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NUMERICAL MODELLING OF STORAGE RACKS SUBJECTED TO EARTHQUAKE

Hervé Degée[1] and Vincent Denoël[1]

[1] University of Liège – Department ArGEnCo
Chemin des Chevreuils, 1 B-4000 Liège (Belgium)
H.Degee@ulg.ac.be, V.Denoel@ulg.ac.be

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Abstract. *This paper presents some aspects of the non linear dynamic time-history analysis of storage racks made of thin-walled steel products commonly used in warehouses when they are subjected to an earthquake action. It focuses essentially on the accounting for structural non-linearities (geometric second order behavior of the structure and non-linear material behavior of the joints) and for the possible relative motion between the rack and the stored goods that occurs as soon as the inertial force exceeds the friction resistance. In particular, it presents the algorithm used of to couple these two problems, followed by some simple applications and comparisons with test results.*

1 INTRODUCTION

Despite their lightness, storage racking systems made of thin-walled cold formed steel products are able to carry very high live load many times larger than the dead load, opposite to what happens in usual civil engineering structures. These racks can also raise considerable height. For these reasons, their use is nowadays very common in warehouses (see fig. 1). However, these structures have to be carefully designed. Indeed many difficulties arise in the prediction of their structural behavior, such as instabilities (global, local and distortional) or modeling problems (beam-upright connection stiffness, base plate anchorages) [1].

Things become even more complicated when a storage rack is installed in a seismic zone where, subjected to an earthquake, it has to withstand horizontal dynamic forces. In that case, in addition to usual seismic global and local mechanisms, another limit state of the system is the fall of pallets with subsequent damages to goods, people and to the structure itself. Indeed the horizontal inertial forces acting on the pallets may be sufficient to exceed the friction resistance. Nevertheless if the amplitude of the sliding movement is not too important, in such a way that pallets remain on the rack, this effect can benefit to the structure as it limits the horizontal forces on the rack to the friction force at the interface between pallet and beams.

Results presented in this paper are part of a wider research project "Seisracks – Storage racks in seismic area" [2] funded by the European Union (RFCS research program). This research program aims at constituting a scientific background document for the drafting of a European Standard [3] and includes therefore many items such as:

- Experimental determination of friction properties of pallets lying on rack beams;
- Statistical evaluation of the rate of occupancy of racks in order to define the design value of horizontal seismic action, which is directly related to the mass of stored goods;
- Experimental study of the cyclic behavior of beam-to-upright joints and of base anchorages;
- Experimental and numerical study of the global dynamic structural behavior of racks subjected to earthquakes including sliding of pallets.

The present paper intends to develop one of the crucial points of this research, namely the non-linear dynamic time-history analysis of rack structures subjected to earthquake, accounting for the global geometrical non-linearities, for the non-linear material behavior of the joints and for the possible sliding of the pallets with respect to the supporting structure. Additional comparisons with test results are also presented.

2 ADVANCED NUMERICAL TOOL

2.1 General

An advanced numerical tool has been developed in order to be able to evaluate accurately the behavior of racks subjected to seismic action with a due account for possible sliding of supported pallets. The tool is included in the non linear finite element software FineLg developed in University of Liège [4]. Indeed this software already allowed performing step-by-step dynamic analysis accounting for geometrical and material non-linearities of the structure. In particular it was possible to study the response of non-linear structures subjected to an earthquake defined by the time-history of the ground acceleration. The only missing feature was the possibility to let the masses slide.

Figure 1: Example of a storage rack

2.2 Basic concept

The starting point of the development of the sliding-mass model is the use of the concept of “mathematical deck” already available in FineLg since its development by FH Yang [5]. The mathematical deck was elaborated to study the dynamic behavior of structures subjected to moving loads or vehicles and particularly to study the bridge-vehicles interaction.

According to this concept, the interactive behavior is obtained by solving two uncoupled sets of equations, respectively for the structure and for the vehicles, and then by ensuring compatibility and equilibrium at the contact points between the structure and the vehicles with an iterative procedure. In this scheme, the mathematical deck acts as an interface element to evaluate the position of the vehicles with respect to the physical deck and to perform the iterative compatibility process (Fig. 2).

