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10th EUROPEAN REGIONAL ASTRONOMY MEETING OF THE IAU

Praha, Czechoslovakia August 24-29, 1987

ASTROPHYSICS

Edited by

PETR HARMANEC

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L U B O Š P E R E K

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THEORETICAL ASPECTS OF NON RADIAL PULSATIONS

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Abstract. An introduction to the theory of non radial modes of oscillations is presented. Evidences of such pulsations can be found in numerous variable stars. We discuss here some new results obtained in stars varying with periods of the [illegible]er or longer than the hour, namely β Cep stars, line profile variable stars and WR stars.

1. Introduction

The first question to ask if one wants to study the stability of a star is the following: "Is the equilibrium configuration representing the star dynamically stable or unstable ?". In a dynamically stable star, a small overall contraction tends to increase the pressure gradient in such a way that the resulting force exceeds the increased gravitational force. So, there is a tendency to come back to the equilibrium configuration. If a perturbation is applied to a dynamically stable configuration, oscillations will take place. The amplitude of these oscillations can eventually increase with time, in which case the configuration is said to be vibrationnally unstable or they can decrease with time, in which case the configuration is vibrationnally stable.

Although extensive studies have been devoted to radial oscillations, as in the case of Cepheid stars for example, the attention has progressively turned towards non radial oscillations. Actually, the Sun has become in the last decade one of the most famous variable star since the discovery in 1960 by Leighton and his co-workers of the phenomenon called the "five-minute" oscillation. But the best candidates for non radial oscillations have since long been the βCep stars. Recently some other groups of stars have been added to the list. We shall consider here only those with periods of the order of or greater than one hour and emphasize line profile variable stars and WR stars.

2. General Equations

We shall consider spherically symmetric equilibrium models and ignore magnetic fields and, in a first step, rotation.

The general method used in stellar stability is the small perturbation method (Ledoux 1958, 1969, Unno et al. 1979, Cox 1980) in which a small perturbation is applied to the model and the problem consists then to follow the evolution of the perturbed model. This is solved by linearizing the general equations of:

- mass conservation
- momentum conservation
- thermal energy conservation
- Poisson

To linearize these equations, one takes each variable, adds a perturbation to it and introduces it in the equation, keeping only the linear terms with respect to the perturbations and taking into account that these equations are satisfied by the unperturbed variables. We assume that the solution of this linear problem is close to the general solution of the complete non linear problem at least in a small domain around the equilibrium state.

In that case, the coefficients of the linearized system are time independent and one can perform a separation between the time variation and the space variation of a perturbed variable f, writing

$$\delta f(\vec{r},t) = \delta f(\vec{r})\, e^{i\sigma t} \qquad (1)$$

The symbol δ will be used for "lagrangian" perturbations while the symbol $'$ will denote "eulerian" perturbations.

This separation of the time variable transforms the initial partial differential problem into an eigenvalue problem, σ^2 being the eigenvalue.

The linearized equations are then:

$$\frac{\delta\rho}{\rho} = \frac{\rho'}{\rho} + \frac{1}{\rho}\frac{d\rho}{dr}\,\delta r = -\nabla\cdot\overrightarrow{\delta r} \qquad (2)$$

mass conservation

$$-\sigma^2\rho\,\delta r = -\nabla P' + \left(\frac{\rho'}{\rho}\nabla P - \rho\nabla\phi'\right) \qquad (3)$$

momentum conservation

$$i\sigma\frac{\delta P}{P} - \Gamma_1\frac{\delta\rho}{\rho} = \frac{(\Gamma_3-1)\rho}{P}\,\delta\left(\varepsilon - \frac{1}{\rho}\nabla\cdot\vec{F}\right) \qquad (4)$$

thermal energy conservation

$$\nabla^2\phi' = 4\pi G\rho' \qquad (5)$$

Poisson

where ρ is the density, P, the pressure, ε, the rate of nuclear energy generation, $\vec{F}$, the flux of energy and ϕ, the gravitational potential. Γ_1 and Γ_3 are the generalized adiabatic coefficients defined by

$$\Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_S \qquad (6)$$

$$\Gamma_3 - 1 = \left(\frac{\partial \ln T}{\partial \ln \rho}\right)_S \qquad (7)$$

the derivations being done at constant entropy S. The symbol δr is the radial component of $\vec{\delta r}$.
Except when σ^2 is very small, a good approximation is to admit that the perturbation is adiabatic and the conservation of thermal energy writes now

$$\frac{\delta P}{P} - \Gamma_1 \frac{\delta \rho}{\rho} = 0 \qquad (8)$$

3. <u>Adiabatic case</u>

We have four equations (2), (3), (8) and (5) with the following boundary conditions:
at the center: $\vec{\delta r}$ is finite,
(9)
at the surface: $\delta P = 0$ and continuity of ϕ' and $\frac{d\phi'}{dr}$

So, one deals with a fourth order in r eigenvalue problem, σ^2 being the eigenparameter. A complete discussion of the solutions is given in Smeyers (1984).

The solution may be expressed in terms of spherical harmonics $Y_l^m(\theta,\varphi)$ of degree l and order m, in the form

$$\delta f(r,\theta,\varphi) = \delta f(r)\, Y_l^m(\theta,\varphi) \qquad (10)$$

for a scalar quantity and

$$\vec{\delta f}(r,\theta,\varphi) = \begin{bmatrix} u(r)\, Y_l^m(\theta,\varphi) \\ \dfrac{v(r)}{r}\; \dfrac{\partial Y_l^m(\theta,\varphi)}{\partial\theta} \\ \dfrac{v(r)}{r\sin\theta}\; \dfrac{\partial Y_l^m(\theta,\varphi)}{\partial\varphi} \end{bmatrix} \qquad (11)$$

for a vectorial one. These modes are the spheroidal modes, corresponding to σ^2 different from zero.

