Solving the gas transmission problem with consideration of the compressors.

Bouchra Bakhouya[1] and Daniel De Wolf[2]

[1] Ieseg, Université Catholique de Lille, rue de la digue, 59.800 Lille
b.bakhouya@ieseg.fr
[2] Institut des Mers du Nord, Université du Littoral Côte d'Opale,
49 Place du Général de Gaulle, BP 5529, 59 383 Dunkerque Cedex 1
daniel.dewolf@univ-littoral.fr

Résumé *In [7], De Wolf and Smeers consider the problem of the gas distribution through a network of pipelines. The problem was formulated as a cost minimization subject to nonlinear flow-pressure relations, material balances and pressure bounds. This model does not reflect any more the current situation on the gas market. Today, the transportation and gas buying functions are separated. This work considers the new situation for the transportation company. The objective for the transportation company is to determine the flows in the network that minimize the energy used for the gas transport. This corresponds to the minimization of the power used in the compressors.*

Mots-Clefs. Optimal dimensionning, Gas transport, non-convex optimisation.

1 Introduction

In De Wolf and Smeers [7], the objective for the transportation company is to satisfy the demand at several points of the network by buying gas at minimal prize. Today, the transportation function and gas buying function are separated. For example, on the Belgian gas market, the transport is devoted to Fluxys company. On the other side, several actors are in charge of gas supplying.

In order to reflect this new situation, a modelization of the compressors is introduced in the model of De Wolf and Smeers. The introduction of the compressors change the nature of the optimization problem. In fact, the problem considered by De Wolf and Smeers [7] was a non convex but separable non linear problem. This problem was solved using successive piecewise linear approximations of the problem.

In the present case, the relation between the pressure increase and the flow in the compressor is non separable. See, for example, Jean André et al [3], Babu et al [5] or Seugwon et al [10].

This non linear non convex problem is solved using a preliminary problem, namely the problem suggested by Maugis [9], which is a convex problem easy to solve. We show, on the example of the Belgian gas network that this auxiliary problem gives a very good starting point to solve the highly non linear complete problem.

Future researches are devoted to introduce this model in a dimensioning model such as [6]. In this case, the gas transmission problem is the second stage problem of a two stage problem, the first stage being the pipes and compressors dimensioning problem.

The paper is organized as follows. The formulation of the problem is presented in section 2, the solution method being discussed next. Section 4 introduces two test problems based on the Belgian and French gas transmission systems. This section also illustrates the utility of the first problem to find a good starting point. Conclusions terminate the paper.

2 Formulation of the problem

The formulation of the problem presented in this section applies thus to a situation where the gas merchant and transmission functions are separated. The transportation company must decide the gas flow in each pipe and the level of compression in each compressor to satisfy fixed demands distributed over different nodes at some minimal guaranteed pressure, the income of gas being also given. For the supply, we have preserved a flexibility close that allows to take gas between 90 % and 110 % of the daily nominal quantity.

The network of a gas transportation company (See Figure **??**) consists of several supply points where the gas is injected into the system, several demand points where gas flows out of the system and other intermediate nodes where the gas is simply rerouted. Pipelines or compressors are represented by arcs linking the nodes.

The following mathematical notation is used. The network is defined as the pair (N, A) where N is the set of nodes and $A \subseteq N \times N$ is the set of arcs connecting these nodes. Since we have preserved the flexibility close, two **variables** are associated to each **node** i of the network : p_i represents the gas pressure at this node and s_i the net gas supply in node i. A positive s_i corresponds to a supply of gas at node i. A negative s_i implies a gas demand $d_i = -s_i$ at node i.

A gas flow f_{ij} is associated with each **arc** (i, j) from i to j. There are two types of arcs : **passive arcs** (whose set is noted A_p) correspond to pipelines and **active arcs** correspond to compressors (whose set is noted A_a).

