Estimations of the Influence of the Non-Linearity of the Aerodynamic Coefficients on the Skewness of the Loading

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ABSTRACT

This paper is devoted to the non linear quasi-steady aerodynamic loading. A linear approximation is often used to compute the response of structures to buffeting forces. Some researchers have however shown that it is possible to account for the non linearity of this loading. This non linearity can come (i) from the squared velocity or (ii) from the shape of the aerodynamic coefficients (as functions of the wind angle of attack).

In this paper, we show that this second origin can have significant implications on the design of the structure, particularly when the non linearity of the aerodynamic coefficient is important (obvious, isn't it?) or when the transverse turbulence is important.

INTRODUCTION

Wind loads acting on bluff bodies like bridge decks are complex functions of the components of the turbulence and of the structural displacements and velocities. In order to simplify the representation of these loads, approached models are generally considered. Since a convenient linear approximation gives accurate results in many cases, such a model has been widely used during the last decades (e.g. Davenport 1962, Scanlan 1978). In its most general formulation, this linear model consists in decomposing the wind loads in three terms: (i) the static wind loading, (ii) the self-excited forces and (iii) the buffeting forces:

$$\begin{aligned} F_D(t) &= F_{D_s}(t) + F_{D_{se}}(t) + F_{D_b}(t) \\ F_L(t) &= F_{L_s}(t) + F_{L_{se}}(t) + F_{L_b}(t) \\ F_M(t) &= F_{M_s}(t) + F_{M_{se}}(t) + F_{M_b}(t) \end{aligned} \tag{1}$$

where $F_D(t)$, $F_L(t)$ and $F_M(t)$ represent respectively the drag and lift forces and the pitching moment. This general definition of the loading involves the well-known flutter derivatives and aerodynamic transfer functions (e.g. Simiu, 1996). After having measured these functions in wind tunnel experiments, the dynamic response and stability studies of the whole structure can be realized.

As a particular case of this linear approximation model, the linear quasi-steady theory (Fig. 1) provides very particular approximations of the flutter derivatives and of the transfer functions in terms of the usual aerodynamic coefficients (C_D, C_L, C_M) and their derivatives with respect to the angle of attack ($C_D^{'}$, $C_L^{'}$, $C_M^{'}$). Even if this model is limited because of its inability to represent transient frequency dependent forces, it can however represent correctly the low frequency motions of the structure. Furthermore this linear quasi-steady theory is also a particular case of another more general model: the non linear quasi-steady theory. Even if it is also limited to low frequency motions, this theory presents however the advantage to give a non linear model of the wind loading. It is thus interesting in the sense that these non linear effects bring new physical phenomena that can't be enlightened with the usual linear models.

In this paper, we focus on some of these phenomena (mainly on the effects of non-linearity of the aerodynamic coefficients with regard to the angle of attack of the incident wind) and on the way to account for the loading terms related to them.

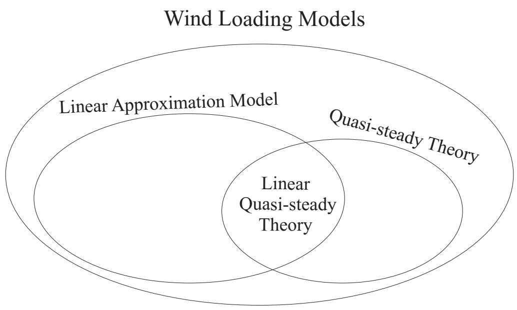

Fig. 1. Schematic view of wind loading models

QUASI-STEADY FORMULATION OF THE WIND LOADING

In order to clarify the context, we first present a short recall of the usual linear quasi-steady theory.

The aerodynamic coefficients of a bridge deck are determined by measuring the aerodynamic forces (F_D, F_L, F_M) acting on a fixed section placed in a wind tunnel:

$$C_D = \frac{F_D}{\frac{1}{2}\rho B V^2} \quad ; \quad C_L = \frac{F_L}{\frac{1}{2}\rho B V^2} \quad ; \quad C_M = \frac{F_M}{\frac{1}{2}\rho B^2 V^2} \tag{2}$$

where ρ, B and V represent respectively the air density, the width of the deck and the constant wind velocity used for the experiment.

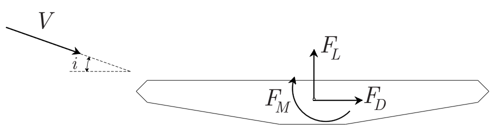

Fig. 2. Aerodynamic forces (drag, lift, moment)

For the range of wind velocities considered in practical applications (high Reynolds number), these aerodynamic coefficients can be considered to be independent of the wind velocity. On the other hand these coefficients are very dependent of the angle of attack i of the wind with respect to the bridge deck. This is illustrated at Fig. 3 which represents aerodynamic coefficients of several famous European bridges. Any of these coefficients is indeed a non-linear function of the angle of the angle of attack. For a convenient comparison, dotted lines represent tangents at the origin.

