

# An investigation on the sliding of pallets on storage racks subjected to earthquake

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**Abstract:** This article presents some basic aspects of a research about the evaluation of the effect of earthquakes on storage racking systems. In particular, it focuses on the possible sliding and friction of the stored good subjected to a horizontal inertial force. In the first part, a theoretical SDOF system is studied to derive general indications on the sliding behavior and to serve as a reference for the validation of a more advanced numerical model. The second part presents then an application of the numerical model to simple rack structures.

**Keywords:** Earthquake engineering, storage racks, friction, sliding

## I. INTRODUCTION

Despite their lightness, storage racking systems made of thin-walled cold formed steel products are able to carry very high live load many times larger than the dead load, opposite to what happens in usual civil engineering structures. These racks can also raise considerable height. For these reasons, their use is nowadays very common in warehouses (see fig. 1).

However, these structures have to be carefully designed. Indeed many difficulties arise in the prediction of their structural behavior, such as instabilities (global, local and distortional) or modeling problems (beam-upright connection stiffness, base plate anchoring) [1].

Things become even more complicated when a storage rack is installed in a seismic zone where, subjected to earthquake, it has to withstand horizontal dynamic forces. In that case, in addition to usual seismic global and local mechanisms, another limit state of the system is the fall of pallets with subsequent damages to goods, people and to the structure itself. Indeed the horizontal inertial forces acting on pallets may be sufficient to exceed the friction resistance.

Nevertheless if the amplitude of the sliding movement is not too important, in such a way that pallets remain on the rack, this effect can benefit to the structure as it limits the horizontal forces on the rack to the friction force at the interface between pallet and beams.

Results presented in this paper are part of a wider research project [2] funded by the European Union and focusing specifically on the seismic behavior of storage racks. This research includes many items such as:

- Experimental determination of friction properties of pallets lying on rack beams;
- Statistical evaluation of the rate of occupancy of racks in order to define the design value of horizontal seismic action, which is directly related to the mass of stored goods;

- Study of the dynamic structural behavior of racks subjected to earthquakes, with a special look at sliding of pallets.

This paper intends to develop three main aspects:

- The study of a simple SDOF sliding system subjected to sinusoidal ground motion, in order to get reference results and general trends;
- The development of an advanced numerical tool to be used for the modeling of whole racking systems;
- A limited study of simplified rack systems intending to show the abilities of the new numerical tool in practical situations.



Fig. 1. Example of a storage rack

## II. SDOF SLIDING SYSTEM

The simple system studied in this chapter is showed at Fig. 2. It consists in a mass  $M$  laid down on a rigid support subjected to a horizontal sinusoidal imposed acceleration. The friction coefficient of the mass on its support  $\mu$  is supposed to be constant. The movement of the support is noted  $u(t)$  while the absolute displacement of the mass is noted  $x(t)$ .

The detailed analytical developments summarized hereafter can be found in Ref. [3].

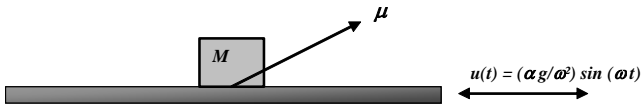


Figure 2 – SDOF system

The behavior of this system can be separated into two phases.

(1) While the inertial force on the mass doesn't exceed the friction force at the interface, the mass is stuck to the support. The governing equations are thus:

$$\begin{cases} \ddot{x}(t) = \ddot{u}(t) = -\alpha g \sin \omega t \\ \dot{x}(t) = \dot{x}(t_0) + \frac{\alpha g}{\omega} (\cos \omega t - \cos \omega t_0) \\ x(t) = x(t_0) + (\dot{x}(t_0) - \alpha g \omega \cos \omega t_0)(t - t_0) \\ \quad + \frac{\alpha g}{\omega^2} (\sin \omega t - \sin \omega t_0) \end{cases} \quad (1)$$

$$\text{while } F(t) = M \ddot{x}(t) < \mu M g \quad (2)$$

Therefore, according to (1.a) and (2), it comes that no sliding of the mass can occur if the non-dimensional maximal acceleration of the support is lower than the friction coefficient ( $\alpha < \mu$ ).

(2) As soon as the inertial force exceeds the friction resistance, the mass starts sliding on the support. The mass is thus subjected to a constant horizontal force equal to the friction force. The governing equations are:

$$\begin{cases} \ddot{x}(t) = \frac{\pm \mu M g}{M} \\ \dot{x}(t) = \dot{x}(t_0) \pm \mu g (t - t_0) \\ x(t) = x(t_0) + \dot{x}(t_0)(t - t_0) \pm \mu g \frac{(t - t_0)^2}{2} \end{cases} \quad (3)$$

The mass starts again sticking to the support as soon as the relative velocity between the mass and the support become equal to zero.