Other displacement fields can also be found. They are the toroïdal modes defined by

$$\vec{\delta f}(r,\theta,\varphi) = \begin{bmatrix} 0 \\ \dfrac{w(r)}{r\sin\theta}\; \dfrac{\partial Y_l^m(\theta,\varphi)}{\partial\varphi} \\ -\dfrac{w(r)}{r}\; \dfrac{\partial Y_l^m(\theta,\varphi)}{\partial\theta} \end{bmatrix} \qquad (12)$$

The eigenvalues are only different from zero in a rotating star. These modes are divergence free:

$$\nabla . \vec{\delta f} = 0 \qquad (13)$$

4. <u>Propagation of waves</u>

For the sake of simplicity, we shall neglect the eulerian perturbation of the gravitational potential.
Let us adopt new variables

$$v = r^2 \delta r\, P^{1/\Gamma_1} \qquad (14)$$

$$w = \frac{p'}{\rho} \rho\, P^{-1/\Gamma_1} \qquad (15)$$

in the case where Γ_1 is constant throughout the star. If Γ_1 is not constant, we define

$$f_1 = \exp\left[\int_0^r \frac{1}{\Gamma_1} \frac{d\ln P}{dr} dr\right] \qquad (16)$$

$$f_2 = \exp\left[\int_0^r \left(\frac{d\ln\rho}{dr} - \frac{1}{\Gamma_1}\frac{d\ln P}{dr}\right) dr\right] \qquad (17)$$

and we have

$$v = f_1 r^2 \delta r \qquad (18)$$

$$w = f_2 \frac{p'}{\rho} \qquad (19)$$

We obtain the second order system

$$\frac{dv}{dr} = \left(\frac{l(l+1)}{\sigma^2} - \frac{r^2}{c^2}\right)\frac{f_1}{f_2}\, w = a w \qquad (20)$$

$$\frac{dw}{dr} = \frac{1}{r^2}\left(\sigma^2 - N^2\right)\frac{f_2}{f_1}\, v = b v \qquad (21)$$

where c is the sound speed and N, the Brunt-Väisälä frequency defined by

$$c^2 = \Gamma_1 \frac{P}{\rho} \qquad (22)$$

$$N^2 = \frac{1}{\rho}\frac{dP}{dr}\left(\frac{d\ln\rho}{dr} - \frac{1}{\Gamma_1}\frac{d\ln P}{dr}\right) = A\frac{1}{\rho}\frac{dP}{dr} = -Ag \qquad (23)$$

Let us suppose that the distance between the nodes of v and w is small when compared to the scale heights. In that case, we may study the propagation of the wave in a limited region of the star and treat the system as a system with constant coefficients in r, which means a and
b constant. Such a system admits solutions of the form

$$v(r) = C_1\, e^{ik_r r} \qquad w(r) = C_2\, e^{ik_r r} \qquad (24)$$

with C_1 and C_2 constant. The coefficient k_r in the exponential is the wave number. Introducing those relations into the second order system, we obtain the following dispersion relation

$$k_r^2 = \frac{1}{\sigma^2 c^2}\left(\sigma^2 - N^2\right)\left(\sigma^2 - \frac{l(l+1)c^2}{r^2}\right) = F(\sigma^2, l) \qquad (25)$$

A wave can propagate in the radial direction if the wave number k_r is real, which means a positive value of k_r . If k_r is negative, the wave number is purely imaginary and the amplitude of the oscillation varies exponentially with r; the wave is said to be evanescent.

We have two critical frequencies:

$$\sigma_g^2 = N^2 \qquad (26)$$

which is the Brunt-Väisälä frequency, and

$$\sigma_a^2 = \frac{l(l+1)c^2}{r^2} = L_l^2 \qquad (27)$$

which is the Lamb frequency.

There is propagation in the radial direction if

$$\sigma^2 < N^2 \quad \text{and} \quad \sigma^2 < L^2 \qquad (28)$$

or

$$\sigma^2 > L^2 \quad \text{and} \quad \sigma^2 > L^2 \qquad (29)$$

This can be seen in a propagation diagram drawn in figure I where the two critical frequencies are plotted in a $(\sigma^2, r/R)$ diagram, R being the radius of the star. On these curves, F is equal to zero.

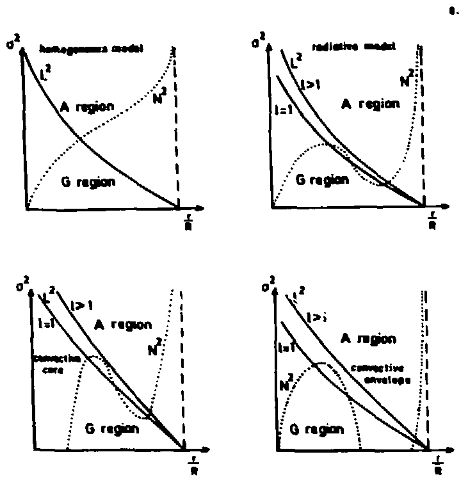

Fig. I
Propagation diagrams

At the surface, we have N^2 increasing to infinity and L^2 decreasing to zero. Those diagrams represents the lowest frequency for propagating acoustic waves and the highest frequency for propagating gravity waves. For a given value of σ^2, one can easily see in which region of the star a wave can propagate and in which region it is evanescent. There are two well defined regions, the A region and the G region where pressure and gravity are respectively the main restoring forces.
In these regions, a propagating wave is mostly reflected at the boundaries, producing a standing wave. Outside these regions, it is spatially damped. So one can see that the oscillation is mainly trapped in some layers of the star.