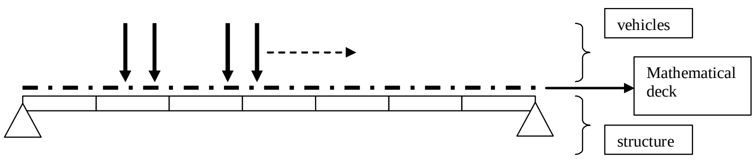

Figure 2: General scheme of the mathematical deck

Regarding the possible movements of the vehicles, the horizontal displacement is imposed according to the speed of the vehicle and to its traffic lane. The vertical displacement, velocity and acceleration are on the contrary the result of a dynamic computation and are obtained from the behavior of the vehicle itself, of the underlying structure and of their possible interaction.

The idea in elaborating the "sliding mass" model is to start from a "moving mass" vehicle without any user-imposed speed and to obtain the horizontal behavior of the mass as the result of a dynamic computation according to a stick/slip model (Fig. 3).

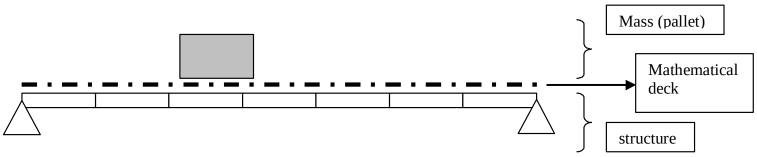

Figure 3: Evolution of the mathematical deck for the sliding mass model (basic scheme)

- "Stick" phase

During this stage, the displacement, velocity and acceleration of both the mass and the underlying structure are identical. The mathematical deck computes thus the horizontal friction force $\boldsymbol{F_h}$ ensuring simultaneously this equality of displacement and the general equations of dynamics for both the mass and the structure, including also the earthquake action (Fig. 4).

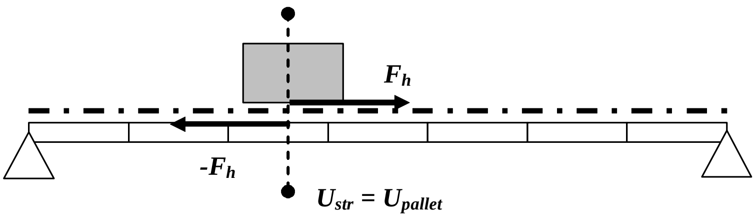

Figure 4: Sliding mass model in "stick" phase

For structures like racks, the supported mass is much more important than the mass of the structure itself ($\boldsymbol{M}$ up to more than 100 times the structural mass). It that case, it is shown in [5] and [7] that the convergence of the iterative procedure to ensure equilibrium and compatibility of the coupled system is very difficult to achieve unless if specific methods are used. In the present case, an Aitken acceleration procedure [6] is developed. The detailed methodology and the way of choosing the best convergence parameters are described in [7].

- "Slip" phase

As soon as the horizontal contact force exceeds the static friction resistance $\boldsymbol{R_{h,st}}$, the mass starts sliding. The dynamic response of the two systems (mass and structure) may then be evaluated separately under a constant contact force equal to the dynamic friction resistance $\boldsymbol{R_{h,dyn}}$ (Fig. 5). During this stage, the pallet moves on the mathematical deck and its position, velocity and acceleration (= $\boldsymbol{R_{h,dyn}/M}$) can be evaluated at any time step. The sliding behavior lasts until the relative velocity between the pallet and the structure becomes equal to zero. From that condition, it can then be evaluated whether and when the next "stick" phase is initiated.

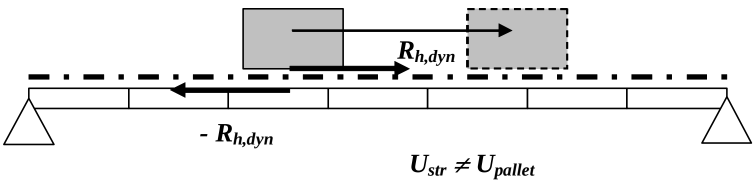

Figure 5: Sliding mass model in "slip" phase

2.3 Validation examples

In order to validate the sliding mass model, a series of very simple systems has been studied with FineLg and compared to equivalent MDOF systems solved with a semi-analytical approach (see ref. [8]). Some of the considered examples are presented in Fig. 6.

