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Stability Problems and Linear Driving

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Abstract. After setting the historical background, the major driving mechanisms in variable stars are reviewed and discussed. Tremendous success has been achieved over the years but some puzzling problems, not unlike those encountered at the beginning of this century, still await the stability specialists.

1. Introduction

Variable stars are known to exist since the 16th century with the observation of *o* Ceti, a Mira variable, by Fabricius (Perdang 1990, Gautschy 1997). It is however the discovery of the periodic variability of δ Cephei by Goodricke in 1784 (Goodricke 1786) which has been the most prominent landmark in the history of variable stars. Since this discovery was nearly simultaneous with the observation of the variability of Algol, readily interpreted as being due to eclipses, *binarity* was the first *variability mechanism* proposed for Cepheids. However, some criticisms arose (Shapley 1914), such as the large radius of Cepheids, derived from their location in the HR diagram, which was not compatible with orbit dimensions, and Plummer (1914) was the first to suggest a *radial pulsation* in a single star.

Eddington (1918) addressed the problem of the long term persistence of pulsations. Instead of being initiated by a cataclysmic event followed by a slow damping, oscillations could be infinitesimal at first, then grow in amplitude and finally, due to viscosity, enter a limit cycle. Using polytropic models, he succeeded in deriving pulsation periods and he obtained a period-density relation. Eddington (1926) proposed an expression for the stability coefficient κ

$$\kappa = \frac{1}{\tau_d} = \frac{1}{2}\frac{< dE/dt >}{< E >} \tag{1}$$

where τ_d is the e-folding time for the dissipation of the pulsation, E is the total pulsation energy, and the brackets indicate averages over a period (Ledoux 1958, J.P. Cox 1980). He discarded driving due to contraction, but turned to *nuclear reactions* as being the main driving source for the pulsation of Cepheids.

Eddington also introduced the concept of a variable opacity related to the pulsation mechanism. This is referred to as the *valve mechanism* which can be seen as a thermodynamic heat engine where the leakage of heat is varied during the cycle. *If the leakage of heat decreases during compression and increases during expansion, then driving of the pulsation is possible.*

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However, this *κ-mechanism* was only required *to reduce the damping effect of the outer layers.* The actual driving effect was located near the center and was due to the nuclear reactions. Ledoux (1941) introduced the notion of a *critical mass* above which homogeneous nuclear burning stars are unstable. He obtained a critical mass of about 100 $M_\odot$ for hydrogen burning stars. The very notion of a critical upper mass for stable main sequence stars is still widely accepted as the best explanation for the upper limit of the mass function.

The case of Cepheids was somewhat different as they were already known not to be main sequence stars but located in the upper part of the the so-called *classical instability strip* in the HR diagram (Sandage 1958). Cox (1955) performed a stability analysis which clearly showed that in such stars the driving was not to be found in a central nuclear burning region but in the external layers. The ionization of an abundant element was a good candidate (Schatzman 1956, Cox 1958). Zhevakin (1953), followed by Cox & Whitney (1958), suggested the second ionization of helium to be responsible for the valve mechanism. A major step was achieved through the work of Baker & Kippenhahn (1962) and Cox (1963) who performed linear non adiabatic computations and beautifully confirmed the helium second ionization layers as the driving mechanism in Cepheids.

2. Pulsation Stability

A variable star must of course be assumed to be dynamically stable. The question is whether or not a small perturbation will be excited or damped. The answer to that problem of the pulsation stability lies in the consideration of the nonadiabatic terms neglected in the dynamical stability analysis. The general analysis will be presented here for radial modes[1].

The eigenvalues as well as the eigenfunctions are complex and we shall write

$$\delta r(r,t) = \delta(r) e^{st} \tag{2}$$

with

$$s = -i\sigma + \sigma' \tag{3}$$

Starting from the linearized perturbed momentum equation, we multiply each term by $4\pi r^2 \bar{\delta r}$, where $\bar{\delta r}$ is the complex conjugate of δr. Integrating over the whole star and introducing the perturbed mass equation, we obtain after some rearrangements

$$s^2 \int |\delta r|^2 dm + \int \left\{ \frac{\delta P}{\rho}\frac{\bar{\delta\rho}}{\rho} + \frac{4r}{\rho}\frac{dP}{dr}|\frac{\delta r}{r}|^2 \right\} dm = 0 \tag{4}$$

Taking the imaginary part of expression (4), we obtain

$$2\mathcal{R}s\mathcal{I}s = -\frac{\mathcal{I}\int \frac{\delta P}{\rho}\frac{\bar{\delta\rho}}{\rho} dm}{\int |\delta r|^2 dm} \tag{5}$$

[1]For further details on radial and nonradial oscillations, we strongly recommend the exhaustive discussion by Ledoux & Walraven (1958) and the more recent reviews or monographs, such as Unno et al. (1979), J.P. Cox (1980), Smeyers (1984), Gautschy & Saio (1995, 1996).