Another interesting diagram is the so called "diagnostic diagram" which is a σ^2 versus k_h^2 diagram, k_h being the horizontal wave number defined by

$$k_h = \frac{\sqrt{\ell(\ell+1)}}{r} \qquad (30)$$

The dispersion relation becomes

$$k_r^2 = \frac{1}{\sigma^2 c^2}(\sigma^2 - N^2)(\sigma^2 - k_h^2 c^2) = \frac{1}{\sigma^2 c^2}\, I \cdot II \qquad (31)$$

Here again, a wave can propagate if k_r is positive.
Figure II shows a schematical diagnostic diagram.

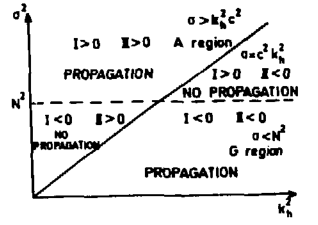

Fig. II
Diagnostic diagram

5. Classification of modes

This discussion clearly shows that there is a distinction between modes. On the one hand, there are the p modes oscillating in the A region and, on the other hand, the g modes oscillating in the G region. This was originally introduced by Cowling (1941) who showed that the eigenvalue problem tends asymptotically towards a second order Sturm-Liouville problem in the cases when $|\sigma^2| \to \infty$ and $|\sigma^2| \to 0$.

I. $|\sigma^2| \to \infty$

The second order equation writes, with $y = \frac{\rho'}{\rho}$

$$\frac{1}{\rho r^2}\frac{d}{dr}\left(\rho r^2 \frac{dy}{dr}\right) + y\left[\frac{\sigma^2}{c^2} + \frac{1}{r^2}\frac{d\left(-\frac{N^2 r^2}{g}\right)}{dr} - \frac{\ell(\ell+1)}{r^2}\right] = 0 \qquad (32)$$

with σ^2 as the eigenvalue.

There is an infinite spectrum of strictly positive discrete eigenvalues with an accumulation point at infinity. They are the p modes, numbered p , p ... and they are dynamically stable.

II. $|\sigma^2| \to 0$

In this case, we have, defining $u = r^2 \delta r$,

$$\frac{1}{\rho}\frac{d}{dr}\left(\rho\frac{du}{dr}\right) + u\left[\frac{N^2}{\sigma^2}\frac{\ell(\ell+1)}{r^2} - \frac{\ell(\ell+1)}{r^2} + \frac{d}{dr}\left(\frac{1}{\Gamma_1}\frac{d\ln p}{dr}\right)\right] = 0 \qquad (33)$$

One can see that σ^2 appears at the denominator and the eigenvalue $1/\sigma^2$ is positive if N^2 is positive and negative if N^2 is negative. There is an infinite spectrum of discrete eigenvalues with an accumulation point at infinity. So σ^2 tends towards zero with positive or negative values depending on the sign of N^2. The stable modes are called g^+ modes. The unstable modes are associated with convective motions and are called g^- modes.
The p modes and g^+ modes are separated by the so-called f mode or fundamental mode whose existence has also been established by Cowling. This mode has normally no node in r and is dynamically stable. It is an extention of the Kelvin mode obtained in the homogeneous uncompressible sphere.
For a main sequence star, the distribution of the dimensionless eigen- frequencies

$$\omega^2 = \sigma^2 \frac{R^3}{GM} \qquad (34)$$

is shown in figure III.

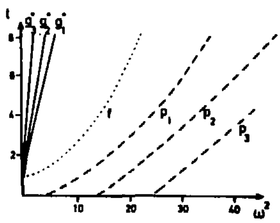

Fig. III
Distribution of ω^2 for a main sequence star

A rigorous mathematical discussion has been done by Gabriel and Scuflaire (1979) in the Cowling approximation. They discuss the behaviour of v and w defined in relations (14) and (15), with respect to polar coordinates Ψ and θ in the form, illustrated in figure IV,

$$v = \Psi \cos \theta \qquad (35)$$

$$w = \Psi \sin \theta \qquad (36)$$

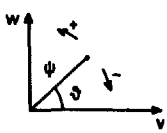

Fig. IV
Count of nodes of δr in the (w,v) plane

Modes can be classified according to the algebraic sum of nodes of δr.
A node is counted positively if is positive and negatively if $\frac{d\theta}{dr}$ is negative. For positive values of σ^2, v has exactly k nodes counted in that way. Positive values of k are associated with p modes and negative values of k with g modes. The fundamental mode has k = 0.

Summary

- Stable p modes, $\sigma_k^2 > 0$, k = 1,2...
  δr_k has k nodes. Accumulation point = ∞
- Fundamental f mode, $\sigma_0^2 > 0$, k = 0
  δr_0 has 0 node.
- If there is a radiative zone:
  stable g^+ modes, $\sigma_k^2 > 0$, k = -1, -2...
  δr_k has k nodes. Accumulation point = 0
- If there is a convective zone:
  unstable g^- modes, $\sigma_k^2 < 0$, k = 1, 2...
  δr_k has (k-1) nodes. Accumulation point = 0

In the (w,v) plane, a high degree p mode and a high degree g^+ mode are illustrated in figure V.

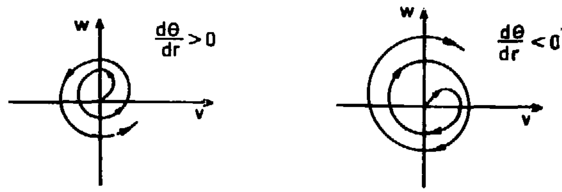

Fig. V
High degree p and g^+ modes in the (w,v) plane.

