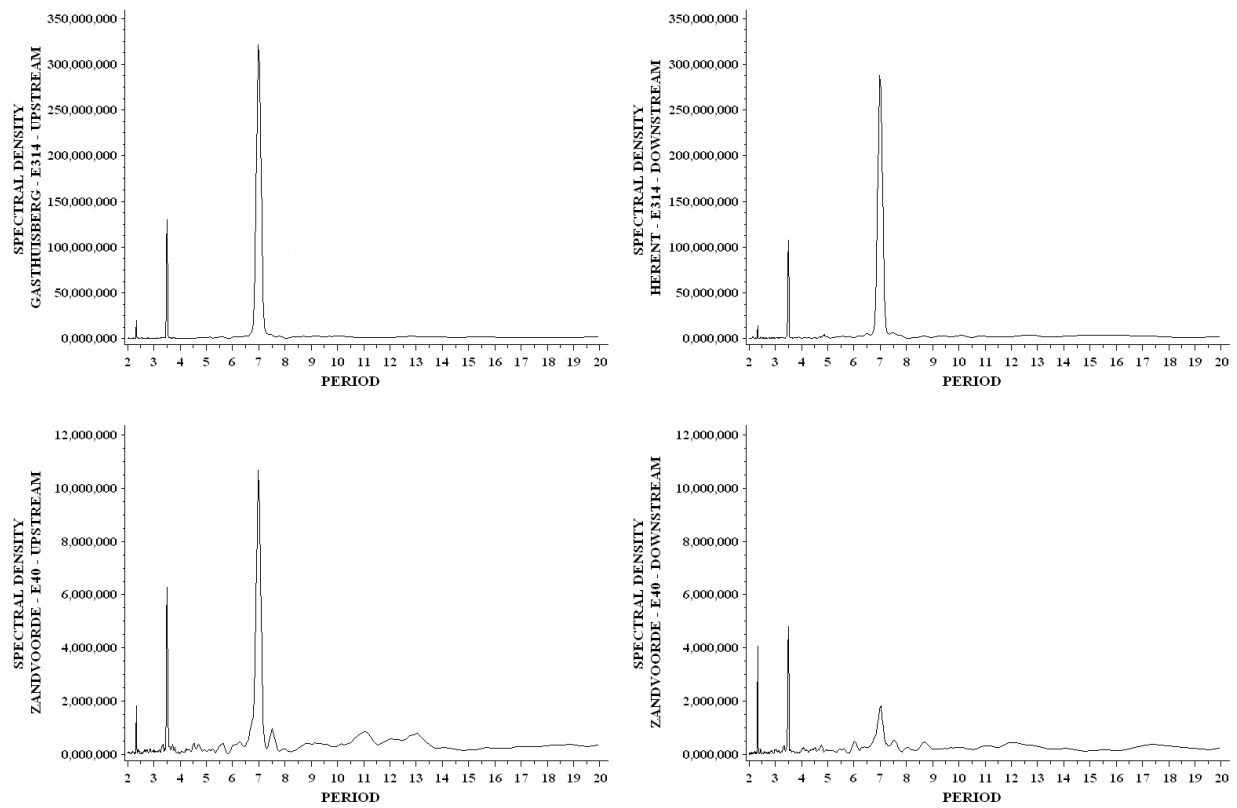


FIGURE 2 Spectral analysis of daily up- and downstream traffic counts for two highways.

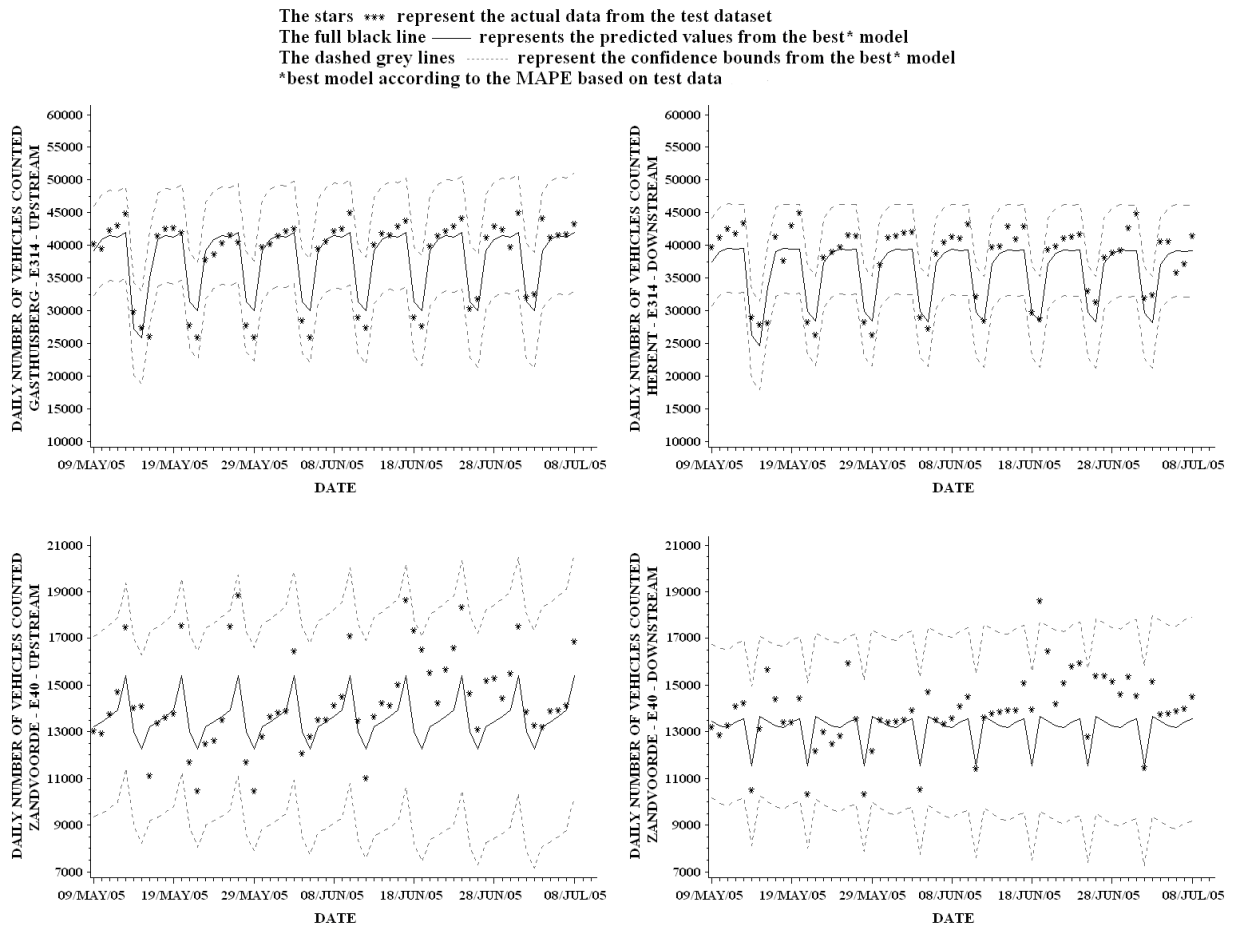


FIGURE 3 Daily traffic counts and their predicted values and confidence bounds.