The results obtained with FineLg and with the reference semi-analytical procedure are found in very good agreement. As illustration, results obtained with FineLg for case 6.c are plotted in Fig. 7 for μ/α = 1.00 (no sliding) and μ/α = 0.5 (μ is the friction coefficient and α is the maximum imposed acceleration referred to gravity). In this second configuration, 4 sliding phases are observed during which the relative displacement between M2 and M3 varies (green curve on Fig. 7).

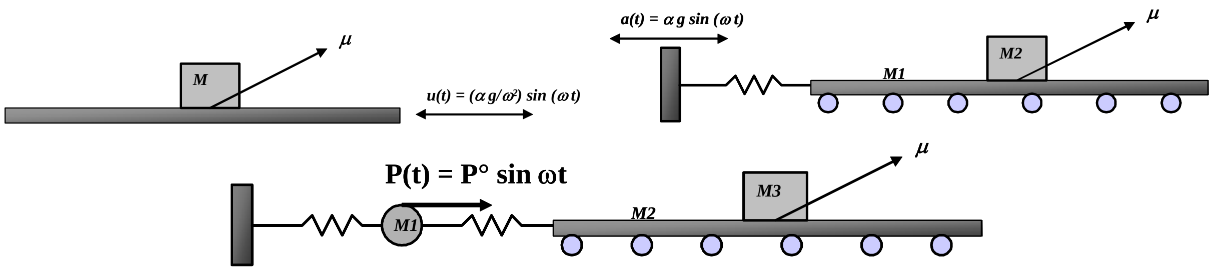

Figure 6: Validation examples (a) 1DOF – (b) 2DOF – (c) 3DOF

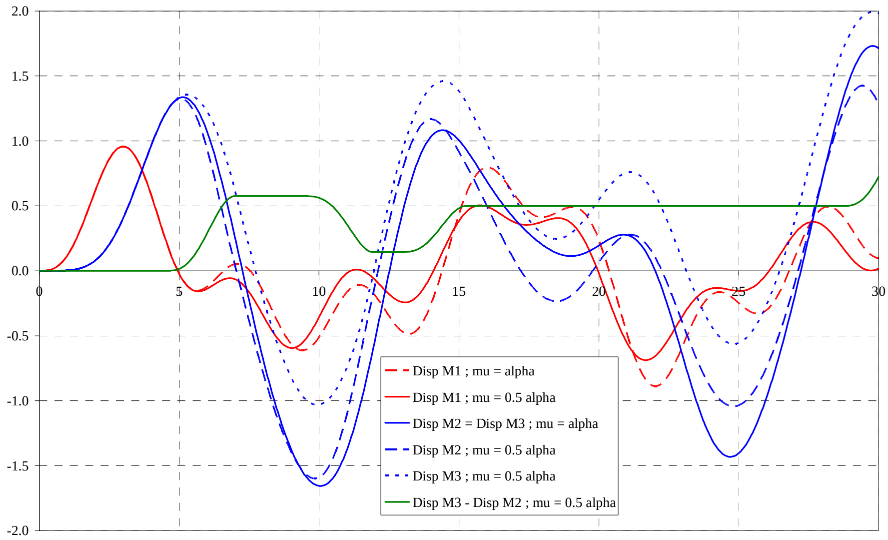

Figure 7: Time-history of the displacements obtained for case 6(c)

3 BEHAVIOR OF A RACK STRUCTURE

3.1 Linear elastic behavior with sliding

This section intends to show an application of the new numerical tool for the step-by-step dynamic analysis of a very simple rack structure subjected to an imposed acceleration of the ground, with account for the possible sliding of the supported masses.

The chosen example comprises two spans and one level with typical dimensions of rack structures (span = 1.8 m – height = 2.0 m; see Fig. 8). The cross section properties of the structural elements are also typical of real rack structures. The beam-to-upright joints and base-anchorages are modeled by springs with appropriate rotational stiffness. Four masses (400 kg) are positioned on the beam. In this basic application, the structure is supposed to behave linearly. This means that no second-order geometrical effects and no yielding of elements are taken into account.