6. Vibrational stability

In order to estimate the effect of the non adiabatic terms neglected in equation (4), one may use a perturbation method (Ledoux 1958). The first order correction to the eigenfrequency is written

$$\sigma^2 = (\sigma_a + i\sigma')^2 \qquad (37)$$

Subscript a means the adiabatic value of σ . The time depedence of the perturbation becomes

$$e^{i\sigma_a t} \; e^{-\sigma' t} \qquad (38)$$

So, an oscillation is amplified when σ' is negative and damped when σ' is positive. The coefficient of vibrational stability σ' writes

$$\sigma' = -\frac{1}{2\sigma_a^2}\,\frac{\int_0^M \left(\frac{\delta T}{T}\right)_a \delta\varepsilon \, dm - \int_0^M \left(\frac{\delta T}{T}\right)_a \delta\left(\frac{1}{\rho}\nabla\cdot\vec{F}\right) dm}{\int_0^M \vec{\delta r}\cdot\vec{\delta r}^{*}\, dm} \qquad (39)$$

$$= -\frac{E_N - E_F}{E_I}$$

In the right member of equation (39), all quantities are expressed in terms of the adiabatic solution.

The nuclear term, E_N , is generally positive and tends to amplify the oscillation. Its contribution comes from the central part of a core burning star or from the intermediate layers of a shell burning model.

The flux term, E_F , is mainly important in the external layers where it is generally positive and thus contributes to the damping of the oscillation.

7. Rotation

For a non rotating star, the eigenfunction and the eigenfrequency σ of a normal mode are independent of the order m of the associated Legendre polynomial $P_l^m(\cos\theta)$ of degree l and order m. There is a (2 l + 1) - fold degeneracy in m, which simply comes from the symmetry of the equilibrium structure around the rotation axis. Rotation is thus able to remove the degeneracy of a non radial mode of oscillation and the modified eigenfrequencies are written, in a rotating frame (Ledoux 1969)

$$\sigma = \sigma_0 - m\, C_\ell\, \Omega \qquad m = -\ell, \dots, +\ell \tag{40}$$

where σ_0 is the eigenfrequency obtained when rotation is ignored, Ω is the angular frequency of rotation and C_1 is a constant depending on the model and on the mode. In a homogeneous model, one has

$$C_\ell = \frac{4\pi G \rho \left(6\sigma_0^2 + 4\pi G \rho \right)}{9\sigma_0^4 + \ell(\ell+1)(4\pi G\rho)^2} \tag{41}$$

Higher order terms in Ω^2 are proportional to m^2, so there is not only a splitting of modes but also a shift in frequency. In a rest frame, relation (40) writes

$$\sigma = \sigma_0 + m\,(1 - C_\ell)\,\Omega \tag{42}$$

8. Observation of non radial oscillations

In an ideal situation, when one can see different parts of the surface of a star, some of them expanding and others contracting, there is no doubt that the star is non radially oscillating. This is possible for the sun and the solar five-minute oscillations are the most evident proof that non radial oscillations do appear in stars. But, we will not treat this case and, for more distant stars, one has to turn to speckle interferometry to resolve the image into a disk or, more generally, to turn to indirect manifestations of a non radial movement.

1. If a star shows a period much longer than the period of the radial fundamental mode, one may suspect this mode to be a g mode, the periods of such modes tending to infinity as the order of the mode increases, in opposite to radial, f and p modes, which have periods smaller than the radial fundamental mode. This is observed in ZZ Ceti stars which we shall not discuss here.

2. The so-called beat phenomenon is also a good indicator of non radial pulsations. If a star displays an amplitude of oscillation modulated periodically, this shows that two different modes are excited. However, if the beat period is much longer than the principal period, one may go further and say that the two modes must be very close in periods. To find really close modes, one can turn towards the rotational splitting of non radial modes, differing solely in m, as defined by relation (40).

3. Periodic variations in line profiles are not rare in variable stars. Of course, radial pulsations would also exhibit variations in line Doppler shifts. But, if the change in line profile appears without any change in the equivalent width, then this must be the signature of movements occuring differently in different part of the surface of the star, in order to keep the general absorption conditions globally constant. This means again a possibility of non radial oscillations.

4. Baade (1926)'s test was used originally to discard non radial oscillations. It is simply based on the notion of total radius, which keeps all its meaning during a radial pulsation for which the relation

$$L = \pi a c R^2 T_e^4 \tag{43}$$

where L is the luminosity, R the total radius and T_e, the effective temperature, is valid throughout the pulsation. When observing a variable star, one may usually draw a light curve, giving L(t) and a radial velocity curve, giving R(t). The change in spectral type gives $T_e(t)$. So, we have two different ways of obtaining the total radius as a function of time. First, we derive it from relation (43), knowing L(t) and $T_e(t)$ and second, we compute it from the radial velocity curve. If the two functions R(t) vary in phase, we have good reasons to believe in a radial pulsation. If not, as has been done by Walker (1954 a,b) for β Cep stars, we may welcome a non radial explanation.

9. β Cep Stars

The β Cep stars are the first variable stars to have been strong candidates for non radial oscillations after Ledoux's suggestion (1951) to explain the multiperiodicity present in some of them. The observational properties of these early-type variables have been reviewed by Struve (1955), by Underhill (1966) and by Lesh and Aizenman (1978). Their spectral types are between B0.5 and B2 and they belong to luminosity classes III and IV. Their periods range from about 3 to 6 hours and the amplitudes of light variations and velocity curves are rather small. Maximum light appears at maximum compression, i. e. at mean velocity on the descending branch of the velocity curve. They are generally slow rotators with rotational velocities smaller than 50 km/s (McNamara and Hansen, 1961), compared to about 200 km/s for typical B stars, although some of them, as has been shown by Shobbrook and Lomb (1972) and by Shobbrook (1972), reach 200 km/s. Their location in the H-R diagram is shown in figure VI.

