Optimal Design and Dimensioning of Hydrogen Transmission Distribution Pipeline Networks

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Abstract

This work considers the problem of the optimal design of an hydrogen transmission network. This design problem includes the topology determination and the dimensioning problem. We define a solution method that simultaneously looks for the least cost topology of the network and for the optimal diameter of each pipe. These two problems were generally solved separately these last years. The application to the case of development of future hydrogen pipeline networks in France has been conducted on several urban areas.

Keywords: Hydrogen, pipelines, graph, optimal design, minimal spanning tree, non linear optimization.

1 Introduction

Taking into account the inevitable drying out of fossil fuels and the environmental problems connected to the emission of polluting gases, the actors of the energy sector are engaged in a general reflection to reconcile energy and sustainable development. The feasibility study of the long-term deployment of an economy of the hydrogen takes place within the framework of this reflection (See Castello et al. [6]). Numerous technological, economic or societal challenges remain before attending the advent of this energy vector. In France, the *Programme d'Action Nationale pour l'Hydrogène* (PAN-H) attempts to investigate these challenges. The ECOTRANSHY project joins in this perspective. The first objective of the project ECOTRANSHY is to develop an economic model for the deployment of hydrogen transmission networks based on hypotheses clearly identified and taking into account the specificity of the transport of the hydrogen as energy vector.

In this paper, we deal essentially with the transport of the hydrogen by pipes. First of all, we present a state of the art of the existing methods allowing to resolve the problems of design and sizing of networks. Having established clearly the physical constraints and the specific costs for the transport of hydrogen by pipes, we present the program of optimization in order to determine the optimal characteristics of the network (topology and sizing). A theoretical important result concerning the topology of the distribution networks by pipelines will then be demonstrated. The fifth part of this article is dedicated to the tested heuristics. Finally, the numerical results on a network supplying the refuelling stations of a European local area are presented before concluding and giving some perspectives of our future works.

2 State of the art

2.1 Methods of design of networks with linear capacities: summary and Limits

In the graph theory (See Dolan et al. [9]), in the classical problem of design of telecommunication networks for example, the capacity of the arcs is defined as an upper bound on the flow through the arc.

If there is an unique choice for the capacity, the topology of a network can be determined by minimal spanning tree where the length of each arc is replaced by its cost. More evolved algorithms allow to determine much more interesting spanning trees by authorizing the passage possibly by intermediate points (Trees of Steiner). In Bang et al.[4], the authors present an algorithm called MST-Steiner (halfway between Minimal Spanning Tree and the Tree of Steiner) allowing to obtain a spanning tree T the total weight of which is not very far from the weight of the tree of Steiner. The authors demonstrate that the tree T supplied by the algorithm MST-Steiner cannot exceed twice the weight of the tree of Steiner (2 - estimate).

2.2 Methods of optimization of the topology of pipe networks

On the pipe networks (water, gas, hydrogen), the capacities are given by the non linear relations linking the flow and the pressures at both ends of the pipe. The first works of design of networks of pipelines were done during the design of collecting networks of gas wells production. So, Rothfarb and al. [17] studied the optimal design of an offshore natural gas network. In Bhaskaran et al. [5], the authors are confronted with a similar problem of optimal design of a network of collection of several wells in a desert environment (Australia). They show that, under certain conditions, the *optimal collecting network is a treelike network*. Let us note that both works Rothfarb et al.[17] and Bhaskaran et al.[5] consider only networks of collection of gas from several wells (multi-sources) but with a unique point of collection. That's defined the value of the flow on each arc. In Walters [20], the author uses the techniques of the dynamic programming to investigate all the possible trees on a water distribution network with several sources (springs) and the multiple wells (with potential fixed to sources and minimal potential in the points of exits). We shall finally note the recent works of Nie [19] on the topology of pipes networks with cycles and multi-sources by means of the use of neuronal networks.