The moment at which this re-sticking happens can be evaluated by solving Eq. (4) obtained by equating the absolute velocity of the mass and of the support. Eq. (4) can't unfortunately be solved analytically. An example of graphical solution is presented at Fig. 3, in which the two red curves are respectively the left-hand and right-hand side of (4). The solution of this equation is noted  $(\omega t_1)$ .

$$K - \frac{\mu}{\alpha} \omega t = \cos \omega t \quad (4)$$

$$\text{with } K = \frac{\mu}{\alpha} \arcsin\left(\frac{\mu}{\alpha}\right) + \sqrt{1 - \left(\frac{\mu}{\alpha}\right)^2}$$

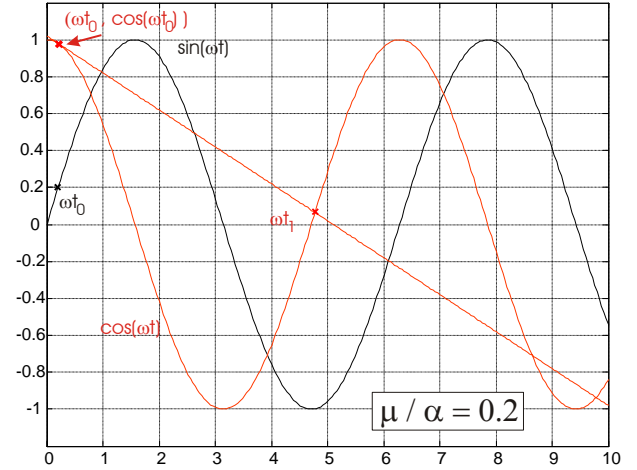


Figure 3 – Graphical evaluation of the re-sticking time

An additional problem to solve consists in evaluating the inertial force at time  $t_1$ . Indeed in order to obtain an effective re-sticking, this force must not exceed the friction force, otherwise the mass starts immediately sliding again. The second condition for re-sticking is thus given by Eq. (5).

$$|\sin \omega t_1| < \frac{\mu}{\alpha} \quad (5)$$

Condition (5) is commented at Fig. 4.a and 4.b. In these figures, the shadowed areas correspond to time-periods for which the condition is verified. Therefore, if the behavior of the system is such that the time  $t_1$  evaluated from Eq. (4) and Fig. 3 falls in a shadowed area, the system exhibits an effective re-sticking (Fig. 4.a -  $\mu/\alpha = 0.6$ ). Otherwise if  $t_1$  is outside one of these areas, the mass goes on sliding. However, it can be demonstrated (see Ref. [3]) that the inversion of the sign of the relative velocity results in an inversion of the sign of the friction force. The sliding sense of the mass is then inverted (Fig. 4.b -  $\mu/\alpha = 0.4$ ).

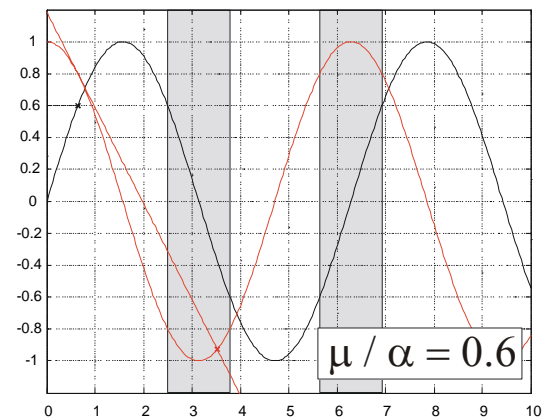


Figure 4.a – Second condition of re-sticking verified

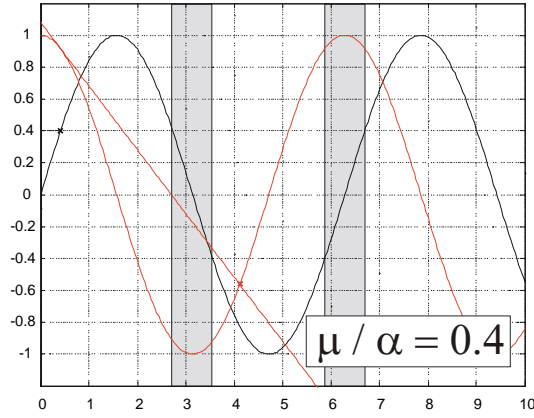


Figure 4.b – Second condition of re-sticking not verified

The analysis of the SDOF system can then be continued according to the principles developed above, in order to evaluate the whole stick-slip behavior of the mass. Figures 5.a and 5.b present one more step for  $\mu/\alpha = 0.6$  and  $\mu/\alpha = 0.4$ .

$\mu/\alpha = 0.6$  (Fig. 5.a):

- Up to  $t_0$ : initial stick phase;
- From  $t_0$  to  $t_1$ : first sliding phase;
- From  $t_1$  to  $t_2$ : stick phase – the mass follows the support;
- From  $t_2$  to  $t_3$  – second sliding phase;
- From  $t_3$  to  $t_4$  – stick phase
- ...

