ULTIMATE STRENGTH ANALYSIS OF STEEL PLATES USING ISO/DIS 1872-2 AND ULSAP

Nguyen Tien Dung¹, P. Rigo² and A. Bayatfar³

¹ Department of Civil Engineering, University of Transport and Communications
  Lang Thuong, Dong Da, Ha Noi, Vietnam
  e-mail: td.nguyen@utc.edu.vn, http://www.utc.edu.vn

² ANAST, University of Liège, Chemin des Chevreuils 1, 4000 Liège, Belgium
  e-mail: ph.rigo@ulg.ac.be, http://www.ulg.ac.be

³ ANAST, University of Liège, Chemin des Chevreuils 1, 4000 Liège, Belgium
  e-mail: a.bayatfar@ulg.ac.be, http://www.ulg.ac.be

ABSTRACT

Ultimate limit states (or ultimate strength) have been assessed that is the much better basis than the allowable working stresses. Today, we design and strength assessment the structural types such as aerospace structures, offshore platforms, ships and land-based structures following the ISO STANDARD.

Aim of this paper is to perform benchmark calculation in order to validating formulas of ISO Standard [1]. An effective approach for plate, using the ISO/DIS 1872-2 standard [1] and ULSAP (Mestro software) [2], are calculated in the current research for studying the ultimate strength problems of plates with six load cases: longitudinal compression, transverse compression, biaxial compression with/without lateral pressure.

The trends of ISO Standard results in some cases agree well to the Maestro results. Other cases are not similar. The values of ISO results were found smaller than Maestro results. For this reason, the ISO results are not more conservative than ULSAP results.

Key words: Ultimate limit states (ULS), Ultimate strength, Plate structures, Combined biaxial compression and lateral, Pressure loads

1. INTRODUCTION

Assessing a ship structure’s strength exactly can reduce the ship’s manufacture and maintenance costs, and increase ship’s life and safety. For these purposes, each ship classification society has provided relatively integrated ship structure strength into assessment methods. Furthermore, also those assessment methods have achieved great successes in practical application, although they are still far from perfection. The unsatisfactory situation of the strength assessment for ship structures is mainly due to the complexity of the structure and its operational environment. A strict assessment process involves much of the computation effort. With the fast development of computer technology, the possibility to accurately assess the ship structural strength
based on the strict principles of mechanic increases. In response to this possibility, International Ship And Offshore Structures Congress proposed a International Standard ISO/DIS 1872-2 [1] for ship structures, which allows integration of all relevant aspects of technology and considers interactions among various factors affecting the ship structural strength.

Many papers have been published since the ISSC regarding the buckling/ultimate strength of the ship structural members and systems [3]. These papers deal with ultimate strength of plates under biaxial compression without lateral pressure and biaxial compression with lateral pressure. Most of them are discussed based on the results of nonlinear FEM analyses and Idealised Structural Unit Method (ISUM) analyses, but some are based on experimental results. In some papers, simplified methods were developed to evaluate the collapse strength. It is hoped that sophisticated and accurate, but simple methods, are developed on the basis of rational formulations to evaluate the buckling/collapse strength and to simulate the collapse behaviour of ship structures.

This paper present the results of an extensive sensitivity analysis carried out by the INTERNATIONAL STANDARD ISO/DIS 1872-2 in the framework of a benchmark on the ultimate strength of plates.

**2. PRINCIPLES OF LIMIT STATE DESIGN**

A limit state is formally defined by the description of a condition for which a particular structural member or an entire structure fails to perform the function that is expected of it. From the viewpoint of a structural designer, four types of limit states are considered for steel structures, namely [1].

\[
C_d \geq D_d \quad \text{or} \quad \eta = \frac{C_d}{D_d} \geq 1
\]  

\[
D_d = \gamma_d D_k : \text{The design value of demand (action)}
\]

\[
C_d = C_k / \gamma_c : \text{The design value of capacity (strength)}.
\]

\[
\eta = \frac{1}{\gamma_c \gamma_d} \frac{C_k}{D_k} \geq 1
\]

This measure of structural adequacy which should be greater than unity to be safe.

