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Investigations on a Level Set based approach for the optimization of flexible components in multibody systems with a fixed mesh grid

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Introduction

State of the art in the structural optimization of multibody systems

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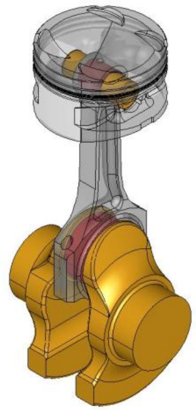
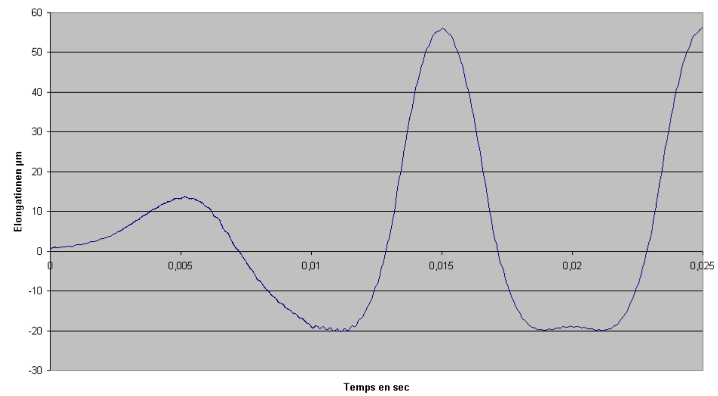
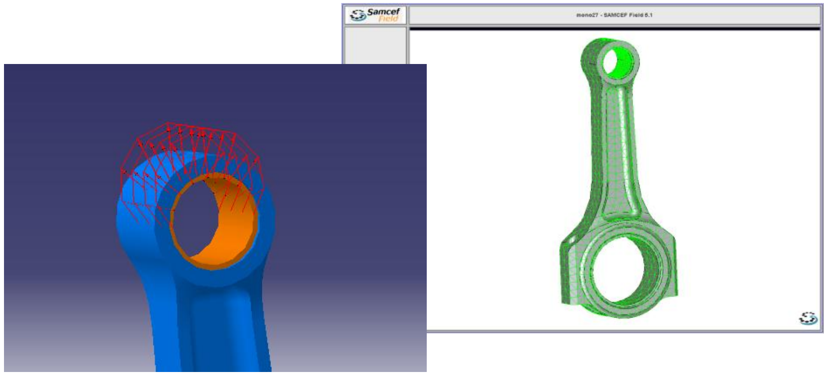


Geometrical modeling



Multibody system dynamics

Loop

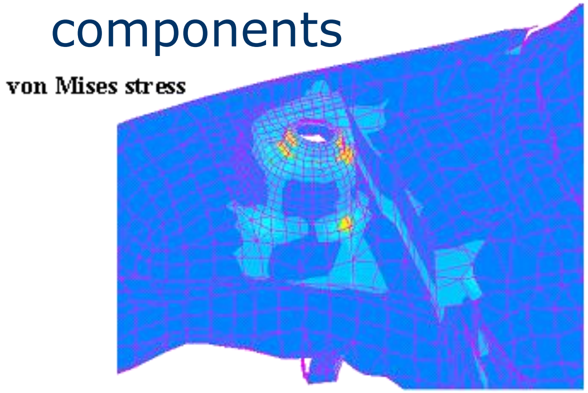


Equivalent static load cases
+Static FE analysis

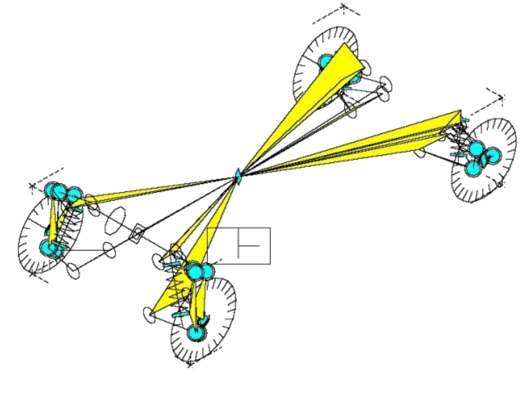
Optimization process

Evolution of virtual prototyping

- Finite Element Method: Structural analysis of components



- Rigid Multibody Systems: Simulation of mechanisms

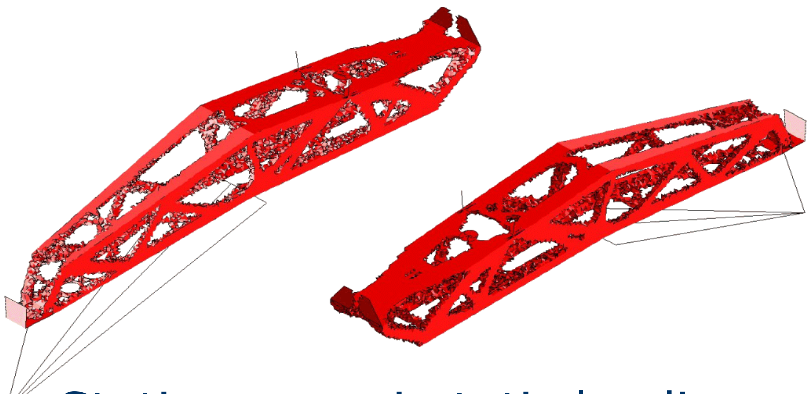


- Flexible Multibody Systems: System approach (MBS) & Structural dynamics (FEM)

Courtesy of LMS-SAMTECH

Evolution of virtual prototyping

- Structural optimization

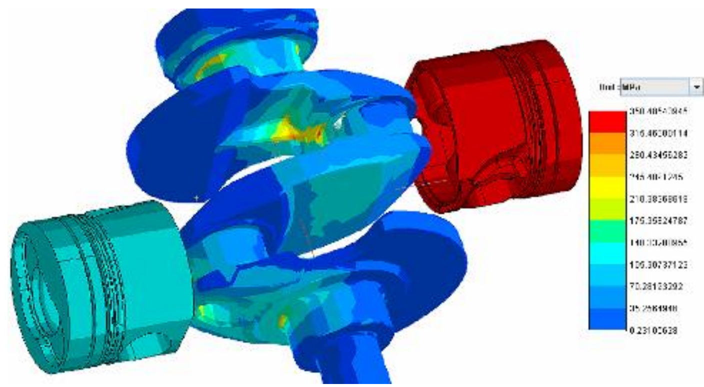


Static or quasi-static loading

- Flexible multibody systems

Dynamic loading

- Integrated optimization of flexible components in multibody systems



Motivations

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- Classical FEM approach
  - (Quasi-) Static load cases (Empirical, experience,…)
- Weak coupling between FEM and MBS
  - Coupling with pre / post processing
  - Define equivalent static load cases (Kang, Park and Arora, 2005)
  - Optimization of isolated components
- Strong coupling between FEM and MBS
  - Define realistic dynamic loadings
  - Take care of the coupling between large overall rigid-body motions and deformations
  - Optimization problem with function depending on time

➔ « Fully Integrated Method »
  - MBS approach based on nonlinear FEM (Samcef Mecano)
  - Coupled with an optimization shell (Boss Quattro)

Finite Element Approach Of Multibody System Dynamics

Equation of FEM-MBS dynamics

- Motion of the flexible body (FEM) is represented by absolute nodal coordinates **q** (Geradin & Cardona, 2001)

- Dynamic equations of multibody system

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{g}^{\text{ext}} - \mathbf{g}^{\text{int}}$$

- Subject to kinematic constraints of the motion

$$\mathbf{\Phi}(\mathbf{q}, t) = 0$$

- Solution based on an augmented Lagrangian approach of the total energy

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}^T(k\boldsymbol{\lambda} + p\mathbf{\Phi}) = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) \\ k\mathbf{\Phi}(\mathbf{q}, t) = 0 \end{cases} \qquad \mathbf{B} = \frac{\partial \mathbf{\Phi}}{\partial \mathbf{q}}$$

$$\mathbf{q}'(0) = \mathbf{q}'_0 \text{ and } \dot{\mathbf{q}}'(0) = \dot{\mathbf{q}}_0$$

