Several years El Niño forecast using a wavelet-based mode decomposition

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9/10/2012

From the Fourier series

A Fourier series decomposes a periodic signal into a possibly infinite sum of sines and cosines functions. From a practical point of view, a signal f is decomposed as a sum of K cosines:

$$f(t) \approx \sum_{k=1}^{K} c_k \cos(\omega_k t + \phi_k).$$

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One thus only gets an approximation of the original signal. Such a decomposition often leads to a decomposition with too many terms, i.e. with a too large K.

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One thus tries to have the following decomposition,

$$f(t) \approx \sum_{j=1}^{J} C_j(t) \cos(\omega_j t + \phi_j),$$

where J should be much smaller than K.

The wavelet spectrum

We use the progressive wavelet

$$\psi(t) = \frac{1}{2\sqrt{2\pi}} \exp(i\Omega t) \exp(-\frac{(2\Omega t + \pi)^2}{8\Omega^2})(\exp(\frac{\pi t}{\Omega}) + 1),$$

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with $\Omega = \pi\sqrt{2/\ln 2}$.
The wavelet spectrum associated to the signal f is defined as

$$\Lambda(a) = E|Wf(\cdot, a)|,$$

where Wf is the continuous wavelet transform of f and E denotes the mean over the time t.

The wavelet decomposition

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The signal f is then decomposed in the following way,

$$f(t) = f_0(t) + \sum_{j=1}^{J} f_j(t),$$

with

$$f_j(t) = |Wf(t, a_j)| \cos\big(\arg Wf(t, a_j)\big),$$

if $j \geq 1$ and $f_0 = f - \sum_{j=1}^{J} f_j$.

An example

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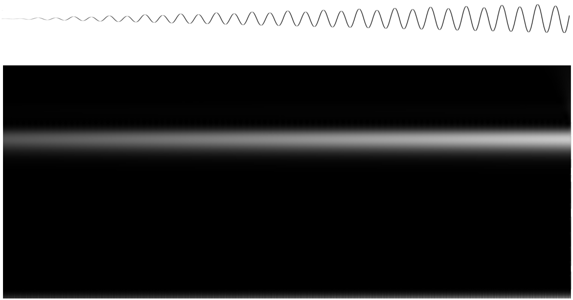

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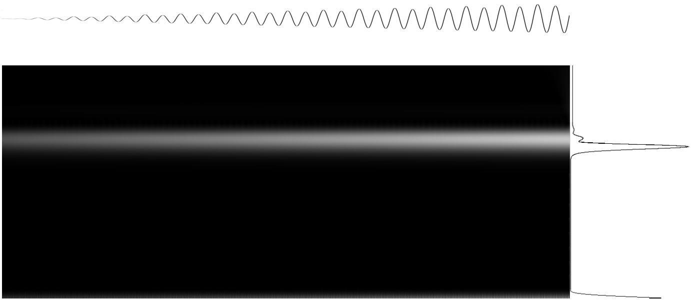

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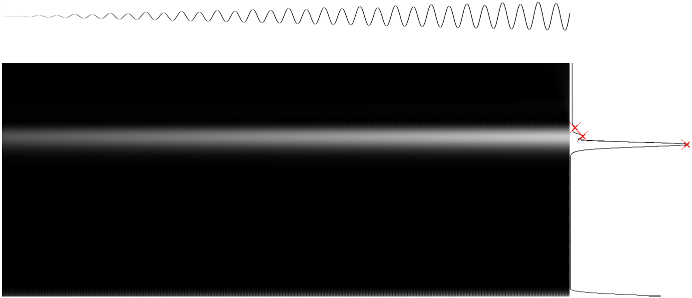

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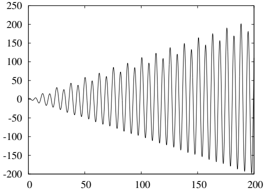