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The physical meaning of the integral appearing in the denominator of the right hand member of relation (5) is quite straightforward. It can ea sily be shown that the total energy of the pulsation takes the form (J.P. Cox 1980)

$$E = \frac{\sigma^2}{2} \int \delta r^2 dm \tag{6}$$

Relation (6) shows that the denominator in expression (5) is related to the *total mechanical energy of the pulsation.*

Let us now turn to the physical meaning of the numerator in expression (5). For a unit mass undergoing a thermodynamic cycle, one can write (Cox 1980, Scuflaire 1995)

$$P(t) = P_0 + \delta P(t) = ae^{i\phi}e^{-i\sigma t} \tag{7}$$

$$\rho(t) = \rho_0 + \delta\rho(t) = be^{i\psi}e^{-i\sigma t} \tag{8}$$

The work done during a whole cycle takes the form

$$w = \oint PdV = \frac{\pi ab}{\rho^2}\sin(\phi - \psi) = \pi\mathcal{I}\left(\frac{\delta P}{\rho}\frac{\bar{\delta\rho}}{\rho}\right) \tag{9}$$

If we figure the star as being composed of a large number of such unit masses, each one being a Carnot-type engine, then integrating over the whole star, one gets the mean power

$$< \frac{dW}{dt} >= \int \frac{w}{\Pi}dm = \frac{\sigma}{2}\mathcal{I}\int \frac{\delta P}{\rho}\frac{\bar{\delta\rho}}{\rho}dm \tag{10}$$

where Π is the period of the thermodynamic cycle. Apart from the coefficient $\sigma/2$, the numerator in expression (5) is the *mean power coming from the whole star undergoing the oscillation.*

Equation (5) can be transformed to show more explicitly the non adiabatic terms. The expression of $\frac{\delta P}{P}$ is now different from its adiabatic form. It contains a nonadiabatic term in δS, and is written as

$$\frac{\delta P}{P} = \Gamma_1\frac{\delta\rho}{\rho} + \frac{(\Gamma_3 - 1)\rho T}{P}\delta S \tag{11}$$

where δS is the perturbed entropy, which can be expressed with the help of the perturbed thermal energy equation

$$\frac{d\delta L}{dm} = \delta\varepsilon - sT\delta S \tag{12}$$

Introducing relations (11) and (12) into equation (5), one gets

$$2\mathcal{R}s\mathcal{I}s = -\frac{\mathcal{I}\frac{1}{s}\int(\Gamma_3 - 1)\frac{\bar{\delta\rho}}{\rho}\left(\delta\varepsilon - \frac{d\delta L}{dm}\right)dm}{\int|\delta r|^2dm} \tag{13}$$

All the eigenfunctions appearing in the right hand member of expression (13) should be computed with the exact nonadiabatic eigenfunctions.

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In the *quasi-adiabatic approximation*, the eigenfunctions are taken to be the solutions of the adiabatic problem. Writing s in the form prescribed in relation (2), relation (13) becomes (Ledoux 1969)

$$\sigma' = \frac{1}{2\sigma^2} \frac{\int (\Gamma_3 - 1)\frac{\delta\rho}{\rho}\left(\delta\varepsilon - \frac{d\delta L}{dm}\right) dm}{\int |\delta r|^2 dm} \tag{14}$$

in the approximation where $|\sigma'/\sigma|$ is much smaller than unity. The integral in the numerator of relation (14) is generally called the *work integral.* The displacement of each mass element is thus nearly periodic and follows a time dependence

$$\delta r(m,t) = \delta r(m) e^{-i\sigma t} e^{\sigma' t} \tag{15}$$

Its amplitude increases or decreases in time according to the sign of σ', i.e. the sign of the numerator in expression (14). *The star is said to be pulsationally stable or unstable if σ' is negative or positive respectively.*

We can write equation (14) in a more physical way, introducing relations (6) and (10). We obtain

$$\sigma' = \frac{1}{2}\frac{< dW/dt >}{E} \tag{16}$$

which is similar to relation (1) proposed by Eddington (1926). Because the total mechanical energy of the pulsation is always positive, the excitation or the damping of the oscillation will come from the sign of $< dW/dt >$. If the integral over the mass of the mean power produced by each mass element is positive, there is an amplification of the oscillation and the star is pulsationally unstable. As $< dW/dt >$ comes from an integral over the mass, each layer contributes positively (negatively) to the excitation (damping) of the oscillation.