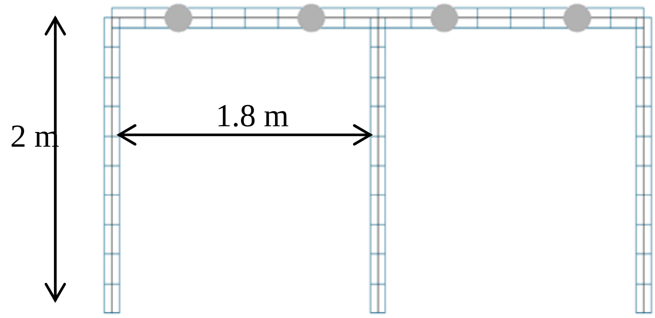

Figure 8: Simple rack structure

The structure is subjected to an imposed acceleration of the ground. The time-history of the imposed acceleration is generated artificially with the software GOSCA [9]. The characteristics of the target response spectrum are:

- EC8 type I spectrum
- PGA = 0.3g
- Soil type C
- Damping ratio ξ = 5 %
- Duration = 15s

The computation is carried out with varying friction properties of the masses, i.e. μ = 0.80, 0.75, 0.60 and 0.30. This covers the normal range of friction coefficient measured for pallets on rack beams [2].

Figures 9.a to 9.d present respectively the evolution of (a) The horizontal displacement of the beam. It is worth noticing that the internal forces in the structure, and in particular the bending moments, are proportional to this displacement – (b) the horizontal acceleration of the beam – (c) the sum of the horizontal contact forces between the beam and the four masses. As the mass of the structure is very small compared to the total additional mass, this contact force may be considered as the total inertial force acting on the structure – (d) the relative displacement of one of the masses with respect to the beam. As the four masses are identical, they exhibit of course the same local displacement. Table 1 summarizes the extreme values derived from the curves of Fig. 9.

The main observations that can be drawn from these figures are the following:

- Racking structures are very flexible. Horizontal displacements are important (δ_h/H = 1/23). This evidence the fact that an accurate analysis of a rack is only possible if second order geometrical effects are duly accounted for;
- The main effect of the sliding of pallets is to limit the horizontal inertial force. With a friction coefficient equal to 0.6, this force is already reduced to 75% of its value without sliding;

- The simple structure studied in this example responds exclusively on its first mode. Therefore the global displacement is directly proportional to the inertial force. The subsequent internal forces are thus also significantly reduced by the limiting effect of sliding;
- Some very sharp peaks are observed for the horizontal acceleration when the friction coefficient decreases. These peaks correspond to strong acceleration or breaking of the structure every time that the masses start or stop sliding;
- The amplitude of the local sliding remains reasonable. Even for very low friction coefficients, the maximum local displacement is less than 5cm.

