Numerical Simulation of a Scroll Expander for Use in a Rankine Cycle

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ABSTRACT

This paper presents two simulation models of scroll expanders for use in heat recovery Rankine cycle applications: a detailed reference model and a simplified model. The reference model associates a detailed geometrical description of the expander with a thermodynamic modeling of the expansion process. Due to large computational effort, the reference model is not appropriate for simulation of the entire Rankine cycle system. The paper shows how a simplified model can be built from the reference model. This model involves a limited number of parameters, which describe the main features of the machine. An experimental study is carried out on a prototype of Rankine cycle working with HCFC-123. Based on measurements, the validation of both models is conducted. It is found that both models are able to predict the mass flow rate, the delivered shaft power and the discharge temperature with a good accuracy.

1. INTRODUCTION

Rankine cycles are particularly suitable for recovering energy from low-grade heat sources, such as waste heat in an industry process, exhaust gases or cooling system of an internal combustion engine or heat produced by solar collectors (Kane, 2002).

In order to improve the performances of such a cycle, two inter-dependent questions are usually asked: what is the best expander design (given the operating conditions of the cycle) and how could the cycle itself be optimized in terms of operating conditions (given the characteristics of the heat source and sink)?

In order to address these questions, two expander simulation models are proposed: a detailed reference model and a simplified model. The first one can be used as a tool for improving the expander design, while the second one can be used as a sub-component of a global Rankine cycle model.

2. EXPANDER REFERENCE MODEL

The reference model associates a detailed geometrical description of the expander with a thermodynamic description of the expansion process. This model is adapted from the flooded scroll expander model proposed by Lemort *et al.* (2008).

2.1. Scroll expander geometry

The scroll expander geometric model is similar to the one proposed by Halm (1997) for scroll compressors. In order to identify the geometric parameters, the geometry of the scroll has been measured by means of a 3-D measuring machine. The geometric parameters have been tuned to match the geometry described by the model to the measured one, what is shown in Figure 1. The tip geometry of the scroll machine under investigation differs obviously from that of a unique circular arc. It was observed that the tip of the scroll can be very accurately described by two circular arcs in series. Parameters of the scroll geometry are summarized in Table 1. These parameters are defined in the nomenclature. Their definitions follow those from Halm (1997).

Table 1: Main geometrical parameters

r_b [mm]	h [mm]	$\varphi_{o,0}$ [rad]	$\varphi_{i,0}$ [rad]	$\varphi_{o,s}$ [rad]	$\varphi_{i,s}$ [rad]	φ_e [rad]	r_o [mm]
3.264	28.65	0	1.3971	1.57	3.5	27.395	5.7

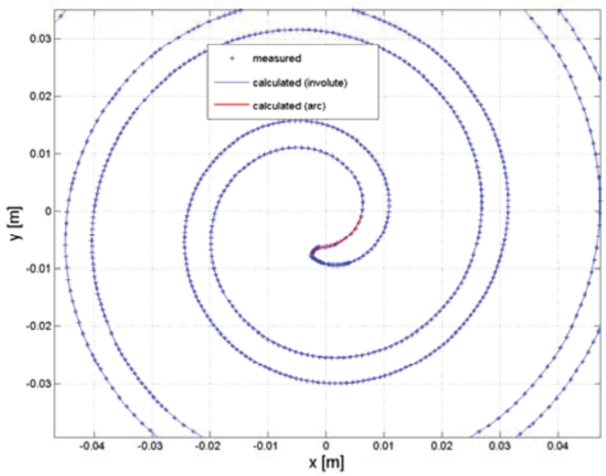

Figure 1: Comparison between the measured geometry and the one predicted by the model

The different working chambers are represented in Figure 2. The evolution of their volumes with the crank angle is shown in Figure 3. The suction chamber, which is in communication with the suction port is labeled chamber 6. This chamber can be fictitiously divided into chamber 1 and chambers 2 and 3. This sub-division of the suction chamber is appropriate to describe the end of the suction process when the sub-chambers communicate only by small gaps and cannot be described anymore by one unique pressure. Chamber 7 is the expansion chamber that develops from chamber 3 and chamber 8 is the expansion chamber that develops from chamber 2. Expansion proceeds successively through chambers 9 and 10 and chambers 11 and 12. The latter only exist for the period $0 - \theta_d$ of one entire revolution (the discharge angle θ_d has been defined by Halm (1997)). Chamber 13 is the discharge chamber that develops from chamber 11 and chamber 14 is the discharge chamber that develops from chamber 12. Chamber 13 and chamber 14 are opened up to the discharge region, called chamber 17. In the case that the pressures in all three chambers 13, 14 and 17 have equalized, the entire control volume is treated as chamber 18 (called also exhaust channel). The discharge process extends from θ_d to 2π, and from 0 to θ_d. It can be observed that the expander under investigation is characterized by a large internal built-in volume ratio, what makes it more appropriate for the Rankine cycle application than a classical refrigeration compressor.