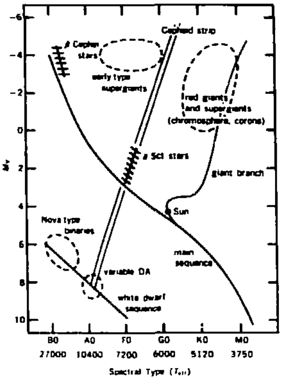

Fig. VI

H-R diagram showing the location of β Cep stars, after Unno(1979).

Most β Cep stars are likely to be radial pulsators with periods of the order of 4 to 6 hours, typical of the radial fundamental mode for a B star. The excitation mechanism for radially pulsating β Cep stars is still unknown although the helium opacity bump has been proposed (Stellingwerf 1978). It is still questionned (Lee and Osaki 1982) whether this mechanism can destabilize the whole star or just act locally.

However, they also show evidence of non radial oscillations. A good discussion of the non radial nature of their pulsations can be found in Unno et al. (1979). The arguments are essentially the line profile variations and radial velocity curves on the one hand and the beat phenomenon on the other hand.

1. Figure VII shows the radial velocity curve and the variation in line profile for σ Sco. There is a strong asymmetry in the velocity curve, the narrower lines occuring during the ascending branch.

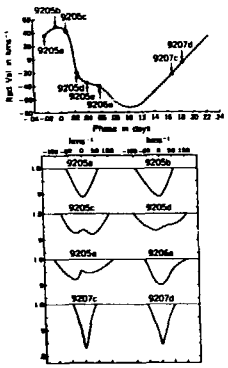

Fig. VII
Radial velocity curve and variations in line profiles for σ Sco, after Huang and Struve (1955)

It is very important to notice that, although the shape of the line changes drastically during the pulsation, the equivalent width remains nearly constant. Ledoux (1951) was the first to suggest that the reason for this was a non radial oscillation coupled with rotation. Since then, detailed line profiles computations have been performed by Christy (1966), Osaki (1971), Stamford and Watson (1976, 1977), Smith (1977) and Kubiak (1978). They tend to confirm that suggestion.

Let us examine a quadrupole oscillation, $l = 2$. Following Unno (1979), the velocity at the surface of a mass element in a non radially oscillating star, is written

$$V_0 = A\left(1,\ k\frac{\partial}{\partial\theta},\ \frac{k}{\sin\theta}\frac{\partial}{\partial\varphi}\right) P_2^m(\cos\theta)\, e^{im\varphi} \qquad (44)$$

Osaki (1971) has shown that the ratio of the transverse to the radial velocity amplitudes is given by

$$k = \frac{GM/R^3}{\sigma^2} = \frac{1}{\omega^2} = \left(\frac{\Omega}{0.116}\right)^2 \qquad (45)$$

Rotation removes the m degeneracy and each quadrupole mode gives five different modes $m = -2, -1, 0, 1, 2$, equally spaced. Because of rotation, there are no standing waves, except for $m = 0$, each wave travelling along or opposite to the rotation, according to the sign of m. Since the eigenfunctions are written in the form

$$f(r,\theta,\varphi,t) = f(r,\theta)\, e^{i(m\varphi+\sigma t)} \qquad (46)$$

the phase velocity is given by

$$\frac{d\varphi}{dt} = -\frac{\sigma}{m} \qquad (47)$$

Let us assume values for l, m and k. We have also to choose a value for i, the angle between the axis of rotation and the line of sight. The star's visible disk is then divided into a large number of surface elements for each of which we compute the velocity resulting from both rotation and non radial oscillation. Each velocity gives a Doppler shift and, adopting a limb darkening law, we may add all the intensities for all the elements to obtain a line profile at time t. The only parameter remaining in this computation is the ratio of the radial amplitude of V_0, A, to the rotational velocity V_R. Figure VIII gives an example of variation of the line profile when the phase varies from 0 to 0.9, computed for $A/V_R = 0.4$, $k = 0.15$ and $i = 90°$ in a $l = 2$, $m = 2$ oscillation.

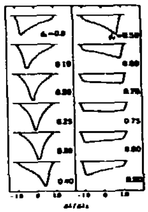

Fig. VIII
Theoretical line profile variations for $A/V_R = 0.4$, after Unno et al. (1979)

A peak on the short wavelength side moves progressively towards the long wavelength side. The line asymmetry is well marked except for $\phi = 0.25$ and $\phi = 0.75$. For values of ϕ greater than 0.50, the profile flattens and two components appear at $\phi = 0.80$, but very weakly. In order to reproduce a more pronounced line doubling as observed in some β Cep stars, one has to choose a greater value of A/V_R, as can be seen in figure IX.

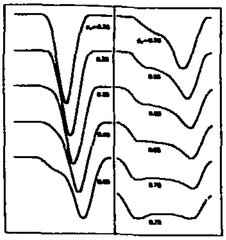

Fig. IX
Theoretical line variations for A/V_R = 1.4, after Unno et al. (1979)

Figure X shows the radial velocity defined from the position of the deepest point in the profile for A/V_R = 0.4.

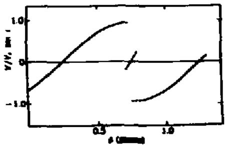

Fig. X
Theoretical radial velocity curve for A/V_R = 0.4, after Unno et al. (1979)

Such discontinuities in the velocity curves are observed in β Cep stars showing line doubling. The variation in line width comes from the variation in the resulting velocity, obtained by adding vectorially V_R and V_o.

More detailed computations of line profiles in the presence of various velocity fields have been described by Campos and Smith (1980). Observational results obtained for γ Peg, β Cep, δ Cet and σ Sco are best reproduced by a radial pulsation in the presence of rotation and shocks. It is difficult to distinguish between a radial mode and a non radial one with m = 0 but, for a random inclination, the amplitude should be much greater in the non radial case. The interesting effect is that a radial pulsation combined with rotation introduces an asymmetry in the line and a variation in the line width, as observed in these β Cep stars. Their radial velocity curves show no discontinuities.