This technique generally gives a good estimate of σ', at least in the cases where the nonadiabatic external layers are not too important. In those layers, the ratio of the thermal time scale, τ_{th}, to the dynamical time scale, τ_d, which is below unity through most of the star, can become very large. A *transition zone* can be defined where this ratio is approximately equal to unity. If we write τ_{th} in the form

$$\tau_{th} = \frac{\Delta m c_V T}{L}, \tag{17}$$

where $\Delta m c_V T$ is the heat capacity of the layer Δm located above the transition zone, and

$$\tau_d = \frac{\Delta r}{c} \approx \Pi, \tag{18}$$

is the time taken by a sound wave to travel through the extent Δm, on the order of the period Π, then one finds an estimate of the transition zone from the relation

$$\Delta m c_V T \approx \Pi L. \tag{19}$$

The star is then composed of three regions : (1) an adiabatic inner part where $\Delta m c_V T \gg \Pi L$, (2) a transition zone and (3) a nonadiabatic outer part where $\Delta m c_V T \ll \Pi L$. In the nonadiabatic region, the heat capacity is so small that δL is very nearly constant. The huge increase of δL computed with the adiabatic

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eigenfunctions is in strong disagreement with such a frozen in value of δL. In the quasi-adiabatic approximation, these nonadiabatic layers are simply dropped out of the integral.

The numerical integration of the nonadiabatic problem is non trivial since the coefficient of δS in the thermal energy equation can be exceedingly large in the inner adiabatic region. On the other hand, the differences in orders of magnitude of σ and σ' can also lead to numerical difficulties (Baker & Kippenhahn 1962, Cox 1963). A major step was taken by Castor (1971) and by Iben (1971) and for nonradial oscillations by Ando & Osaki (1975). A short but enlightening description of Castor's method can be found in J.P. Cox (1980).

3. Driving Mechanisms

For the sake of simplicity, we shall start this discussion in the quasi-adiabatic approximation. The use of adiabatic eigenfunctions in the right hand member of equation (14) leads to another form of this relation, i.e.

$$\sigma' = \frac{1}{2\sigma^2} \frac{\int \frac{\delta T}{T} \delta\varepsilon dm - \int \frac{\delta T}{T} \frac{d\delta L}{dm} dm}{\int |\delta r|^2 dm} \tag{20}$$

Amplification of the oscillation, i.e. a positive value of σ', can be achieved through a nuclear contribution (first integral in the numerator) or an energy transfer contribution (second integral in the numerator).

3.1. ε-mechanism

Nuclear reactions generally take place in the inner regions of the star, where the adiabatic approximation is valid. Writing ε in the form

$$\varepsilon = \varepsilon_0(X, Y, Z)\rho^\mu T^\nu \tag{21}$$

one obtains

$$\frac{\delta\varepsilon}{\varepsilon} = \mu\frac{\delta\rho}{\rho} = +\nu\frac{\delta T}{T} \tag{22}$$

and the nuclear contribution is then

$$E_N = \int \frac{\delta T}{T}\delta\varepsilon dm = \int \left(\frac{\delta T}{T}\right)^2 \varepsilon \left[\frac{1}{\Gamma_3 - 1}\mu + \nu\right] dm \tag{23}$$

This term is always positive and contributes to the amplification of the oscillation. It should be noted that the μ and ν values in relation (22) can be different from their equilibrium values, depending on the time scale of the oscillation (Schatzman 1953, Ledoux and Walraven 1958).

Physically, the significance of the ε-mechanism comes from the fact that, at maximum compression, the temperature and thus the energy production rate is higher than at equilibrium. In the layers where nuclear reactions take place, *energy is gained at compression while the opposite happens during expansion.* This is exactly what is required to gradually increase the amplitude of the oscillation.