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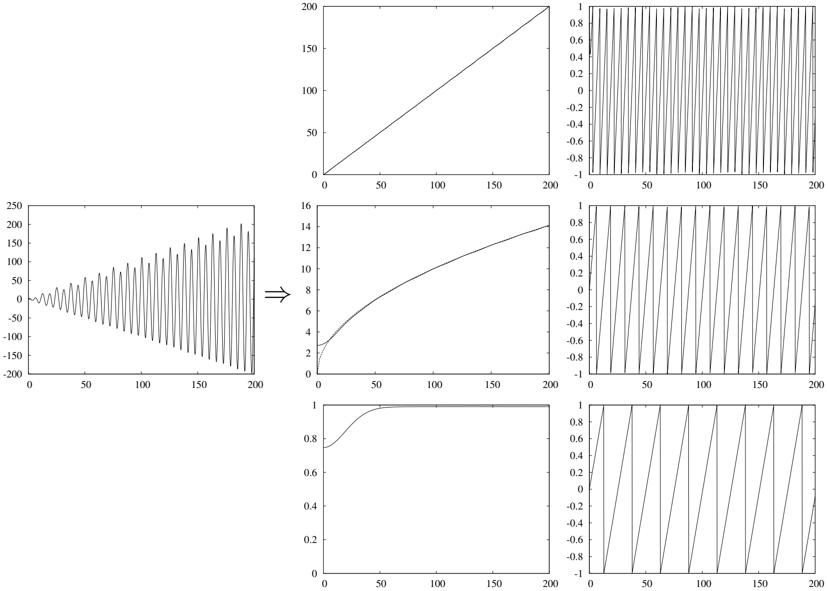

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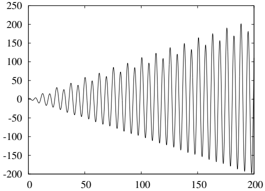
250
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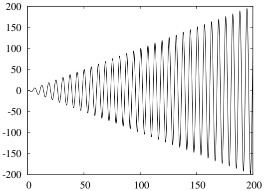
⇒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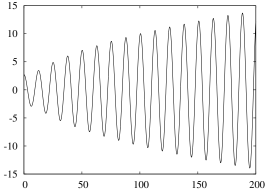
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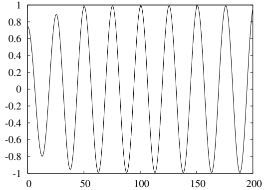
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0.2
0
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-1
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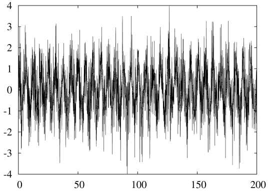
150
200

Noise-resistance



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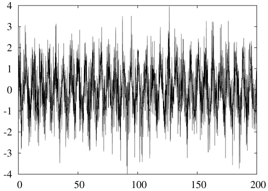

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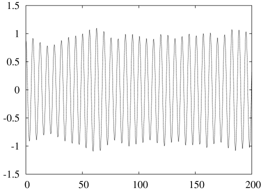

Approximation of the signal

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If we assume that the mean frequency of a component f_j is constant, the reconstructed signal has the following form,

$$f(t) \approx \hat{f}(t) = \sum_{j=1}^{J} C_j(t) \cos(\omega_j + \phi_j),$$

where ω_j corresponds to the mean (angular) frequency.

Extrapolation

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Summing the so-obtained extrapolated values related to each component defines the forecast; more precisely, it corresponds to a forecast of the reconstructed signal $\hat{f}$.

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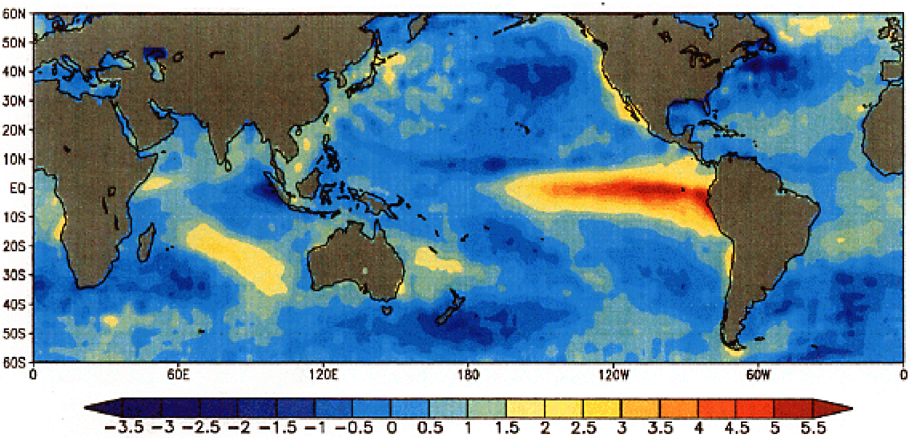
Summing the so-obtained extrapolated values related to each component defines the forecast; more precisely, it corresponds to a forecast of the reconstructed signal $\hat{f}$.
The efficiency of the predictions was tested with probing hindcasts.