Some β Cep stars could be radial pulsators and others non radial pulsators. It should be interesting to investigate whether the non radial candidates could be found preferentially in a binary system or not.

2. About one half of the stars of the Cep group shows a beat phenomenon, which is generally correlated with variations in line width. Although other explanations have been proposed, such as an accidental degeneracy between the radial fundamental mode and the non radial l = 2, f mode (Chandrasekhar and Lebovitz, 1962), but questioned by Osaki (1975), or a tidal deformation produced by a companion (Fitch 1967, 1969), we shall only retain the non radial oscillation in a rotating star.

Osaki (1971) has computed the rotational velocity from the difference between the two beating frequencies, giving rise to a beating period P_b with l = 2, for m = 2 and 0, that is from

$$\Delta \sigma = \frac{2\pi}{P_b} = 2 C_2 \Omega \qquad (48)$$

This relation is derived from the rotational splitting formula (40). The angular frequency of rotation is simply V_R/R, the total radius R being computed from the luminosity and the effective temperature. The comparison with values of V_R obtained from the fitting of line profiles is generally good except for β CMa for which other values of the parameters could explain the discrepancy.

10. Line Profile Variable Stars

High precision spectroscopy has permitted the discovery of a new class of early type variable stars, the so-called line profile variable stars. Their variation appears in the profiles of photospheric absorption lines, with time scales of hours or days, but only very weakly in luminosity or radial velocity, which distinguish them from their sisters, the β Cep stars. In the H-R diagram, they surround the β Cep stars, their spectral type varying from B5 to O8 and their luminosity classes from II to V.

Line profile variable stars subdivide into two classes (Osaki and Shibahashi 1986, Osaki 1985).

The 53 Per stars (Smith 1977)

They are sharped lined variables, slowly rotating, with periodic variations in line width and line skewness. They are located in the H-R diagram at the edge of the classical β Cep region. Smith (1977) has shown that it was possible to explain their variation in line profiles with low l, equal to 2 or 3, non radial pulsation modes of travelling wave type.

The ζ Oph stars or SCAS stars (Vogt and Penrod 1983a)

These stars are rapid rotators of spectral type B or Be and luminosity classes III to V. They seem to undergo phases of ejection of matter leading to the formation of rings, during which their brightness decreases. They show variations in line profiles where travelling bumps are seen to move across the rotationally enlarged line. Vogt and Penrod (1982) have suggested that this type of variations could well be explained with non radial sectoral modes for higher values of l, about 6 to 10. The choice of sectoral modes is rather arbitrary. Some authors dare say that it is just easier to work with l= m . One generally beleived that it was unlikely to observe non radial modes with such high values of l because too many varying regions on the visible disk would cancel each other and this would put the variation of light and radial velocity under the noise limit. It was also said before that rapid rotation would prevent the observation of line profile variations because it would give a diffuse aspect to the absorption lines. It turns out that rapid rotation helps the observation of high l modes since it resolves the star's disk into different components in the line profile, through rotational Doppler shift by the method called "Doppler Imaging".

Vogt and Penrod (1983a), Penrod (1986) and Osaki (1986) have suggested that the Be phenomenon might be episodical in a B star. As a result of non radial pulsations, angular momentum is injected in the atmospheric layers. A mass ejection in the equatorial plane follows and non radial pulsations are then damped. After a while, the whole mechanism is repeated. Non radial pulsations in B and Be stars have been shown to affect the structure of the atmospheric layers in such a way that a stellar wind can appear (Willson and Bowen 1985).

It may well be that those two subclasses are different aspects of only one class, the observed values of l depending on the rotational velocity. Smith (1980) has suggested that β Cep stars might only well be the "tip of the iceberg" in a global class of line profile variable stars. Even non variable stars in this region of the H-R diagram could be variable in line profile if high enough spectroscopic resolution were achieved.

Among this global class of variable stars, Spica has the peculiar property to having lost a four hour radial pulsation mode about ten years ago. Spica is the primary of a 4.01 day binary. The disappearance of radial pulsations could result from a redistribution of angular momentum within the star. Another hypothesis has been advanced by Balona (1985). This mode could be a non radial l=2, m=0 mode whose pulsation axis, assumed to coincide with the rotation axis, precess with a period of about 200 yr. The observed light and velocity curves would then vary with time. This star has been extensively studied by Smith (1985a, 1985b). The observation of Spica seems to be very difficult from the northern hemisphere and some photometric results have been questionned by Sterken et al. (1986). Its spectral line profiles exhibit very complex variations, with bumps and travelling spikes. Bumps are typical of non radial oscillations and it has been possible to reproduce the variations in line profiles by assuming the simultaneous excitation of four sectorial modes.

1. l = m = 2. This is an equilibrium tidal mode with a 48.2 hour period.

2. A quasi-toroidal mode, responsible for the travelling spike pattern, with an apparent period, measured in a rest frame, of 8.03 hours.

3. l = -m = 8 ± 1. This sectorial mode has an apparent period of 6.51 ± 0.02 hours.

4. l = -m = 16 ± 2. The apparent period is 3.20 ± 0.016 hours.

The ratio of the apparent periods of the two last modes is nearly exactly two, which is also the ratio of their l values. In other words, we can say that when two modes are excited simultaneously in a star, their phase velocities, given by relation (47), tend to be identical. So sectorial modes commensurable in apparent periods are also commensurable in m and l. Such a commensurability has been observed in other β Cep stars although this property seems to be less and less certain, the only candidate remaining could well be σ Sco.