The **constraints** of the model are as follows. At a **supply node** i, the gas inflow s_i must remain within take limitations specified in the contracts. A gas contract specifies an average daily quantity to be taken by the transmission company from the producer. Depending on the flexibility of the contract, the transmission company has the possibility of lifting a quantity ranging between a lower and an upper fraction (e.g. between 0.90 and 1.1) of the average contracted quantity. Mathematically :

$$\underline{s_i} \leq s_i \leq \overline{s_i}$$

At a demand node, the gas outflow $-s_i$ must be greater or equal to d_i, the demand at this node.

The gas transmission company cannot receive gas at a pressure higher than the one insured by the supplier at the entry point. Conversely, at each exit point, the demand must be satisfied at a minimal pressure guaranteed to the industrial user or to the local distribution company. Mathematically :

$$\underline{p_i} \leq p_i \leq \overline{p_i}$$

The flow conservation equation at node i for the flow conservation at a supply node i) insures the gas balance at node i. Mathematically :

$$\sum_{j|(i,j)\in A} f_{ij} = \sum_{j|(j,i)\in A} f_{ji} + s_i$$

Now, consider the constraints on the arcs. We distinguish between the passive and active arcs. For a **passive arc**, the relation between the flow f_{ij} in the arc (i,j) and the pressures at the two ends of the pipe p_i and p_j is of the following form (see O'Neill and al.[11]) :

$$sign(f_{ij})f_{ij}^2 = C_{ij}^2(p_i^2 - p_j^2), \ \forall (i,j) \in A_p$$

where C_{ij} is a constant which depends on the length, the diameter and the absolute rugosity of pipe and on the gas composition. Note that the pipes flow variables f_{ij} are unrestricted in sign. If $f_{ij} < 0$, the flow $-f_{ij}$ goes from node j to node i.

For an **active arc** corresponding to a compressor, the following expression of the power used by the compressor can be found, for example, in André et al [3], Babu et al [5] or Seugwon et al [10]) :

$$W_{ij} = \frac{1}{k\eta_{ad}} \frac{100}{3600} \frac{P_0}{T_0} T_i \frac{Z_m}{Z_0} \frac{\gamma}{\gamma - 1} f_{ij} \left(\left(\frac{p_j}{p_i} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \quad (1)$$

with the following parameters :
P_0 $= 1.01325$ bar,
T_0 $= 273.15$ K,
γ $= 1.309$,
Z_0 $= 1$,
k $= 0,95 \times 0,97 \times 0,98$,
Z_m = mean compressibility factor of the gas,
η_{ad} = the efficacity factor,
T_i = the gas temperature at the entry of the compressor.

Note also that in (1), the gas flow is expressed in m^3 per hour, the dissipated power in kW, the pressures p_i and p_j are in bar. Using mean values for theses factors (Z_m = 0,9, T_i = 288,15 K, η_{ad} = 0.75 for a turbo-compressor, 0.8 for a moto-compressor), the energy dissipated can be written as :

$$W_{ij} = \gamma_1 f_{ij} \left(\left(\frac{p_j}{p_i} \right)^{\gamma_2} - 1 \right) \quad (2)$$

with $\gamma_1 = 0.167$ for a turbo-compressor, 0.157 for a moto-compressor and $\gamma_2 = 0.236$.

The power used in the compressor must be lower than the maximal power, noted $\overline{W_{ij}}$:

$$W_{ij} \leq \overline{W_{ij}}$$

There is also an upper bound on the maximal pressure increase ratio :

$$\frac{p_j}{p_i} \leq \gamma_3$$

The constant $\gamma_3 = 1.6$ in most of practical cases.