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3.2. κ-mechanism

The energy transfer contribution to the numerator of σ' (relation 20) is written

$$E_F = -\int \frac{\delta T}{T}\frac{d\delta L}{dr}dr \tag{24}$$

This contribution is positive (destabilizing) if a positive value of δT is associated with a decrease of δL towards the surface. This means that instability occurs if *matter gains energy at the stage of high temperature, i.e. in the compression phase, and loses energy during expansion.* This term is generally the most important one since the amplitudes of the eigenfunctions increase in the outer layers.

For the sake of simplicity, we shall neglect convection. Writing the opacity law in the form

$$\kappa = \kappa_0 \rho^m T^n \tag{25}$$

one gets, from the radiative transfer energy equation, after introducing the perturbed mass equation,

$$\frac{\delta L}{L} = \frac{\frac{d(\delta T/T)}{dr}}{\frac{d\ln T}{dr}} - \frac{4r}{3}\frac{d(\delta r/r)}{dr} + \left[4 - n - \frac{m+\frac{4}{3}}{\Gamma_3 - 1}\right]\frac{\delta T}{T} \tag{26}$$

Neglecting the first two terms in the right hand member of relation (26) and assuming that the radiative luminosity is constant in the outer layers, one gets

$$\frac{d\delta L}{dr} = L\frac{d}{dr}\left[4 - n - \frac{m+\frac{4}{3}}{\Gamma_3 - 1}\right]\frac{\delta T}{T} \tag{27}$$

With Kramers type values for m and n ($m = 1$, $n = -3.5$), the bracket in relation (27) is of the order of 4, i.e. positive. With $\delta T/T$ increasing outward, E_F will be negative and the outer layers have in that case a damping effect. To obtain a destabilizing effect, n must change sign. Actually, *the opacity must increase at compression so that energy is gained, and decrease at expansion so that energy is lost* which is again the condition found in a thermodynamic heat engine.

Positive values of n can be found in regions where H^- is a major contributor to the opacity. The ionizations of H and He have the same effect. In these cases, destabilization is enhanced by the small values of $(\Gamma_3 - 1)$. This effect is called the *γ-mechanism* and it occurs simultaneously with the κ-mechanism.

For most variable stars, the excitation mechanism is the κ-mechanism acting in layers where the second ionization of helium takes place. The modes the most likely to be excited are those whose periods are of the order of the thermal time scale of the excitation zone.

The first big success of the κ-mechanism is the understanding of the instability strip in the HR diagram. Two problems were in fact related to well known stars located in this strip, namely the classical Cepheids and the RR Lyrae stars : first, an explanation of the exact location of this strip as well as of its extent in the HR diagram (Cox 1967) and, second, the phase lag discrepancy showing up in the observation of these variable stars (Castor 1968, 1971).

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4. κ-Mechanism in Variable Stars

The major excitation mechanism is the κ-mechanism. It is responsible for the pulsation of most variable stars, namely Cepheids, RR Lyrae, δ Scuti, SPB, and β Cephei stars (Gautschy & Saio 1996). The great majority of these stars have been studied by Art Cox (for Cepheids, see A.N. Cox 1980, 1983) and his collaborators who pointed out many troublesome problems and advanced solutions, sometimes invoking additional processes such as diffusion, levitation, and mixing. Most of these problems, which have puzzled the stability specialists during about two decades, have been essentially solved by the introduction of new opacity data, namely OPAL and OP opacities (Rogers & Iglesias 1992, Seaton et al. 1994).

All classes of variable stars pulsating as a result of the κ-mechanism are widely discussed during this meeting.

5. ε-Mechanism in Variable Stars

As we have seen in section 3.1, the ε-mechanism is generally inefficient as it takes place in the inner regions where the amplitudes are usually much lower than in the damping outer regions. When the mass increases, however, the importance of the radiation pressure increases and Γ_1 decreases. For the fundamental radial mode, the ratio ξ_s/ξ_c of the amplitude at the surface to the amplitude at the center becomes smaller and tends towards unity for a constant value of Γ_1 equal to 4/3. These simple arguments show that there exists a *critical mass* above which a pulsational instability due to the ε-mechanism occurs (Ledoux 1941). This limiting mass was thought to be at the origin of an upper limit in the mass function and the first value proposed by Ledoux (1941) was of the order of 100 $M_\odot$ for hydrogen burning stars. The value of the critical mass is rather sensitive to the opacity. With the new OPAL opacities, Stothers (1992) has shown that the critical mass is increased to 121 $M_\odot$ for $Z = 0.02$.