2.3 Methods for optimal dimensioning of pipe networks

Because of the laying constraints of pipelines in industrial nations, the optimal topology of networks of pipelines problems gradually left the place with problems of the sizing of the diameters of the distribution networks with fixed topology. We shall note in this domain, the works of Osiadack and Gorecki [14] and De Wolf and Smeers [7] where a method of Lagrangean relaxation gave good results for a new trunkline of the Belgian network. Zhang and Zhu [22] took into account the discreet aspect of the commercial diameters by proposing the distribution of the sizes on every section. More recently, we shall note the works of Babonneau and Vial [2] on the design of the networks of water flowing due to the gravity. Finally, the reinforcement problems of existing networks (by doubling some pipes) appeared with the works of André et al. [1].

2.4 View on the future networks of hydrogen and added value of the article

In forward-looking studies imagining the future modes of transportation of the hydrogen if a large-scale hydrogen economy is deployed, most of the studies are considering the topology of networks by estimating the total length of the network from ratios of the already existing ones on the natural gas network. We shall quote in this category the studies of Castello et al.[6] on the European case and of Smit et al.[18] on the Dutch case. Other models based on geographical information systems (GIS), estimate the real distances between a production point and a consumption point localized on the map. The search for a topology is reduced to look for a set of pipelines leaving the source. In this category, we shall quote Ball [3] with the development of the model MOREHyS and its application on the German case. This approach tends to overestimate the costs because the approach of network construction cannot allow to take into account the economy of the passage of several demands in the same pipeline. Intermediate models estimate the lengths of networks by assuming given the shape of networks. In Yang et al.[21] , the authors envisage a system of concentric rosettes in an ideal city. Finally more detailed models are using minimal spanning algorithms to search for the optimal total length. We shall quote the work of Lin et al.[13] who justify this choice by indicating that the total length of the pipeline so obtained is the most significant factor of the transportation costs. Also, Patay et al.[16]

use the same techniques. Regarding the arcs dimensioning, it is supposed in all the approaches quoted above, that we shall put the same diameter on all the sections of a network of the same level: national transport, regional transport, local distribution. This standardization of diameters allows to obtain average costs for one kilometer (See Smit et al. [18]).

With regard to this state of the art, this paper has for ambition to go further on both aspects of the design and dimensioning of networks of pipeline of hydrogen. On one hand, we wish to go beyond the simple algorithms of minimal spanning to elaborate a method determining an optimal topology dedicated to pipelines. On the other hand, we wish to avoid the simplification of the standardization of diameters by proposing a method adjusting diameters by section in order to reduce the costs. The third objective is to couple both optimizations topology / dimensioning which are strongly connected.

3 Mathematical formulation

3.1 Equations of head losses for the hydrogen

Note by A the set of all arcs of the network corresponding to a pipe.

When we consider a fluid flowing in a pipeline, the difference of pressure between two ends of the pipe has for origin the friction of the fluid on the internal wall of the pipe. We call them head losses (See Joulié [12]). These regular[1] losses of energy depend on physical properties of the fluid (density, viscosity) and on the geometry of the pipe (diameter, length and roughness).

The literature concerning the head losses proposes several formulae which differ according to the brought level of precision. For the present work, we have used the equation used by De Wolf and Smeers [7] relating the gas flow Q_{ij} (m^3/ hour), the pressure at the entry of the pipe p_i (bar) and the pressure at the exit of the pipe p_j (bar):

$$Q_{ij} = K(D_{ij})\sqrt{p_i^2 - p_j^2}, \quad \forall (i,j) \in A \tag{1}$$

where the coefficient $K(D_{ij})$ can be computed by the following formula:

$$K(D_{ij}) = 0,0129\sqrt{\frac{D_{ij}^5}{\lambda \cdot Z_m \cdot T_m \cdot L_{ij} \cdot d}}$$

with D_{ij} = the pipe diameter (mm),
λ = the dimensionless coefficient of friction,
Z_m = the dimensionless compressibility factor,
T_m = the gas mean temperature (Kelvin),
L_{ij} = the pipe diameter (km),
d = the relative density of the gas with regard to the air.

[1] We don't consider the effect of the gravity. We take only into account the regular losses. We don't consider the singular losses (such as the elbow effect of the effect of diameter changes).