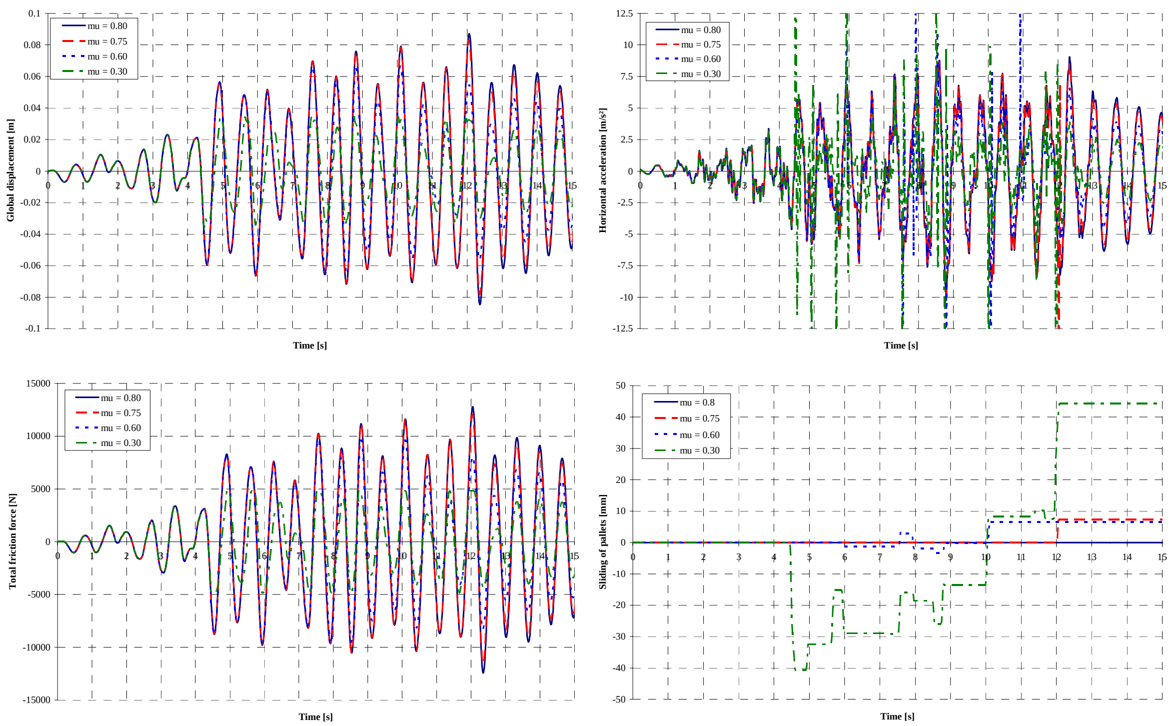

Figure 9: Time-history results for the simple frame – (a) Horizontal displacements – (b) Horizontal acceleration – (c) Total friction force – (d) sliding displacements

μ	Disp. [m]	Acc. [m/s²]	Force [N]	Sliding [mm]
0.80	0.087	9.03	12788	0.00
0.75	0.083	8.74	12000	7.32
0.60	0.067	31.82	9600	6.85
0.30	0.035	69.53	4800	44.34

Table 1: Extreme values from Figure 9(a) to 9(d)

3.2 Non linear behavior

The numerical model presented above has been used to reproduce test results obtained on the shaking table of the Laboratory of Earthquake Engineering of the NTU Athens. Figure 10 shows the tested specimen (2 bays – 3 levels unbraced structure) and the corresponding numerical model.

It is obviously not possible to describe in this paper the whole series of test results and the numerous variations of the different parameters of the model that have been taken into consideration. Results for one intermediate level of acceleration are presented (i.e. peak ground acceleration of the table equal to 0.45g) and only the two parameters having the most important impact on the rack's behavior are commented. The full comparison can be found in [2].