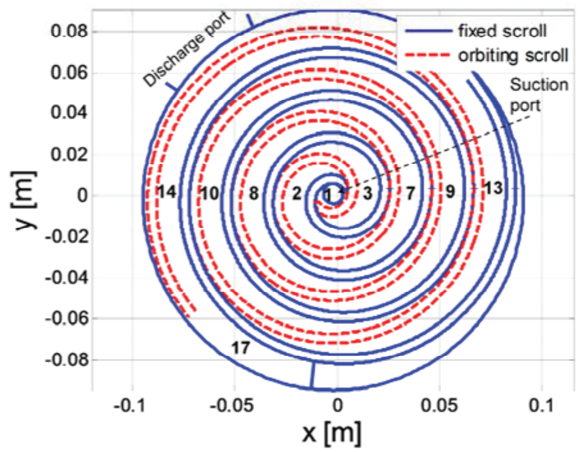

Figure 2: Definition of the expander working chambers

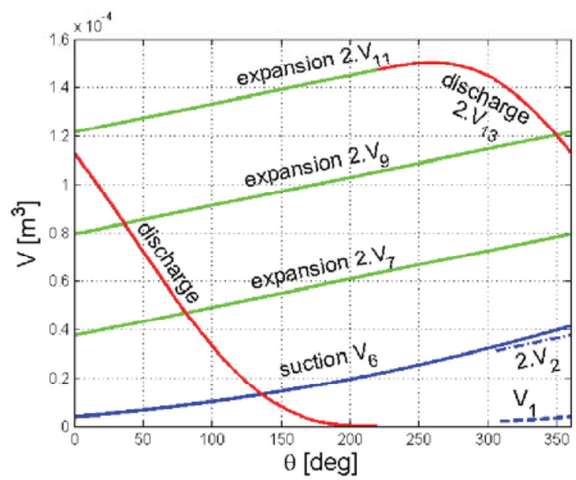

Figure 3: Evolution of the volumes of the different chambers with the crank angle

2.2. Expansion process

The expansion process is modeled with each expander chamber considered as a control volume for which the governing equations of conservation of mass and energy are established and numerically solved. Using these equations, instead of the derivatives of the pressure and the temperature is more appropriate if a two phase state has to be described.

$$\frac{dM}{d\theta} = \frac{1}{\omega}\left(\sum \dot{M}_{su} - \sum \dot{M}_{ex}\right) \tag{1}$$

$$\frac{dU}{d\theta} = \frac{\dot{Q}}{\omega} - P\frac{dV}{d\theta} + \frac{1}{\omega}\sum \dot{M}_{su}.h_{su} - \frac{h}{\omega}\sum \dot{M}_{ex} \tag{2}$$

Thermodynamics properties of refrigerant R123 are described by working equations *P=f(T,v)* and *h=f(T,v)* proposed by Baehr and Tillner-Roth (1995). Within their range of validity, these working equations are almost as accurate as the fundamental equations from which they are derived.

The overall heat transfer model of the expander is similar to the one proposed by Halm (1997). The two scrolls and the shells are lumped into one unique mass element for which the steady-state energy balance equation is established; this allows the introduction of external heat transfer and mechanical losses $\dot{W}_{loss}$. Internal heat transfers (suction and discharge heat transfers and scroll-refrigerant heat transfers) are modeled as described by Bell *et al.* (2008). Mechanical losses are defined by introducing a fictitious friction torque T_{loss} which is assumed to be independent of the expander speed ω:

$$\dot{W}_{loss} = \omega T_{loss} \tag{3}$$

3. EXPANDER SIMPLIFIED MODEL

Due to large computational effort, the reference model is not convenient for global simulation of the entire Rankine cycle. This paper presents a simplified expander model involving a limited number of parameters, which describe the main features of the machine. This model has been developed and validated using EES software (Klein, 2007). This model is deduced from a previous one proposed by Winandy et al. (2002) for hermetic scroll compressors. It has been partially validated by tests with water steam (Lemort et al., 2006). The conceptual scheme of the expander model is shown in Figure 4. Based on results generated by the reference model, only the main thermodynamics characteristics are retained in the simplified model; they are described by 8 physical parameters: the expander displacement V_s, its internal built-in volume ratio $r_{v,in}$, the suction and discharge heat transfer coefficients AU_{su} and AU_{ex}, a global heat transfer coefficient between the expander and the ambient AU_{amb}, an equivalent suction diameter d_{su} (to describe the suction pressure drop, since the reference model made appear that it is not negligible), a lumped leakage area A_{leak} and a friction torque T_{loss}.