\[C_k : \text{The characteristic value of capacity (strength).} \]

\[D_k : \text{The characteristic value of demand (actions). The partial safety factors could be obtained by probabilistic analysis involving associated uncertainties.}\]

\[\gamma_d, \gamma_c : \text{Partial safety factors of capacity or demand, respectively, in association with the uncertainties of capacity or demand, which must be greater than unity.}\]

Within the frame work of ULS design or ULS-based safety check, therefore, the \(\eta\) values in association with ULS must be computed accurately and efficiently. Within the
frame work of ULS design or ULS-based safety check, therefore, the Ck values in association with ULS must be computed accurately and efficiently. The contribution of the present series study is to develop some useful insights of ISO Standard [1] application for predicting the η values for plate elements and stiffened panels under combined biaxial compression and lateral pressure actions which are typical types of actions in ships and ship-shaped offshore structures.

![Figure 1: A stiffened plate structure[1]](image1)

![Figure 2: Plate notation and applied actions [1]](image2)

3. METHODS OF ULS ASSESSMENT

This paper uses two methods to assess the Ultimate Limit State of the plate which compared to make conclusions.

- ALPS/ULSAP (2006)[2]


- Plates subject to compression without lateral pressure.

Plates subject to compression should satisfy:

\[
σ_e ≤ \frac{f_c}{γ_{R,e}}
\]  

(3)-(10.3-4ISO)

- \(σ_e\): is the compressive stress from factored actions, \(σ_{cx}\) for longitudinal compression and \(σ_{cy}\) for transverse compression.

- \(f_c\): is the representative compressive strength, in stress units, \(f_{cx,R}\) for restrained plates and \(f_{cx,u}\) for unrestrained plates subject to longitudinal compression and \(f_{cy,R}\) for restrained and \(f_{cy,u}\) for unrestrained plates subject to transverse compression.

- \(γ_{R,e}\): is the partial resistance factor for plate compressive strength.

The utilization of a plate subject to compression, \(U_m\), shall be calculated from:
\[
U_m = \frac{\sigma_c}{f_c / \gamma_{R,c}} \leq 1
\] (4)-(10.3-5ISO)

\[
\left( \frac{\sigma_{cx}}{f_{cx,u} / \gamma_{R,c}} \right)^2 + \left( \frac{\sigma_{cy}}{f_{cy,u} / \gamma_{R,c}} \right)^2 \leq 1
\] (5)-(10.3-5ISO)

**Remark:**

When only Longitudinal axial compression:

\[
\left( \frac{\sigma_{cx}}{f_{cx,u} / \gamma_{R,c}} \right)^2 \leq 1
\] (6)

\[
\frac{\sigma_{yu}}{\sigma_y} = \frac{f_{cx,u} / \gamma_{R,c}}{\sigma_{cx,\max}} = \frac{f_{cx,u}}{\gamma_{R,c}}
\] (7)

When only Transverse axial compression:

\[
\left( \frac{\sigma_{cy}}{f_{cy,u} / \gamma_{R,c}} \right)^2 \leq 1
\] (8)

\[
\frac{\sigma_{yu}}{\sigma_y} = \frac{f_{cy,u} / \gamma_{R,c}}{\sigma_{cy,\max}} = \frac{f_{cy,u}}{\gamma_{R,c}}
\] (9)

° Plates subject to compression combined with lateral pressure.

\[
\sigma_{cp} \leq \frac{f_{cp}}{\gamma_{R,cp}}
\] (10)-(10.4-3 ISO)

\(\sigma_{cp}\): is the compressive stress in the presence of lateral pressure from factored actions, \(\sigma_{cp,x}\) for longitudinal compression and \(\sigma_{cp,y}\) for transverse compression.

\(f_{cp}\): is the representative compressive strength, in the presence of lateral pressure of lateral pressure in stress units, \(f_{cp,x}\) for plates subject to longitudinal compression and \(f_{cp,y}\) for plates subject to transverse compression.