This result is somewhat different from the value obtained by Glatzel & Kiriakidis (1993). These authors showed that a stronger instability related to *strange modes* (Glatzel 1997) leads to a much lower value of about 60 $M_\odot$.

For homogeneous helium burning stars, the critical mass due to the ε-mechanism was found to be about 16 $M_\odot$, using the Los Alamos opacities (Stothers & Simon 1970, Noels & Masereel 1982). The physical relevance of such stars is however not straightforward. They may result from the evolution of a massive star with sufficient mass loss to have peeled off all the layers surrounding the once convective hydrogen burning core. Should a small mass fraction of this envelope remain, the rise in mass concentration rapidly increases the critical mass. A mass fraction of the order of 10^{-4} leads to an upper limit of 80 $M_\odot$ (Noels & Magain 1984).

Actually, the evolution of massive stars ($\geq 40 M_\odot$) with mass loss is the scenario now adopted to explain the formation of WR stars (Conti 1976, Noels et al. 1980). At the onset of central helium burning, such stars can be well above the critical mass and instability driven by the ε-mechanism has indeed been found (Noels & Gabriel 1981, Maeder 1985, Cox & Cahn 1988) with very short periods, of the order of one hour. Although this cannot be the origin of

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the WR phenomenon since it needs a previously large mass loss rate for the instability to occur, this could hasten the transformation of a WN into a WC star.

Variability has indeed been observed in some WR stars (Vreux 1986, Gosset et al. 1989, van Genderen et al. 1990) with periods of the order of a few hours and instability towards large period nonradial g-modes was searched for. In a quasi-adiabatic analysis, Noels & Scuflaire (1986) found a marginal instability towards low order g-modes of $\ell = 1$.

The subsequent development of an intermediate convective zone inside the hydrogen shell sharpens the peak in the Brunt-Väisälä frequency and this can lead to *trapped modes*, i.e. modes having an oscillatory nature inside the peak and an evanescent behaviour on both sides. Again marginal instability was found towards g- and p-modes for $\ell = 10$ (Scuflaire & Noels 1986).

In a similar, but nonadiabatic analysis, Cox & Cahn (1988) did not find any unstable g-modes. The reason of the difference is not clear. It could be due to the nonadiabatic treatment in Cox & Cahn (1988) although some of the trapped modes in Scuflaire & Noels (1986) have large amplitudes *only* in the vicinity of the hydrogen burning shell. With vanishingly small amplitudes in the external layers, it is difficult however to relate them to an observed variability. Such an instability would more likely induce an additional mixing in the μ-gradient zone.

6. The Status of 51 Pegasi, or Back to Square One

Recently, Mayor & Queloz (1995) and Marcy et al. (1997) observed periodic wobbles in the radial velocity curve of 51 Pegasi, a Sun-like star. They attributed these variations to a Jupiter-like planet orbiting the star. The spectrum of this star has however been monitored since 1989 by Gray (1997) who concluded, after a careful analysis of his data, that the variations were mostly in the *shape* of the lines. His proposed explanation is an intrinsic nonradial oscillation.

An almost immediate answer came from the planet advocates. The main points of their argumentation against a nonradial pulsation are the following : (1) other analyses of 51 Pegasi's spectrum have not shown evidence of a changing shape in the spectral lines, (2) should the Doppler shifts be interpreted as an intrinsic pulsation of the star, a change in brightness would be expected, (3) the star is strictly monoperiodic with no changes in amplitude nor in velocity phase and (4) other 51 Peg-like stars have different periods although they are probably very similar.

A planet explanation can easily answer all these points. It is even possible, if line shape variations were confirmed, to reconcile them with a planet explanation by invoking a tidal effect.

So, we are back to square one. *Is the variability of 51 Peg-like stars due to intrinsic pulsations or to a binary system involving the possible discovery of a planet?* At the beginning of this century, it was conceptually more easy to accept a binarity explanation. Now it is perhaps just much more fun...

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7. Conclusions

Considerable progress has been made during this century and especially during the last fifty years. Huge efforts had to be made in developing adiabatic and nonadiabatic, linear and nonlinear codes and Art Cox has played a particularly important role. Most of the problems, acute in the 70's, are now, thanks to new opacity data, on their way to being solved. The biggest challenges in a near future are to introduce diffusion processes and to correctly treat the interaction between convection and pulsation.

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