Mon. Not. R. Astron. Soc. **368,** 1941–1948 (2006) doi:10.1111/j.1365-2966.2006.10275.x

Modelling the components of binaries in the Hyades: the dependence of the mixing-length parameter on stellar mass

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Accepted 2006 March 3. Received 2006 February 28; in original form 2005 September 23

ABSTRACT

We present our findings based on a detailed analysis of the binaries of the Hyades, in which the masses of the components are well known. We fit the models of the components of a binary system to observations so as to give the observed total V and $B - V$ of that system and the observed slope of the main sequence in the corresponding parts. According to our findings, there is a very definite relationship between the mixing-length parameter and the stellar mass. The fitting formula for this relationship can be given as $\alpha = 9.19(M/\mathrm{M}_{\odot} - 0.74)^{0.053} - 6.65$, which is valid for stellar masses greater than $0.77\,\mathrm{M}_{\odot}$. While no strict information is gathered for the chemical composition of the cluster, as a result of degeneracy in the colour–magnitude diagram, by adopting $Z = 0.033$ and using models for the components of 70 Tau and θ^2 Tau we find the hydrogen abundance to be $X = 0.676$ and the age to be 670 Myr. If we assume that $Z = 0.024$, then $X = 0.718$ and the age is 720 Myr. Our findings concerning the mixing-length parameter are valid for both sets of the solution. For both components of the active binary system V818 Tau, the differences between radii of the models with $Z = 0.024$ and the observed radii are only about 4 per cent. More generally, the effective temperatures of the models of low-mass stars in the binary systems studied are in good agreement with those determined by spectroscopic methods.

Key words: stars: abundances – binaries: eclipsing – stars: evolution – stars: interiors – open clusters and associations: individual: Hyades.

1 INTRODUCTION

Open clusters and binary systems have their own essential roles in almost all branches of astrophysics. While we obtain information on the fundamental properties (such as mass, radius and luminosity) of the components of binary systems from observations (see e.g. Andersen 1991), the age of stars of an open cluster can be taken fairly accurately as the main-sequence (MS) lifetime of its brightest (*normal*) MS star. The age and the fundamental properties of a system are complementary to each other for the purposes of a better understanding of stellar structure and evolution. Therefore, binary systems in clusters are invaluable. In this respect, the Hyades open cluster is an absolute treasure: in addition to very precise observational data for the distances and photometric measurements of its members (de Bruijne, Hoogerwerf & de Zeeuw 2001; Perryman et al. 1998), the masses of the components of its five double-lined binaries are also known. These binaries are V818 Tau (Peterson & Solensky 1988), 70 Tau (Fin 342), 51 Tau, θ^2 Tau and θ^1 Tau (Torres, Stefanik & Latham 1997c,b,a).

The cluster itself and its binaries have been the subject of innumerable papers. Recently, Pinsonneault et al. (2003), Lebreton, Fernandes & Lejeune (2001) and Lastennet et al. (1999) have researched (some or all of) these binaries in detail by comparing the observational results with models for the internal structure of the component stars. Lastennet et al. (1999) tested the validity of three independent sets of stellar evolutionary tracks, using good photometric data for V818 Tau, 51 Tau and θ^2 Tau. Lebreton et al. (2001) focused on the determination of the helium abundance by considering in detail all of the five binaries, and derived the helium abundance as $Y = 0.255$, while Pinsonneault et al. (2003) found $Y = 0.271$ from the calibration of the components of the eclipsing binary V818 Tau. Both of these studies stress the difficulty of calibrating the radii of the components of V818 Tau. Furthermore, Lebreton et al. (2001) take note of the stellar mass dependence of the mixing-length parameter (α).

The mixing-length parameter $\alpha = l/H_P$ is an unknown in stellar modelling and is often chosen to be constant and equal to the solar

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value. As discussed in many papers, however, there is no good reason for keeping it constant; it may change from star to star (see below) and from phase to phase (Castellani, Degl'Innocenti & Prada Moroni 2001; Chieffi, Stranerio & Salaris 1995).

For the stellar mass dependence of α, contradictory results are obtained from studies on binaries by different investigators. In studies on eclipsing binaries (Lastennet et al. 2003; Ludwig & Salaris 1999; Lebreton et al. 2001), it is reported that α is an increasing function of the stellar mass. Such a dependence is also found by Morel et al. (2000) in their studies on the visual binary ι Peg. However, two other possibilities are reported for visual binaries: while Fernandes et al. (1998) state that the solar value can be used to model components of the four visual binaries they studied, Pourbaix, Neuforge-Verheecke & Noels (1999) find from the calibration of α Cen that α is a decreasing function of the stellar mass. This complexity for the visual binaries may be a result of the low precision of the values of their components (this is also the case for the 70 Tau binary system). On the other hand, hydrodynamical simulations of convection (Ludwig, Freytag & Steffen 1999; Trampedach et al. 1999) also give that α is a decreasing function of effective temperature (or stellar mass). This contradiction needs to be explained.

The determination of the chemical composition of a star is another difficult matter. Although it is assumed that the members of an open cluster have the same age and chemical composition, the numerous different abundance determinations of various elements in the Hyades stars mean that it is not possible to assign a single value for its heavy element abundance Z (see Section 2). Therefore, in the present study, the hydrogen abundance (X) and Z are considered as unknowns.