If one accepts the idea of commensurability, one can derive an interesting result, showing the presence of a differential rotation inside the star. Let us apply relation (42) using the apparent frequency of the l = 16 mode, $\sigma_{a\,16}$, with an angular frequency of rotation at the surface, Ω_s derived from estimations of R, V_R sin i and i. This gives the intrinsic value σ_{16}. This mode is the more superficial of the modes appearing here, so Ω_s may enter relation (42). Owing to the commensurability property, we can derive σ_8 simply by dividing by two the value of σ_{16}. From σ_8 and from the apparent frequency of the l = 8 mode, $\sigma_{a\,8}$, we then derive Ω_8, which might be the angular frequency of rotation in the interior of the star. It turns out that the period of rotation at the surface is of the order of 2.3 days while it is only of 2 days in the interior. So a gradient in rotational velocity seems to exist but let us remember that this argument strongly rests on the commensurability of periods, which might be wrong.

A toroidal mode can only be observable in the presence of rotation because its frequency would be strictly zero in a non rotating star. These modes are the Rossby modes observed in the Earth's atmosphere and oceans (Greenspan 1969). The angular frequency σ for a toroidal mode l, m is given, in the rest frame, by (Papaloizou and Pringle 1978)

$$\sigma = m\,\Omega\,\left(-1 + \frac{2}{\ell(\ell+1)}\right) \qquad (49)$$

So, a particular period can as well be obtained by a spheroidal or a toroidal mode for the same values of l and m if l is large enough since the term $-m\,\Omega$ dominates in relations (49) and (42). This term appears only in the rest frame. So, another interpretation, instead of a high l and m toroidal mode, would be a spot on the star's disk observed in a rest frame. Variations in line profiles in rapidly rotating stars have been attributed by Vogt and Penrod (1983b) to such dark spots. A new method applicable to the "Doppler Imaging" technique has been derived by Vogt (1987) and Vogt, Penrod and Hatzes (1987) which permits to obtain a two dimensional image of a star. This method has been applied to [illegible] stars and Ap stars.

The presence of a quasi-toroidal mode in Spica is based upon the existence of a travelling spike. The problem is now: are we sure that we can distinguish between a moving bump and a travelling spike? Osaki (1985) has investigated the possibility of Rossby or toroidal modes to explain line profile variations. From figure XI, we see that it is impossible to distinguish between a non radial sectorial spheroidal mode and a Rossby mode.

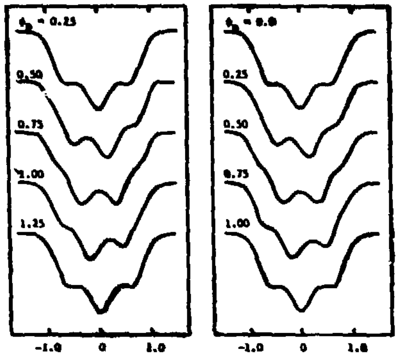

Fig. XI

Line profiles produced by a l = −m = 8(a) spheroidal mode and (b) Rossby mode, after Osaki (1985)

A judicious choice of the ratio of the amplitude of the oscillation's velocity to the rotational velocity has to be made to obtain such a similarity. Rossby modes have the advantage of suppressing the so-called K-problem, dealing with the value of k appearing in relation (44). This parameter has to be small to reproduce line profiles with spheroidal modes, otherwise there should be a strong effect on the wings of the lines, which is not observed. Observations suggest that it must be large since, according to relation (45), the observed frequencies of oscillations are generally low. With toroidal mode, there are only horizontal components in the velocity at the surface and the K-problem may disappear.

Up to now, there are no clear theoretical reasons invoked to explain the excitation of a non radial mode in these stars. A few mechanisms have been advanced, linked with rotation (Osaki 1974) or with the presence of a companion (Kato 1974).

11. δ Scu stars

These stars, located near the main sequence, in the Cepheid instability strip, have spectral types A5 to F5. Their periods range from one to a few hours. They show complicated light curves whose amplitude vary with time. This property has generally been attributed to a beat phenomenon between two or more oscillations.

Most δ Scu stars are assumed to be radial pulsators on the basis on their period ratios. Evidences of radial oscillations have been advanced in δ Scu itself and in some others. Dziembowski (1975) has shown that models of main sequence stars which are unstable towards radial modes are also unstable towards non radial modes.

12. WR stars

Classical WR stars are especially known for their peculiarities in surface abundances and do not generally appear in any catalog of variable stars. The evolutionary scenario proposed by Conti (1976) for single stars explains very nicely the abundance anomalies as a result of mass loss through stellar wind. We shall not talk here about WR stars in binary systems (de Loore 1980), although we are fully aware that such situations are frequent but in the context of non radial pulsations, attention has mostly been paid to single objects. The main idea in the evolutionary picture is simply that mass loss removes all the unburnt material out of the star, leaving at the surface matter which has been previously processed by nuclear burning. Evolutionary computations have been made for numerous values of the initial mass. A review of the most important effects of mass loss has been made by Maeder and de Loore (1982).

A massive star evolves from the zero age main sequence losing mass through stellar wind and becomes an Of star. Once it has lost all the material above the maximum extension of the convective core, the chemical composition at the surface changes drastically, revealing abundances of C, N and O typical of CNO burning (Noels et al. 1980).
The star has now most features of a WR star although it is still difficult to fix precisely the hydrogen abundance it must have when receiving its new name. Mass loss rate is then increased, up to values observed in WR stars. Such changes can take place during core hydrogen burning (Noels and Gabriel 1981) or during hydrogen shell burning (Maeder 1985) if the initial mass is high enough, of the order of about 60 $M_\odot$ or more. The star can also go through a Hubble–Sandage phase (Maeder 1985) near the end of core hydrogen burning, with very violent mass loss. This accelerates the transformation of an O star into a WR star.