$\mu/\alpha = 0.4$  (Fig. 5.b):

- Up to  $t_0$ : initial stick phase;
- From  $t_0$  to  $t_1$ : first sliding phase;
- From  $t_1$  to  $t_3$ : second sliding phase (inverted sense of the sliding);
- ...

Fig. 6 summarizes the general behavior of the system for different values of the ratio  $\mu/\alpha$ . As illustrative examples, Fig. 7 presents the displacement, velocity and friction force for three particular values of the ratio. The following conclusions can be drawn from these two figures:

- If  $\mu/\alpha$  is greater than 1.0, the mass remains stuck to the support all along the loading;
- If  $\mu/\alpha$  is between 0.537 and 1.0, the movement of the mass is an alternation of stuck and sliding phases. For what regards the sliding phases, the sign of the friction force changes from a phase to the following one. The movement is thus oscillatory around its equilibrium position (see figure 7, case 1). The distance between the initial position of the mass and the equilibrium position is related to the friction coefficient. The smaller the coefficient, the more distant the equilibrium position.
- If  $\mu/\alpha$  is smaller than 0.537, the movement of the mass is a first stuck phase followed by alternated slidings corresponding to alternate changes in the sign of the friction force. These changes of the sign are represented by the solid lines on Fig. 6. In this case, it is observed that the general movement of the

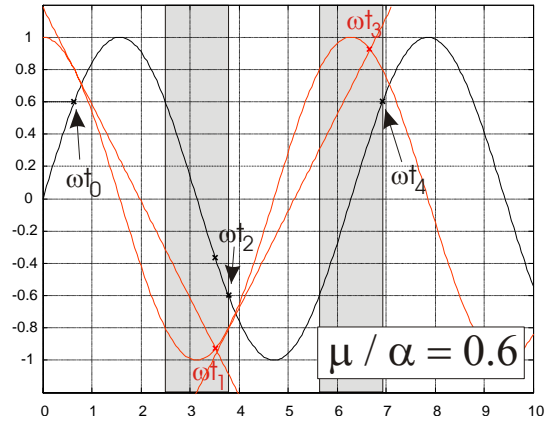


Figure 5.a – Behavior of the SDOF system -  $\mu/\alpha = 0.6$

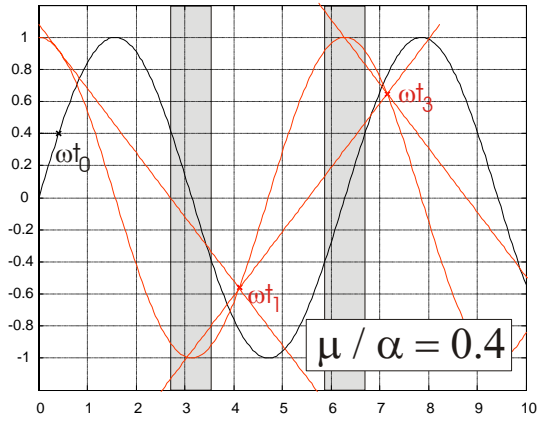


Figure 5.b – Behavior of the SDOF system -  $\mu/\alpha = 0.4$

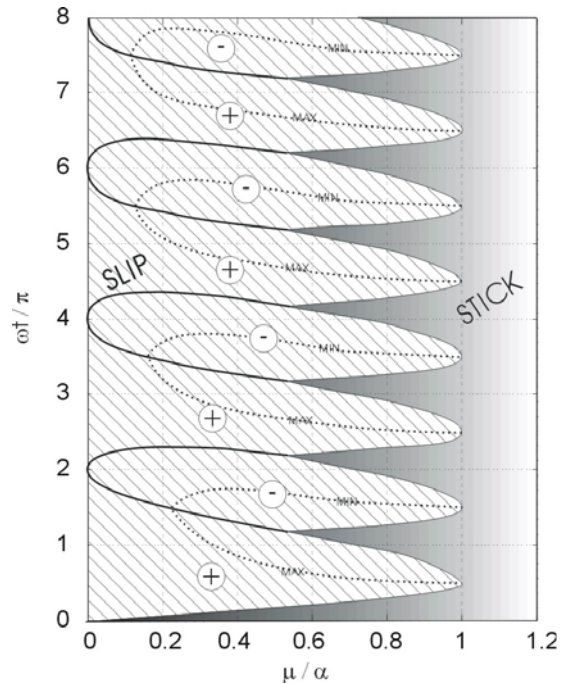


Figure 6 – Summary of the behavior of the SDOF system as a function of the ratio  $\mu/\alpha$



mass is an oscillation around an equilibrium position that is moving along the supporting device. However this equilibrium position tends to stabilize after a sufficient duration, this duration being an inverse function of the friction coefficient.