For the calibration of a well-known binary with late-type components, the number of constraints on the models of its components is four. For the luminosity (L) (or absolute magnitude) and the radius (R) (or colour) of each component, we can write down two equations:

$$L_{\rm obs} = L_{\rm o} + \frac{\Delta L}{\Delta X}\delta X + \frac{\Delta L}{\Delta Z}\delta Z + \frac{\Delta L}{\Delta \alpha}\delta \alpha + \frac{\Delta L}{\Delta t}\delta t, \tag{1}$$

$$R_{\rm obs} = R_{\rm o} + \frac{\Delta R}{\Delta X}\delta X + \frac{\Delta R}{\Delta Z}\delta Z + \frac{\Delta R}{\Delta \alpha}\delta \alpha + \frac{\Delta R}{\Delta t}\delta t, \tag{2}$$

where $L_{\rm o}$ and $R_{\rm o}$ are values from the reference model with fixed values of X, Z, age (t) and α (solar values, for example, except for t). The number of unknowns for models of the component stars is five: X, Z, t, $\alpha_{\rm A}$ and $\alpha_{\rm B}$. Thus, if we have no extra constraint on the binary system, there is in principle no unique solution for it. On the other hand, for a visual binary in which the component stars are not well known, we can write two equations similar to equations (1) and (2) for the total V and $B-V$ of the system.

The remainder of the present paper is organized as follows. In Sections 2 and 3, respectively, the observed chemical composition of the cluster and the properties of the binary systems studied are summarized. The model properties of the binaries and their components are presented in Section 4, and concluding remarks are given in Section 5.

2 CHEMICAL COMPOSITION OF THE HYADES

There are many papers devoted to the determination of abundances of heavy elements of Hyades stars of different classes (from A- to K-type stars). For the abundance of iron, Boesgaard, Beard & King (2002) and Hui-Bon-Hoa & Alecian (1998) find enhancement relative to the solar abundance (0.16 dex for G-type and 0.13 for A-type stars in the Hyades). Recently, Paulson, Sneden & Cochran (2003) derived an abundance with a very small formal error [0.13 ± 0.01 dex; see also Yong et al. (2004)]. In contrast to these findings, Varenne & Monier (1999) find the mean value of iron abundances from the spectra of 29 F-type stars as −0.05 dex. Thus, the abundances found from the stellar spectra by different research groups are in general not in agreement with each other. Three-dimensional calculations for the stellar atmosphere (see, for example, Asplund et al. 2004) may solve such problems.

The customary consideration of iron abundance as a good tracer of the total Z of a star is, however, highly debatable. Iron is not among the most abundant heavy elements, and there is no one-to-one relationship between the abundances of iron and the most abundant heavy elements (for example oxygen). This fact can be seen, for example, in fig. 10 of Bensby, Feltzing & Lundström (2004): for the stars in the Galactic disc that have an [O/H] value of about zero, [Fe/O] abundance varies between −0.4 and +0.1. Thus, it is not reasonable to take [Fe/H] = [O/H].

Indeed, abundances of oxygen and nitrogen have been determined by many spectroscopists. Takeda et al. (1998) find the oxygen abundance to be 0.10 dex for F stars in the Hyades, while King & Hiltgen (1996) determine it as 0.15 dex from the spectra of the two dwarfs. Takeda et al. (1998) also give the abundance of nitrogen as 0.30 dex.

It could be claimed that the scattering of this amount for a given element is a result of the diffusion process, whose rate varies from star to star. However, contradictory results are found for the oxygen abundance of Hyades member HD 27561: while Garcia Lopez et al. (1993) find −0.14 dex, Clegg, Lambert & Tomkin (1981) give 0.15 dex as the oxygen abundance of this star. This leads us to conclude that the difference between the abundances determined by different studies is the result of different techniques. Consequently, it is a very difficult job to estimate the value of the heavy element abundance of the Hyades. In the present paper we therefore consider Z as an unknown parameter.

3 OBSERVED PROPERTIES OF THE BINARIES OF THE HYADES

Several studies have been devoted to determining the fundamental properties of the components of V818 Tau (McClure 1982; Schiller & Milone 1987; Peterson & Solensky 1988). Peterson & Solensky (1988) found the masses of the primary and the secondary components of V818 Tau as $1.072 \pm 0.010\,{\rm M}_\odot$ and $0.769 \pm 0.005\,{\rm M}_\odot$, respectively. Recently, Torres & Ribas (2002) also found the masses of the components: $M_{\rm A} = 1.0591 \pm 0.0062\,{\rm M}_\odot$ and $M_{\rm B} = 0.7605 \pm 0.0062\,{\rm M}_\odot$. Although the individual masses found by these two studies are very close to each other, V found from the models with the former masses is in better agreement with the observed V than that of the latter. Therefore, in our model computations for this system, we use the masses found by Peterson & Solensky (1988). Owing to the activity feature of the system pertaining to the late-type stars, the measured values of V and $B-V$ are dispersed. Therefore, we compare theoretical results with their minimum values, $V = 8.28$ and $B-V = 0.73$, observed by Yoss, Karman & Hartkopf (1981). The radii of the components are found by Peterson & Solensky (1988) as $R_{\rm A} = 0.905 \pm 0.029\,{\rm R}_\odot$ and $R_{\rm B} = 0.773 \pm 0.015\,{\rm R}_\odot$. Torres & Ribas (2002) also find values very similar to these.

70 Tau is a close visual binary in the Hyades cluster. Torres et al. (1997b) determined the masses of the primary and secondary components as $1.363 \pm 0.073\,{\rm M}_\odot$ and $1.253 \pm 0.075\,{\rm M}_\odot$, respectively. Torres et al. (1997a) carried out a similar study for 51 Tau, and

Table 1. The individual masses of the component stars and total V and $B-V$ of five binary systems. The slopes of the main sequence ($S_{\rm MBV}$) near V818 Tau and 70 Tau are computed from the data in de Bruijne et al. (2001) by a least-squares method. The values of the slopes computed from the binary data are given in parentheses.