\(\gamma_{R,cp}\): is the partial resistance factor for plate compressive strength in the presence of lateral pressure.

\[
\left( \frac{\sigma_{cp,x}}{f_{cp,x,u} / \gamma_{R,cp}} \right)^2 + \left( \frac{\sigma_{cp,y}}{f_{cp,y,u} / \gamma_{R,cp}} \right)^2 \leq 1
\] (11)-(10.3-5ISO)
Remark:

When only Longitudinal axial compression with lateral pressures:

\[
\left( \frac{\sigma_{cpx}}{f_{cpx,u} / \gamma_{R,ey}} \right)^2 \leq 1 \quad (12)
\]

\[
\sigma_{yu} = \frac{f_{cpx,u}}{\gamma_{R,ey}} ; \sigma_{yu} = \sigma_{cpx,\text{max}} = \frac{f_{cpx,u}}{\gamma_{R,ey}} \quad (13)
\]

When only Transverse axial compression with lateral pressures:

\[
\left( \frac{\sigma_{cpy}}{f_{cpy,u} / \gamma_{R,ey}} \right)^2 \leq 1 \quad (14)
\]

\[
\sigma_{ypu} = \frac{f_{cpy,u}}{\gamma_{R,ey}} ; \sigma_{ypu} = \sigma_{cpy,\text{max}} = \frac{f_{cpy,u}}{\gamma_{R,ey}} \quad (15)
\]

3.2. ALPS/ULSAP Method

ALPS/ULSAP method is based on semi-analytical approaches and provides ULS computations of plate elements and stiffened-plate structures. Because the theoretical details of the ALPS/ULSAP method are found in [4, 5, 6], a brief description is given herein for the ULS computations of plate elements.

The membrane stress distribution inside a plate element surrounded by support members (or stiffeners) is evaluated using the solutions of nonlinear governing differential compatibility and equilibrium equations of the plate element, involving elastic large deflection behaviour. In this process, the plate element is modelled that all (four) plate edges are simply supported and kept straight, considering that the degree of rotational restraints and interacting relations with adjacent plate elements along support member locations is relatively small. This presumption of the plate boundary condition is well adopted for practical design purpose of continuous stiffened panels in maritime industry.

It is assumed that the plate element will collapse if any part of plate edges having been kept straight yields. This is due to the fact that the straight plate edge can no longer resist membrane-tension actions after the inception of yielding at the corresponding location of the plate edge. Three possible conditions of plate-edge yielding are relevant, namely plate corners, longitudinal (long) plate edges and transverse (short) plate edges.

In ALPS/ULSAP method, the three failure conditions are expressed as functions of applied membrane stresses, initial imperfections (plate initial deflections and welding residual stresses) together with geometric and material property parameters, and the minimum value among the three solutions obtained from each of the three failure conditions will be the real ultimate strength of the plate element. Various shapes of plate
initial deflection including buckling mode and hungry horse mode can be dealt with as parameters of influence in the three failure conditions. Welding residual stresses, structural damages (e.g., corrosion wastage, fatigue cracking damage, denting damage) and cut-outs with circular, elliptical or rectangular shape) are also treated as parameters of influence. The effect of impact pressure actions is considered.

4. STRUCTURAL MODELING FOR ANALYSIS

This paper summarizes the results of the benchmark study on the ULS assessment of ship steel plates under single types of loads or combined loads by using ISO formulas and Meastro software. From these points of view, bottom plating of bulk carriers and deck plating of VLCC are selected as study cases and the thickness of plating as well as sizes are varied. Finally results are compared and given comments the formulas of the ISO standard.

4.1. Geometric and Material Properties

Ship mild steel plates surrounded by longitudinal stiffeners and transverse frames are considered. The material yield stress is 313.6 MPa Young’s modulus, $E = 205.8$ GPa, the Poisson ratio is $\nu = 0.3$.

The geometrical dimensions of plates are $a$ (plate length), $b$ (plate breadth) and $t$ (plate thickness) as shown in Figure 4.

Model 1: $a \times b \times t_p = 850 \times 2, 550 \times (9.5, 11, 13, 16, 22, 33)$ mm (Bottom plating of bulk carriers).

Model 2: $a \times b \times t_p = 950 \times 4, 750 \times (11, 12.5, 15, 18.5, 25, 37)$ mm (Deck plating of VLCC).

4.2 Fabrication-induced Initial Imperfections

Fabrication-induced initial imperfections include initial geometrical deflections and welding residual stresses. These can vary significantly in distribution and magnitude but measurement programmes have helped define typical ranges of values, broadly classified as small, average and severe. For both analysis and assessment, average values are typically adopted.

For the analysis of plates, initial deflections and compressive residual stresses should be specified. For the analysis of stiffened panels, plate initial deflections, column-type initial deflections of stiffeners, and sideways (torsional) initial deflections of stiffeners, and compressive residual stresses in the plating and in the stiffener web should be defined. For the analysis of primary support members, web initial deflections and web compressive residual stresses should be specified.