El Niño and its impact on the guano harvest

Mining guano in the Central Chincha Islands off the southwest coast of Peru (1860)

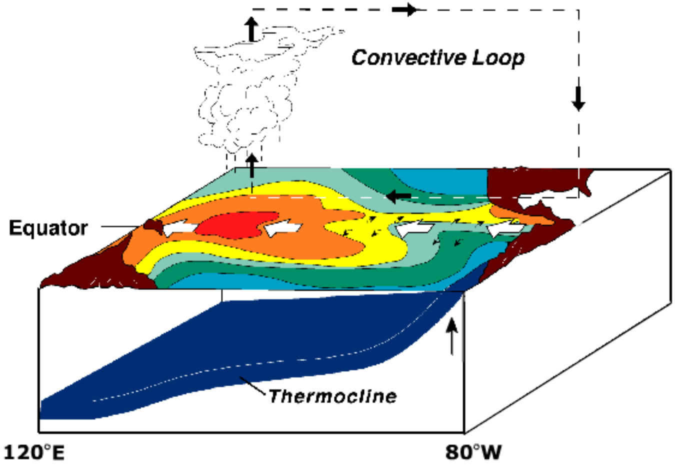
SST anomaly during the El Niño phenomenon in 1997



Circulation

Normal Pacific pattern: Equatorial winds gather warm water pool toward west. Cold water upwells along South American coast.

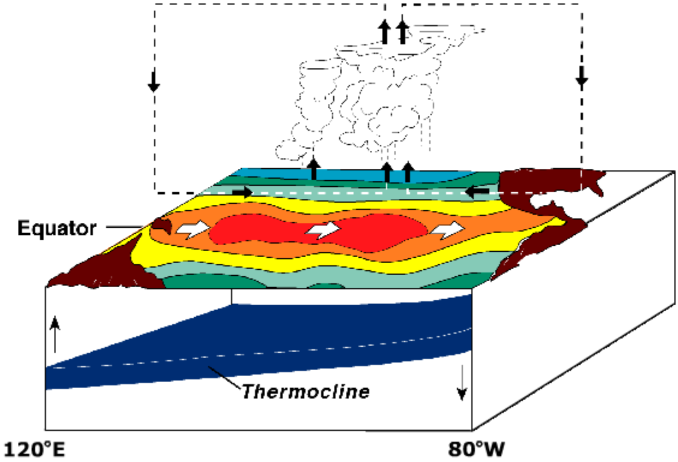
Circulation



Circulation

El Niño Conditions: Warm water pool approaches South American coast. Absence of cold upwelling increases warming.

Circulation



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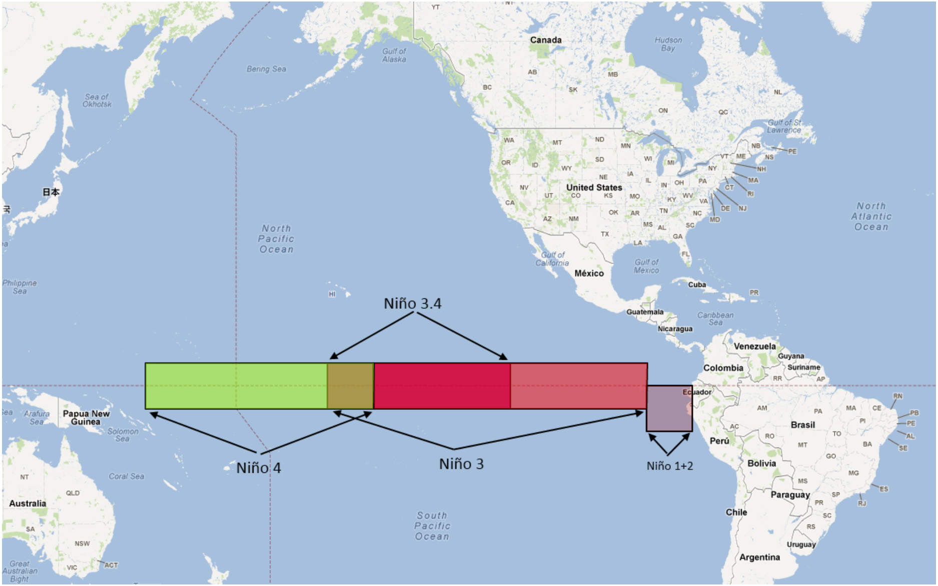
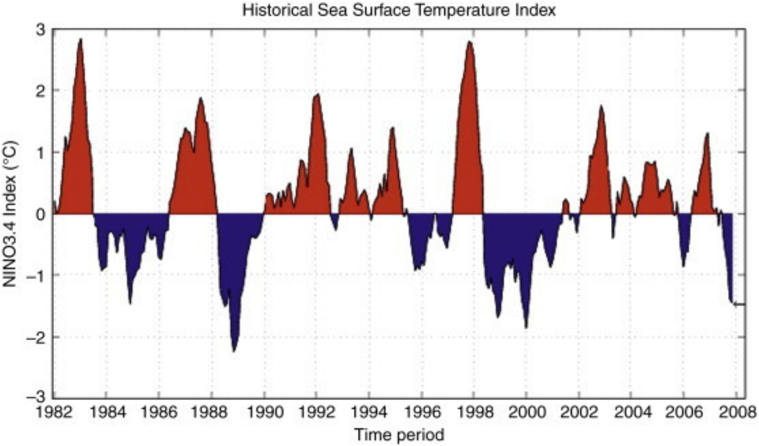
- The accepted definition is a warming or cooling of at least 0.5 °C averaged over the east-central tropical Pacific Ocean.
- Typically, this anomaly happens at irregular intervals of 3–7 years and lasts nine months to two years.
- When this warming or cooling occurs for only seven to nine months, it is classified as El Niño/La Niña "conditions"; when it occurs for more than that period, it is classified as El Niño/La Niña "episodes".