Figure 7 (cases 2 and 3) illustrates this behavior. In case 2, the equilibrium position is obtained rather quickly, while in case 3, the global movement of the mass is much more significant before to find a stabilized state. It can also be noticed that the ratio between the global displacement of the equilibrium position and the amplitude of the oscillatory component is increasing when the friction coefficient decreases.

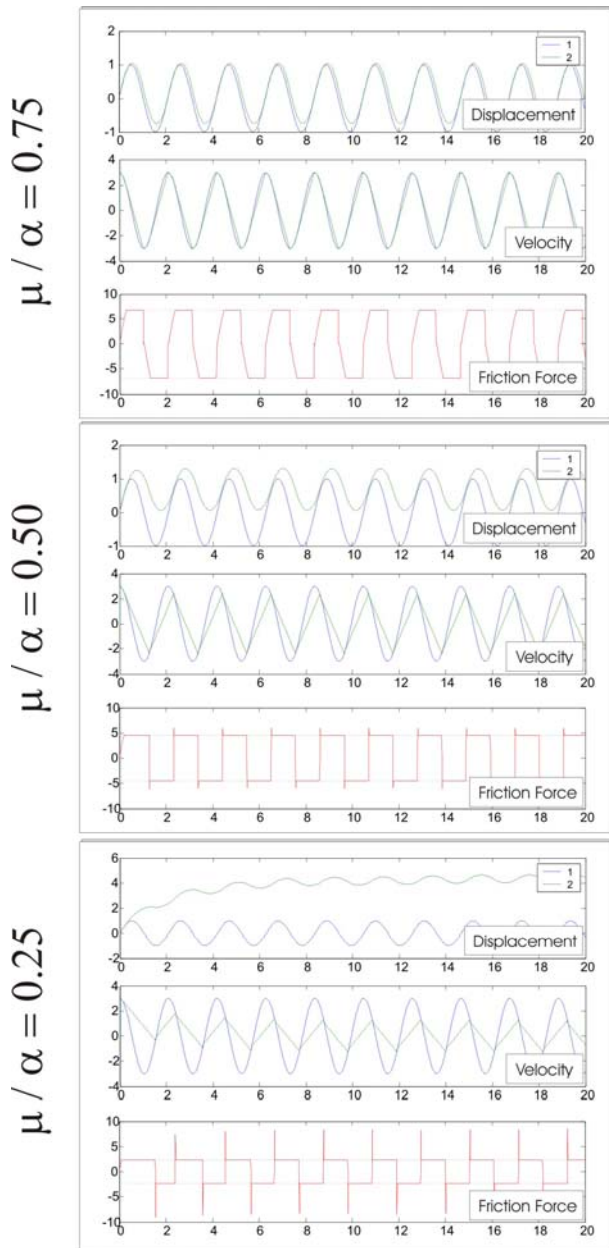


Figure 7 – Displacement, velocity and friction force for the SDOF system with  $\mu/\alpha$  equal 0.75 (case A), 0.5 (case B) and 0.25 (case C) – 1: support; 2: supported mass

### Consequences on the behavior of pallets on racks

As far as a pallet may be considered as a simple mass, two main conclusions can be drawn.

- The general movement of a pallet exhibiting ‘stick and slip’ behavior is a global displacement in the direction of the first sliding. This conclusion is in agreement with experimental observations made during tests carried out on the shaking table of NTUA Athens [2].
- The ratio  $\mu/\alpha$  - where  $\alpha$  is the relative acceleration of the support referred to  $g$  - is the main parameter allowing evaluating whether the pallet will slide during an earthquake or not. In the case of a pallet lying on a rack,  $\alpha$  is the relative horizontal acceleration of the supporting beam. Its value can be estimated from both the movement of the soil during the earthquake and the dynamic properties of the rack. The main difficulty is that almost all the mass of the system is actually the mass of the pallets. Therefore as soon as the pallets start sliding, the dynamic properties of the rack may be significantly modified, with consequences on the relative acceleration  $\alpha$ . The procedure to evaluate the possible sliding should thus either be based on the above conclusions on SDOF systems, coupled with an iterative procedure to evaluate the relative acceleration of each supporting beam, or make use of advanced structural dynamics computational tools.

### III. ADVANCED NUMERICAL MODEL

In order to be able to evaluate precisely the behavior of racks subjected to seismic action with a due account for possible sliding of supported pallets, an advanced numerical tool is developed. The tool is included in the non linear finite element software FineLg [4]. Indeed this software already allows performing step-by-step dynamic analysis accounting for geometrical and material non linearity's of the structure. In particular it is possible to study the response of a structure subjected to an earthquake defined by the time-history of the ground acceleration. The only missing feature is the possibility to let the masses slide.