Time Integration

- The set of nonlinear DAE is solved using the generalized-α method by Chung and Hulbert (1993)
- Definition of a pseudo acceleration vector **a**:

$$(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n = (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f \ddot{\mathbf{q}}_n$$

- Newmark integration formulae

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h(1-\gamma)\mathbf{a}_n + h\gamma\mathbf{a}_{n+1}$$

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_{n+1} + h^2(1/2-\beta)\mathbf{a}_n + h\beta\mathbf{a}_{n+1}$$

- Solve iteratively the dynamic equation system (Newton-Raphson)

$$\begin{cases} \mathbf{M}\Delta\ddot{\mathbf{q}} + \mathbf{C}_t\Delta\dot{\mathbf{q}} + \mathbf{K}_t\Delta\mathbf{q} + \mathbf{B}^T\Delta\boldsymbol{\lambda} = \Delta\mathbf{r} \\ \mathbf{B} = \mathbf{0} \end{cases} \qquad \mathbf{r} = \mathbf{M}\ddot{\mathbf{q}} - \mathbf{g} + \mathbf{B}^T\boldsymbol{\lambda}$$

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The Level Set Description

Principle (Sethian & Osher, 1988)

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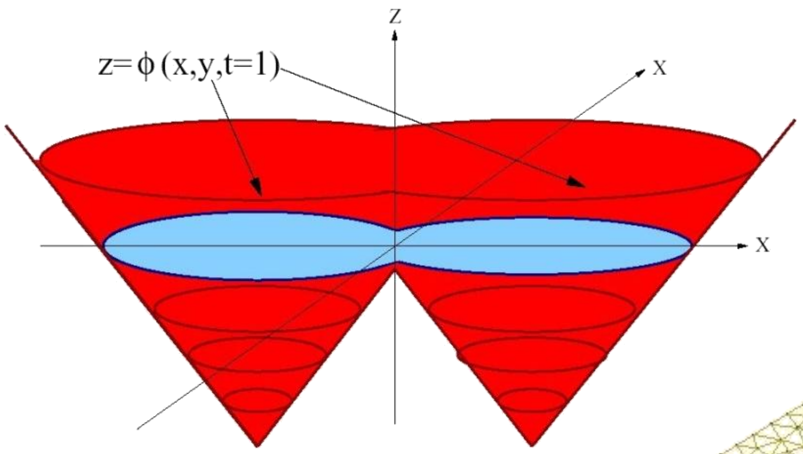
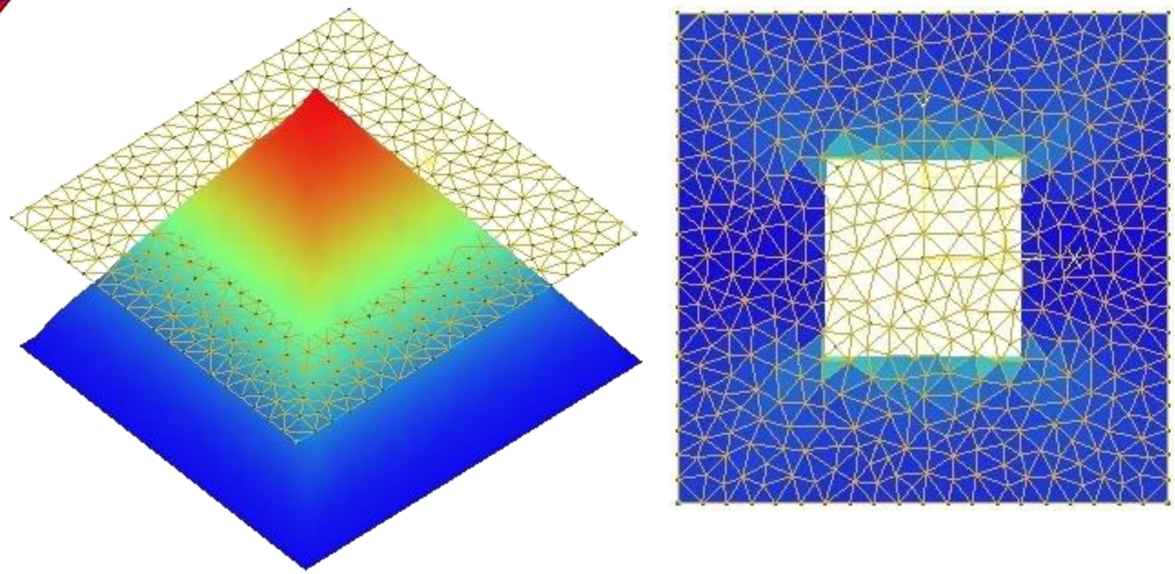
- Numerical technique for tracking interfaces
  - Introduce a higher dimension function
  - Implicit boundary representation $\psi(x,t)=0$
  - Interface = the zero level of the function



A 2D application of the level set method:
Square plate with a square hole



Advantages - Drawbacks

Advantages

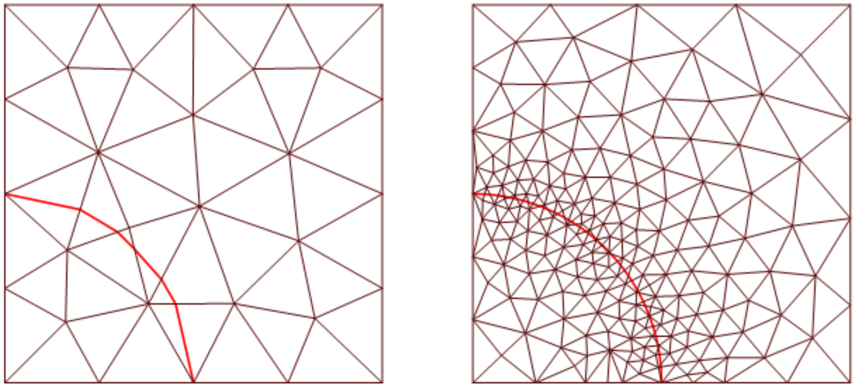
- Easiness of combining entities (min, max,…)
  - Remove entities
  - Separate entities
  - Merge entities

➔ Topology modifications

- Extension 2D/3D
- Nicely coupled to XFEM

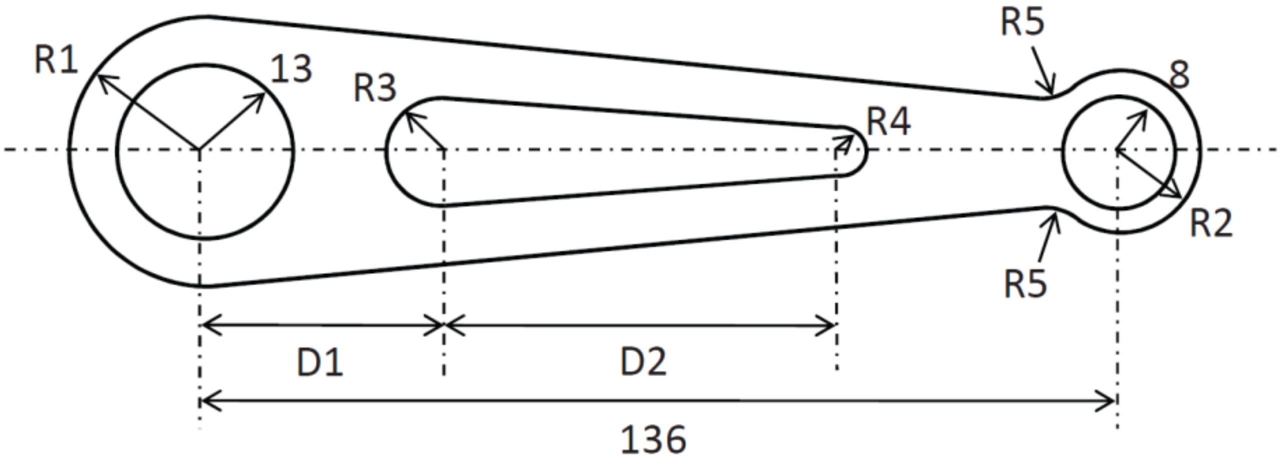
Drawbacks

- Construction:
  - Specific tools
  - Analytical functions
  - Point set – NURBS
- Mesh “adaptation” necessary but not in the method proposed here