System	$M_{\rm A}/{\rm M}_\odot$	$M_{\rm B}/{\rm M}_\odot$	$R_{\rm A}/{\rm R}_\odot$	$R_{\rm B}/{\rm R}_\odot$	V	$B-V$	$S_{\rm MBV}$	$\log \frac{L_{\rm A}}{L_{\rm B}}$	Ref.
V818 Tau	1.072 ± 0.010	0.769 ± 0.010	0.905 ± 0.029	0.773 ± 0.015	8.28	0.73	4.6 (4.8)	0.71	1, 2, 3, 8
70 Tau	1.363 ± 0.073	1.253 ± 0.075	. . .	. . ..	6.46	0.49	6.6 (5.2)	. . ..	4, 8
51 Tau	1.80 ± 0.13	1.46 ± 0.18	. . .	. . ..	5.65	0.28	. . .	. . ..	5, 8
θ^2 Tau	2.42 ± 0.30	2.11 ± 0.17	. . .	. . ..	3.40	0.18	. . .	. . ..	6, 8
HD 27149	1.096 ± 0.002	1.010 ± 0.002	. . .	. . ..	7.53	0.68	. . .	. . ..	7

(1) Peterson & Solensky (1988), (2) Yoss et al. (1981), (3) Schiller & Milone (1987), (4) Torres et al. (1997b), (5) Torres et al. (1997a), (6) Torres et al. (1997c), (7) Tomkin (2003), (8) Lebreton et al. (2001).

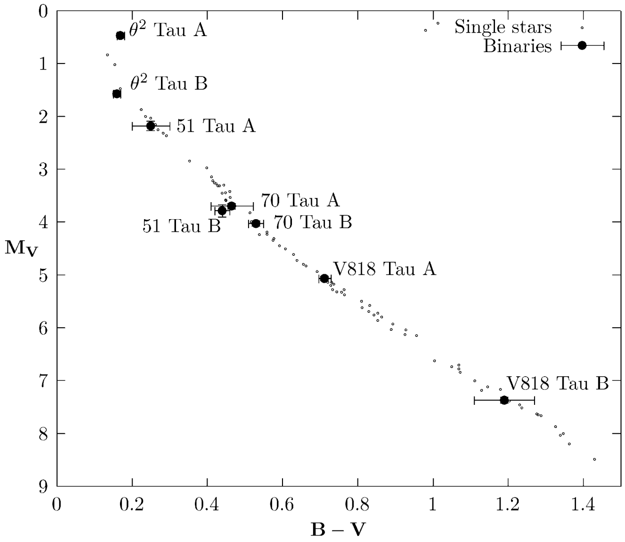

Figure 1. Colour–magnitude diagram for the single stars (dots) and the components of four binaries in the Hyades.

obtained the masses of its components as $M_{\rm A} = 1.80 \pm 0.13\,{\rm M}_\odot$ and $M_{\rm B} = 1.46 \pm 0.18\,{\rm M}_\odot$. They confirmed that 51 Tau A is a fast rotator: $v_{\rm A} \sin i = 97$–125 km s^{-1}.

θ^2 Tau is another spectroscopic binary system. Its primary component is one of the brightest stars in the Hyades and is a δ-Sct-type variable star. Furthermore, the binary system consists of an evolved and a main-sequence star. This binary system is therefore very important for testing the different evolutionary stages. Torres et al. (1997c) found the masses of the components to be $M_{\rm A} = 2.42 \pm 0.30\,{\rm M}_\odot$ and $M_{\rm B} = 2.11 \pm 0.17\,{\rm M}_\odot$. They also found the rotational velocities of the components. According to their results, both of the components of θ^2 Tau are rapid rotators: $v_{\rm A} \sin i = 80$ km s^{-1} and $v_{\rm B} \sin i = 90$–170 km s^{-1}.

HD 27149 is also a spectroscopic binary in the Hyades. The minimum masses for its components have been found by Tomkin (2003) as $M_{\rm A} = 1.096 \pm 0.002\,{\rm M}_\odot$ and $M_{\rm B} = 1.010 \pm 0.002\,{\rm M}_\odot$. Tomkin confirmed that these minimum masses are unexpectedly large for the spectral type of the stars, thus suggesting the possibility of eclipses. By comparing appropriate models with the observations we find the masses of the components and then the inclination angle of the system (see Section 4.4).

In Fig. 1, absolute magnitudes of the components of V818 Tau and the other binaries, as given by Lebreton et al. (2001), are plotted with respect to their colours. For comparison, the single stars with accurate data, given by de Bruijne et al. (2001), are also plotted in this figure (dots). The basic data of the systems, needed for our calibration process, are listed in Table 1.

We also use the slopes of the Hyades MS as constraints in this process. The middle and the lower parts of the MS have different slopes. By applying a least-squares method to the data given by de Bruijne et al. (2001), we obtain the slope of the lower part, which contains the components of V818 Tau:

$$S_{\rm MS1} = \frac{\Delta M_V}{\Delta(B-V)} = 4.6. \qquad (3)$$

Similarly, for the upper part, which contains the components of 70 Tau, we find

$$S_{\rm MS2} = \frac{\Delta M_V}{\Delta(B-V)} = 6.6. \qquad (4)$$

When we use all the available data in the WEBDA[1] data base, we obtain very similar results: $S_{\rm MS1} = 4.4$ and $S_{\rm MS2} = 6.4$. However, from their absolute magnitudes and colours, we obtain the derivative for V818 Tau (Schiller & Milone 1987) as $S_{\rm MS1} = 4.8$ and for 70 Tau (Torres et al. 1997b) as $S_{\rm MS2} = 5.2$. The derivative for components of 70 Tau is significantly less than the values obtained from the data of other similar stars in the cluster. This may arise from the fact that it is not easy to distribute correctly the total V and $B-V$ of a visual binary among its components.