For assessment, when fabrication-induced initial geometrical deflections are within the tolerances required by the quality assurance provisions of ISO 18072-1, the initial geometrical deflections given in Equations 9.5-1 ISO to 9.5-3 ISO may be taken to
represent average levels of fabrication-induced initial imperfections. From an assessment perspective, these geometrical deflections, when used in conjunction with the strength formulations for plates, stiffened panels and hull girder in Clauses 10, 11 and 14 (ISO) [1], may be assumed to account for the effect of both geometrical deflections and welding residual stress.

Minimum ultimate compressive strengths are normally realised when the plate initial imperfection model is affine with the buckling Model. To determine the buckling model, an eight-value solution should be performed considering all the in-plane actions in their correct proportions but excluding lateral pressure. Most if not all models will involve a single half-wave in the transverse (short) direction.

For plates, the average value of initial geometrical deflection is:

\[ w_{opl} = 0.1\beta^2t \]  

(16)

Where: \( \beta = \frac{b}{t_p} \sqrt{\frac{\sigma_y}{E}} \); b: plate width for \( a \geq b \); a: plate length; t: plate thickness.

Alternatively, for plates subject to biaxial in-plane compression, the number of half-waves in the longitudinal direction, m, can be found as the minimum integer satisfying:

\[ \left( \frac{m^2 / a^2 + 1/b^2}{m^2 / a^2 + c/b^2 - 1/b^2} \right)^2 \leq \left( \frac{(m+1)^2 / a^2 + 1/b^2}{(m+1)^2 / a^2 + c/b^2} \right)^2 \]  

(17)-(10.2-1ISO)

Where: \( c = \frac{\sigma_y}{\sigma_x} \); ratio of biaxial in-plane compression actions.

When only transverse axial compression is applied, i.e. \( \sigma_x = 0 \), \( m = 1 \).
When only longitudinal axial compression is applied, i.e. $\sigma_y = 0$, $c = 0$, Equation (17) simplifies to

$$\frac{a}{b} \leq \sqrt{m(m+1)}$$  \hfill (18)

### 4.3. Load Properties

The actions shown in Figure 2 are: Longitudinal axial stress $\sigma_x$; Transverse axial stress $\sigma_y$; In-plane shear stress $\tau$; Longitudinal in-plane; Bending stress $\sigma_{bx}$; Transverse in-plane bending stress $\sigma_{by}$; Lateral pressure $p$.

Where plates are subject to non-uniform in-plane actions, for the application of the provisions provided in this clause, the non-uniform actions shall be resolved into a pure in-plane bending component and a uniform axial component. The in-plane bending component shall be added to any coexisting in-plane bending stress, accounting appropriately for the sign of the components and the uniform axial component shall be added to any coexisting axial stress, again accounting appropriately for the sign of the components.

### 4.3. Boundary Condition

In maritime engineering structure, we are often assumed that the boundary condition for plates is that they are simply supported at their edges (Figure 4), although the edges are surrounded by support members and thus are neither simply supported nor clamped.

The plate is supported at its four edges by beam members, e.g., longitudinal stiffeners and transverse frames. The bending rigidities of the boundary support members are usually quite large compared to that of the plate itself. This implies that the displacements of the support members normal to the plane of the plating are very small even up to plate collapse, and thus it is presumed that all (four) of the plate edges remain in-plane. The rotational restraints along the plate edges depend on the torsional rigidities of the support members, and these are neither zero nor infinite.

Numerical results are available from Dowling et al. [7] and Valsgard [8]. In these cases, the boundary conditions are defined precisely and one is able to assess their effect on the plate strength. Restrained conditions correspond to the situation where the in-plane displacements perpendicular to the edge are zero, while in constrained conditions the edge is specified to remain straight, but is free to pull-in. Unrestrained conditions imply that all in-plane displacements are free. Therefore, within the scope of my thesis is used the restrained boundary condition.
5. COMPUTATIONAL RESULTS AND DISCUSSIONS

The results of ULSAP-Mastro software and the ISO, two ultimate strength analyses of plates from which I comment and validate some formulas in ISO standard. These plates cover a wide range of geometries (two plate models). Six types of loads are applied to the plates and in some cases combined with lateral pressure. The 52 ISO results were compared the ultimate strength with the computer program-ULSAP (MAESTRO Software).