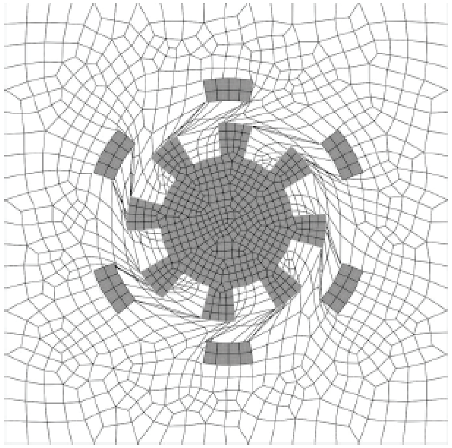
Shape optimization

- Necessary to have an initial design of the component
- Parametric model
- Design variables = Geometrical parameters of flexible body shape



Shape optimization

- The finite element mesh is modified to follow the shape modifications.



  - ➔ It leads to mesh distortion. Major Problem!
  - ➔ The quality of the mesh decreases and the solution accuracy of the FEA decreases after the first iteration.

    Re-meshing techniques exist to avoid this problem.
  - ➔ Velocity field for the sensitivity analysis

- The adopted parameterization influences the performance of the optimization process.

Topology optimization

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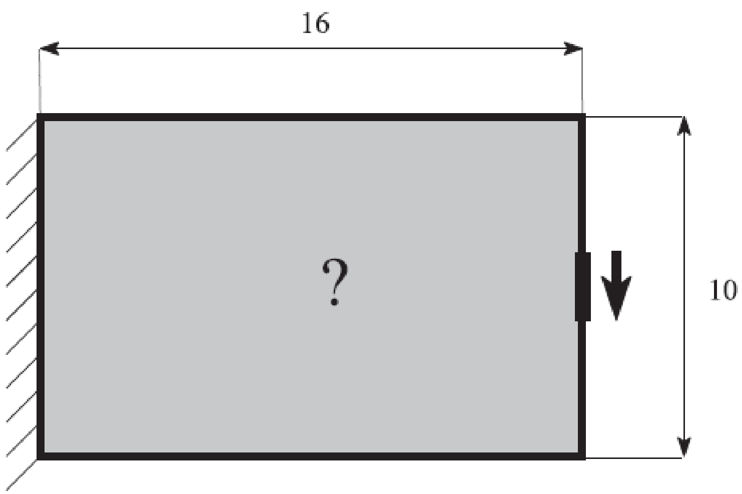
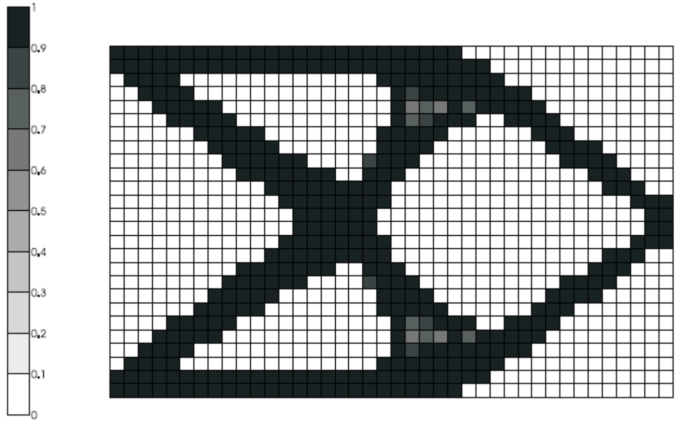
- Can be seen as an optimal material distribution within a design domain
- No initial knowledge on the component
  - Only have to define:
    - The design domain
    - The loading
    - The boundary conditions
    - A volume constraint
- The optimization process gives the best design for these pieces of information.



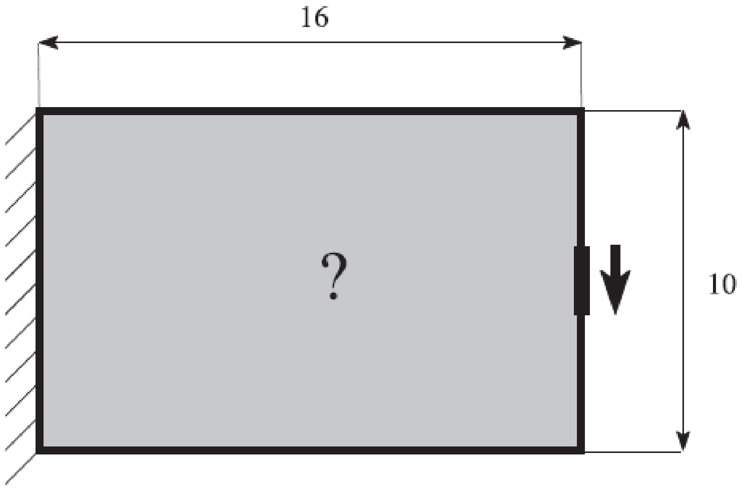


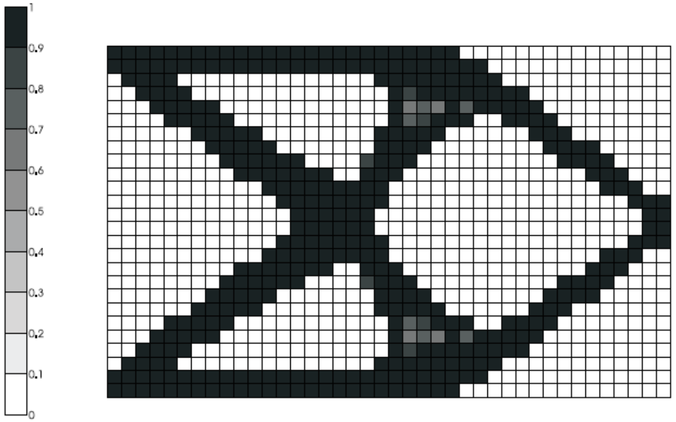
Topology optimization

- The design variables are the density of each finite element.
  Large number of variables – local optima
- Feasibility of manufacturing:
  Difficulties to determine structural boundary shape from the topology optimization results.

But…

- Fixed mesh grid





Goals of this work

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- The *Level Set Description* of the geometry leads to an intermediate type of optimization between shape optimization and topology optimization.
  - Fixed mesh grid: No mesh distortion
  - The geometry is based on CAD entities: can easily be manufactured.
  - Remove, separate, merge entities: Modification of the topology
  - Design variables: parameters of the level sets (quite small number)
- The proposed approach is different from the topology optimization based on the level set (in static optimization) proposed by G. Allaire because we use the level set for the description of the component geometry.