In this model, the evolution of the fluid state through the expander is decomposed into the following steps: suction pressure drop (su→ su,1), suction cooling-down (su,1 → su,2), isentropic expansion to the "adapted" pressure imposed by the internal built-in volume ration of the expander (su,2 → ad), expansion at a constant machine volume V (ad → ex,2), mixing between the suction and leakage flows (ex,2 → ex,1) and exhaust cooling-down or heating-up (ex,1 → ex). The model is able to compute "primary" variables such as the mass flow rate, the delivered shaft power and the discharge temperature as well as "secondary" variables, such as the suction heating-up, the discharge cooling-down, the ambient losses and the mechanical losses.

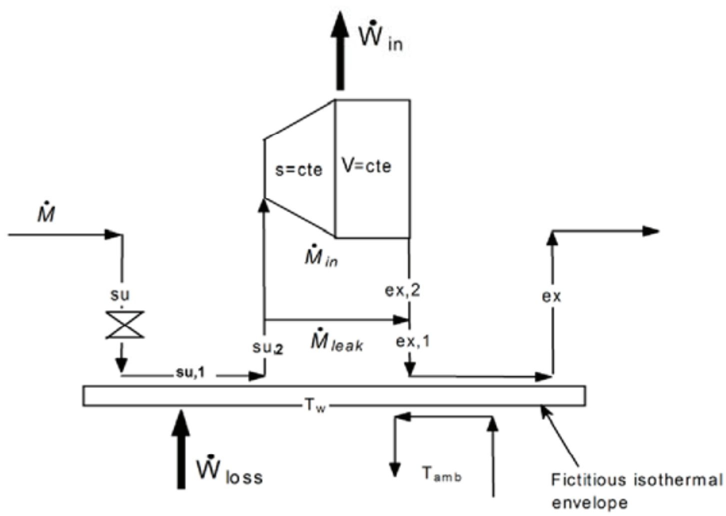

Figure 4: Conceptual scheme of the expander simplified model

4. EXPERIMENTAL INVESTIGATION

In order to validate both expander models, an experimental study has been performed on a prototype of Rankine cycle working with HCFC-123. The expander is originally an oil-free open-drive air scroll compressor. As shown in Figure 5, the expander drives an asynchronous machine through two belt-and-pulley couplings and a torque-meter. The asynchronous machine imposes the expander rotation speed. The latter is set to different values by modifying the pulley ratio. The boiler is made up of three plate heat exchangers HX1, HX2 and HX3 in series supplied with hot air. The condenser is made up of two plate heat exchangers in parallel supplied with cold water. A diaphragm pump drives the refrigerant through the cycle. Its displacement can be adjusted, what allows the control of the refrigerant flow rate. The expander mechanical power is determined by measuring simultaneously the rotation speed and the torque developed on the torque-meter shaft. The refrigerant flow rate is measured by a Coriolis flow meter, located at the outlet of the pump. Temperatures and pressures are measured at the main points of the cycle.
In total, 39 measuring points are achieved. They are characterized by different expander rotation speeds (1771, 2296, 2660 rpm), supply and exhaust pressures and supply temperatures.