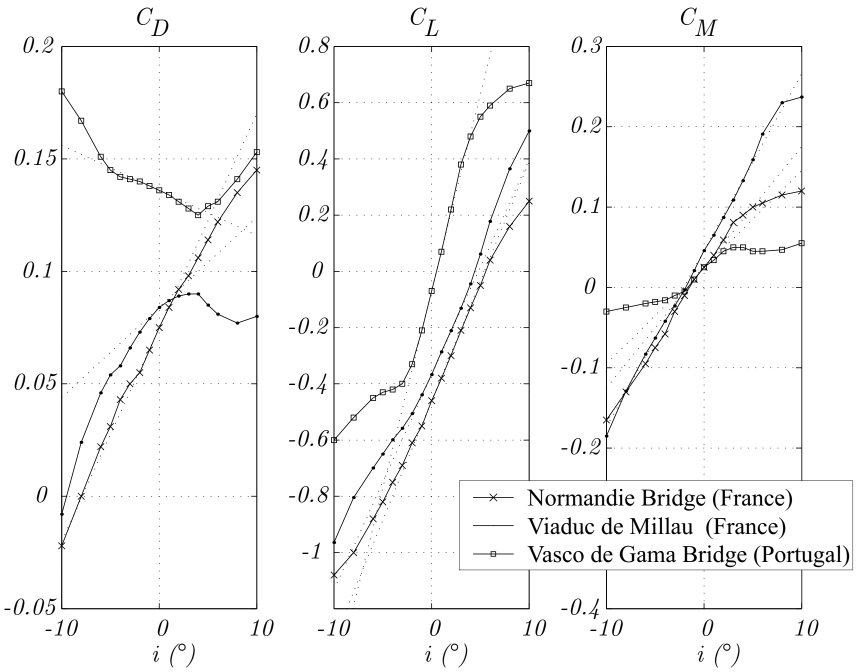

Fig. 3. Examples of aerodynamic coefficients

Provided the displacements of the structure are slow or their amplitude remains small, Equs. 2 may be used to estimate the buffeting forces acting on a bridge deck. For example, the drag force can be estimated by:

$$F_D(t) = \frac{1}{2} C_D\left[i(t)\right] \rho B V^2(t) \tag{3}$$

where the aerodynamic coefficient C_D is now time dependent (trough the angle of attack i) and where the wind velocity is also time dependent since it depends of the mean wind speed (U) and of the turbulence components (u and v). In order to partially account for a fluid-structure interaction, relative values must be considered for both the wind angle of attack and the wind velocity.

With notations of Fig. 4, these quantities can be expressed by:

$$\begin{aligned} i(t) &= ArcTan\left(\frac{v(t) - \dot{h}(t)}{U + u(t) - \dot{p}(t)}\right) - \alpha(t) \\ V^2(t) &= \left(U + u(t) - \dot{p}(t)\right)^2 + \left(v(t) - \dot{h}(t)\right)^2 \end{aligned} \tag{4}$$

where the upper dot denotes time derivatives.

Introducing Equs. 4 into Equ. 3 gives the non linear quasi-steady expression of the wind loading. As introduced before, it can be seen that this expression is a complex function of the components of the turbulence and the motion of the structure.

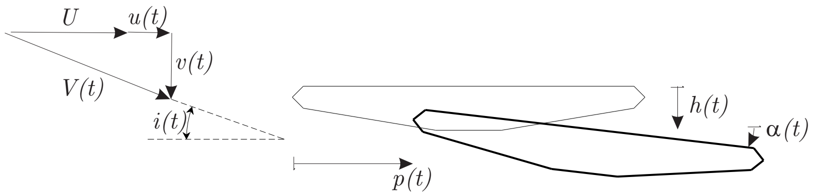

Fig. 4. Displacements of the structure

The components of turbulence are known in a probabilistic way only (Simiu, 1974). The most common methods that can be used to compute the dynamic response of a bridge are therefore:

- A stochastic analysis procedure (see e.g. Clough, Penzien, 1993), which is based, in its most basic formulation, on the computation of the power spectral density (PSD) of the response of the bridge as a function of the PSD of the turbulence components and of the mechanical properties of the bridge;
- The use of Monte Carlo simulations, which consists in generating wind histories and solving several times a deterministic problem by means of step-by-step analyses. This method allows accounting for the complete non linear expression but is rather slow since many generations are needed for a good accuracy. Thus this method should be used essentially to check results obtained by a stochastic analysis.

Linearization of the aerodynamic loading

A couple of decades ago, the field of application of stochastic procedures was limited by the abilities of computation means: probabilistic properties of structural response could thus be determined up to the second order only (variances), and therefore this response was generally considered as Gaussian.