  (G. Allaire *et al.*, A level-set method for shape optimization. C.R. Acad. Sci. Paris, 334:1125–1130, 2002)

The proposed approach

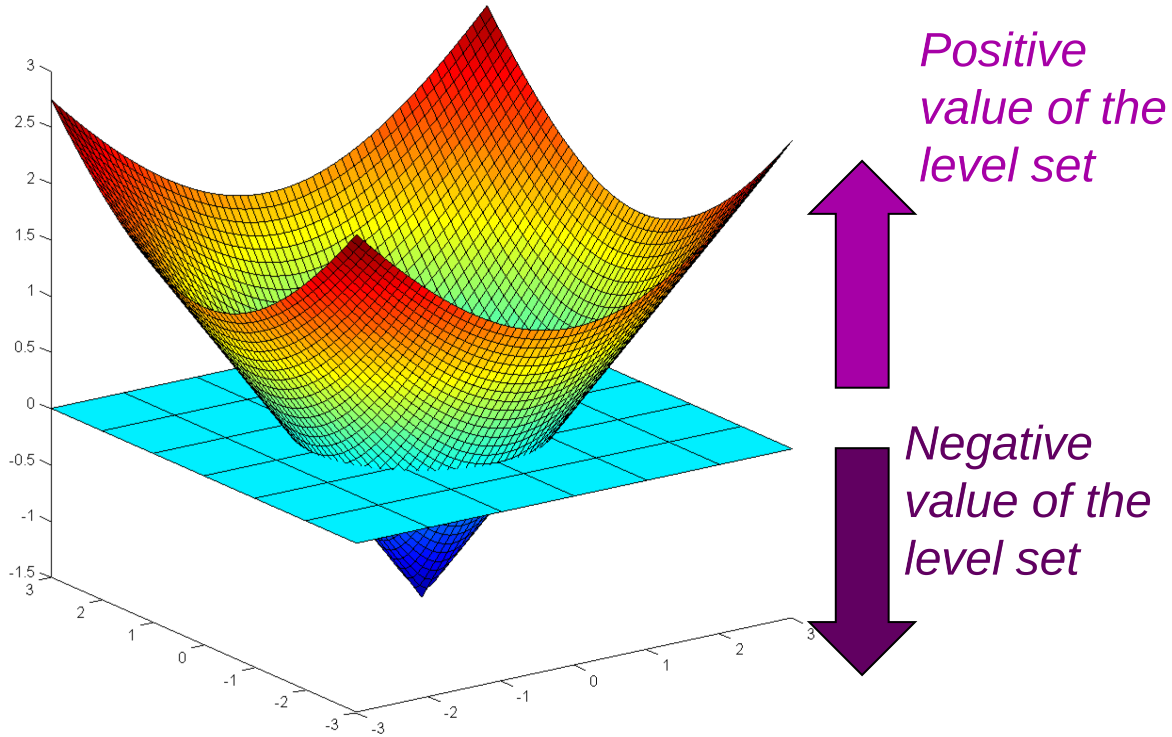
The method: Square plate with a hole

- Mesh definition (fixed during all the process) + Level Set definition:

  Fixed mesh grid: 6*6 elements

  Level Set: a cone

- No element is removed to create the hole but the properties of elements are modified: the density and the Young modulus.



The method: Square plate with a hole

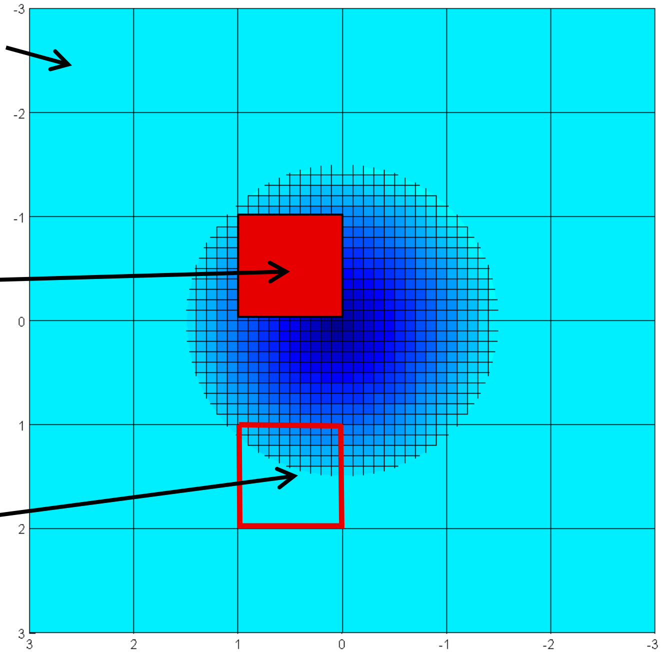
- For each node: Computation of the level set value.
- Different possibilities can happen for each element:
  - 4 positive nodal values: Solid material

$$\rho = \rho_0 \text{ and } E = E_0$$

  - 4 negative nodal values: void

$$\rho = 10^{-3}\rho_0 \text{ and } E = 10^{-9}E_0$$

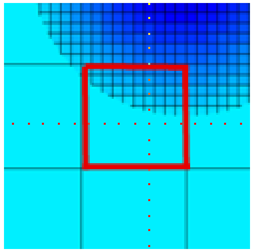
  - Positive and negative nodal values = boundary element



The method: Square plate with a hole

- For the boundary elements → SIMP law
  - Introduction of a pseudo-density

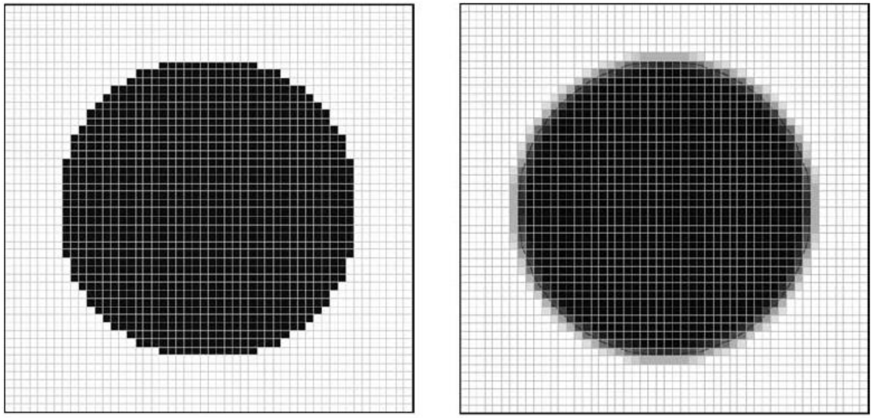
$$\mu = \frac{\text{Volume of material}}{\text{Volume of the element}}$$



  - SIMP law

$$\rho = \mu \rho_0 \text{ and } E = \mu^3 E_0$$

- Consequences:
  - Smooth transition
  - Sensitivity analysis:
    Fixed number of elements



(a) (b)

Formulations Of Flexible Multibody System Optimization Problem

General form of the optimization problem

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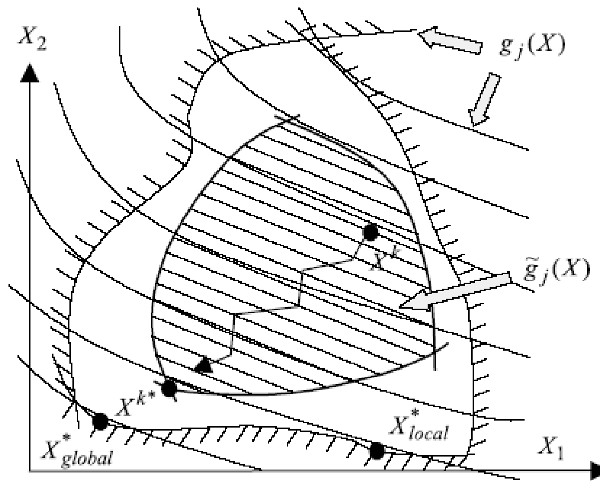
- Design problem is cast into a mathematical programming problem

$$\min_{\mathbf{x}} g_0\left(\mathbf{x}\right)$$

$$s.t. \begin{cases} g_j\left(\mathbf{x}\right) \leq \overline{g}_j, & j = 1, \ldots, m \\ \underline{x}_i \leq x_i \leq \overline{x}_i, & i = 1, \ldots, n \end{cases}$$



- Provides a general and robust framework to the solution procedure

- Efficient solver :
  - Sequential Convex Programming (Gradient based algorithm)

    ➔GCM (Bruyneel et al. 2002)

Sensitivity analysis

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- Gradient-based optimization methods require the first order derivatives of the responses