4 MODELLING THE COMPONENTS OF THE BINARIES OF THE HYADES

The characteristics of our code are described in Yıldız (2000, 2003) (see also references therein), and therefore we shall not provide full details here. Our equation of state uses the approach of Mihalas et al. (1990) in the computation of the partition functions. The radiative opacity is derived from Iglesias, Rogers & Wilson (1992), and is completed by the low-temperature tables of Alexander & Ferguson (1994). For the nuclear reaction rates we use the analytic expressions given by Caughlan & Fowler (1988), and we employ the standard mixing-length theory for convection (Böhm-Vitense 1958).

For comparison of the theoretical and observational values of the binary systems, we compute the theoretical V and $B-V$ of any system. We first construct models for its components with given masses and then find M_V and $B-V$ using tables for model atmospheres (Bessell, Castelli & Plez 1998). Using the parallax of the binary, and M_V and $B-V$ of models for the individual stars, we find the combined total V and $B-V$ of the system.

4.1 Solutions from V818 Tau and 70 Tau

For these binary systems, we have seven equations (V and $B-V$ of the systems 70 Tau and V818 Tau, $L_{\rm A}/L_{\rm B}$ in V818 Tau, and

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[1] http://www.univie.ac.at/webda

Table 2. The observable quantities of V818 Tau and 70 Tau and their components computed from the models with Set A.

L	R	$T_{\rm eff}$	M_V	$B-V$	$U-B$	V	$B-V$	$S_{\rm MBV}$	$\log\frac{L_{\rm A}}{L_{\rm B}}$
0.849	0.950	5692	5.007	0.701	0.203	8.263	0.744	4.603	0.692
0.173	0.741	4327	7.362	1.213	1.176				
2.766	1.297	6543	3.643	0.457	0.020	6.459	0.483	6.953	
1.834	1.145	6285	4.107	0.524	0.010				

the slopes in the middle and lower parts of the MS in the Hyades cluster), similar to equations (1) and (2), with seven unknowns (X, Z, t and values of α for four stars). For the solution of these seven equations, we need the derivatives of seven quantities with respect to independent variables (unknowns). These derivatives are computed by using the reference models with solar composition ($X = 0.705$, $Z = 0.02$) and $\alpha = 2.0$ for all the components at $t = 1.0 \times 10^9$ yr. The solution we find for this case is the following (Set A)

$$X = 0.679,\ Z = 0.0319,\ \alpha_{\rm V818A} = 1.89,\ \alpha_{\rm V818B} = 1.01,$$

$$\alpha_{\rm 70TauA} = 2.43,\ \alpha_{\rm 70TauB} = 2.33,\ t = 590\ \rm Myr.$$

The observable properties of these binary systems themselves and their components obtained from the models with Set A are listed in Table 2. The theoretical visual magnitudes and colours are very close to the observed values for both systems.

Is the result we found from the solution of seven equations a unique one? Unfortunately, the answer is not simply yes. One of the main reasons for this is that the numerical derivatives are not constant but depend on the intervals of the variables (or on the reference model). To confirm how the derivatives depend on the intervals, we re-evaluate the numerical derivatives from the models with

$$X = 0.69,\quad Z = 0.032,\quad \alpha_{\rm V818A} = 2.11,\quad \alpha_{\rm V818B} = 1.3,$$

$$\alpha_{\rm 70TauA} = 2.66,\quad \alpha_{\rm 70TauB} = 2.45,\quad t = 1100\ \rm Myr.$$

Using these derivatives, we resolve the seven equations simultaneously and find the following values for the seven unknowns (Set B):

$$X = 0.695,\ Z = 0.0298,\ \alpha_{\rm V818A} = 2.17,\ \alpha_{\rm V818B} = 0.94,$$

$$\alpha_{\rm 70TauA} = 2.52,\ \alpha_{\rm 70TauB} = 2.25,\ t = 1030\ \rm Myr.$$

The values of the observables of the binaries found from the models with this set, given in Table 3, are in perfect agreement with the observations. However, we do not try to fit model radii directly to the observed radii of the components of V818 Tau. The difference between them is very small, about 3 per cent. Indeed it is possible to remove this difference, and then the agreement between the theoretical and observational values of $B - V$ and $S_{\rm MBV}$ (the corresponding slope of the MS) of the system will disappear.

Table 3. The observable quantities of V818 Tau and 70 Tau and their components computed from the models with Set B.

L	R	$T_{\rm eff}$	M_V	$B-V$	$U-B$	V	$B-V$	$S_{\rm MBV}$	$\log\frac{L_{\rm A}}{L_{\rm B}}$
0.838	0.929	5736	5.013	0.687	0.182	8.289	0.727	4.385	0.730
0.156	0.748	4200	7.578	1.272	1.241				
2.828	1.316	6531	3.619	0.460	−0.018	6.443	0.489	6.343	
1.847	1.165	6240	4.103	0.536	0.021				

The very striking common feature of the solutions with Set A and Set B is that $\alpha_{\rm V818A} \sim 2\,\alpha_{\rm V818B}$. Although both sets (A and B) give very similar results, the values for the age of the cluster in the two sets are very different from each other. Therefore, we shall consider a star that evolves faster than these stars to fix the age: this star is θ^2 Tau A.

4.2 Properties of θ^2 Tau and the age of the cluster

We shall first see which one of the models of θ^2 Tau A is in better agreement with the observations. As mentioned above, θ^2 Tau A rotates rapidly, and this effect should be taken into account. If the rotation is included in the model computations, the problem arises of angular momentum distribution inside early-type stars. We should first specify how the inner regions of this star rotate. Because this is a very complicated matter, and is indeed one of the essential problems in stellar astrophysics, we study two typical cases: (i) solid rotation; and (ii) the rotation profile as determined by contraction. The models for the latter case are differentially rotating (DR) models: the central regions are rotating much faster than the outer regions (Yıldız 2003, 2004).