For active arcs, the direction of the flow is fixed :

$$f_{ij} \geq 0,\ \forall (i,j) \in A_a$$

There are also some valves in the network. Note A_v the set of arcs corresponding to valves. The role of a valve is to decrease the pressure :

$$p_i \geq p_j, \forall (i,j) \in A_v$$

Note also that the direction of the flow is fixed for valves :

$$f_{ij} \geq 0,\ \forall (i,j) \in A_v$$

The **objective function** of the gas transmission company is to minimize the energy used in the compressor. This can be written :

$$\min z = \alpha \sum_{(i,j)\in A_a} \frac{1}{0,9\eta_{therm}} W_{ij} \tag{3}$$

where α is the unitary energy price (in Keuro/kW) and η_{therm} is the thermic efficacity of the compression station.

The gas transmission problem can thus be formulated as follows :

$$\min z(f,s,p,W) = \alpha \sum_{(i,j)\in A_a} \frac{1}{0,9\eta_{therm}} W_{ij}$$

$$\text{s.t.} \begin{cases} \displaystyle\sum_{j|(i,j)\in A} f_{ij} = \sum_{j|(j,i)\in A} f_{ji} + s_i \ \forall i \in N & (4.1) \\ sign(f_{ij}) f_{ij}^2 = C_{ij}^2 (p_i^2 - p_j^2) \quad \forall (i,j) \in A_p & (4.2) \\ W_{ij} = \gamma_1 f_{ij} \left(\left(\dfrac{p_j}{p_i} \right)^{\gamma_2} - 1 \right) \quad \forall (i,j) \in A_a & (4.3) \\ \underline{s_i} \leq s_i \leq \overline{s_i} \quad \forall i \in N & (4.4) \\ \underline{p_i} \leq p_i \leq \overline{p_i} \quad \forall i \in N & (4.5) \\ f_{ij} \geq 0 \quad \forall (i,j) \in A_a, A_v & (4.6) \\ \dfrac{p_j}{p_i} \leq 1.6 \quad \forall (i,j) \in A_a & (4.7) \\ W_{ij} \leq \overline{W_{ij}} \quad \forall (i,j) \in A_a & (4.8) \\ p_i \geq p_j \quad \forall (i,j) \in A_v & (4.9) \end{cases} \tag{4}$$

It can be seen that the problem (4) is separable for equation (4.2) but not separable for equation (4.3).

3 Solution Procedure

We now consider the solution of the problem (4). As already seen in De Wolf and Smeers [7], the formulation of the gas flow-pressure relation in pipes (4.2) is clearly nonconvex. The problem without the compressors modelization (4.3) was separable allowing to solve the problem by successive piecewise linear approximations of the problem. Now the compressors modelization (4.3) renders the problem non separable.

Some procedure is thus required in general to tackle the nonconvexity of the problem, if only to find a local solution. The approach proposed here is to proceed by successively solving two problems, the first one being expected to produce a good initial point for the second one.

Convergence in nonlinear programming may indeed crucially depend on a good choice of the starting point and this is especially true when the problem is nonconvex. Our first problem is obtained by *relaxing the pressure constraints and eliminating all compressors in the full model.*

The solution of this problem is conjectured to provide a good starting point for the **second problem** which is the complete model with pressure bounds and compressors. The same procedure was already proposed for solving the problem of an integrated transmission and merchant gas company by De Wolf and Smeers [7]. But here, we shall show on examples that the *solution of first problem is a very good starting solution for the complete problem.* In fact, we shall prove that they have similar objective functions (namely to minimize the energy dissipated in the network to transport the gas).

3.1 First problem : find a good initial point.

Consider the following convex problem which only accounts for pressure losses along the pipelines :

$$\begin{aligned} \min h(f,s) = & \sum_{(i,j)\in A} \frac{|f_{ij}|f_{ij}^2}{3C_{ij}^2} \\ \text{s.t.} \quad & \sum_{j|(i,j)\in A} f_{ij} - \sum_{j|(j,i)\in A} f_{ji} = s_i \ \forall i \in N \\ & \underline{s_i} \le s_i \le \overline{s_i} \qquad \forall i \in N \end{aligned} \tag{5}$$