As the components of the turbulence can be considered as Gaussian, the classical procedure consists in simplifying the expression of the loading to a linear function of the turbulence. The loading is then Gaussian, and hence, if the structure can be assumed to behave linearly, the structural response is Gaussian too.

After having linearized Equs. 4, introduced the resulting equations into Equ. 3, replaced the exact expression of aerodynamic coefficient by a linear approximation, and finally removed the subsequent quadratic terms, the linear quasi-steady formulation can be obtained:

$$F_D(t) = \frac{1}{2}\rho B U^2 \left[C_{D_0} - \left(2C_{D_0}\frac{\dot{p}(t)}{U} + C'_{D_0}\frac{\dot{h}(t)}{U} + C'_{D_0}\alpha(t) \right) + \left(2C_{D_0}\frac{u(t)}{U} + C'_{D_0}\frac{v(t)}{U} \right) \right] \tag{5}$$

It can be seen that this expression is a particular case of Equs. 1 since each term is a linear expression of the displacements (or velocities) of the structure or of the components of the turbulence. Since the vertical structural velocity ($\dot{h}(t)$) and the rotation of the deck ($\alpha(t)$) are present in this expression of the drag force, Equ. 5 shows that a coupling between vertical, horizontal and torsional motions could exist.

Higher degree polynomial approximation of the aerodynamic loading

Even if the theoretical possibilities of studying the dynamic response of structures subjected to non-Gaussian loading, i.e. to forces expressed by non-linear functions of the wind turbulence, is quite old now (Lutes 1986, Soize 1978), recent researches have developed new means of applying this theory in practical cases. Two main ways can be distinguished:

- Third and fourth order characteristics can be represented by bispectra and trispectra, similarly as second order characteristics can be represented by PSDs (Gurley 1997, Kareem 1998);
- Another possibility consists in solving a particular set of equations (the moment equations) derived from the Fokker-Planck-Kolmogorov equations (Di Paola, 1990). The main disadvantage of this method is that the force must be represented by a Markov chain; but when this condition is fulfilled, this second method is much faster that the first one since probabilistic characteristics of the response can be estimated by solving a simple set of algebraic equations.

Both of these methods allow accounting for a non linear loading provided it is expressed as a polynomial approximation of the actual loading. So instead of linearizing Equ. 3, a higher order polynomial approximation can be used :

$$F_D(t) = \frac{1}{2}\rho B U^2 \left[-2C_{D_0}\frac{\dot{p}(t)}{U} - C'_{D_0}\frac{\dot{h}(t)}{U} - C'_{D_0}\alpha(t) + \sum_{l=0}^{\infty}\sum_{k=0}^{\infty} F_{kl}\frac{u^l(t)v^k(t)}{U^{l+k}} \right] \tag{6}$$

Table 1: Values of the parameters F_{kl}

F_{kl}	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
$l=0$	C_{D_0}	C'_{D_0}	$C_{D_0} + \frac{C''_{D_0}}{2}$	$\frac{2C'_{D_0}}{3} + \frac{C'''_{D_0}}{6}$	$\frac{C''_{D_0}}{6} + \frac{C''''_{D_0}}{24}$
$l=1$	$2C_{D_0}$	C'_{D_0}	0	$\frac{-2C'_{D_0}}{3} - \frac{C'''_{D_0}}{6}$	$\frac{-C''_{D_0}}{3} - \frac{C''''_{D_0}}{12}$
$l=2$	C_{D_0}	0	0	$\frac{2C'_{D_0}}{3} + \frac{C'''_{D_0}}{6}$	$\frac{C''_{D_0}}{2} + \frac{C''''_{D_0}}{8}$
$l=3$	0	0	0	$\frac{-2C'_{D_0}}{3} - \frac{C'''_{D_0}}{6}$	$\frac{-2C''_{D_0}}{3} - \frac{C''''_{D_0}}{6}$
$l=4$	0	0	0	$\frac{2C'_{D_0}}{3} + \frac{C'''_{D_0}}{6}$	$\frac{5C''_{D_0}}{6} + \frac{5C''''_{D_0}}{24}$

This expression is however obtained by neglecting non linear terms of the structural motions. This means that only non-linear components of buffeting forces can be accounted for in these approaches. The values of the coefficients F_{kl} in Equ. 6 are given in Table 1 for the first values of k and l.. These coefficients are expressed in terms of the first four derivatives of $C_D(i)$ with respect to the angle of attack; these derivatives are defined by :

$$C_D(i) = C_{D_0} + iC'_{D_0} + \frac{i^2}{2!}C''_{D_0} + \frac{i^3}{3!}C'''_{D_0} + \frac{i^4}{4!}C''''_{D_0} + \dots \tag{7}$$

It can be seen for example that the curvature of the aerodynamic coefficient C''_{D_0} is present in a second order term of the loading (k=2, l=0). It can be also checked that Equ. 5 is a particular case of Equ. 6.