This equation can be rewritten in the following way:

$$\pi_i - \pi_j = k' Q_{ij}^2 \frac{L_{ij}}{D_{ij}^5}, \forall (i,j) \in A \tag{2}$$

with π_i = the square of pressure at the entry of the pipe,
π_j = the square of pressure at the exit of the pipe.

3.2 Calculation of the costs of a gas transportation pipeline

The costs of a gas pipeline breaks down into capital expenditures (Capex) and operating costs (Opex). Capital expenditures, widely dominating, divide into two main posts: the material costs and the installation cost. The operating costs are considered as a percentage of the capital cost. That explains that we use as criterion of costs only pipe investment costs including the installation cost.

For the hydrogen, it is generally allowed that we can adapt the costs models of the natural gas (See Castello et al.[6] and Parker [15]). The costs functions linking diameters D (mm) to the costs by units of length (€/ km) for natural gas available in the literature are sometimes linear (See Castello [6]) but most of the time are quadratic (See De Wolf and Smeers [7], Hafner[11] or Parker [15]). That is why, for the needs of the ECOTRANSHY project, we shall use a quadratic costs equation containing 3 terms:

$$C(D_{ij}) = a_0 + a_1 D_{ij} + a_2 D_{ij}^2 \tag{3}$$

3.3 Mathematical model

For the problem of optimal design and sizing of a hydrogen transportation network, the complete mathematical model that we propose is the following one:

$$\begin{aligned} &\text{Look for the set of arcs } A \subset N \times N \text{ such that} \\ &\min C(\pi, D, Q) = \sum_{(i,j)\in A} \left(a_0 + a_1 D_{ij} + a_2 D_{ij}^2\right) L_{ij} \\ &\text{subject to} \begin{cases} \pi_i - \pi_j = k' Q_{ij}^2 \dfrac{L_{ij}}{D_{ij}^5}, \forall (i,j) \in A \\ s_i + \displaystyle\sum_{k|(k,i)\in A} f_{ki} = \sum_{j|(i,j)\in A} f_{ij} + d_i, \forall i \in N \\ D_{min} \le D_{ij} \le D_{max}, \forall (i,j) \in A \\ \pi_{min} \le \pi_i \le \pi_{max}, \forall i \in N \end{cases} \end{aligned} \tag{4}$$

with N = set of all gas supply and gas consumption nodes,
s_i = the gas supply at node i,
d_i = the gas demand at node i.

By satisfying the constraints of head losses, the node flow conservation equations, the minimal and maximal pressures and minimal and maximal diameters available on the market, this model looks for the optimal topology of a network (i.e. the set A) and for optimal diameters of each used arc (i.e. the value of D_{ij}). This program is first of all an *integer program* because of the binary choice of opening the arcs (the choice of $A \subset N \times N$). Secondly, this program is *nonlinear* because of the head losses constraints. It is thus a problem difficult to exactly solve.

Note that the direction of flow is chosen in (4) in order to have that $\pi_i > \pi_j$.

4 Characteristics of the optimal topology

We demonstrate here that with the choice made within the framework of the ECOTRANSHY project for the investment objective function and for the head losses equation, the optimal networks are trees by using the following result of Bhaskaran and al. [5]:
Lemma. *Considering the following head losses equation*

$$D_{ij} = NQ_{ij}^{\beta_1} \left(\frac{L_{ij}}{\pi_i - \pi_j} \right)^{\beta_3} \tag{5}$$

and the following investment objective function:

$$\min COST = \sum_{(i,j) \in A} L_{ij} C(D_{ij}). \tag{6}$$

If the following condition is satisfied

$$D_{ij} \frac{C''(D_{ij})}{C'(D_{ij})} < \frac{1-\beta_1}{\beta_1} \tag{7}$$

then **the optimal network is a tree.**
Proof: See Bhaskaran et Salzborn [5].