Since the problem is strictly convex in the flow variables, its optimal solution is unique. The first constraints of (5) insure that the solution is also unique in the supply variables. It is easy to prove the following results for problem (5) :

Proposition 1. *The unique optimal solution of the problem (5) satisfies the nonlinear flow pressure relation (4.2).*

Proof : Let π_i be the dual variable associated to the gas balance constraint at node i. The Karush-Kuhn-Tucker necessary conditions (See Bertsekas [4, page 284]) satisfied at the optimum solution of the problem (5) can be written as :

$$sign(f_{ij})\frac{f_{ij}^2}{C_{ij}^2} = \pi_i - \pi_j \quad \forall (i,j) \in A$$

This is exactly the nonlinear flow pressure relation (4.2) if we define :

$$\pi_i = p_i^2, \ \forall i \in N$$

As remark in De Wolf and Smeers [7], there is no sign constraint on the π variables since π_i is the Lagrange multiplier associated to an equality constraint, namely the gas balance equation at node i. Thus directly replacing $\pi = p_i^2$ is not allowed. Let $\underline{\pi}$ be the value of the lowest dual variable :

$$\underline{\pi} = \min_{i \in N} \{\pi_i\}$$

Define now

$$p_i^2 = \pi_i - \underline{\pi}.$$

We obtain exactly the flow pressure relations (4.2). QED.

The optimal solution of (5) satisfies thus all the constraints of (4) except the pressure bounds constraints (4.5) and the compressors modelization (4.3), (4.7) and (4.8).

It can be shown (See De Wolf and Smeers [8]) that problem (5) has a physical interpretation. Namely, its objective function is the mechanical energy dissipated per unit of time in the pipes. This implies that the point obtained by minimizing the mechanical energy dissipated in the pipes should constitute a good starting point for the complete problem.

Definition 1. *The* mechanical energy *is defined as the energy necessary for compressing f_{ij} from pressure p_i to pressure p_j.*

Extending the work of Maugis for distribution network, we have the following proposition.

Proposition 2. *The objective function of problem (5) corresponds up to a multiplicative constant to the mechanical energy dissipated per unit of time in the pipes*

Proof : At node i, the power W_i given by a volumetric outflow of Q_i units of gas per second at pressure p_i can be calculated in the following manner. The total energy released by the gas when changing pressure from p_i to the reference pressure p^o is :

$$W_i = \int_{p_0}^{p_i} Q(p)dp$$

By using the perfect gas state relation ($p^0Q^0 = pQ$), we can write :

$$W_i = \int_{p^0}^{p_i} p^0 Q^0 \frac{dp}{p} = p^0 Q^0 log(\frac{p_i}{p^0})$$

Denote the volumetric flow going though arc (i, j) under standard conditions by Q^0_{ij} and the pressures at the two ends of the arc by p_i and p_j. The power lost in arc (i, j) can be calculated by :

$$W_{ij} = W_i - W_j = Q^0_{ij} p_0 log(\frac{p_i}{p_j}) = Q^0_{ij} \frac{p_0}{2} log(\frac{p_i^2}{p_j^2})$$

This power loss can be expressed through the head discharge variable $H_{ij} = p_i^2 - p_j^2$ as follows. Introducing the mean of square of pressure p_M defined by

$$p_M^2 = \frac{p_i^2 + p_j^2}{2},$$

the power discharge can be rewritten as

$$\begin{aligned} W_{ij} &= Q^0_{ij} \frac{p^0}{2} log(\frac{p_M^2 + \frac{H_{ij}}{2}}{p_M^2 - \frac{H_{ij}}{2}}) \\ &= Q^0_{ij} \frac{p^0}{2} \left[log(1 + \frac{H_{ij}}{2p_M^2}) - log(1 - \frac{H_{ij}}{2p_M^2}) \right] \end{aligned}$$