In Fig. 2, the DR models of θ^2 Tau A with Set A (model AADR; diamonds) and Set B (ABDR; stars), and the model rotating like a solid body (ABSB; ×) are plotted in the HR diagram. Their $B - V$ values are in good agreement with the observed one (0.17) (de Bruijne et al. 2001) at t = 675, 699 and 661 Myr, respectively. Their equatorial velocities at their corresponding ages are $v_{\rm eq} \sim 100$ km s^{-1}. It seems that the subgiant phase of θ^2 Tau A is compatible with the observed position, and the DR models with both sets of chemical composition are in much better agreement with the observed values than the models rotating like a solid body. Even though the solid-body rotation cannot be totally ruled out by such an analysis, we shall consider only the DR models hereafter. What is important here in our analysis is that the ages of models with the two rotation types are very similar to each other.

The evolutionary track of the DR model of θ^2 Tau B with Set B is also plotted in Fig. 2. Although its evolutionary track passes through the observed position of θ^2 Tau B in the HR diagram, the time of

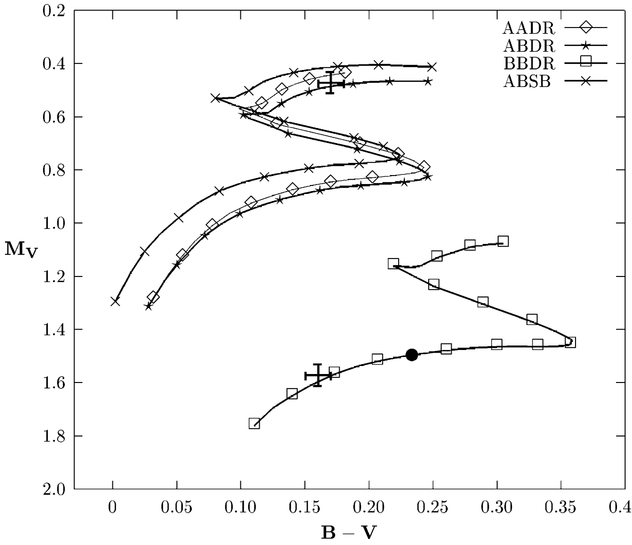

Figure 2. Colour–magnitude diagram for θ^2 Tau A and B. The first letter in the key indicates the component (A or B), and the second letter denotes the chemical composition (Set A or B) of the model. The third and fourth letters specify the type of rotation (solid-body or differential rotation).

agreement ($t = 444$ Myr) is not the same as that of θ^2 Tau A ($t = 699$ Myr). The position of θ^2 Tau B at the latter time is marked by a filled circle, and is far from the observed position of θ^2 Tau B in the diagram. An internal rotation, more complicated than the rotation as determined by the contraction, may cause this discrepancy.

4.3 Solutions derived from models of θ^2 Tau A and 70 Tau A and B

As shown in the previous sections, two models having the same mass but different chemical compositions may have the same (or very near) location in the HR diagram. Because of this degeneracy, we calibrate models of θ^2 Tau A and the components of 70 Tau to the observations for fixed values of Z. For $Z = 0.028$, we have (Set 28)

$$X = 0.699,\ \alpha_{\rm 70TauA} = 2.31,\ \alpha_{\rm 70TauB} = 2.21,\ t = 705\ \text{Myr}.$$

We made similar computations for $Z = 0.033$ and obtain the following results (Set 33):

$$X = 0.676,\ \alpha_{\rm 70TauA} = 2.29,\ \alpha_{\rm 70TauB} = 2.20,\ t = 676\ \text{Myr}.$$

For $Z = 0.024$, using the values of α in Set 28, we find $X = 0.718$ and $t = 721$ Myr (Set 24).

The differences between the models of any star with different sets are negligibly small. Therefore, these sets are equivalent to each other (see Table 4).

We also build models (with Set 28) with the microscopic diffusion process for 70 Tau A to test its influence on the observable properties of such stars. While the difference between the absolute magnitudes of the models with and without diffusion is $\Delta M_V = -\ 0.0022$, the difference between the colours is $\Delta\ (B - V) = 0.0031$. These differences are small enough, in comparison to the uncertainty in the observed magnitude and colour of the system, that the diffusion process can be ignored.

For these sets, we find that the mixing-length parameters for the components of V818 Tau are $\alpha_{\rm V818A} = 2.04$ and $\alpha_{\rm V818B} = 0.99$. In Fig. 3, the models for the components of θ^2 Tau, 70 Tau and V818 Tau with Set 24 (filled circles) are plotted in the HR diagram among the single stars with very precise parallaxes (dots). Except for θ^2 Tau B (see above), all the models of the components are qualitatively in good agreement with the general trend identified by the single stars with very accurate observational data.

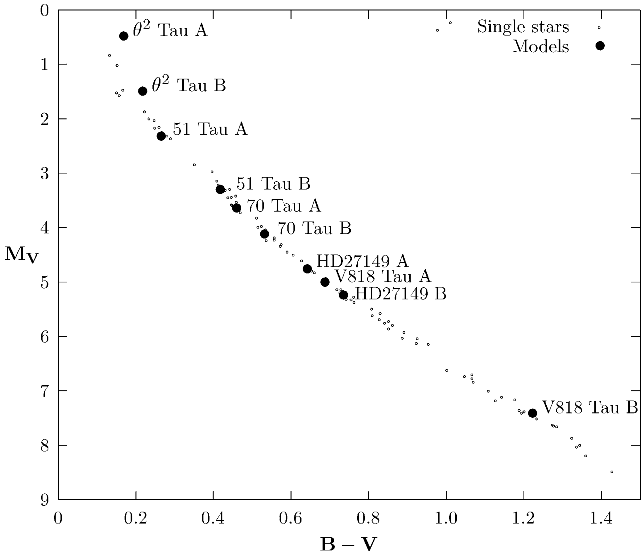

Figure 3. Colour–magnitude diagram for the single stars (dots) and the models of the components of five binaries (filled circles).