The objective function for investment chosen in the ECOTRANSHY project is the following:

$$C(D_{ij}) = a_0 + a_1 D_{ij} + a_2 D_{ij}^2 \tag{8}$$

The head losses equation used in the ECOTRANSHY is the following:

$$\pi_i - \pi_j = k' \frac{L_{ij} Q_{ij}^2}{D_{ij}^5}.$$

Solving this equation for D_{ij}, we obtain:

$$D_{ij} = k'' Q_{ij}^{\frac{2}{5}} \left(\frac{L_{ij}}{\pi_i - \pi_j} \right)^{\frac{1}{5}} \tag{9}$$

By comparison with (5), we can conclude that: $\beta_1 = \frac{2}{5}$. Compute now the first and second derivatives of our investment objective function:

$$\begin{aligned} C'(D_{ij}) &= 2a_2 D_{ij} + a_1 \\ C''(D_{ij}) &= 2a_2 \end{aligned}$$

Compute now:

$$D_{ij}\frac{C''(D_{ij})}{C'(D_{ij})} = D_{ij}\frac{2a_2}{2a_2D_{ij}+a_1} = \frac{2a_2D_{ij}}{2a_2D_{ij}+a_1}$$

Let us consider now the two following cases:

Case 1. $\mathbf{a_1 = 0.}$ In this case, condition (7) becomes:

$$D_{ij}\frac{C''(D_{ij})}{C'(D_{ij})} = \frac{2a_2D_{ij}}{2a_2D_{ij}} = 1 < \frac{1-\beta_1}{\beta_1}$$

What is equivalent to say that:

$$\beta_1 < 1-\beta_1 \Leftrightarrow \beta_1 < \frac{1}{2}$$

In our case, we have seen here above that $\beta_1 = \frac{2}{5}$. The condition is thus satisfied.

Case 2. $a_1 > 0$**.** In this case, the condition (7) becomes :

$$D_{ij}\frac{C''(D_{ij})}{C'(D_{ij})} = \frac{2a_2D_{ij}}{2a_2D_{ij}+a_1} = \epsilon < \frac{1-\beta_1}{\beta_1}$$

with $\epsilon \in]0,1[$. What is equivalent to:

$$\epsilon\beta_1 < 1-\beta_1$$

or:

$$\beta_1(\epsilon+1) < 1 \Leftrightarrow \beta_1 < \frac{1}{\epsilon+1}$$

Examine the two limit cases:

$$\begin{array}{lll} \epsilon = 0 & \text{the condition becomes} & \beta_1 < 1 \\ \epsilon = 1 & \text{the condition becames} & \beta_1 < \frac{1}{2} \end{array}$$

Thus, forall $\epsilon \in]0,1[$, if $\beta_1 < \frac{1}{2}$, the condition is satisfied. In our case, $\beta_1 = \frac{2}{5}$. The condition is thus satisfied.

This completes the proof. So, we have demonstrated that with the choice of investment objective function, the result of Bhaskaran[5] remains valid in our case. The structure of the optimal network which we wish to conceive is thus a tree.

5 Proposed approach

In this section, we present the approach which we develop for the design and the sizing of a transportation or distribution hydrogen network. Indeed, we adapted the algorithms available in the literature and improved some of them. The proposed methodology of design contains three main subroutines that we present here below.

5.1 Initializations

Taking advantage of the result indicating that the optimal network is treelike, we use a classical algorithm of determination of the minimal MST-Steiner spanning tree (See Bang et al.[4]). We give as input the geographical coordinates of the nodes of the network to determine the topology of minimal length of the studied network.

5.2 Sizing of the continuous diameters on a tree

When the topology of the tree is fixed, this module is dedicated to the optimal sizing of the diameters of a treelike network. So, the problem of optimization written in (4) is reduced to the program (10) below. The *flows are no more variables* because they are perfectly determined by the treelike structure.

$$\begin{aligned} \min C(\pi, D) &= \sum_{(i,j)\in T} \left(a_0 + a_1 D_{ij} + a_2 D_{ij}^2\right) \cdot L_{ij} \\ \text{s.t.} & \quad \begin{cases} \pi_i - \pi_j = k' Q_{ij}^2 \dfrac{L_{ij}}{D_{ij}^5}, \forall (i,j) \in T \\ s_i + \displaystyle\sum_{k|(k,i)\in A} f_{ki} = \sum_{j|(i,j)\in A} f_{ij} + d_i, \forall i \in N \\ D_{min} \le D_{ij} \le D_{max}, \forall (i,j) \in T \\ \pi_{min} \le \pi_i \le \pi_{max}, \forall i \in N \end{cases} \end{aligned} \tag{10}$$

In that case, the set T only includes the arcs of the spanning tree and no longer from the complete graph. As output, we have a list of optimal diameters minimizing the setup costs and satisfying the head losses constraints.