Note that since H_{ij} is small with respect to p_M^2, we obtain the following first order approximation

$$W_{ij} \approx Q^0_{ij} p^0 \frac{H_{ij}}{2p_M^2} = Q^0_{ij} p^0 \frac{p_i^2 - p_j^2}{2p_M^2}$$

The power discharge through the whole network is thus (we suppose that p_M is similar for each arc (i, j) and can be factored out in the following sum) :

$$W \approx \frac{p^0}{2p_M^2} \sum_{(i,j)\in A} Q^0_{ij}(p_i^2 - p_j^2) = \frac{p^0}{2p_M^2} \sum_{(i,j)\in A} \frac{(Q^0_{ij})^3}{C_{ij}^2} sign(Q^0_{ij})$$

which corresponds, up to a multiplicative constant, to the first term of the objective of problem (5). We can thus conclude that the function h corresponds, up to a multiplicative constant, to **the mechanical energy dissipated per unit time in the network due to the flow of gas in the pipes**. QED

This proposition was suggested to us by Mr. Zarea, direction de la recherche of Gaz de France.

Problem (5), being a convex separable problem, can be solved by any convex non linear optimization method. We have solve this problem using GAMS/CONOPT. The resolution with GAMS leads to a solution (f^*, s^*) with associated Lagrange multipliers π^* which are used as starting point for the complete problem.

3.2 Second problem : the complete problem.

The solution to the first problem satisfies all the constraints of problem (4) except for the pressure bounds and the compressors modelization equations. We now return to problem (4) and solve it using also GAMS/CONOPT. As we shall see in the following section, the flow variables founded in the first problem are the (local) optimal values of the complete problem. Thus the first problem give a very good starting point for the second problem.

This is not a surprizing since the objective function of problem (5) is the mechanical energy dissipated in all the network since the objective of the complete problem (4) is to minimize the energy dissipated in the compressors. The only task for CONOPT for solving the complete problem (4) from the solution of problem (5) is to restore the bounds on pressure.

4 Application to the Belgium gas network

To illustrate the utility of the first problem (5), we apply our suggested solution procedure to the schematic description of the Belgium gas network given in De Wolf and Smeers [7]. Recall that Belgium has no domestic gas resources and imports all its natural gas from the Netherlands, from Algeria and from Norway. The Belgium gas transmission network carries two types of gas and is therefore divided in two parts. The high calorific gas (10 000 kilocalories per cubic meter), comes from Algeria and Norway. The gas from Algeria comes in LNG form at the Zeebrugge terminal and the gas from Norway is piped through the Netherlands and crosses the Belgian border at Voeren. The gas coming from the Netherlands is a low calorific gas (8 400 kilocalories per cubic meter). We consider only to the high calorific network.

The reader is referred to De Wolf and Smeers [7] for a more detailed description of the Belgium network. For each compressor, the maximal power is given in in Table 1. Note that the compressor in Berneau corresponds to a large

Localization	$\overline{W_{ij}}$ (kW)
Berneau	20 888
Sinsin	3 356

Tab. 1. Compressors description

compression station.

4.1 Optimal solution

The energy used in the compressors founded by GAMS/CONOPT is :

$$z^* = 6\ 393.825 \text{ kW}$$

Arc	from	to	Flow (10^6 SCM)
1+2	Zeebrugge	Dudzele	11.594
3+4	Dudzele	Brugge	19.994
5	Brugge	Zomergem	16.076
6	Loenhout	Antwerpen	4.800
7	Antwerpen	Gent	0.766
8	Gent	Zomergem	-4.490
9	Zomergem	Péronnes	11.586
10 +11	Voeren	Berneau	19.344
12 +13	Berneau	Liège	19.344
14+15	Liège	Warnand	12.979
16	Warnand	Namur	10.838
17	Namur	Anderlues	8.718
18	Anderlues	Péronnes	9.918
19	Péronnes	Mons	22.464
20	Mons	Blaregnies	15.616
21	Warnand	Wanze	2.141
22	Wanze	Sinsin	2.141
23	Sinsin	Arlon	2.141
24	Arlon	Pétange	1.919

Tab. 2. Optimal flows

The optimal flow pattern is given in table 2. Note that we have converted each couple of two parallel arcs in one equivalent larger pipe (See appendix **??**).