- Finite differences $$\frac{\partial \mathrm{f}}{\partial x} \approx \frac{\mathrm{f}(x+\delta x)-\mathrm{f}(x)}{\delta x}$$

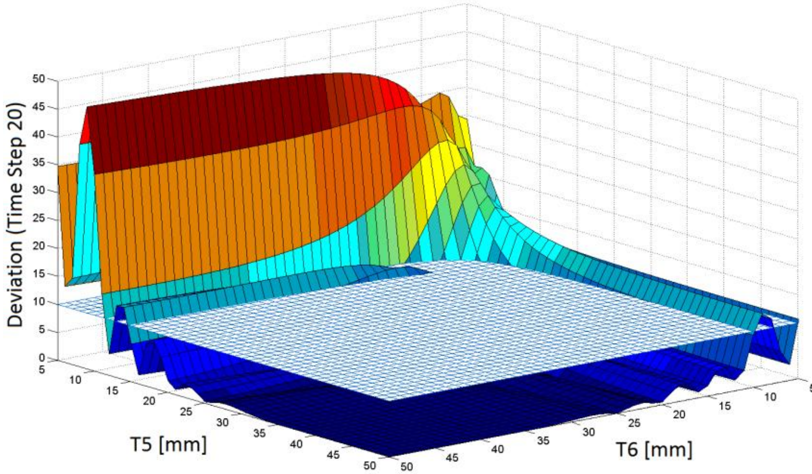
  Perturbation of design variables
  ➔ Additional calls to MBS code

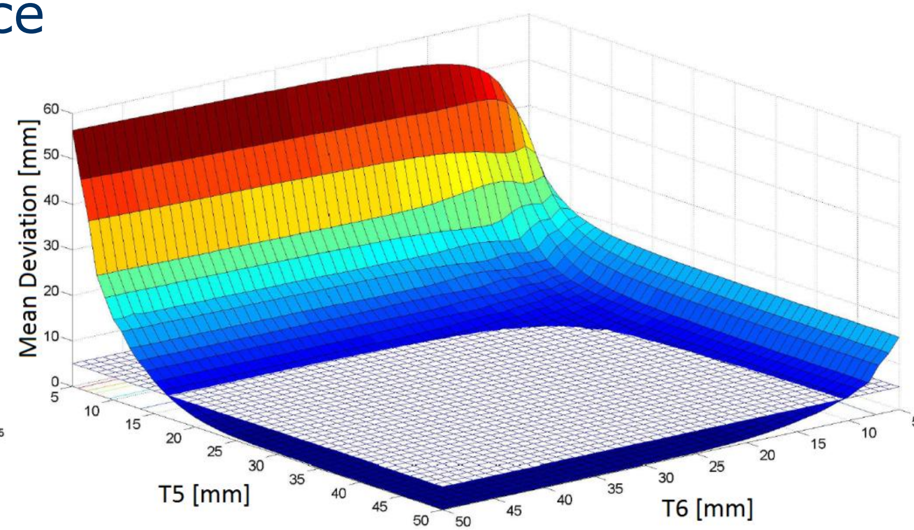
- Semi-analytical approach (Not yet developed)

$$\frac{\partial \mathbf{r}}{\partial x} \approx \frac{\mathbf{r}(x+\delta x)-\mathbf{r}(x)}{\delta x} \qquad \frac{\partial \boldsymbol{\Phi}}{\partial x} \approx \frac{\boldsymbol{\Phi}(x+\delta x)-\boldsymbol{\Phi}(x)}{\delta x}$$

The formulation

- The formulation is a key point for this type of problems:
  Very complex nonlinear behavior
- Impact on the design space





- Extremely important for gradient based algorithm
- Genetic algorithms
  - Do not necessary give better results
  - Computation time much more important

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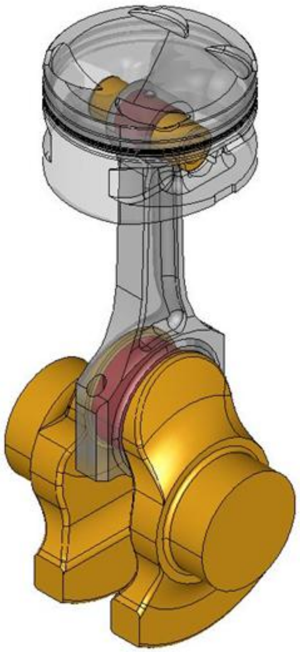
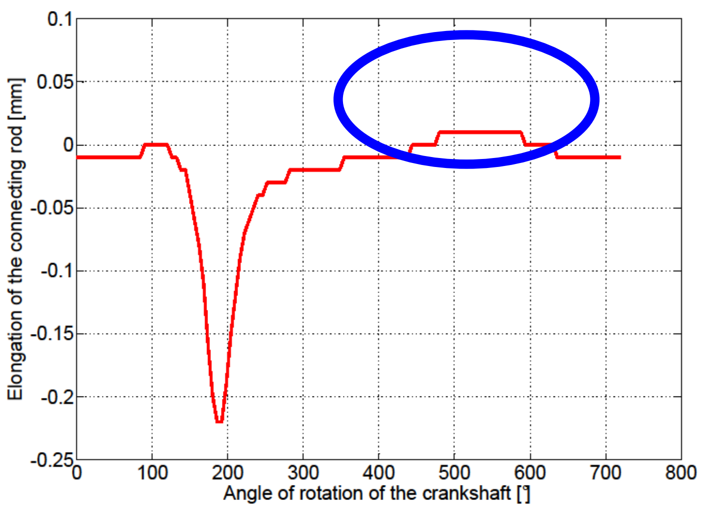
Numerical Applications

Connecting rod optimization

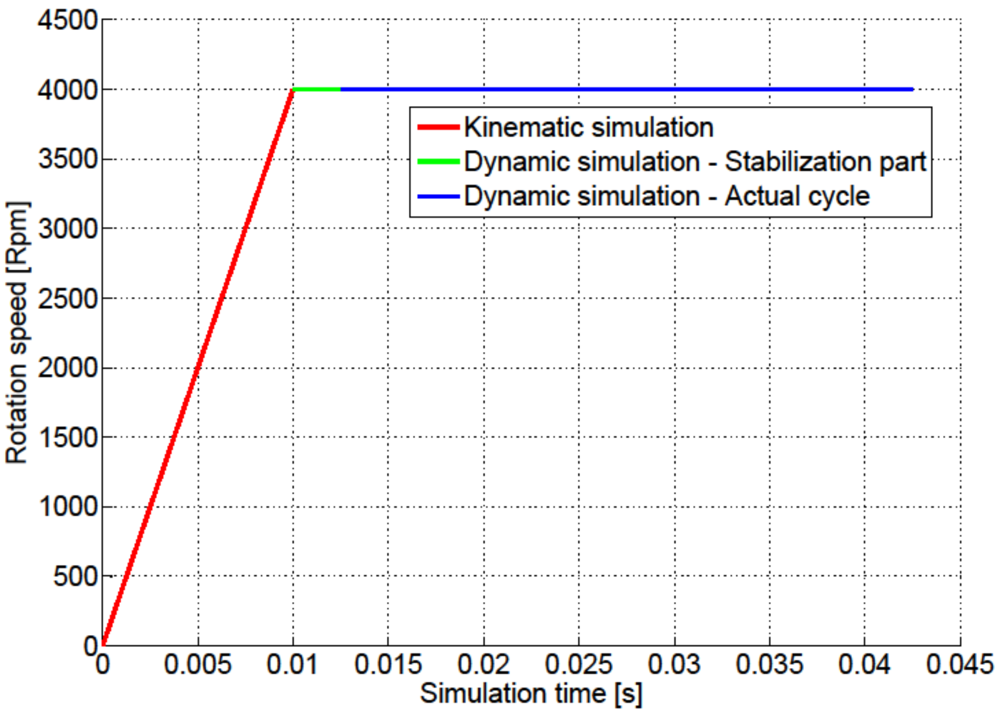
- Minimization of the connecting rod mass in a real combustion engine (Diesel).
- During the exhaust phase, the connecting rod elongates which can destroy the engine.
  ➔Collision between the piston and the valves.
- Constraints imposed on the elongation





Modeling of the connecting rod

- Consideration of one single complete cycle as the behavior is cyclic (720°) for the optimization
- Rotation speed 4000 Rpm
- Gas pressure taken into account.