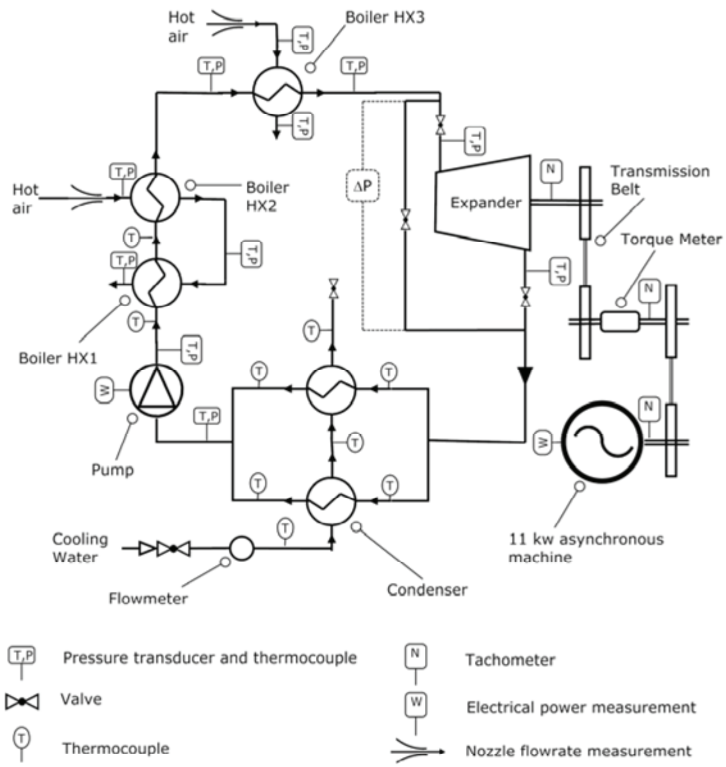

Figure 5: Schematic representation of the Rankine cycle test rig

The maximum developed shaft power is 1.82 kW and the maximum achieved isentropic effectiveness is 68%. This effectiveness is defined by the ratio of the measured shaft power and the isentropic power. The latter is the product of the measured flow rate by the expansion work associated to an isentropic expansion from supply conditions to the exhaust pressure.

5. MODELS VERIFICATION

5.1. Fitted parameters

Based on detailed measurements collected from the experimental study, the reference and the simplified models are validated. This validation is conducted by imposing some measurements as model inputs, by tuning the parameters and by comparing the outputs to the other measurements. The inputs are the supply pressure P_{su}, the exhaust pressure P_{ex}, the supply temperature T_{su}, the expander rotation speed ω and the ambient temperature T_{amb}. The outputs are the displaced mass flow rate $\dot{M}$, the developed shaft power $\dot{W}_{sh}$ and the discharge temperature T_{ex}.

For both the reference and the simplified model, parameters are tuned in order to minimize the error on the prediction of the mass flow rate, of the shaft power and of the discharge temperature. For the reference model, the identified parameters are the friction torque T_{loss}, the flank and radial gap clearances (δ_f and δ_r) and the external heat transfer coefficient AU_{amb} (Lemort *et al.*, 2008). These parameters are summarized in Table 2. The fitted parameters for the simplified model are given in Table 3. It is found that the friction torque and the external heat transfer coefficient identified for the simplified model yield good results with the reference model.

Table 2: Identified parameters for the reference model

δ_f [µm]	δ_r [µm]	AU_{amb} [W/K]	T_{loss} [N.m]
10	40	6.38	0.4

Table 3: Identified parameters for the simplified model

V_s [cm^3]	AU_{su} [W/K]	AU_{ex} [W/K]	AU_{amb} [W/K]	A_{leak} [mm^2]	$r_{v,in}$ [-]	d_{su} [mm]	T_{loss} [N.m]
36.39	$21.22\left(\frac{\dot{M}}{0.12}\right)$	$34.2\left(\frac{\dot{M}}{0.12}\right)$	6.38	4.858	4.005	5.91	0.4

5.2. Mass flow rate prediction

Figures 6 and 7 compare the evolutions of the mass flow rate measured and predicted by the two models with the expander supply specific volume, for the three different rotation speeds. The mass flow rate decreases with the specific volume of the refrigerant and increases with the rotation speed. It can be observed than the agreement between the measurement and the predictions of each model is very good. For the reference model, the maximal deviation between the model predictions and the measurements is 3.1%. For the simplified model, this maximum deviation is 2.5%.