#### A. Basic concept

The starting point of the development of the sliding-mass model is the use of the concept of “mathematical deck” already available in FineLg since its development by FH Yang [5]. The mathematical deck was elaborated to study the dynamic behavior of structures subjected to moving loads or vehicles and particularly to study the bridge-vehicles interaction.

According to this concept, the interactive behavior is obtained by solving two uncoupled sets of equations, respectively for the structure and for the vehicle, and by ensuring conditions of geometrical compatibility and equilibrium at the interaction points between the structure and the vehicle using an iterative procedure. In this scheme, the mathematical deck acts as an interface element to evaluate the position of the vehicles with respect to the physical deck and to perform the iterative compatibility process (Fig. 8).

Regarding the possible movements of the vehicles, the horizontal displacement is imposed according to the speed of the vehicle and to its traffic lane. The vertical displacement, velocity and acceleration are at the contrary the result of a dynamic computation and are obtained from the behavior of the vehicle itself, of the underlying structure and of their possible interaction.

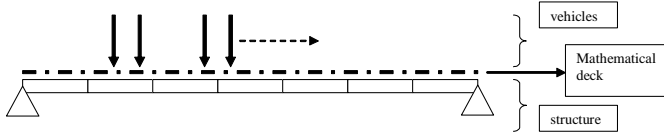


Figure 8 – General scheme of the mathematical deck

The idea in elaborating the "sliding mass" model is to start from a "moving mass" vehicle without any user-imposed speed and to make the horizontal behavior of the mass be the result of a dynamic computation according to a stick/slip model (Fig. 9).

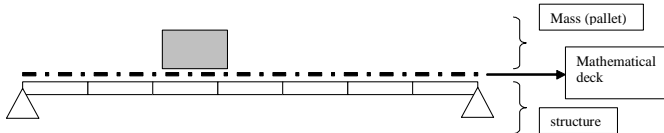


Figure 9 – Evolution of the mathematical deck for sliding mass – basic scheme

#### 1) Stuck phase

During this stage, the displacement, velocity and acceleration of both the mass and the underlying structure are the same. The mathematical deck computes thus the horizontal friction force  $F_h$  necessary to ensure simultaneously the compatibility and the general equations of dynamics including the whole set of external actions (Fig. 10.a).

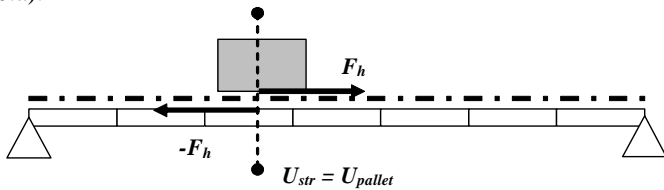


Figure 10.a – Sliding mass model in stuck phase

For structures like racks, the supported mass is much more important than the mass of the structure itself ( $M$  up to more than 100 times the structural mass). In that case, the experience shows that the convergence of the iterative procedure for ensuring equilibrium and compatibility of the coupled system is rather difficult to achieve unless using specific methods. Therefore the particular iterative approach chosen here is an Aitken acceleration procedure (see [5] and [6]).

#### 2) Sliding phase

As soon as the horizontal contact force exceeds the static friction resistance  $R_{h,st}$ , the mass starts sliding. The dynamic response of the two systems (mass and structure) may then be

evaluated separately under a constant contact force equal to the dynamic friction resistance  $R_{h,dyn}$  (Fig. 10.b). During this stage, the pallet moves on the mathematical deck and its position, velocity and acceleration ( $= R_{h,dyn}/M$ ) can be evaluated at any time step. The sliding behavior lasts until the relative velocity between the pallet and the structure becomes equal to zero. From that condition, it can then be evaluated whether and when the pallet starts sticking again.

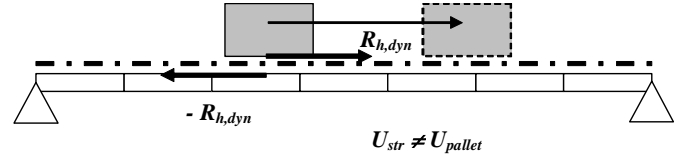


Figure 10.b – Sliding mass model in sliding phase

#### B. Validation examples

In order to validate the sliding mass model, a series of very simple systems has been studied with FineLg and compared to equivalent MDOF systems solved with the semi-analytical approach followed in part II of the paper. Some of the considered examples are described in the next paragraphs.

##### 1) Example 1 – mass on rigid support with imposed sinusoidal acceleration

This example is the SDOF system studied analytically in the first part of the paper (Fig. 2). The results obtained with FineLg are perfectly similar to the curves of Figure 7.