This program, although containing no more combinatorial aspects, is strongly nonlinear and nonconvex because of the presence of the head losses equations. To solve this program, we used a nonlinear solver SNOPT developed by Stanford's university (See Gill et al.[10]). Let us note that this solver only supplies local optima when the program is nonconvex.

5.3 Delta change: a heuristics of improvement of trees

This algorithm is inspired by the works of Rothfarb and al. [17]. It is initialized by the final solution supplied by the sizing process (optimization of diameters) by using the MST-Steiner algorithm. This heuristic tries to improve the cost of the network of the initial minimal spanning tree by making local changes on arcs, and thus on the topology. Table 1 illustrates the pseudo-code of the delta change subroutine.

Contrary to the approach of Rothfarb and al. [17], the assessment for the cost of every trees is done by use of the module *"Sizing of the continuous diameters"* and not by means of dynamic programming.

Figure 1 illustrates the *Delta change* subroutine on a simple example.

Pseudo-code of the delta change subroutine
- Starting from an initial tree and its corresponding cost;
- Choose arbitrarily a node n_1 of the tree;
- Select $n_{1,1}$, the closest node not connected to n_1;
- Add arc $(n_1, n_{1,1})$ and determine the cycle so created;
- Eliminate one by one the other arcs of the cycle.
- As soon as the cost is improved, we adopt the new network

Table 1: Delta change subroutine

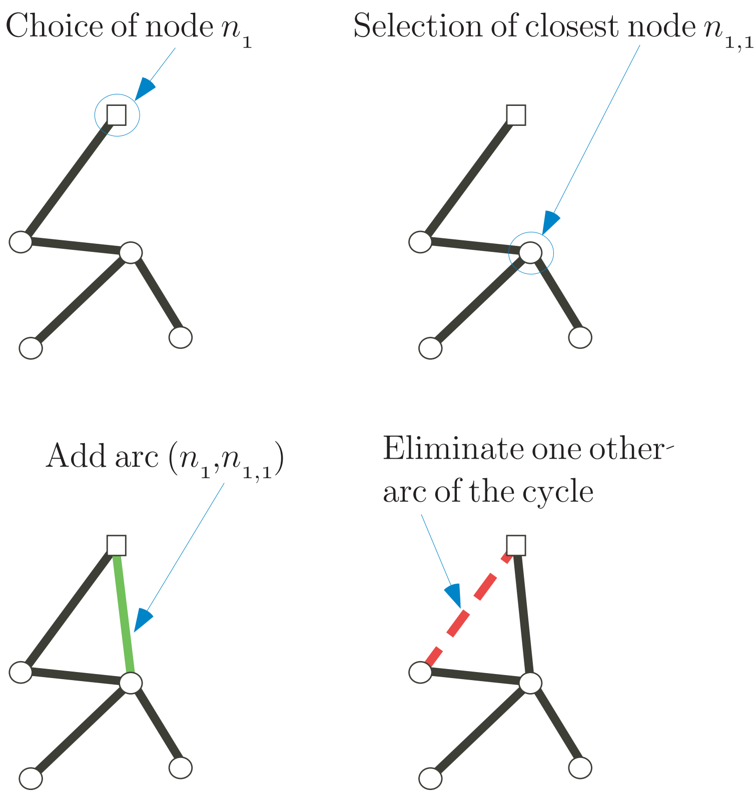


Figure 1: Illustration of the Delta change

6 Results

6.1 Test examples

Two small networks were first built to test the performances of the delta change algorithm and of its gain with regard to a minimal spanning tree algorithm. The two test examples are composed of a set of consumption nodes (nodes 1 to 6) provided by a single hydrogen factory (node 7). The pressure at the exit of the plant is 40 bar and the required pressure in the demand nodes is 36 bar.