Local formulation

$$\min_{\mathbf{x}} m(\mathbf{x})$$

$$s.t. \qquad \Delta l(\mathbf{x}, t_i) \leq \Delta l_{max}$$

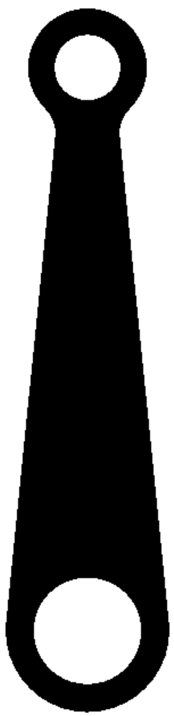
$$\text{with } i = 1, \ldots, \text{nbr time step}$$

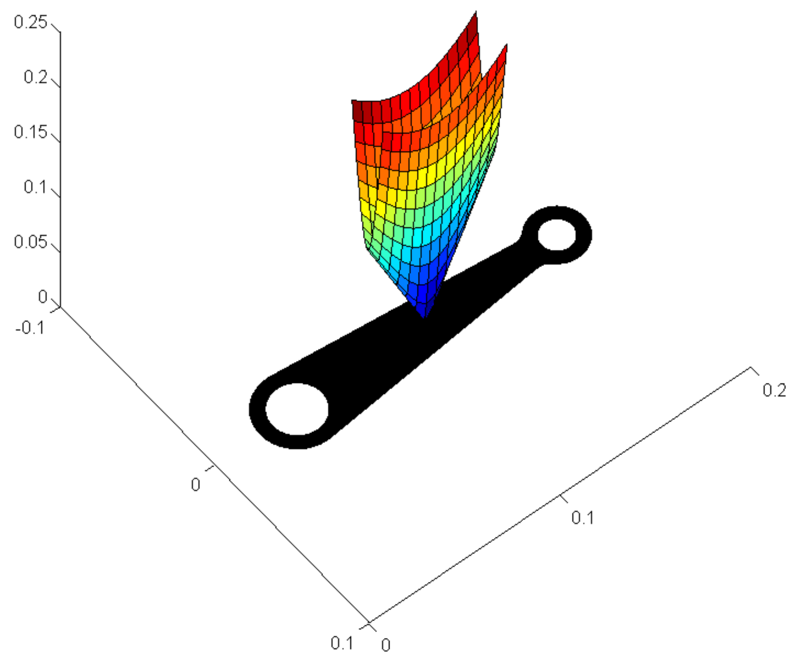
- The constraint on the elongation $\Delta l(\mathbf{x}, t_i)$ is considered at each time step.
  - ➔ As many constraints as the number of time steps (134)

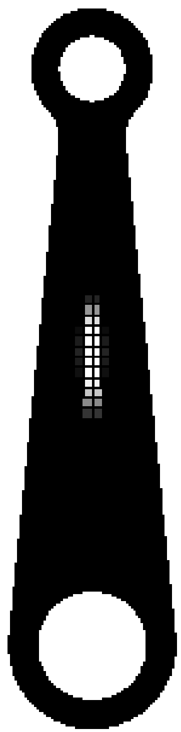
First application – 1 level set

- The level set is defined in order to have an ellipse as interface.
- 5 different design variables: a, b, c_x, c_y and d. Here only d is chosen.

$$\Phi(x,y) = \frac{(x-c_x)^2}{a^2} + \frac{(y-c_y)^2}{b^2} - d = 0$$







Results

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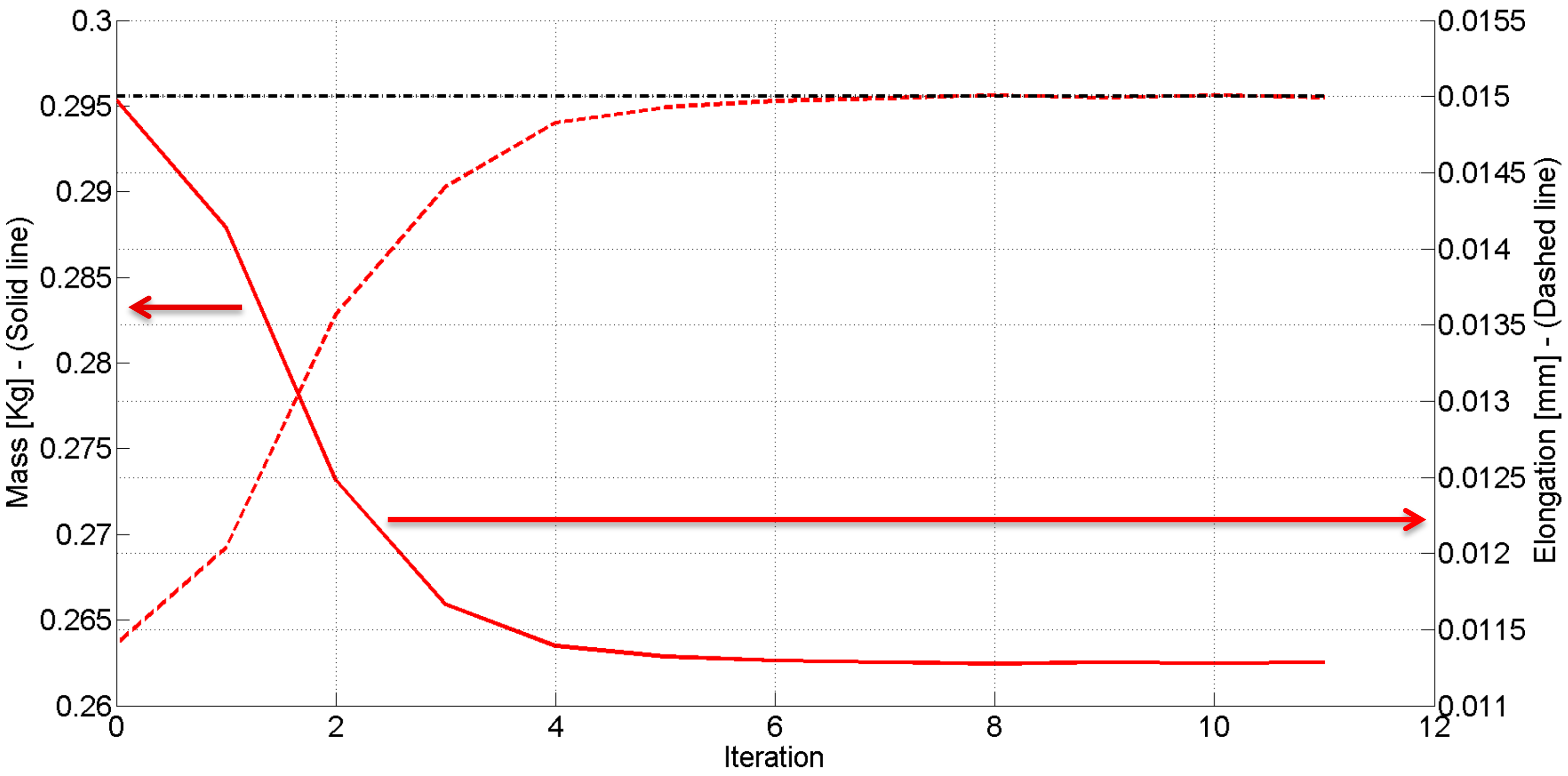
- Convergence obtained after 12 iterations
- Monotonous behavior of the optimization process