##### 2) Example 2 – 3DOFs system subjected to a sinusoidal force applied on one of the DOFs

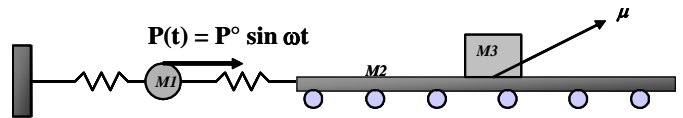


Figure 11.a – Simple example n° 2

The results obtained with FineLg and with the reference semi-analytical procedure are found in very good agreement. FineLg results are plotted in Fig. 11.b for  $\mu/\alpha = 1.00$  (no sliding) and  $\mu/\alpha = 0.5$ . In this last case, 4 sliding phases are observed during which the relative displacement between M2 and M3 varies.

##### 3) Example 3 – 2DOFs system subjected to an imposed acceleration of the support

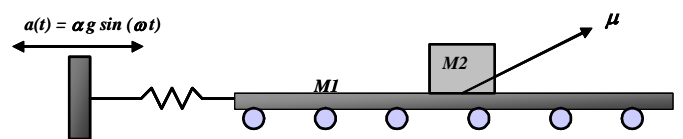


Figure 12.a – Simple example n° 3

Results are once again found in perfect agreement. Displacement of mass M1 is plotted on Fig. 12.b for decreasing values of the friction coefficient.

As expected, it is observed that accounting for the possible sliding of the supported mass M2 limits the inertial action on

the supporting structure M1 and reduces its maximal displacement in consequence.

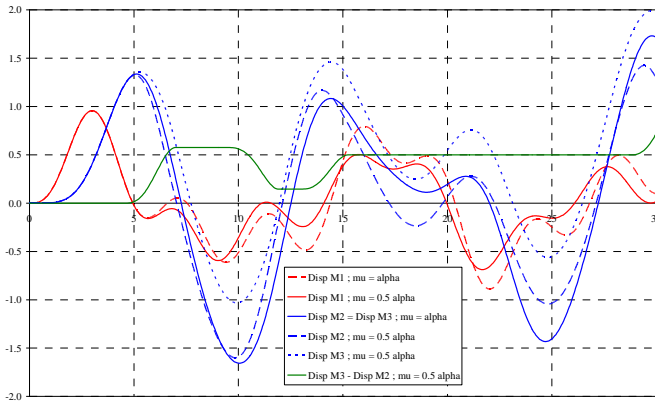


Figure 11.b – Results of the simple example n°2

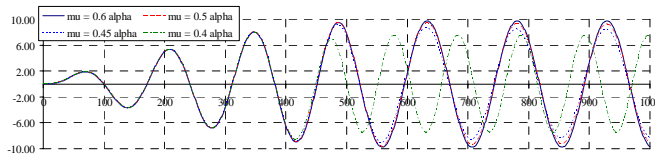


Figure 12.b – Results of the simple example n°3

#### IV. BEHAVIOR OF A SIMPLE RACK STRUCTURE

This chapter intends to show the application of the new numerical tool for the step-by-step dynamic analysis of a simple rack structure subjected to an imposed acceleration of the ground, with account for the possible sliding of the supported masses.

##### A. Definition of the example

The chosen structure comprises two spans and one level with typical dimensions of rack structures (span = 1.8 m – height = 2.0 m; see Fig. 13). The cross section properties of the structural elements are summarized in Table 1. Four masses (400 kg) are positioned on the beam.

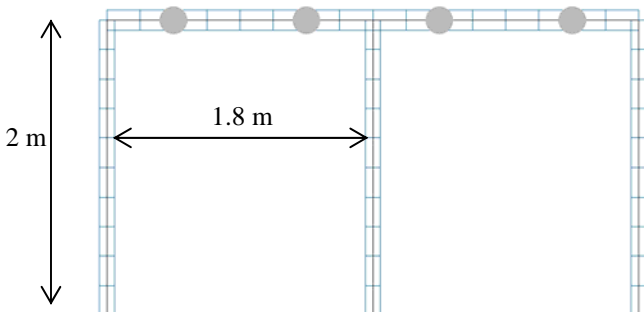


Figure 13 – Simple rack structure

TABLE 1

CROSS SECTION PROPERTIES OF THE STRUCTURAL COMPONENTS OF THE RACK

Element	Area	Inertia
<i>Beams</i>	605.8 mm <sup>2</sup>	93.8 cm <sup>4</sup>
<i>Uprights</i>	488.9 mm <sup>2</sup>	41.3 cm <sup>4</sup>
<b>Equivalent spring stiffness</b>		
<i>Beam-to-upright connection</i>	160 kNm	
<i>Base anchorage</i>	160 kNm	

In this application, the structure is supposed to behave linearly. It means that no second-order geometrical effects and no yielding of elements are taken into account.

##### Eigenmodes

The eigenmodes of the structure are computed with FineLg before performing the step-by-step dynamic analysis. The only relevant mode for horizontal seismic action – with 99.6% of collaborating mass – is plotted on Fig. 14.