Helium burning may eventually start in a star nearly homogeneous in chemical composition and devoid of hydrogen. If its total mass is then higher than the critical mass for He burning stars, of the order of 16 $M_\odot$ (Boury and Ledoux 1965, Noels-Grötsch 1967, Simon and Stothers 1969, Stothers and Simon 1970, Noels and Masereel 1982), a vibrational instability towards radial pulsations occurs, with periods less than the hour (Noels and Gabriel 1981,1984, Maeder 1985). It still remains to detect some variability in the most massive WR stars.

Such variability in the emission line profiles of some WR stars has been discovered by Weller and Jeffer (1979), Vreux (1985) and Vreux et al. (1985). The periods, of the order of a few hours, are longer than the fundamental radial mode and the best candidates for variability seem to be found among the WN stars, i. e. stars with hydrogen still present in the external layers. WC stars are devoid of hydrogen and their spectra show abundances of C and O typical of He burning. This gives two reasons to discard radial pulsations because, apart from the disagreement in the periods, the presence of a hydrogen rich envelope enhances the vibrational stability of a helium star (Simon and Stothers 1970, Noels and Magain 1984). Vreux (1985) had already suggested the presence of non radial oscillations in those stars. For such modes of pulsations, it is worthwhile looking for a vibrational instability in shell burning models since the amplitudes of these modes tend to zero at the center (Simon 1957). Noels and Scuflaire (1986) have found a phase of vibrational instability towards g modes of degree l=1, with periods of the order of a few hours. The e-folding time is about 1000 yr but the duration of the unstable phase is rather short, only about 5000 yr. For higher values of l, of the order of 5 to 10, an instability occurs, some modes being trapped in the hydrogen burning shell (Scuflaire and Noels 1986). Periods are similar to the observed ones but here again the instability is rather mild and cannot by itself explain the variability suspected in WR stars.

Attention has recently been paid to non radial oscillations, not only in WR stars but also in Of stars and this has suggested a possible explanation for the variability in their P-Cygni profiles. Scuflaire and Vreux (1987) have presented a method to compute the profile of P-Cygni lines when the wind has neither spherical nor axial symmetry. The method has been applied to resonance lines only. Therefore the results cannot be used directly in WR stars as these lines are saturated. It would be interesting to investigate Of stars where resonance lines are not saturated.

The detection of variability in those stars is extremely difficult. First, most of them are in binary systems and it is not always easy to distinguish between an orbital period and an intrinsic period of variation. A possible way to discard orbital periods would be to find a period slowly varying with time (Remy 1987). Secondly, although the IUE satellite has revealed numerous variations in line profiles, it is still not clear whether these variations are periodic or not. Much work remains to be done in this promising field.

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Discussion

Pecker: I feel that we should be very careful in predicting waves (G-waves) behaviour from the B.V. frequency as derived in models of stars with deep and thick convective zones: We have now only very first approximation models for the convective zone, which are generally assumed to be adiabatic, whenever they certainly never act strictly in this assumed adiabatic way!

Walker: We observe Spica every spring and certainly find $l=8$ dominant. We have an indication that there was a significant contribution from $l=16$ this year which, if true, would support Smith's finding. We find also that $k \cong 1$.

Fullerton: I have two comments on this very clear review. The first is that it is not at all certain that we can relate the observations of nonradial pulsations in intermediate and rapidly rotating stars with the theoretical expectations derived from "slowly" rotating models. Dr. Maurice Clement of the University of Toronto has been tackling the extremely difficult problem of establishing eigensolutions for rapidly rotating models, and his results are discouragingly complicated. The m values derived by observers are surely a valuable description of the pulsational activity, but it will probably be some time before they can be linked with "modes" that theoreticians find.

My second comment concerns the perception that the preference observers express for sectorial modes (i.e. $l=|m|$) is arbitrary. This is not the case, at least for those stars with detectable photometric amplitudes (e.g. Epsilon Per). Only sectorial modes can produce such variations - other patterns have too much cancellation across the disk. Furthermore, there is evidence which Gordon Walker presented on Monday that the pulsations are confined to the equatorial band - again, this is a property of sectorial modes. To be rigorous we should check for photometric variations of appropriate amplitude for each nonradially pulsating star - a lot of this data exists for B and Be stars, but not so much for O stars, but my point is that sectorial modes are a necessity, not a choice.

Walker: It is very hard to reproduce the amplitude and sharpness of the sub-features in the line profiles for Spica unless $l=|m|$.

Sterken: What is the situation concerning the light variability of Spica now? Do you know if your colleagues of Nice Observatory have any recent observations of it?

Baglin: The last campaign on Spica took place in April/May 1987 but was a complete failure due to bad wheather conditions at Pico de la Veleta Observatory.

Bolton: At this stage in our understanding of variable phenomena in early-type stars, it is well to keep in mind all possible models. In that spirit, I welcome your emphasis on Rossby waves. However, I think this is one of the easier models to reject. As I understand it, Rossby waves with subsonic velocities, such as are required to explain the observed line profile variations, should produce no photometric variations. But many of the best studied line profile variables also have photometric variations of one to a few percent with the same period.

Hall: [illegible]

Van Paradijs: I recall having seen a paper recently in Astrophys. J. Letters on short-term, small radial-velocity variations in Arcturus, but I can't remember the detailed number.

Bartolini: About the line profile variable stars, do you think that periods shorter than one hour are possible? At Bologna Observatory we carried out spectroscopic observations of a B3 star that seems to display line-profile variations on a time scale of 20 to 30 minutes. Could sectorial modes with $l=-m=32$ or 64 be involved?

Baglin: It should be a p mode.

Kubát: How can non-radial pulsations start?

Baglin: Like a radial pulsation. If they are self sustained, any small perturbation will grow and start the motion. If - like in the Sun - they are forced (probably by convection), they just decay slowly till the next kink.

Kubát: Should not the Alfvén waves be included into the theory?

Baglin: Alfvén waves certainly exist in the magnetic configuration at the surface of the object, but they are not global coherent perturbations.