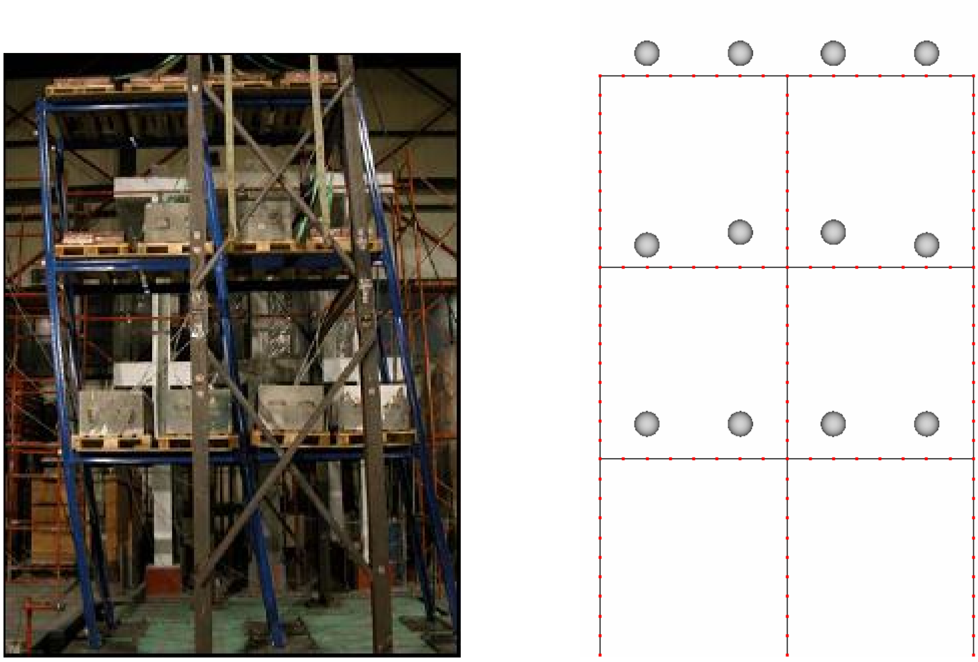

Figure 10: Test specimen and corresponding numerical model

In the model, the second-order geometrical non-linearities of the structure and the material non-linear behavior of the joints are considered. The two main governing parameters have been identified as being (1) the plastic resistance of the base-anchorages and (2) the sliding properties of the masses, even if this second parameter exhibits less influence in the present case of moderate acceleration.

Figure 11 shows the comparison of the displacement of the top level obtained for the test and for the numerical model with different assumptions on the base resistance. In this comparison, the friction coefficient is assumed rather high (0.6) so that no sliding was predicted by the numerical model. The input signal is the time-history of the acceleration recorded on the table during the test. Table 2 summarizes some of the important numerical values of the comparison.

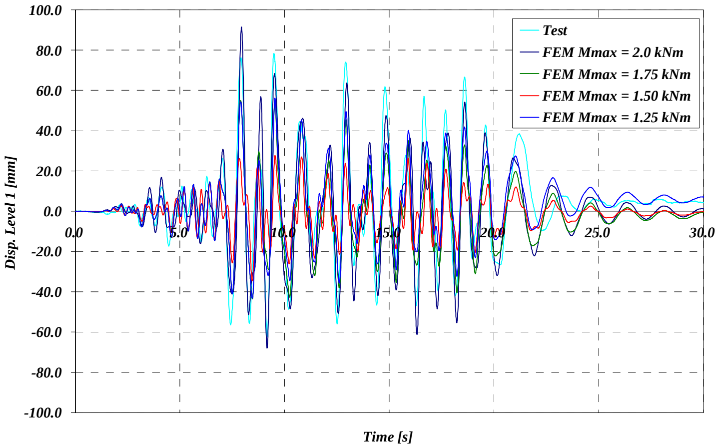

Figure 11: Effect of the column base characteristics on the structural response

	Test	Model (Base resistance = 2.00 kNm)	Model (Base resistance = 1.75 kNm)	Model (Base resistance = 1.50 kNm)	Model (Base resistance = 1.25 kNm)
Maximum displacement [mm]	78.3	91.4	91.4	91.4	90.3
Minimum displacement [mm]	62.2	68.0	71.0	75.9	75.0
Residual displacement [mm]	4.7	0.4	1.5	0.9	7.1

Table 2: Effect of the column base characteristics on the structural response

From these results, the following observations can be drawn:

- The general shape of the time-history response of the structure is correctly predicted by the model.
- It is necessary to account for the non-linear behavior of the joints otherwise the residual displacements of the structure can't be explained.
- Displacements are slightly overestimated (about 15%). However in this comparison, no sliding was accounted for, while some sliding occurred during the test and brought some additional damping to the structure.

Figure 12 shows the comparison of the numerical results obtained for different assumptions on the sliding coefficient (no sliding, μ = 0.40 and μ = 0.30). Fig. 12-a compares the horizontal displacement of the structure, Fig. 12-b compares the contact friction force and Fig. 12-c compares the local sliding. As a matter of additional comparison, Fig. 12-d also presents the sliding recorded for 2 masses during the tests. Table 3 summarizes some interesting values.