The corresponding period is equal to 0.65s.

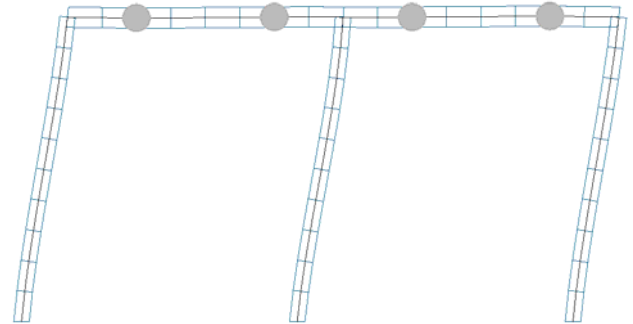


Figure 14 – First horizontal eigen-mode of the structure

##### Loading

The structure is subjected to an imposed acceleration of the ground. The time-history of the imposed acceleration is generated artificially with the software GOSCA [7]. The characteristics of the target response spectrum are:

- EC8 type I spectrum
- PGA = 0.3g
- Soil type C
- $\eta = 5\%$
- Duration = 15s

The generated accelerogram, the corresponding response spectrum and the target spectrum are given in Fig. 15.a and 15.b.

##### Friction properties

The computation is carried out with varying friction properties of the masses, i.e.  $\mu = 0.80, 0.75, 0.60$  and  $0.30$ . This covers the normal range of friction coefficient measured for pallets on rack beams.

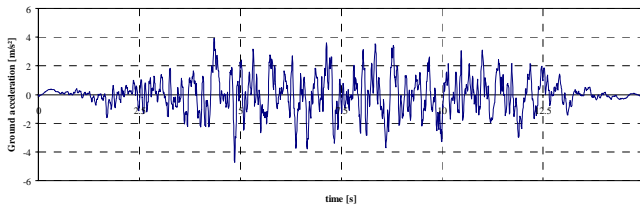


Figure 15.a – Imposed acceleration of the ground

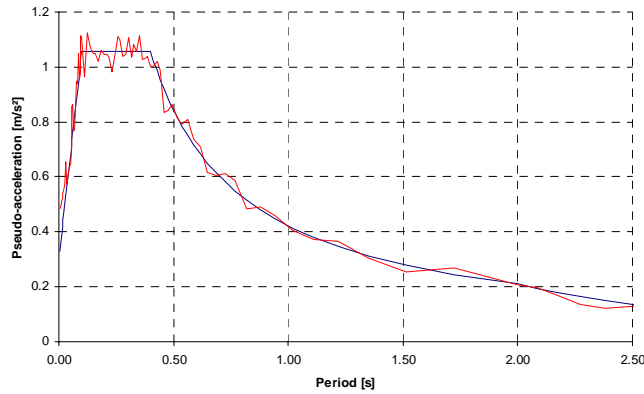


Figure 15.b – Corresponding response spectrum and target EC8 spectrum

## B. Results

Figures 16.a to 16.d present respectively the evolution of

- The horizontal displacement of the beam. It is worth noticing that the internal forces in the structure, and in particular the bending moments, are proportional to this displacement;
- The horizontal acceleration of the beam;
- The sum of the horizontal contact forces between the beam and the four masses. As the mass of the structure is very small compared to the total additional mass, this contact force may be considered as the total inertial force acting on the structure;
- The relative displacement of one of the masses with respect to the beam. As the four masses are identical, they exhibit of course the same local displacement.

Table 2 summarizes the extreme values derived from the curves of Fig. 16.