RESPONSE OF STRUCTURES TO NON GAUSSIAN LOADINGS

Amongst both available methods to compute the response of a structure to a non Gaussian loading we follow the one proposed by Kareem and Spanos. The theoretical basics of this method won't be presented in the paper (see e.g. Gurley 1997). We will just present some interesting results that have been obtained with it.

Let us consider the equation of motion governing the dynamics of a single degree of freedom system subjected to a particular loading (Equ.8):

$$\ddot{x}(t) + 2\xi\varpi\dot{x}(t) + \varpi^2 x(t) = \gamma\left(U^2 + 2Uu(t) + u^2(t)\right) \tag{8}$$

where ξ and ϖ represent the structural characteristics, $\gamma = \rho C_D B / 2$ is supposed to be constant and $u(t)$ represents a zero-mean Gaussian and Ornstein-Uhlenbeck process of which PSD is given by:

$$S_u(\omega) = \frac{\alpha}{\pi}\frac{\sigma_u^2}{\alpha^2 + \omega^2} \tag{9}$$

where $\sigma_u^2 = I_u^2 U^2$ is the variance of the process, I_u is the turbulence intensity and α is a frequency-shaping parameter.

Second order response

Even if the loading is non linear and hence non Gaussian, its PSD ($S_F(\omega)$) can be expressed in terms of the PSD of the turbulence ($S_u(\omega)$) (e.g. Floris, 2002). After multiplication by the transfer function of the system, the PSD of the response ($S_u(\omega)$) can be obtained and finally, after integration along the frequencies, the variance of the displacement $\sigma_{x_{quad}}^2$ can be obtained.

As a comparison, we also compute the variance of the response ($\sigma_{x_{lin}}^2$) obtained by neglecting the non-linear terms of the loading.

The ratio between these two variances is thus a measure of the influence of the quadratic term of the loading on the second order characterization of the structural motion. An approached expression of this ratio is given at Equ. 10, where Ψ represents the ratio between the quasi-static components of the PSD and the total variance of the response.

$$\frac{\sigma^2_{x_{quad}}}{\sigma^2_{x_{lin}}} \approx \Psi\left(1+\frac{I_u^2}{2}\right)+(1-\Psi)\left(1+I_u^2\,\frac{1+(\varpi/\alpha)^2}{4+(\varpi/\alpha)^2}\right) \tag{10}$$

Fig. 5 illustrates this ratio of variances in the most realistic case (which appears to be also the worst case), namely when ϖ/α is much larger than unity.

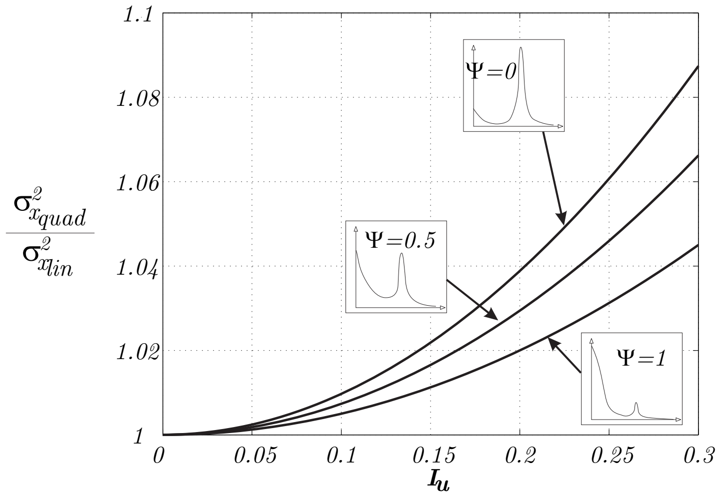

Fig. 5. Influence of the quadratic term of the loading on the variance of the displacement (as a function of the dispatching of energy between quasi-static and dynamic contributions)

This figure shows that the non linear term of the loading produces a more significant influence when the response of the structure is essentially dynamic and not quasi-static (i.e. when $\Psi = 0$). It can be seen that for usual values of the wind intensity ($I_u \approx 15\%$ to $I_u \approx 20\%$), the quadratic term of the loading doesn't affect significantly the variance of the response. This observation should however be mitigated if the coefficient of $u^2(t)$ in the loading would have been larger than the coefficients of U^2.

This figure shows also that the ratio of the variance of the response for $\Psi = 0$ and $\Psi = 1$ is equal to $\left(1+I_u^2\right)\Big/\left(1+I_u^2/2\right)$. Furthermore, for any Ψ this ratio could be estimated by:

$$f(\Psi) = \frac{\sigma^2_{x_{quad,\Psi}}}{\sigma^2_{x_{quad,QS(\Psi=1)}}} = \frac{\Psi\left(1+\frac{I_u^2}{2}\right)+(1-\Psi)\left(1+I_u^2\right)}{1+\frac{I_u^2}{2}} = 1+(1-\Psi)\frac{\frac{I_u^2}{2}}{1+\frac{I_u^2}{2}} \tag{11}$$

This observation is interesting since it shows that the exact variance of the response under quadratic loading $\sigma^2_{x_{quad}}$ can be estimated by multiplying the quasi-static contribution of the variance by a simple function of Ψ and I_u.