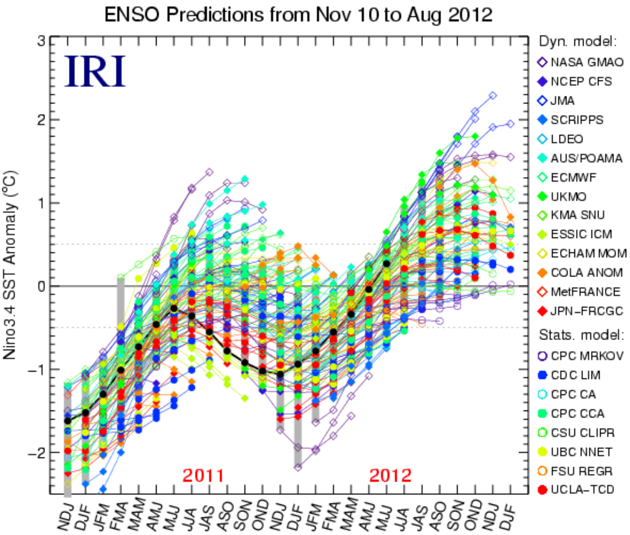
The Nino 3.4 index



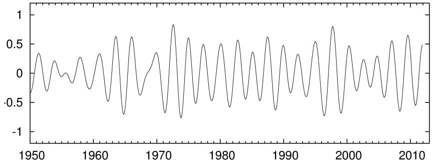
Nino 3.4 index from 1982 to 2008



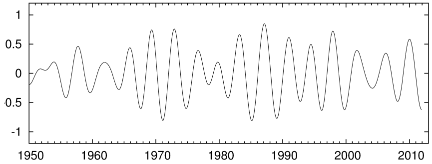
Other methods



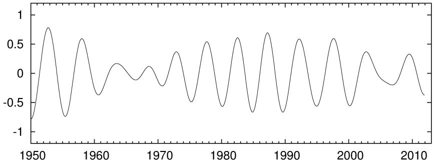
The modes of Nino 3.4



30.6 months

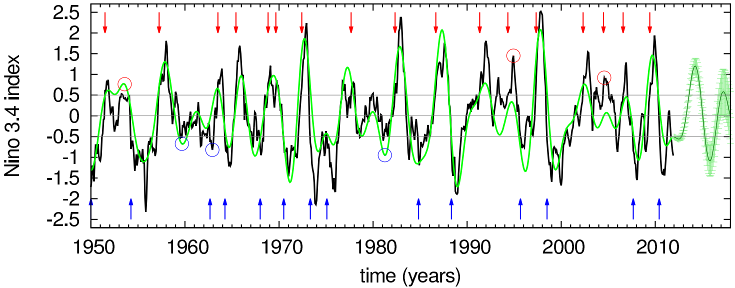


40.8 months

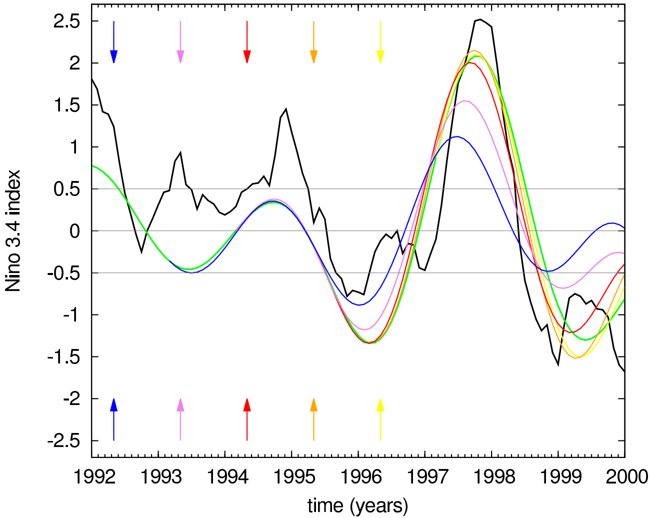