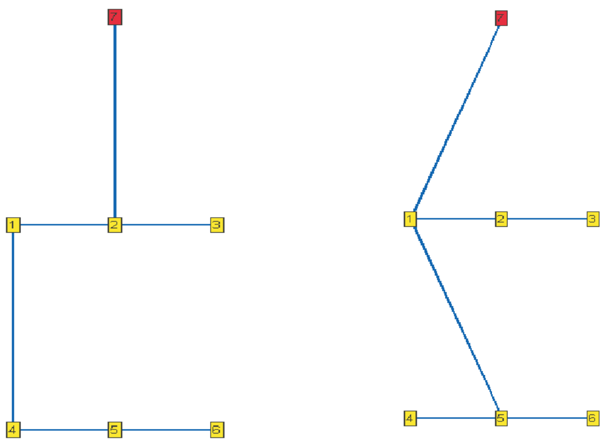
The demands of hydrogen are identical on all the consumption points (47 214 m^3 per day). Two tests of geographical configuration were realized:

- the *test number 1* where the consumption are concentrated in a square;
- the *test number 2* where the consumption are scattered on a rectangle.

We notice that on both tests, the delta change allows to reduce significantly the total costs of investment of the network (approximately 7 % on the test 1 and 18 % on the test 2) which justifies the use of this tool to design the network.

Furthermore, contrary to the intuition collectively accepted, the decrease in cost is made in two cases *while the total length of the proposed network increases of 5 km on the test 1 and more than 30 km on the test 2.* So, the decrease of the costs results only from the *decline of the proposed diameters.*

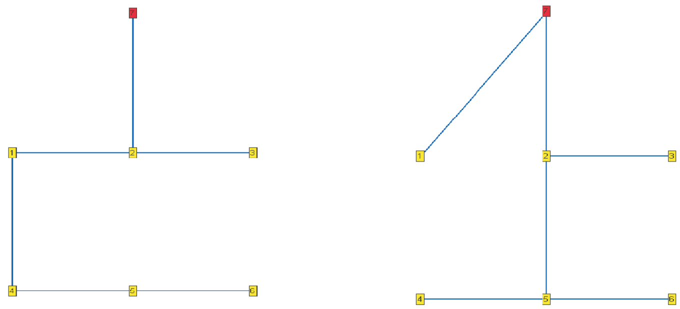
Finally, the topology of the optimal networks is not known in advance. Indeed, the topology obtained with the delta change by starting from the same minimal spanning tree is strongly different from the case 1 in the case 2 (only 3 from the 6 arcs are the same).



a) Minimal spanning tree
Length = 182.4 km,
Cost =7,365,756 €

b) Tree after delta change
Length = 187.9 km,
Cost = 6,868,273 €

Figure 2: Test Network number 1 with 40 bar at the exit of the hydrogen factory



a) Minimal spanning tree Length = 253.4 km, Cost =10,149,111 €

b) Tree after delta change Length = 283.8 km, Cost = 8,316,807 €

Figure 3: Test Network number 2 with 40 bar at the exit of the hydrogen factory

6.2 Real network

We present here the results obtained on a real network of refuelling stations in a European city. It contains 82 nodes including an unique source.

The initialization of the sizing algorithm is made in the following manner. We first determine a minimal spanning tree by calling the MST algorithm. Then we fix diameters large enough to allow the flow of the desired volumes of hydrogen by testing the feasibility of the network in view of the constraints of head losses. The sizing module then obtains diameters included between 20 and 292 mm by respecting the constraints of minimal and maximal imposed pressures, namely 36 and 71 bar (identical to those of Yang et al. [21]). The cost associated to this structure is 25,337,207 €.

Then, we apply the delta change algorithm to this network to decrease its investment cost. We have tested the impact on the cost and the computation time of two parameters:

- the number of tested nodes n_1 in the *Delta change*,
- the number of not directly connected nodes $n_{1,1}$ visited around the tested node.