The corresponding pressure and supply optimal patterns are given in table 3. Note that in Voeren, there is one unit of gas that is taken and which remains at Voeren. This is possible because there is no price associated to the gas taken at the source nodes.

The two compressors located at Berneau and Sinsin are in use. Table 4 give the used powers and the compression ratio p_j/p_i.

4.2 Role of the first problem

Consider now the utility by resorting to the first problem. First of all, we have asked GAMS/CONOPT to solve directly the complete problem (4) from scratch. Due to the nonconvexity of the problem, CONOPT can not find a feasible solution to the problem. This fact already justifies the utility of the first problem.

But if we consider the solution, in term of flows, of the first problem and of the complete problem, we see that the **optimal solution of problem (5) is the optimal solution of problem (4) !**

Note that there is some work for CONOPT to solve the complete problem (4) starting from the solution of the first problem (5). In fact, if we consider the pressure computed from the dual variables of the first problem, we can see that

Node	Town	Supply (10^6 SCM)	Demand (10^6 SCM)	Pressure (Bars)
1	Zeebugge	11.594		56.710
2	Dudzele	8.400		56.678
3	Brugge		3.918	56.532
4	Zomergem			54.869
5	Loenhout	4.800		56.174
6	Antwerpen		4.034	54.090
7	Gent		5.256	54.053
8	Voeren	20.344	1.	50.000
9 in	Berneau			49.579
9 out	Berneau			57.820
10	Liège		6.365	56.126
11	Warnand			55.140
12	Namur		2.120	53.893
13	Anderlues	1.200		53.110
14	Péronnes	0.960		52.982
15	Mons		6.848	51.653
16	Blaregnies		15.616	50.000
17	Wanze			54.326
18 in	Sinsin			47.300
18 out	Sinsin			58.726
19	Arlon		0.222	27.520
20	Pétange		1.919	25.000

Tab. 3. Optimal supplies and pressures

Localization	W_{ij} (kW)	$\overline{W_{ij}}$ (kW)	p_j/p_i	$\overline{p_j/p_i}$
Berneau	4 973.991	20 888	1.166	1.6
Sinsin	780.452	3 356	1.242	1.6

Tab. 4. Compressors energy used

they are not feasible (See Table 5). As can be seen, the work of GAMS/CONOPT is to satisfy the upper bound on pressure variables using the compressors at a minimum level.

Perhaps this phenomena is due to the *arborescent structure of the Belgian gas network*. We have try to apply the same methodology on a **a part of the French gas network which has many cycles** . The solution of the first and second problem are now different but the preprocessing throught the first problem allow to solve the second problem by CONOPT without difficulty. The main result of the application of our solution procedure is the fact that the first problem give the right direction of use of each arc, which is specially important for the actives arcs.

Node	Town	Pressure end first problem	Pressure end problem (4)	Maximum pressure
1	Zeebugge	86.664	56.710	77.0
2	Dudzele	86.634	56.678	77.0
3	Brugge	86.500	56.532	80.0
4	Zomergem	84.975	54.869	80.0
5	Loenhout	86.171	56.174	77.0
6	Antwerpen	84.263	54.090	80.0
7	Gent	84.230	54.053	80.0
8	Voeren	88.019	50.000	50.0
9 in	Berneau	87.686	49.579	66.2
9 out	Berneau	87.686	57.820	66.2
10	Liège	86.127	56.126	66.2
11	Warnand	85.223	55.140	66.2
12	Namur	84.084	53.893	66.2
13	Anderlues	83.370	53.110	66.2
14	Péronnes	83.254	52.982	66.2
15	Mons	82.048	51.653	66.2
16	Blaregnies	80.555	50.000	66.2
17	Wanze	84.479	54.326	66.2
18 in	Sinsin	78.139	47.300	63.0
18 out	Sinsin	78.139	58.726	66.2
19	Arlon	36.505	27.520	66.2
20	Pétange	25.000	25.000	66.2