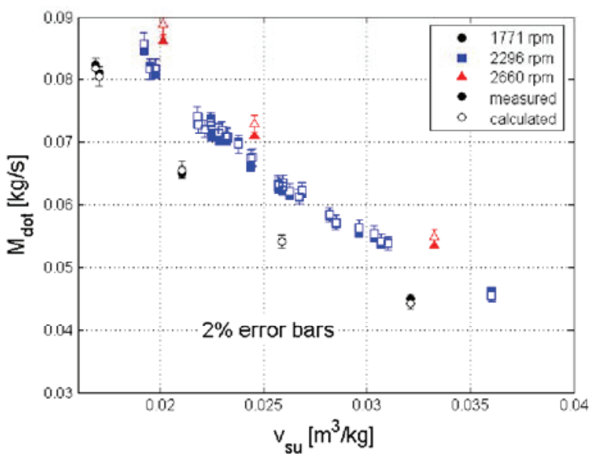

Figure 6: Verification of the mass flow rate (reference model)

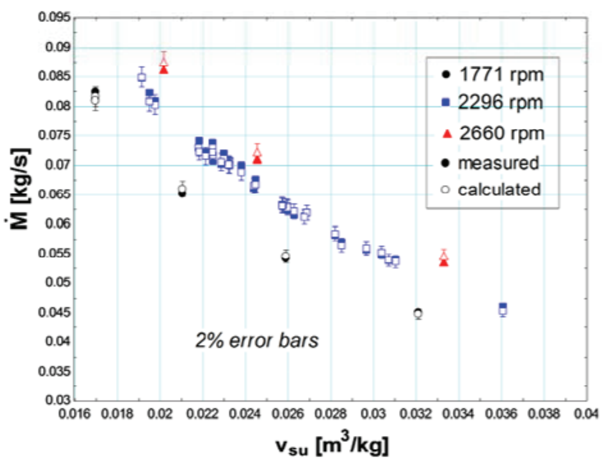

Figure 7: Verification of the mass flow rate (simplified model)

5.3. Shaft power prediction

A similar comparison is carried out for the shaft power. Figures 8 and 9 compare the evolutions of the measured and computed shaft powers as a function of the pressure ratio imposed to the expander.

For almost all the points, the shaft power is predicted within 5%. The maximal deviation between the model predictions and the measurements is 9.7% for the reference model and 9.2% for the simplified one. This largest deviation is associated to the same point and seems to be due to measuring problems associated to a two-phase regime at the expander supply.

5.4. Discharge temperature prediction

Comparisons of the discharge temperature predicted by each model and the measured discharge temperature are given in Figures 9 and 10. The maximal deviation between the model predictions and the measurements is 3.9 K for the reference model and 3.8 K for the simplified one. Both models present the same trend: the lowest discharge temperatures are under-predicted and the simplified model over-predicts the highest temperatures. The reason may be that the external heat transfer cannot be calculated by reference to one unique temperature node as it is done in both models.

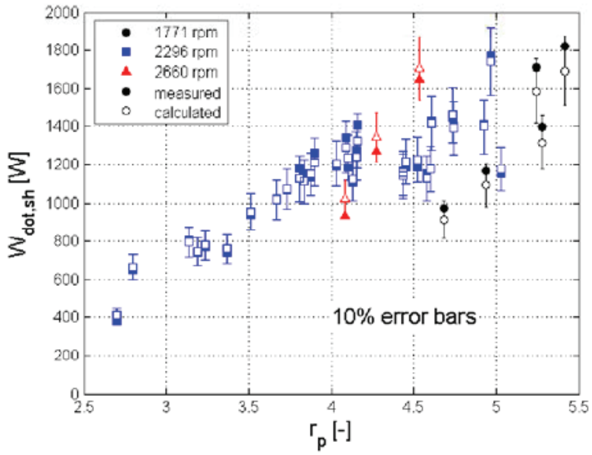

Figure 8: Verification of the shaft power (reference model)

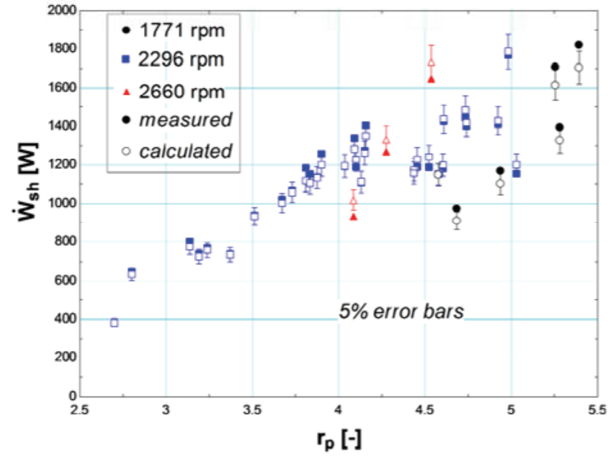

Figure 9: Verification of the shaft power (simplified model)