Results – Optimal design

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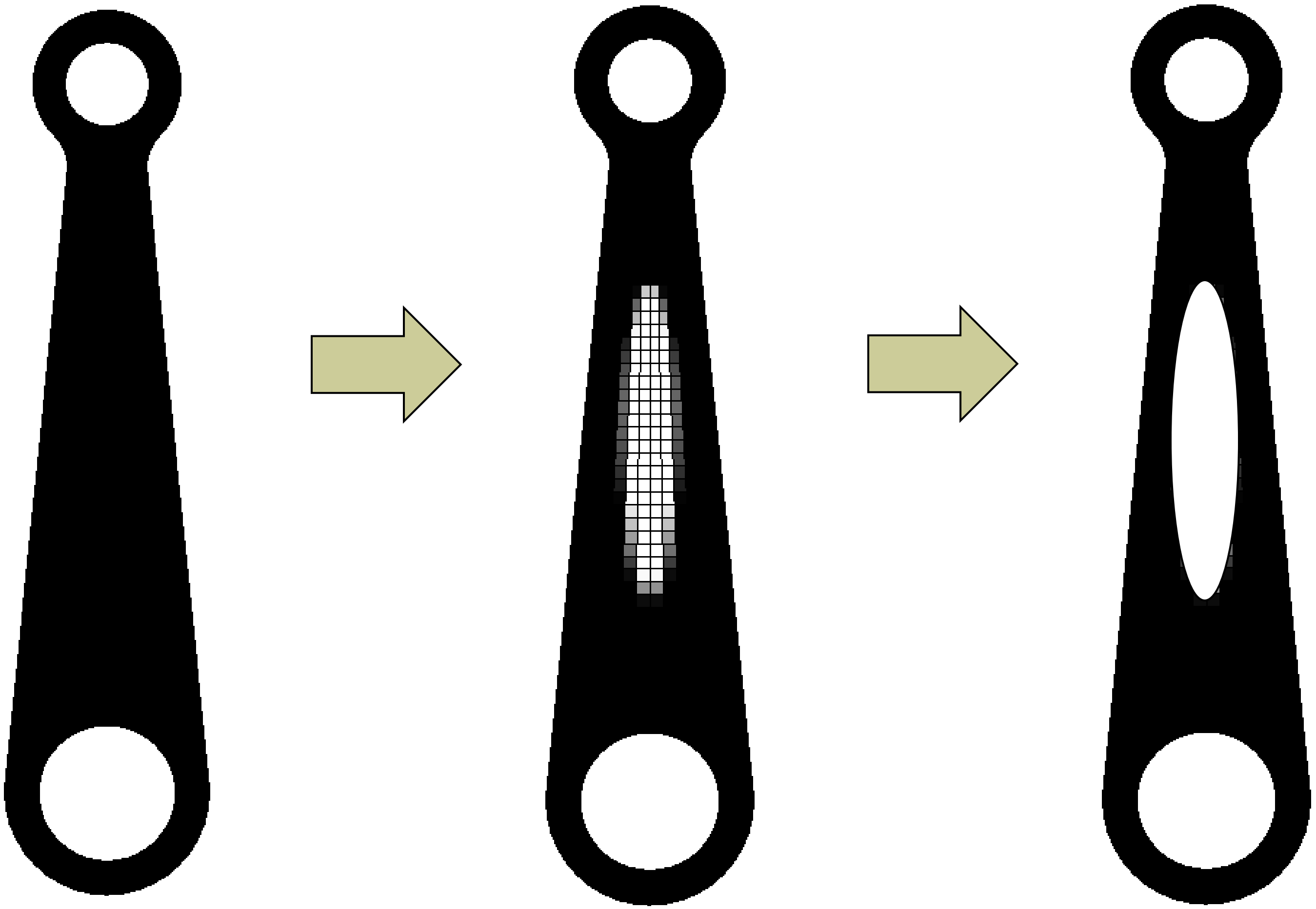
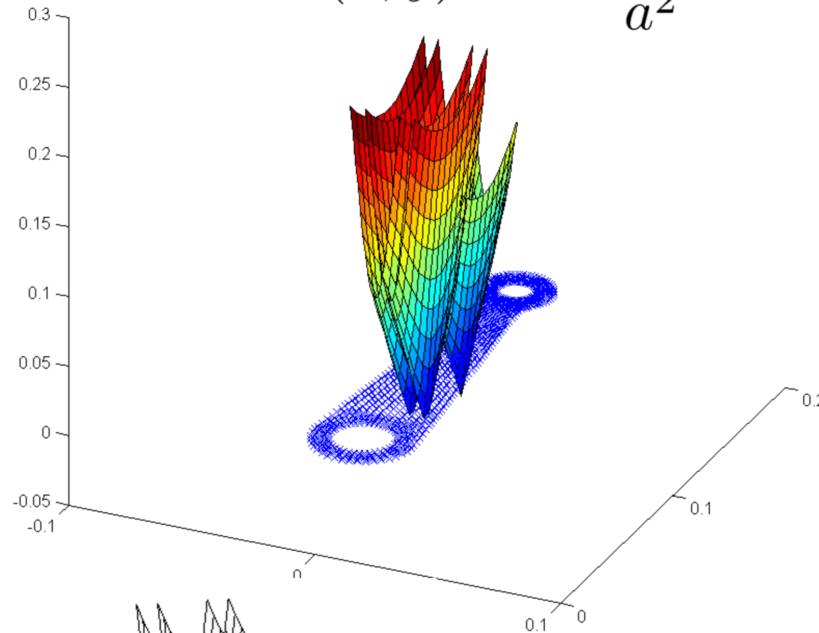
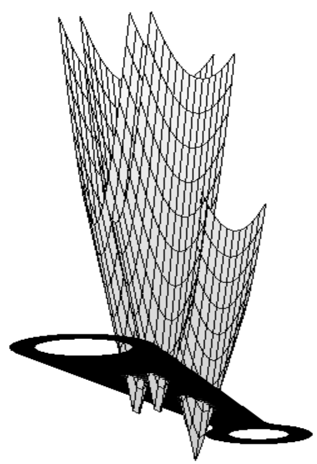
- As the boundary is defined by a CAD entity, the connecting rod can be directly manufactured without any post processing.

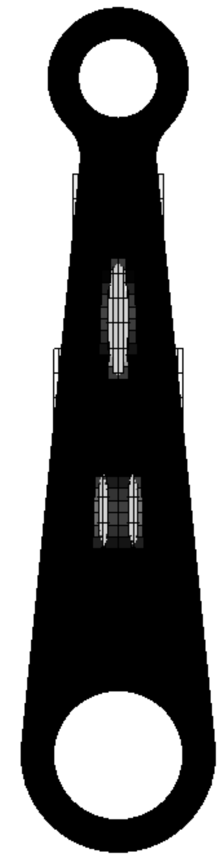


Second application – 3 level sets

- 3 ellipses are defined. $\Phi(x,y) = \frac{(x-c_x)^2}{a^2} + \frac{(y-c_y)^2}{b^2} - d = 0$