5.1. Plate Subject to Compression WITHOUT Lateral Pressure

5.1.1. Longitudinal axial compression
The ultimate strength (Figures 5 & 6) of the plates in two from analyses are in the similar trends but the results seem quite different at the lowest & highest plate, the ISO results are smaller than the results were analyzed by ULSAP.

The average variances in the Model 1 (8.0%) are larger in the Model 2 (7.8%). The max variances at two heads of the curves, tp=9.5, 33 mm (Model 1), tp=11, 37 mm (Model 2)

The minimum variance at tp=13 mm-Model 1(4.0%) and tp=15 mm-Model 2(3.9%).

5.1.2 Transverse axial compression
Like the cases of the longitudinal compression, Figures 7 & 8 show that they are the similar trends, the ULSAP results are larger than the ISO results. At two heads of curves are the maximum variances, $t_p=9.5, 33$ mm (Model 1), $t_p=11, 37$ mm (Model 2). The minimum variance at $t_p=16$ mm-Model 1(15.9\%) and $t_p=25$ mm-Model 2(36.7\%).

The opposite of the longitudinal compression, in this case the average variances in the Model 2 (49.8\%) are larger in the Model 1 (24.8\%).

5.1.3. Biaxial compression

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9}
\caption{Ultimate strength of the plates (Model 1) under Biaxial compression were analyzed by ISO standard and ULSAP (Maestro software)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure10}
\caption{Ultimate strength of the plates (Model 2) under Biaxial compression were analyzed by ISO standard and ULSAP (Maestro software)}
\end{figure}

As discussed above, we would not be surprised when the variances in the transverse compression are larger in the longitudinal compression (Figure 9 & 10), Maximum variance: Model 1 with $t_p=33$ (mm); Model 2 with $t_p=37$ (mm). Evidently, the ULSAP curves are higher ISO curves, and they are in a similar trend.

5.2. Plate Subject to Compression WITH Lateral Pressure

5.2.1. Longitudinal axial compression with lateral pressure

The ultimate strengths (Figures 11 & 12) of the plates in two form analyses are nearly similar trend. This is easily understandable because the theoretical calculation of ISO & ULSAP are nearly similar.

The ISO results are smaller than the ULSAP results, the average variances in these cases are 14.1\%-Model 1 & 14.1\%-Model 2. The minimum variances: $t_p= 11$ mm-13.9\%-Model 1; $t_p= 12.5$ mm-13.9\%-Model 2. The different results of two form analyses are based on the partial resistance factor ($\gamma_R=1.15$), this factor is small.

The same as discussed above, the obtained curve of ultimate strength of $t_p= 9.5$ mm intersect the curves of $t_p=11,13$ mm (Model 1), the curve $t_p=11$mm intersect the curves
tp=12.5, 15 mm (Model 2). That is to say: At p=0.25 MPa then the ULS of tp=9.5 mm is larger than the ULS of tp=11, 13, 16 mm (Model 1-Figure 11); the ULS of tp=11 mm is larger than ULS of tp=12.5, 15, 18 mm (Model 2-Figure 12), that seems unreasonable.

5.2.2. Transverse axial compression with lateral pressure

Like discussion above, the minimum variances: tp= 16 mm-14.4%-Model 1; tp= 25 mm-17.3%-Model 2. The curve of ultimate strength of tp= 9.5 mm intersects the curves of tp=11, 13, 16, 22 mm (Model 1), the curve tp=11mm intersects the curves tp=12.5, 15,18 mm (Model 2).
5.2.3. Biaxial compression with lateral pressure

As discussed above, the variances in the transverse compression with lateral pressure are larger in the longitudinal compression with lateral pressure (Figure 15 & 16). Maximum variance: Model 1 with \( tp=33 \) (mm); Model 2 with \( tp=37 \) (mm).

Normally, the ultimate strength of plate is much increasing when the thickness plate of plate is increasing BUT in the cases of compression loading WITH lateral pressure (section 5.2) is not so, they have been proven in section 5.2.1 & 5.2.2.