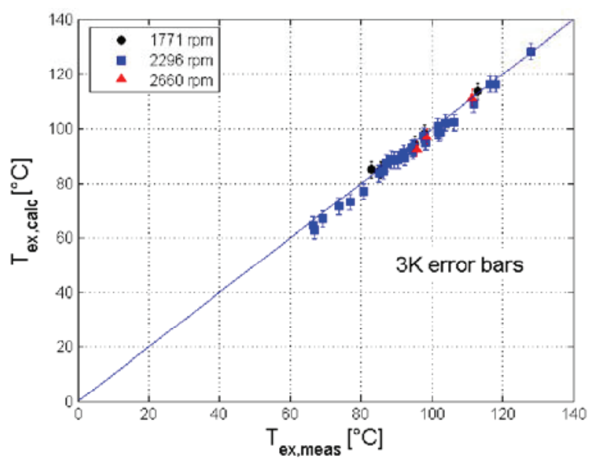

Figure 10: Verification of the discharge temperature (reference model)

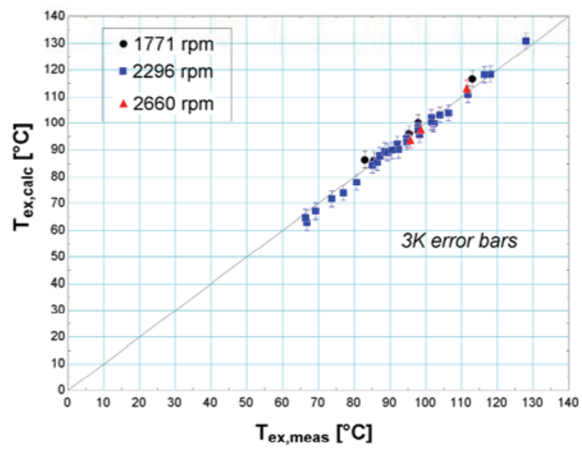

Figure 11: Verification of the discharge temperature (simplified model)

8. CONCLUSIONS

Two different scroll expander simulation models have been presented. The reference model aims at investigating the influence of design changes on the scroll expander performance. The simplified model is appropriate for modeling the expander as a sub-component of a global Rankine cycle simulation model.

Through experimental validation, it is found that both models are able to predict the mass flow rate, the delivered shaft power and the discharge temperature with a good accuracy. In the reference model, the heat transfer schema could still be improved in order to attenuate the small deviations between measured and predicted discharge temperatures.

The reference scroll model can be easily adapted to other scroll geometries and other working fluids. Accordingly, the proposed model could be used for studying other applications of the scroll expander than the Rankine cycle.

The reference model necessitates fewer parameters to identify than the simplified model, but requires the exact knowledge of the scroll geometry. For example: the suction pressure drops are indirectly described by the geometry of the suction chambers; the expander displacement and its internal built volume ratio are imposed by the expander geometric parameters; the internal leakage flow rates are computed on the basis of measured leakage lengths and radial and flank leakage gaps (which could be given by the manufacturer); internal heat transfers areas are computed by the model and convective heat transfer coefficients between the fluid and the scrolls are estimated by correlations (Bell *et al.*, 2008). Up to now, the friction losses are still described in a similar way than in the simplified model, but a description of the mechanical losses could be implemented.

On the other hand, the parameters of the simplified model can only be identified on the basis of performance data. However, the reference model could be used to extrapolate the performance of the expander when modifying its design. Parameters of the simplified model could then be identified on the basis of data generated by the reference model.

NOMENCLATURE

A	area	(m^2)
AU	heat transfer coefficient	(W/K)
d	diameter	(m)
δ	gap	(μm)
h	scroll height	(m)
h	specific enthalpy	(J/kg)
$\dot{M}$	mass flow rate	(kg/s)
N	rotation speed	(rpm)
ω	rotation speed	(rad/s)
P	pressure	(Pa)
φ	involute angle	(rad)
$\dot{Q}$	heat transfer rate	(W)
r_b	involute basic circle radius	(m)
r_o	orbiting radius	(m)
r_p	pressure ratio	(-)
r_v	volume ratio	(-)
T	torque	(N-m)
T	temperature	(°C)
U	Internal energy	(J)
V	volume	(m^3)
v	specific volume	(m^3/kg)
$\dot{W}$	power	(W)

Subscripts

amb	ambient
calc	calculated
e	end
ex	exhaust
f	flank
i	inner involute
in	internal
leak	leakage
meas	measured
o	outer involute
r	radial
s	start, isentropic
sh	shaft
su	supply

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