Third order analysis

Several authors (Soize 1978, Gurley 1997) have shown that the extreme values of non Gaussian processes depend of their higher order statistical characteristics, namely their skewness and kurtosis defined by :

$$\gamma_3 = \frac{m_x^{(3)}}{\sigma_x^3} \qquad ; \qquad \gamma_4 = \frac{m_x^{(4)}}{\sigma_x^4} \tag{12}$$

where $m_x^{(n)} = E\left[\left(x - \bar{x}\right)^n\right]$ represents the n^th^ centred moment of the process $\boldsymbol{x}$.

Gurley and Kareem propose to compute the extreme values as if the process was Gaussian and then to multiply this first estimation by a correcting factor that accounts for the non Gaussianity (see Fig. 6). This factor is expressed as a function of the mean crossing rate (ν) and of the duration of the observation T. The correcting factor is of course equal to unity for a Gaussian process, for which $\gamma_3 = 0$ and $\gamma_4 = 3$.

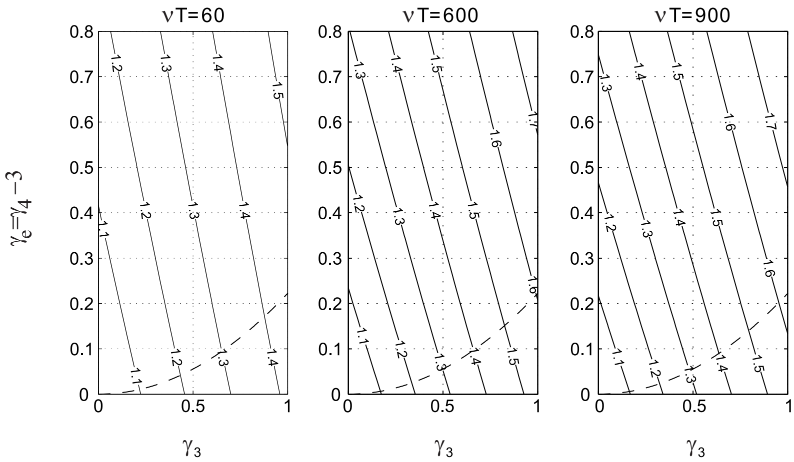

Fig. 6. Correcting factor proposed by Gurley and Kareem to account for the effect of non Gaussianity on extreme values.

If it is desired to estimate the effects of the non linearity of the loading on the extreme values of the displacement, it is thus necessary to compute higher order statistical characteristics of the response. Exactly as the variance can be obtained by integration of the PSD, the third centred moment can be estimated by integration of another mathematical quantity: the bispectrum. Since, in our developments, the loading (Equ. 8) is considered as a polynomial form of a Gaussian process $\boldsymbol{u(t)}$, the bispectra of the loading and of the response can be determined in an analytical way.

If the non linear term of the loading should be neglected, the response would be Gaussian and the skewness coefficient would be equal to zero. The ratio $\gamma_{3x_{quad}} / \gamma_{3x_{lin}}$, similar to the ratio used in the previous paragraph to characterize the second order response, is thus meanless. It is then chosen to evaluate, instead of this ratio, the importance of the non linearity on the skewness of the response, related to the skewness of the loading. Fig. 7

represents the ratio between these values for several damping coefficients and eigen frequencies.

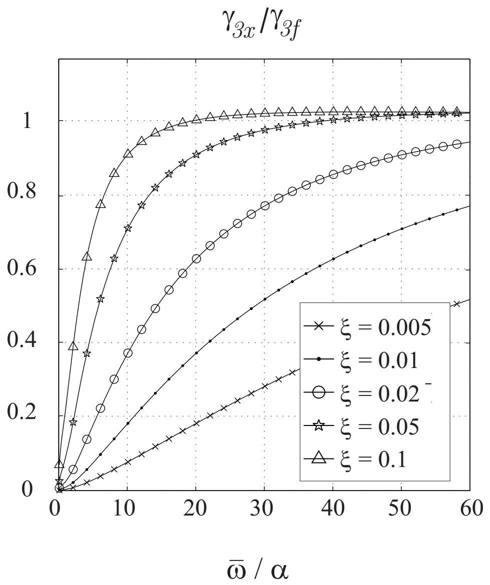

Fig. 7. Importance of the skewness of the response as a function of the skewness of the loading, the damping coefficient and the eigen frequency (closed form)

It can be observed that :

- the skewness of the response is always smaller than the skewness of the loading;
- the skewness of the response is small for lightly damped or soft structures;

These observations can be justified by considering that the response of the structure is composed of two contributions: (i) the quasi-static contribution, shaped like the applied force and thus characterized by the same skewness coefficient, and (ii) the dynamic component, rather shaped like a Gaussian process. It seems thus obvious to obtain an intermediate skewness coefficient for the response, this coefficient being smaller for structures having an important dynamic contribution, i.e. for lightly damped and soft structures.