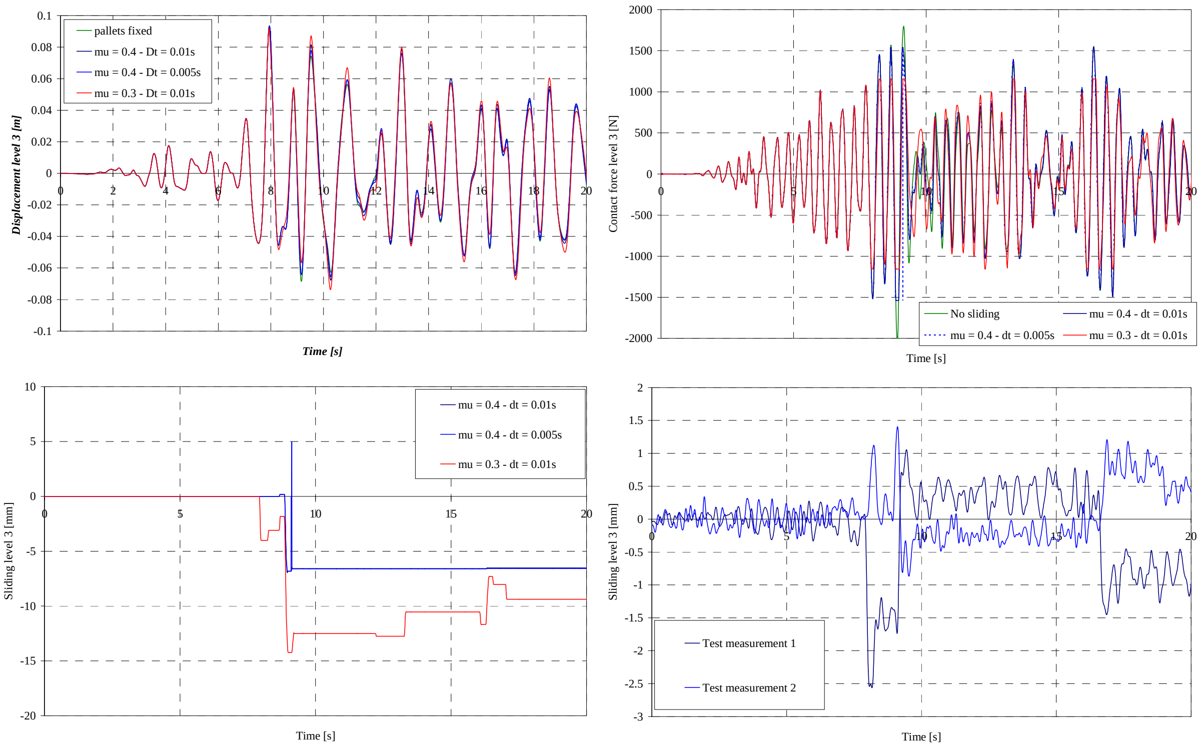

Figure 12: Effect of the sliding of pallets on the structural behavior

	Test	Model without sliding	Model with μ = 0.40	Model with μ = 0.30
Maximum displacement [mm]	78.3	93.4	93.4	91.9
Minimum displacement [mm]	62.2	68.5	67.8	73.8
Maximum friction force [N]	1751	1799	1540	1155
Minimum friction force [N]	1759	2055	1540	1155
Maximum sliding displacement [mm]	2.54	0	6.95	14.23

Table 3: Effect of the sliding of pallets on the structural behavior

From these results, the following observations can be drawn:

- The main effect of the sliding is to limit the contact force transmitted from the pallets to the structure (Fig. 12-b). However for the moderate level of imposed acceleration considered in this example, this limitation is activated only during very short periods corresponding to peaks in the acceleration time-history of the non-sliding system. The consequent effect on the displacement is therefore not significant.
- The moments where some sliding occurs is correctly predicted by the numerical model in comparison with the test results (see Fig. 12-c and 12-d). Even if the order of magnitude is correct (a few millimeters), the amplitude of each individual sliding stage is strongly overestimated, but this should be improved by using slightly higher values of the friction coefficient. This requirement for a higher friction coefficient is also confirmed by the maximum friction forces roughly evaluated from the acceleration measured during the test (see Table 3). It is also important to notice that the maximum sliding displacement is very small (2.5 mm) and corresponds thus to an acceptable motion of the pallets with respect to the structure, from a point of view of "falling of pallets" limit-state.

4 CONCLUSION AND PERSPECTIVES

In this paper, one presented a numerical model able to reproduce in a satisfactory manner the behavior of storage racks subjected to earthquakes, in comparison with analytical examples and with test results. It also emphasized that the main parameters on which it is possible to act for calibrating the model are the behavior of the column bases, the friction coefficient of the pallets and the viscous damping (even if this last aspect was not commented into details in the paper). It is now possible to consider extensive parameter studies with variations of these parameters, in the perspective of developing backgrounds for design recommendations leading to an improved safety of the storage areas for both industrial and commercial applications.

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