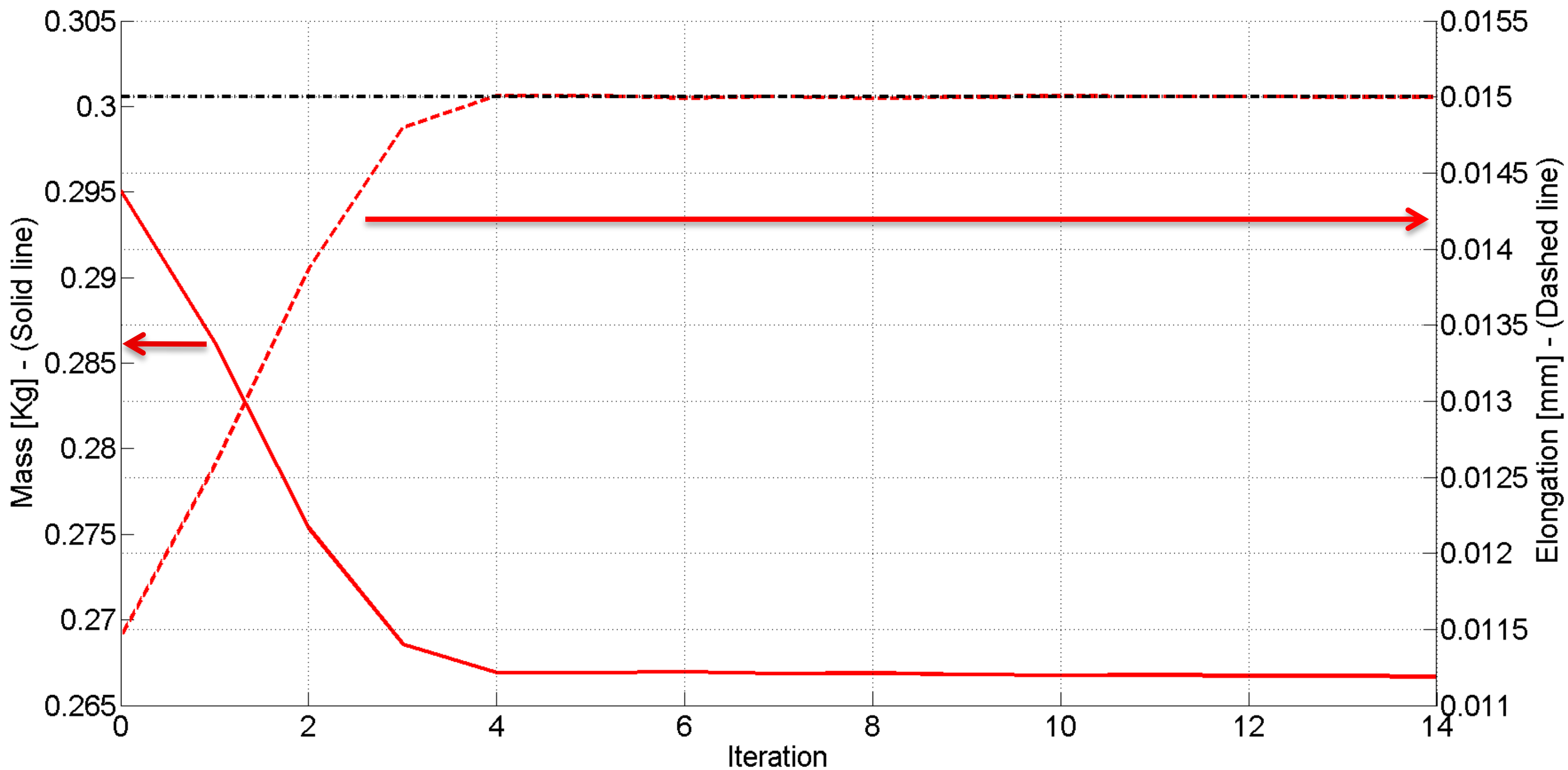






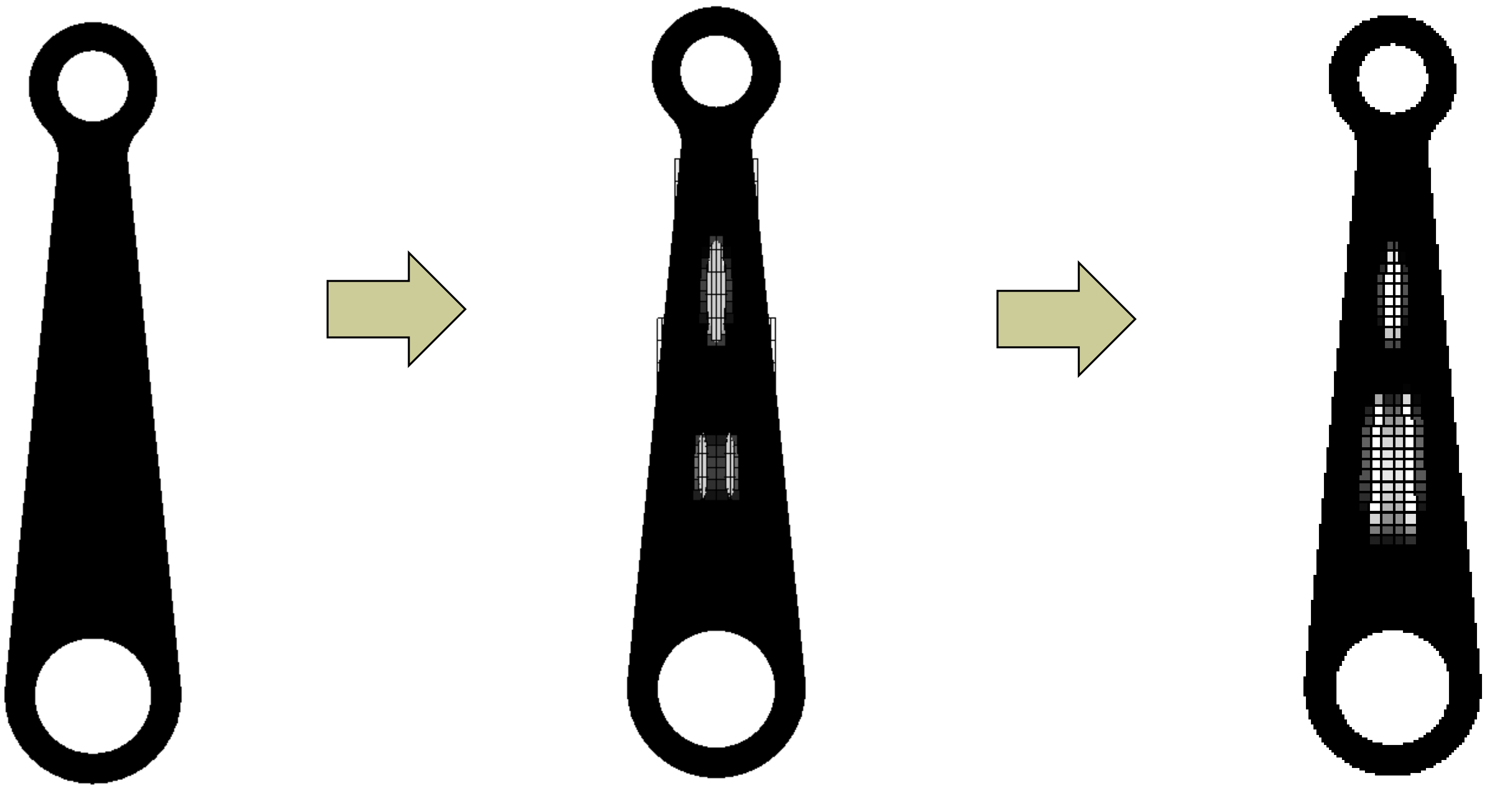
Results

- Convergence obtained after 15 iterations
- Monotonous behavior of the optimization process
- Even better than the simpler case



Results – Optimal design

- Modification of the topology during the evolution of the optimization process



- It should be interesting to add the position of the level sets in the design variable set.

Conclusions and Perspectives

Conclusions

- Optimization of flexible components carried out in the framework of flexible dynamic multibody system simulation
- Type of optimization between shape optimization and topology optimization
- Validation of the proposed approach
- Combine the advantages of both methods:
  - No mesh problem
  - Possibility of changing the topology but must be introduced before the optimization.
  - The geometry is expressed by CAD entities
    - ➔ Can be directly manufactured.

Perspectives

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- Semi-analytical approach for the derivatives

$$\delta u_m = \frac{1}{A_m} \int_C \mathbf{V}^t \mathbf{n} \, d\Gamma$$

Nam H. Kim,
Youngmin Chang,
2005

Need to establish the relation between the velocity field of the level set boundaries and the design variables.

- Progress on the optimization problem formulation

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Thank You Very Much For Your Attention

Contact

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