The models for the components of 51 Tau with Set 24 are also placed in Fig. 3. Their positions are good enough in comparison with the positions of the single stars (de Bruijne et al. 2001). The model of 51 Tau A is a DR model with an equatorial velocity of 130 km s^{-1}, and the mixing-length parameter for 51 Tau B is 2.35.

4.4 Properties of HD 27149

Because the angle between the line of sight and the orbital plane of HD 27149 is observationally not known, we find the individual masses of the components by fitting V of the models for a given chemical composition and time to $V_{\rm obs}$ of the system. For Set 24 and Set 28, we find that $M_{\rm A} = 1.118$ and $M_{\rm B} = 1.030$. Then, from the calibration of models of these masses with both sets, the mixing-length parameters are derived as $\alpha_{\rm A} = 2.10$ and $\alpha_{\rm B} = 1.95$. These different values of the mixing-length parameters for the different stellar masses prove once more the mass dependence of α.

From these values of individual masses, we can obtain the value of the inclination angle by using the observed value of $M_{\rm A}\ \sin^3 i$ (Tomkin 2003):

$$M_{\rm A} \sin^3 i = 1.118 \sin^3 i = 1.096. \tag{5}$$

Equation (5) gives the inclination of the system as $i = 83\overset{\circ}{.}4$. The minimum value of i for eclipsing to occur is, however, $i_{\rm c} = 88\overset{\circ}{.}9$. Thus, HD 27149 is not an eclipsing binary (see Tomkin 2003).

4.5 The stellar mass dependence of the mixing-length parameter

The values of the mixing-length parameter for the components of the binaries derived from the calibration are listed in Table 4. They are plotted in Fig. 4 as a function of the stellar masses. The uncertainties in α of each model are computed assuming an uncertainty of $\Delta(B - V) = 0.01$ (uncertainty in $B - V$ of the binary systems; see Lebreton et al. 2001) for each star. Then,

$$\delta \log \alpha = \frac{\Delta(B - V)}{\partial(B - V)/\partial \log \alpha}, \tag{6}$$

where the partial derivatives are computed from the models. In Table 4, radii and effective temperatures of the models with Set 24 and Set 28 are also listed. In the last column, the fractional difference between the models with these sets are given for the components of V818 Tau and 70 Tau. Because these differences are very small, we deduce that the mass dependence of α is independent of the (solution) sets.

The curve in Fig. 4 is the fitting formula

$$\alpha = 9.19(M/{\rm M}_{\odot} - 0.74)^{0.053} - 6.65 \tag{7}$$

derived using the data given in Table 4. It is valid for stellar masses greater than 0.77 M$_{\odot}$. The curve, surprisingly, also covers α of the Sun ($\odot$ in Fig. 4). Moreover, the αs of the components of α Centauri from equation (7) explain the stellar parameters of these stars well (Yıldız, in preparation).

One might also want to ascertain whether the effective temperatures of the models with variable α are consistent with the spectroscopic measurements. For this task, similar to fig. 2 in Pinsonneault et al. (2004), we plot M_V of the models (Set 24) of the late-type components of the three binary systems against $T_{\rm eff}$ (filled circles) in Fig. 5. The observational effective temperatures and absolute magnitudes are taken from Paulson et al. (2003) and de Bruijne

Table 4. The mixing-length parameters of the calibrated models of the components of binaries V818 Tau, 70 Tau and HD 27149 with Set 24. In order to derive a fitting formula, models are also computed for the masses 0.80, 0.85, 0.90 and 0.95 $\mathrm{M}_\odot$ (see Fig. 4). The uncertainties in α of each model are computed assuming an uncertainty of $\Delta(B{-}V) = 0.005$ (see text). For comparison, radii and effective temperatures of the models with Set 24 and Set 28 are also given. The fractional differences between the model radii of the components of V818 Tau and 70 Tau with these sets are listed in the last column.

Star	$M/\mathrm{M}_\odot$	α	T_{eff}(Set 24)	$R/\mathrm{R}_\odot$(Set 24)	T_{eff}(Set 28)	$R/\mathrm{R}_\odot$(Set 28)	$\delta R/R$
V818 Tau B	0.769	0.99 ± 0.03	4305	0.739	4295	0.741	0.003
V818 Tau A	1.072	2.04 ± 0.09	5718	0.940	5720	0.937	-0.003
70 Tau B	1.253	2.21 ± 0.09	6252	1.154	6248	1.162	0.007
70 Tau A	1.363	2.31 ± 0.18	6527	1.306	6514	1.318	0.009
HD 27149 B	1.030	1.95 ± 0.07	5583	0.896	...	...	...
HD 27149 A	1.118	2.10 ± 0.08	5876	0.985	...	...	...
–	0.800	1.20 ± 0.05	4500	0.749	...	...	...
–	0.850	1.50 ± 0.06	4764	0.768	...	...	...
–	0.900	1.70 ± 0.06	5086	0.799	...	...	...
–	0.950	1.80 ± 0.07	5329	0.836	...	...	...
Sun	1.000	1.88					