6. CONCLUDING REMARKS

1. The objective of the study results in this paper was to check the accuracy of ULS of plates under six loading cases in ISO standard in comparison with the results of Maestro software (ALPS/ULSAP). The dimensions and material properties of a real ship panel were selected as the standard panel for testing purposes, and a wider range of plating dimensions was considered. Two types of plate, namely, Bottom plating of bulk carriers (Model 1), deck plating of VLCC (Model 2) were considered. Different loading conditions, including longitudinal compression, transverse compression, biaxial compression, with and without pressure loads, were applied.

2. The trends of the ISO results in some cases agree well to the Maestro results. Other cases are not similar (Figure 11, Figure 12). The ISO results were found smaller than Maestro results. For this reason, the ISO result does not approach to the actual ULS of plates. This should be considered when the value of the partial resistance factors is defined.

3. The ultimate compressive strength is given by Eqs.(10.3-6) - (10.3-10) for longitudinal compression and by Eqs.(10.3.12) and (10.3.13) for transverse compression. On the other hand, the ultimate compressive strength under the combined
action of longitudinal or transverse compression and lateral pressure is given by Eqs.(10.4-5) - (10.4-9) and Eq.(10.4-10) - (10.4.14), respectively. These formulas should give the same ultimate strength when the pressure is zero, but they do not.

4. The ISO standard does not clearly explain the concept of “restrained boundary” and “unrestrained boundary”, for this reason, made serious difficulties for user.

5. Should the partial resistance factor for plate compressive strength $\gamma_{R,e} = 1.15$ be correct?

- This valuable is a generic coefficient and does not represent the actual working of the structure.

- Should we choose a value is smaller than 1.15?

6. Should we separate the resistance factor for the longitudinal compression cases and transverse compression cases? Each case should use a respective resistance factor. We need through research on the resistance factor.

7. Many formulas are given in the ISO without showing the application limits of each formula. The application limit or the range of structural dimensions such as the slenderness ratio of plates should be clearly described. Theoretical calculations for thick plate and thin plate are different but the standard does not explain so to make some variances in values.

8. We need to clarify the meaning of biaxial compression cases that is loading interaction chart (Figure 17). That’s to say:

- A load point inside the interactional chart has enough ultimate strength. Example: Point A.

- A load point outside the interactional chart has not enough ultimate strength. Example: Point B.

9. Fundamental condition for ultimate limit state assessment is expressed as Eq. (1). This equation is explained in a different way:

$$\frac{\sigma_u}{\gamma_a} \leq \sigma_u \gamma_v$$

(19)

In the ISO standard was not described which methods calculated action stress. Values of the partial safety factors are given for each case, but the basis of the specified values is not clear.
Some rational basis for partial safety factors as well as concrete description should be specified together with the ultimate strength formulas if Eq.(1) is applied for the limit state assessment $\sigma_a, \sigma_r, \gamma_a, \gamma_r$ should be considered as a set.

However, if the document is concentrated only to give the formulas to evaluate buckling/ultimate strength, it is still acceptable.

![Ultimate strength interaction relationship between biaxial compressive loads for the plate](image)

Figure 17: Ultimate strength interaction relationship between biaxial compressive loads for the plate

10. The ISO standard has not explained causes of initial deflection in the plate as buckling mode, welding, production, etc.

11. Should the ISO standard or ULSAP(Maestro software) and other numerical methods(Nonliner FEM, Mesh-Free Method, Semi-analysis method, etc) be exact?. We need some test results for comparison.

**REFERENCES**

## APPENDIX

Table 1: Dimensions of the considered plates (Model 1 - axb=2550x850) and ultimate strength values of the plates under Biaxial compression, according to two analytical methods.

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<tr>
<td></td>
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<tr>
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<tr>
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Table 2: Dimensions of the considered plates (Model 2- axb=4750x950) and ultimate strength values of the plates under Biaxial compression, according to two analytical methods.

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<td></td>
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<td>σyu/σy</td>
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Table 3: Dimensions of the considered plates (Mode 1- axb=2550x850) and ultimate strength values of the plates under Biaxial compression with lateral pressure, according to two analytical methods.

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<th>ISO</th>
<th>ULSAP</th>
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</tr>
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Table 4: Dimensions of the considered plates (Mode 2- axb=4750x950) and ultimate strength values of the plates under Biaxial compression with lateral pressure, according to two analytical methods.
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