Tab. 5. Optimal pressures for first and complete problems

5 Conclusions

In this paper we have updated the gas transmission model of De Wolf and Smeers [7] to the new situation in several european countries. Namely the fact that the merchant and the transportation function are now separated. The consequence of this new situation is the necessity of the introduction of the compressors modelization. This is the first objective of the present paper. The second is to present a procedure to solve this non linear non convex non separable problem. We have seen, on the example of the Belgium gas network that the preprocessing trough the Maugis problem [9] is very efficient in this case. Namely, on the presented examples, the solution in term of flow variables of the preliminary problem is the optimal solution of the gas transmission problem. We have given a physical interpretation to the objective function of the first problem (5), namely the power used in the network due to the flows in the pipes. This constitutes a justification to this phenomena.

Futures research are devoted to introduce this model in a dimensioning model such as [6]. Such a model will consider the trade-off between the minimization of capital expenditures (as in [2]) and the minimization of operational expenditures. In other terms, this model could balance any decrease in investment of pipelines with an increase of compressor power (and conversely) regarding the costs. In fact, if we increase the pipe diameters, this leads to minus head loses and perhaps the use of one compressor can be avoided.

Références

1. Jean ANDRE, Note sur les modéles simplifiés de stations de compressions, privated communication, Direction de la recherche de Gaz de France, April 2006.
2. Jean ANDRE, Frédéric BONNANS and Laurent CORNIBERT, Planning reinforcement of gas transportation networks with optimization methods, Proceedings of the 19th Mini EURO Conference (ORMMES 06), 6-9 September 2006
3. Jean ANDRE, Laurent CORNIBERT, A tool to optimize the Reinforcement Costs of a Gas Transmission Network, Pipeline Simulation Interest Group, 38th Annual Meeting, Williamsburg (VA), October 2006.
4. BERTSEKAS, D. 1995, Nonlinear Programming, *Athena Scientific*, Belmont.
5. B.V. BABU, Rakesh ANGIRA, Pallavi G. CHARKOLE and J. H. Syed MUBEEN, Optimal Design of Gas Transmission Network, Using Differential Evolution, Department of Chemical Engineering, Birla Institute of Technology Science, Pilani, India, 2003.
6. Daniel DE WOLF and Yves SMEERS, Optimal Dimensioning of Pipe Networks with Application to Gas Transmission Networks, Operations Research,Vol 44, No 4, July-August 1996, pp 596-608.
7. Daniel DE WOLF and Yves SMEERS, The Gas Transmission Problem Solved by an Extension of the Simplex Algorithm, Management Sciences, Vol. 46, No 11, November 2000, pp 1454-1465
8. Daniel DE WOLF, Mathematical Properties of Formulations of the Gas Transmission Problem, submitted april 2004 to RAIRO in the field "Operations Research".
9. MAUGIS, M.J.J. 1977, Etude de Réseaux de Transport et de Distribution de Fluides, *R.A.I.R.O.,* Vol.11, No 2, pp 243-248.
10. Seugwon AN, Qing LI, Thomas W. GEDRA, Natural Gas and Electricity Optimal Power Flow, Oklahoman State University, IEEE Paper, 2003.
11. O'NEILL, R.P., WILLIARD, M., WILKINS, B. and R. PIKE 1979, A Mathematical Programming Model for Allocation Of Natural Gas, *Operations Research 27,* No 5, pp 857-873.
12. WILSON, J.G., WALLACE, J. and B.P. FUREY 1988, Steady-state Optimization of Large gas transmission systems, *in Simulation and optimization of large systems,* A.J. Osiadacz Ed, Clarendon Press, Oxford.