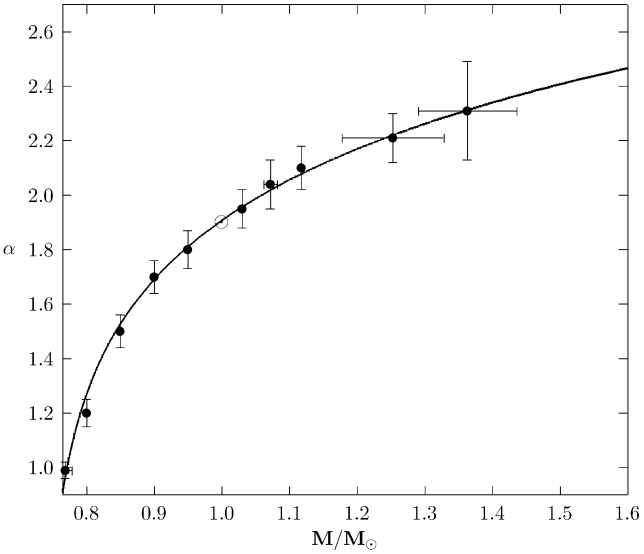

Figure 4. The mixing-length parameter for the models of the components of three binaries (V818 Tau, 70 Tau and HD 27149) as a function of mass. In order to fill the gap, models are also computed for the masses 0.80, 0.85, 0.90 and 0.95 $\mathrm{M}_\odot$, taking care to keep the model in the sequence of stars with data of high quality in the colour–magnitude diagram. For the method of computation for the uncertainty in α, see the text.

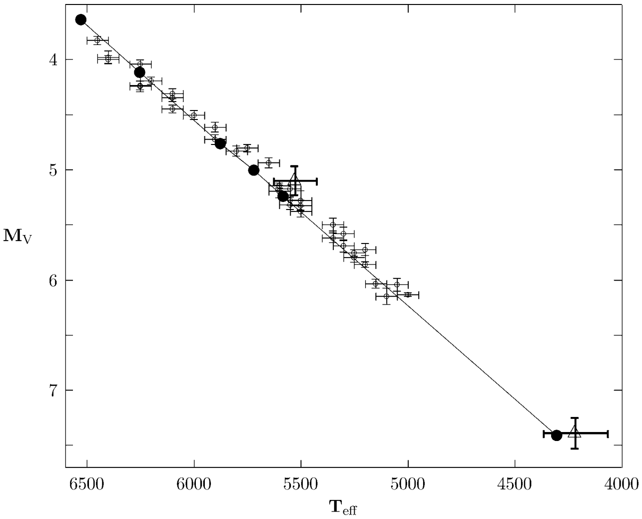

Figure 5. The models of the components of three binaries (V818 Tau, 70 Tau and HD 27149) with Set 24 (filled circles) are plotted in a M_V vs. T_{eff} diagram. The error bars with circles represent the observations: while the effective temperatures are spectroscopicallly derived by Paulson et al. (2003), the absolute magnitudes are from de Bruijne et al. (2001). The error bars with triangles are for the components of V818 Tau based on the photometric data (Torres & Ribas 2002).

et al. (2001), respectively. We include photometric data (triangle with thick error bars) for the components of V818 Tau (Torres & Ribas 2002), because of a lack of observational data for stars with a T_{eff} of less than 5000 K. It can be seen that the theoretical and the observational effective temperatures are in very good agreement.

We also compare the model radii with the observed radii of stars in well-known eclipsing binaries. In Fig. 6, the observed radii, taken from Andersen (1991) (circles) and Lopez-Morales & Ribas (2005) (stars), are plotted against the stellar mass. The filled circles represent the models of the components of V818 Tau, 70 Tau and HD 27149 with Set 24. We confirm a very good agreement between the observed and the model radii. The models for the *unevolved* late-type stars in the Hyades cluster are very near to the zero-age main-sequence line, and therefore occur on the left side of the main sequence formed by the stars in the well-known eclipsing binaries.

4.6 The colour–colour diagram

In Fig. 7, $U-B$ and $B-V$ colours derived from our models with Set 24 are plotted. For comparison, the observed colours for the Hyades stars are also plotted. The filled circles and boxes represent the colours computed from the tables of Bessell et al. (1998) and Lejeune, Cuisinier & Buser (1998), respectively, for solar composition. Although both of the tables are in general in good agreement with the observations, the $U-B$ colours of models of the early-type stars (51 Tau A, θ^2 Tau A and B) derived from Lejeune et al. (1998) are in better agreement with the observed $U-B$ than those derived from Bessell et al. (1998). The colours of some models are also computed from Lejeune et al. (1998) for the metallicity as given in Paulson et al. (2003), namely [Fe/O] = 0.13 dex (triangles). As the metallicity of a model is increased, the model moves towards the bottom-right part of the colour–colour diagram, as expected. The majority of the stars are located between the colours of

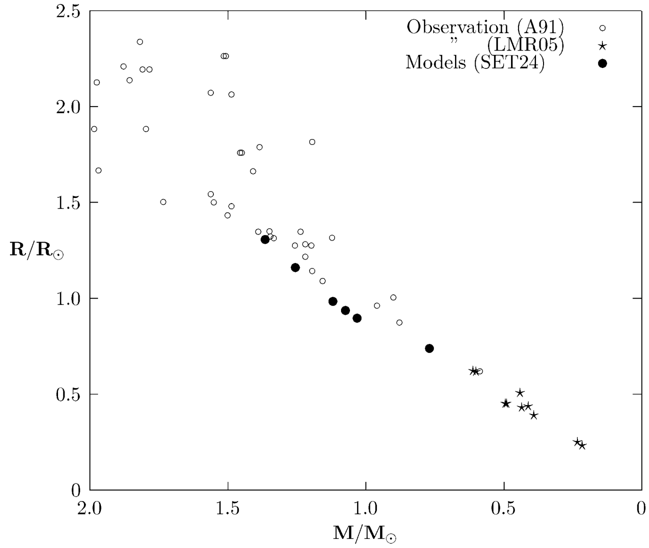

Figure 6. The models of the components of three binaries (V818 Tau, 70 Tau and HD 27149) with Set 24 (filled circles) are plotted in a R vs. M diagram. The circles and stars represent the observed stellar radii taken from Andersen (1991) and Lopez-Morales & Ribas (2005), respectively.