61.2 months

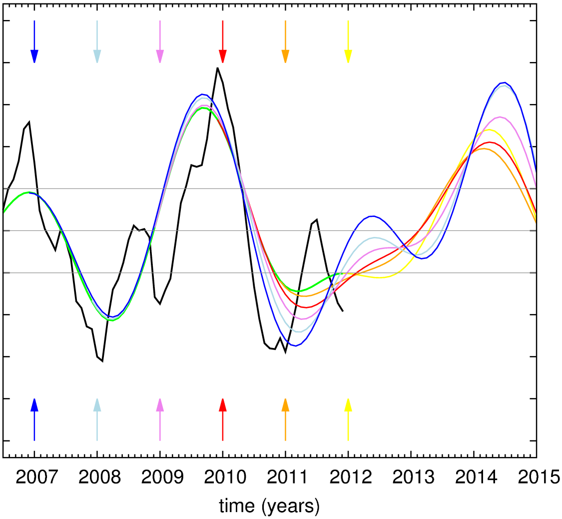
Application to the Nino 3.4 index



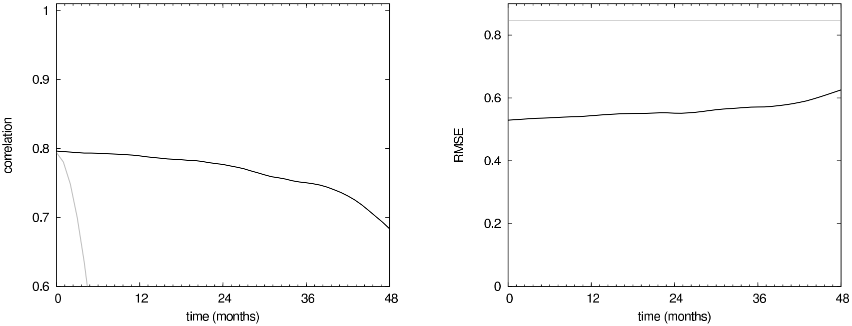
Forecast of the 1997–1998 El Niño event



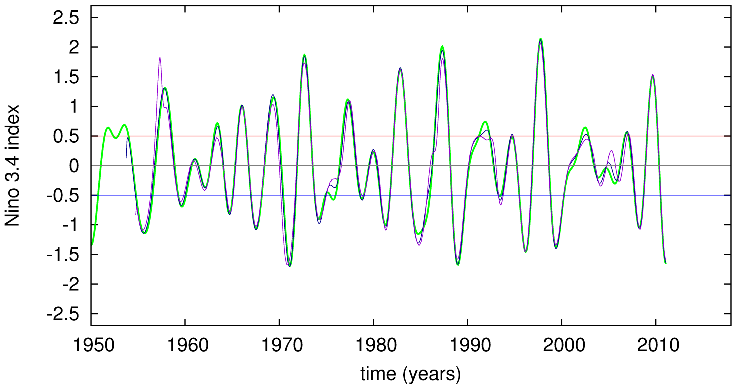
What about the next El Niño event



Anomaly correlation and RMSE



Retroactive probing forecast for 12 (blue) and 24 (purple) months



The end

Thanks for your attention!