We have also tested the strategy of investigation of the node to be tested. To select the investigated node, we envisaged two possible strategies:

- *random choice* of the node n_1,
- choice first the node n_1 *closest to the source.*

All these parameters were tested and the results obtained are illustrated by tables 2 and 3. We give for every test the cost in € of the proposed network and the computation time in seconds.

The first conclusions are that the computation time increases *linearly* with the number of tested nodes n_1 and with the number nodes $n_{1,1}$ visited in the neighbors of this node.

number of $n_{1,1}$	Percentage of nodes n_1 explored			
	5 %	10 %	50 %	100 %
2	25 330 902 €	25 230 987 €	23 355 855 €	22 461 563 €
	206 seconds	212 seconds	885 seconds	1 684 seconds
3	24 944 776 €	25 193 221 €	23 904 006 €	22 257 381 €
	80 seconds	285 seconds	1 264 seconds	2 466 seconds
4	25 279 491 €	24 732 752 €	23 208 930 €	22 319 028 €
	81 seconds	226 seconds	1 549 seconds	3 800 seconds
5	25 250 787 €	24 530 474 €	23 371 519 €	23 363 733 €
	184 seconds	220 seconds	1 696 seconds	4 524 seconds
6	25 321 374 €	22 796 209 €	22 744 885 €	21 335 215 €
	211 seconds	544 seconds	2 182 seconds	4 939 seconds

Table 2: Random choice of the node n_1

Number of $n_{1,1}$	Percentage of nodes n_1 explored			
	5 %	10 %	50 %	100 %
2	25 337 207 €	24 216 508 €	24 216 508 €	22 682 712 €
	92 seconds	227 seconds	227 seconds	1 868 seconds
3	25 337 207 €	24 216 508 €	24 216 508 €	22 135 428 €
	146 seconds	336 seconds	339 seconds	2 955 seconds
4	24 785 647 €	24 664 725 €	23 673 283 €	23 087 430 €
	241 seconds	458 seconds	1643 seconds	4 341 seconds
5	24 225 351 €	24 184 386 €	23 125 643 €	23 087 430 €
	275 seconds	557 seconds	3 233 seconds	4 355 seconds
6	24 225 351 €	23 974 081 €	22 894 056 €	22 564 011 €
	334 seconds	750 seconds	3 265 seconds	6 870 seconds

Table 3: Choice first the node n_1 closest to the source.

Secondly, substantial gains are obtained with regard to the cost of the initial tree supplied by MST-Steiner algorithm (until 16 % of economy). The network cost

- *decreases significatively* with the number of tested nodes n_1,
- *decreases slowly* with the number of not directly connected nodes $n_{1,1}$ visited in the neighbors of n_1.

Thirdly, it seems that the random choice of the node can produce *very variable costs* (between 21.33 and 25.33 Millions of €). On the other hand, the ordered choice of node n_1 gives cost strictly decreasing with the number of node n_1 and with the number of node $n_{1,1}$ explored. This phenomenon can be explained by the fact that savings are obtained on nodes closest to the source and that it is not interesting to continue more downstream. On the other hand, the *random choice* of node n_1 gives slightly better solution when considering 100 % of nodes n_1 explored and a maximum of nodes $n_{1,1}$ visited around this node.

7 Conclusions and perspectives

In this paper, we presented a new methodology for the simultaneously determination of the topology and of the diameters of hydrogen transport network. Theses two problems were generally solved separately. For example, the determination of the optimal topology is generally based on some basic criteria such as the minimum length of the network.

Our solution method were tested on a network of refuelling gas stations in a middle size European city. This study show that the two stages approach generally used (first look for a network topology of minimal length and then optimize the diameters for this fixed topology) is not the best one. In contrary, our case study has showed that increasing the total length of the network can help to decrease the network cost by using smaller diameters for some pipes.

We plan to consider in a near future the network of other cities by considering also several sources of hydrogen. This makes the problem more complicated to solve and this implies also to consider the facility location problem.

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