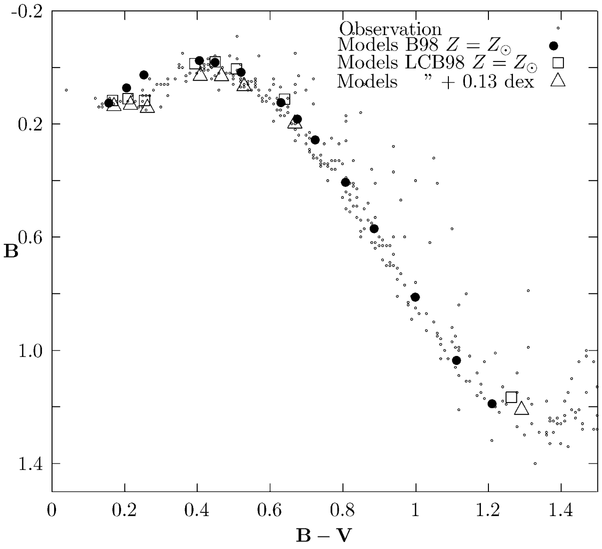

Figure 7. Colour–colour diagram for the Hyades cluster. The models of the components of V818 Tau, 70 Tau and HD 27149 are constructed with Set 24. The filled circles represent the colours of the model computed from Bessel (1998) and the box is for the colours from Lejeune et al. (1998) for the solar composition. The triangle is for the colours from Lejeune et al. (1998) for the metallicity [Fe/O] = 0.13 dex.

the models derived for the solar composition and for the metal-rich composition, at least for stars with $B - V < 0.6$. This result may be interpreted as meaning that the Hyades cluster is slightly more metal-rich than the Sun. More precise results on the metallicity of the cluster depend on which solar mixture is used. Therefore, new tables with the recently calculated solar composition (Asplund et al. 2004) for colours of stars are required.

5 CONCLUSION

By constructing models for the components of the Hyades binary systems whose masses are observationally determined, we have reached very important conclusions concerning the detailed physical structure as well as the fundamental properties of the cluster itself. The most striking outcome of the present study is that we discover a smooth dependence of the mixing-length parameter on the stellar mass. Although we calibrate the models in order to obtain the measured quantities of the corresponding binary system rather than those of the individual component stars, the models of each star are in good agreement with the observed properties: while the difference between the model and data radii is about 3–4 per cent for the components of V818 Tau, the effective temperatures of the models of the late-type components of V818 Tau, HD 27149 and 70 Tau are in perfect agreement with the $T_{\rm eff}$s derived from spectroscopic measurements by Paulson et al. (2003). Because V818 Tau is an active binary, the fundamental properties of its components should be re-evaluated using much more precise observational data than are used at present.

The relationship we derived between α of the components of the Hyades binaries and their masses also gives the solar value of α. This result should, however, be examined. It is possible that the age and chemical composition differences just happen to counterbalance each other, and that therefore the relationship gives the solar value coincidentally. The fact that the α values for the components of α Centauri from this relationship yield models in agreement with the observed stellar parameters leads us to adopt it as a prevalent relationship (Yıldız 2005). However, a time variation of this relationship should not be ruled out.

From 2D and 3D hydrodynamical simulations of convection, Ludwig et al. (1999) and Trampedach et al. (1999) find that α is a decreasing function of effective temperature (or stellar mass). It is noteworthy that stellar evolution codes and simulation codes give opposite results for this relationship (see also Ludwig & Salaris 1999 for α values of the components in the eclipsing binary AI Phe). This contradiction may be a result of the variation of α in the layers of convective envelopes (see e.g. Deupree & Varner 1980) or in time.

The second important outcome concerns the structure of the early-type stars: the differentially rotating models for the components of θ^2 Tau and 51 Tau are in better agreement with the observations than the non-rotating models and models with solid-body rotation.

The fundamental properties of the Hyades cluster that we have derived are not unique but can be given in terms of the metal abundance. If $Z = 0.024$, then $X = 0.718$ and the age of the cluster is $t = 721$ Myr; if $Z = 0.033$, then $X = 0.676$ and $t = 676$ Myr. It should be pointed out here, however, that the mass dependence of α is valid regardless of the value assigned to Z.

From these fundamental properties of the cluster we derive the masses of the components in the binary system HD 27149 by fitting the brightness of the models to the observed value: $M_{\rm A} = 1.118\,{\rm M}_\odot$ and $M_{\rm B} = 1.03\,{\rm M}_\odot$. Using the observed lower value for the masses, we show that the inclination of its orbit is about $i = 83\overset{\circ}{.}4$. This value of i is smaller than the critical value for the occurrence of eclipsing ($i_{\rm c} = 88\overset{\circ}{.}9$), and hence this system is not an eclipsing one.

Unless the metal abundance of the cluster is observationally determined very precisely, we cannot specify its helium abundance. In order to be able to do this, abundances of the most abundant chemical species, in particular oxygen, nitrogen, carbon and neon, should be found from the spectrum of its stars. Otherwise, we can only give the helium abundance as a function of Z: while $Y = 0.258$ for $Z = 0.024$, $Y = 0.291$ for $Z = 0.033$. With very precise data on the colours of the stars, it is possible, however, to specify the heavy element abundance from the colour–colour diagram.

ACKNOWLEDGMENTS

We thank Ayşe Lahur Kırtunç and Rachel Drummond for their suggestions, which improved the language of the manuscript. The anonymous referee is acknowledged for her/his useful comments. This research was supported by The Scientific and Technological Research Council of Turkey (TÜBİTAK).

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