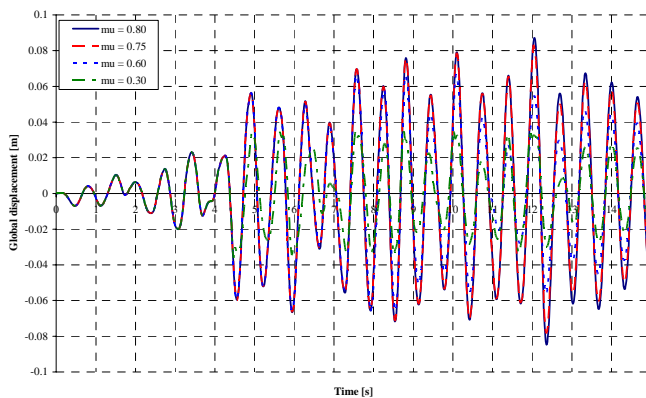


Figure 16.a – Horizontal displacement

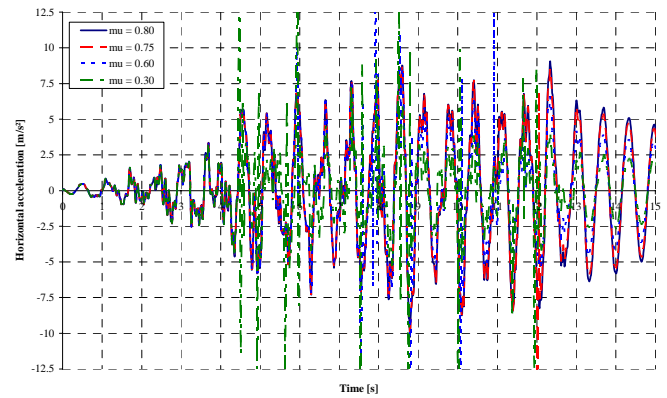


Figure 16.b – Horizontal acceleration

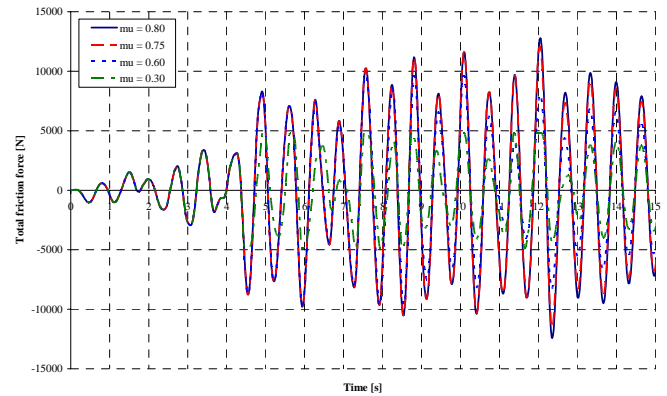


Figure 16.c – Total friction force

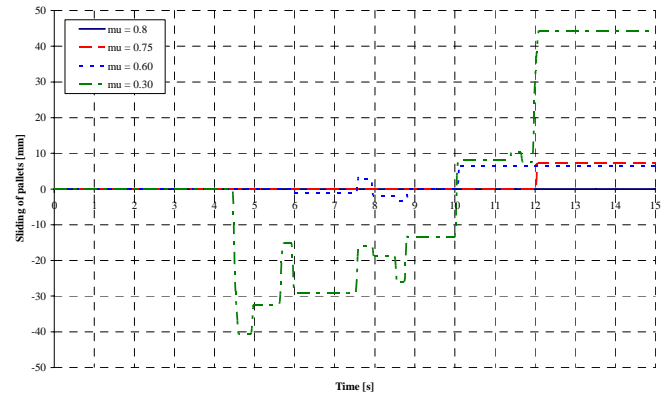


Figure 16.d – Local displacements

TABLE 2

EXTREME VALUES FROM FIG. 16

$\mu$	Disp.	Acc.	Force	Sliding
0.80	0.087	9.03	12788	0.00
0.75	0.083	8.74	12000	7.32
0.60	0.067	31.82	9600	6.85
0.30	0.035	69.53	4800	44.34

## Observations

- Racking structures are very flexible. Horizontal displacements are important ( $\delta_h/H = 1/23$ ). A complementary non linear computation accounting for geometrical second order effects should be necessary.

However the results presented here are very significant at a qualitative point of view;

- The main effect of the sliding of pallets is to cut the horizontal inertial forces. With a friction coefficient equal to 0.6, the force is already reduced to 75% of its non sliding value;
- The simple structure studied in this paper responds exclusively on its first mode. Therefore the global displacement is directly proportional to the inertial force. The subsequent internal forces are thus also significantly reduced by the cutting effect of sliding;
- Some very important peaks are observed for the horizontal acceleration when the friction coefficient decreases. These peaks correspond indeed to strong acceleration or breaking of the structure every time that the masses start or stop sliding;
- The amplitude of the local slidings remains reasonable. Even for very low friction coefficients, the maximum local displacement is less than 5cm.
- The conclusion of part II of the paper can be commented again with regard to this example. The maximum relative acceleration  $\alpha$  of the supporting beam is here equal to  $9.03/g \approx 0.9$ . The sliding condition defined by Eq. (2) for sinusoidal excitation of the support forecast thus sliding of the pallets if the friction coefficient is under 0.9. However on the example, no sliding is observed above 0.75. It seems then that the use of the simplified verification formula is safe-sided regarding the initiation of the sliding.

## V. CONCLUSIONS AND PERSPECTIVES

This paper has presented a numerical tool for the simulation of the sliding of masses on structures subjected to earthquakes, with a particular emphasis on the sliding of pallets on storage racks. The numerical developments have been validated by comparisons with analytical and semi-analytical results. An example of simple structure has then been treated, leading to interesting observations on the prediction of the sliding of pallets and on the favorable effect of a controlled sliding on the internal forces in the structure.

The next steps of this research should consist in accounting for the non linear geometrical effects and in investigating structures comprising a greater number of storage levels requiring thus the participation of more than one single eigenmode for the evaluation of the dynamic response.

## VI. ACKNOWLEDGEMENTS

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