The analytical relations used to draw figure 7 can be useful in several applications. Let us imagine for instance that the actual PSD of the turbulence could be approached by an Ornstein-Uhlenbeck process (e.g. by fitting parameter α). Instead of establishing and integrating the bispectrum of the response, these analytical relations give directly the values of the skewness coefficient of the response, which can be used to estimate the non Gaussianity of the response and hence its extreme values.

NON LINEAR LOADING INCLUDING THE NON LINEARITY OF THE AERODYNAMIC COEFFICIENTS

Let us come back to the analysis of the buffeting response of structures subjected to a turbulent wind flow. The polynomial approximation of Equ. 6 is here limited to the second order, leading to:

$$\frac{F_D(t)}{\frac{1}{2}\rho B U^2} = C_{D_0} + 2C_{D_0}\frac{u(t)}{U} + C'_{D_0}\frac{v(t)}{U} + C_{D_0}\frac{u^2(t)}{U^2} + C'_{D_0}\frac{u(t)v(t)}{U^2} + \left(C_{D_0} + \frac{C''_{D_0}}{2}\right)\frac{v^2(t)}{U^2} \tag{13}$$

This means that the non linearity of the aerodynamic coefficient can be taken into account up to its second order derivative only. We also suppose, as it is often the case, that the components of the turbulence $u(t)$ and $v(t)$ are independent Gaussian processes:

$$\begin{gathered} E\left[u(t)\right] = 0; \quad E\left[u^2(t)\right] = \sigma_u^2 \\ E\left[v(t)\right] = 0; \quad E\left[v^2(t)\right] = \sigma_v^2 \\ E\left[u(t)v(t)\right] = 0; \end{gathered} \tag{14}$$

With these notations and assumptions, the statistical properties of the loading can be computed up to the third order:

$$\begin{aligned} \frac{\bar{F}_D(t)}{\frac{1}{2}\rho B U^2} &= C_{D_0} + C_{D_0}\frac{\sigma_u^2}{U^2} + \left(C_{D_0} + \frac{C''_{D_0}}{2}\right)\frac{\sigma_v^2}{U^2} \\ \frac{\sigma_{F_D}^2}{\left(\frac{1}{2}\rho B U^2\right)^2} &= \left(4C_{D_0}^2\frac{\sigma_u^2}{U^2} + C_{D_0}^{'2}\frac{\sigma_v^2}{U^2}\right) + \left(2C_{D_0}^2\frac{\sigma_u^4}{U^4} + C_{D_0}^{'2}\frac{\sigma_u^2\sigma_v^2}{U^4} + 2C_{D_0}^2\frac{\sigma_v^4}{U^4}\right) \\ &\quad + C''_{D_0}\left(2C_{D_0} + \frac{C''_{D_0}}{2}\right)\frac{\sigma_v^4}{U^4} \\ \frac{m_{F_D}^{(3)}}{\left(\frac{1}{2}\rho B U^2\right)^3} &= 3\left(8C_{D_0}^3\frac{\sigma_u^4}{U^4} + 4C_{D_0}C_{D_0}^{'2}\frac{\sigma_u^2\sigma_v^2}{U^4} + C_{D_0}^{'2}\left(2C_{D_0} + C''_{D_0}\right)\frac{\sigma_v^4}{U^4}\right) + \\ &\quad + \left(8C_{D_0}^3\frac{\sigma_u^6}{U^6} + 6C_{D_0}C_{D_0}^{'2}\frac{\sigma_u^4\sigma_v^2}{U^6} + 3C_{D_0}^{'2}\left(2C_{D_0} + C''_{D_0}\right)\frac{\sigma_u^2\sigma_v^4}{U^6}\right) \\ &\quad + \left(2C_{D_0} + C''_{D_0}\right)^3\frac{\sigma_v^6}{U^6} \end{aligned} \tag{15}$$

In the following these relations will be considered as reference values. Fig. 8 illustrates these three first non-dimensional statistical moments for several values of the wind intensities $I_u = \sigma_u / U$ and $I_v = \sigma_v / U$. These are computed for the drag coefficient of the Viaduct of Millau (see Fig. 1). The coefficients of the quadratic approximation are obtained by a least square fit with Gaussian weight distribution[1]:

[1] It can be proved that the actual statistical distribution of the angle of attack is almost Gaussian when both wind intensities are almost the same (Denoel, 2005)

$$C_{D_0} = 0.0830 \quad ; \quad C'_{D_0} = 0.202 \quad ; \quad C''_{D_0} = -3.40 \tag{13}$$

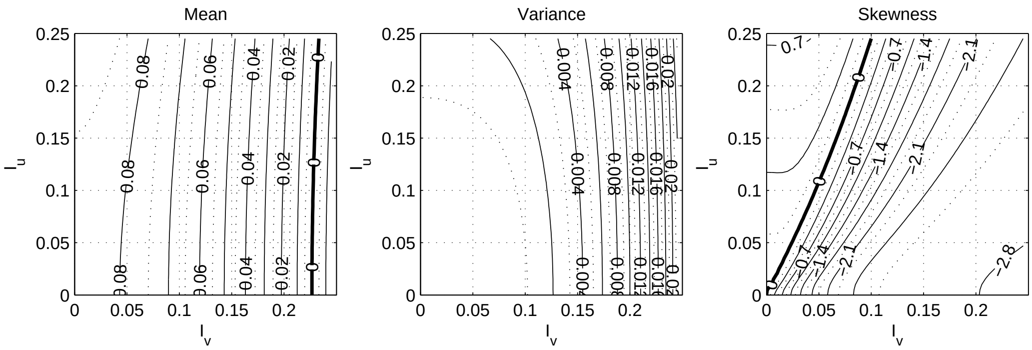

Fig. 8. “Exact” statistical characteristics of the loading as a function of the wind intensities

From these reference values of the statistical moments of the loading, two particular approximations can be derived.

First approximation : linearization of the aerodynamic coefficient

As a first approximation, it could be supposed that the aerodynamic coefficient is linear ($C''_{D_0} = 0$). Since non linear terms coming from the expression of the squared velocity are kept, the subsequent expression of loading remains non-linear and hence non Gaussian. Only the last term is removed from Equ. 13. The approached statistical values obtained under this hypothesis are determined by imposing $C''_{D_0} = 0$ in Equs. 15.

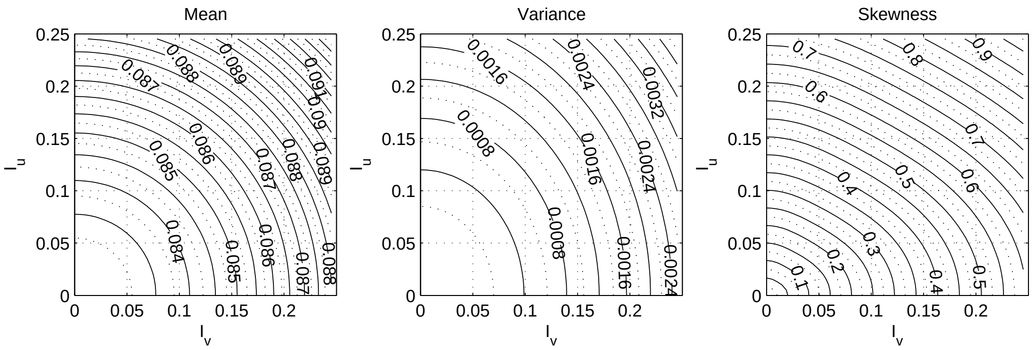

Fig. 9. Approached statistical characteristics of the loading as a function of wind intensities

The first developments concerning the non Gaussianity of the aerodynamic loading correspond to this hypothesis. Many authors (Grigoriu 1986, Benfratello 1996, Kareem 1998, Gusella 2000, Floris 2002) have studied the effects of the wind intensity on the skewness of the loading. Their developments were mainly based on a one-dimensional turbulence field ($I_v = 0$). In this case, the comparison of Figs. 8 and 9 shows that the exact and approached means, variances ad skewnesses are exactly the same, as only the values along the vertical axis have to be compared. From a theoretical point of view, this means that the aerodynamic coefficients can be linearized provided the transverse wind intensity is equal to zero.

In practical applications, this transverse intensity I_v is however generally not equal to zero and the vertical axis of Figs. 8 and 9 is no more sufficient to represent the actual statistical moments. Indeed, the comparison of these two figures shows that a small transverse intensity modifies drastically the values of the statistical moments. For example, for $I_u = 10\%$ and $I_v = 10\%$, the exact and approached non dimensional variances are respectively equal to $12.1E-4$ and $6.9E-4$.

Reagrding the skewness coefficient, it can be significantly affected by the non linearity of the aerodynamic coefficient. Indeed, Fig. 8 exposes a negative skewness zone (for large I_v) that can not be explained with the approached model. Consequently, the non linearity of the aerodynamic coefficient can turn the positively-skewed probability density function into a negatively-skewed one. In terms of extreme values, this may have very important consequences.

Second approximation : linearization of the aerodynamic loading

As a second approximation, we consider the equations of the linear quasi-steady loading. In this method, only linear terms of the loading must be considered: the first three terms of Equ. 13 are thus only accounted for. The non dimensional statistical characteristics of the loading are now much simpler:

$$\frac{\bar{F}_D(t)}{\frac{1}{2}\rho B U^2} = C_{D_0} \quad ; \quad \frac{\sigma_{F_D}^2}{\left(\frac{1}{2}\rho B U^2\right)^2} = 4 C_{D_0}^2 \frac{\sigma_u^2}{U^2} + C_{D_0}^{\prime 2} \frac{\sigma_v^2}{U^2} \quad ; \quad \frac{m_{F_D}^{(3)}}{\left(\frac{1}{2}\rho B U^2\right)^3} = 0 \qquad (16)$$

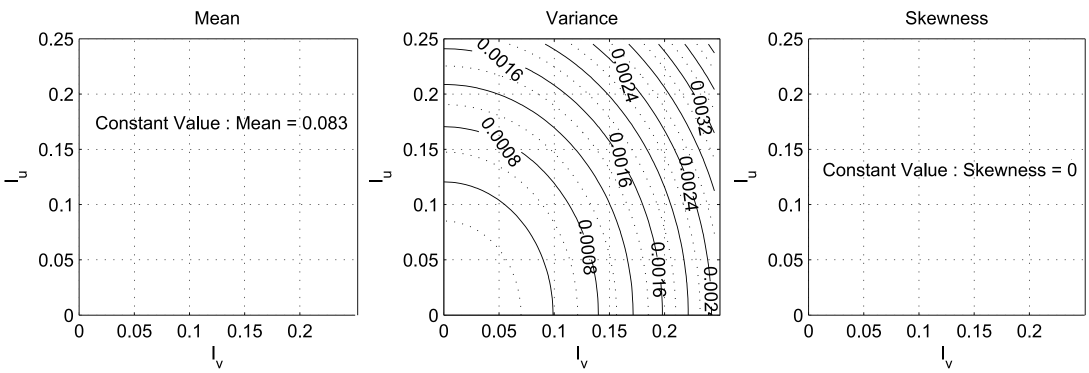

Fig. 10. Approached statistical characteristics of the loading as a function of wind intensities

These values are plotted on Fig. 10. It can be checked that this method provides inaccurate results, even on the vertical axis. Furthermore since the loading is now Gaussian, this method gives a skewness coefficient obviously equal to zero.

ESTIMATION OF THE INFLUENCE OF NON LINEARITY OF THE LOADING ON THE RESPONSE OF THE STRUCTURE

Several famous researchers have presented advanced methods to compute exactly the statistical moments of the response of a structure subjected to a non Gaussian loading (see previous paragraph). In this paper, it is desired to give an estimation of the skewness of the response with a minimum computation effort.

Let us suppose that the reference statistical values of the loading have been computed following the developments of the previous paragraph. Note that this computation implies simple algebraic operations only.

If the structure would be very stiff, its response would be mainly quasi-static and, in this case, the variance of the structure could be obtained by dividing the variance of the loading by the squared stiffness. In practical applications, the ratio Ψ between the quasi-static and the dynamic components of the PSD is however not equal to zero. In this more general case, Equ. 11 can be used to give an estimation of the complete variance, i.e. the sum of the quasi-static and dynamic contributions. This result is not rigorous since the function $f(\Psi)$ was computed for an Ornstein-Uhlenbeck process and not for the actual PSD of the wind loading. This approximation presents however the advantage to give very fast results.

Concerning the skewness coefficients, we have seen (Fig.7) that the skewness of the response is smaller than the skewness of the loading. Hence, the skewness coefficient of the loading could be used as boundary value for the design of the structure. If the dynamic component of the response is more important, its skewness coefficient can be significantly reduced. Fig. 7 can thus help giving a better approximation of the skewness coefficient of the response. This value can be estimated by the knowledge of:

- the skewness coefficient of the loading,
- the damping coefficient of the structure,
- the eigen frequency of the structure,
- an equivalent shaping factor α

This equivalent shaping factor α must be determined in accordance with the actual PSD of the loading. Several authors (Muscollino 1995, Floris 2002) have proposed some formulations for this equivalence which consists in replacing the actual PSD by an Ornstein-Uhlenbeck one.

CONCLUSIONS

This paper has presented two important features in the analysis of structures subjected to non Gaussian loadings.

The first one consists in the determination of the statistical characteristics of the loading. They are commonly established by supposing that the aerodynamic coefficients can be linearized. We have seen that this linearization can lead to a significant inaccuracy on the statistical characteristics. The developments presented in this paper were limited to the second order derivative of the aerodynamic coefficients with respect to the angle of attack but the same reasoning could be easily extended to higher order polynomial approximations.

As a second step, we have seen how the statistical characteristics of the loading could be used to determine estimations of the statistical characteristics of the response. In this part, the heavy rigorous analysis method has been avoided and a simpler approach, based on simple and fast computations, has been retained. This very simple method can give estimations of the response that could be used, for instance, for a pre-design of a structure.

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