NONLINEAR DYNAMICS OF A DRILL BIT UNDER PERCUSSIVE ACTIVATION

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Summary The dynamics of the percussive drilling process is investigated with a 1-DOF discrete model, periodic impulsive force loading and a realistic bit/rock interaction model. In particular, the limit cycling behavior of this dissipative piecewise-smooth model is explored with the aid of analytical and numerical tools to perform a bifurcation analysis. The existence of optimal activation frequencies, that depend on the rock characteristics, is demonstrated.

INTRODUCTION

This paper discusses the steady-state response of a drill bit under periodic impulsive activation, unilaterally constrained by a bilinear contact law representing the bit/rock interface law. This model is an extreme simplification of the process of down-the-hole percussive drilling, that has widespread utilization in the industry of earth resources exploitation. Its study aims at merging the independent approaches of the scientific community which has separately focused on the bit dynamics under harmonic activation [1] or on the bit/rock interaction model [2]. While the former works have dealt with the motion of a bit subject to unilateral viscoelastic sliding contact, the latter studies have shown that a realistic representation of the contact force at the bit/rock interface is the rate-independent bilinear law shown in Figure 1, relating the contact force to the bit penetration. Our study meets that of Ajibose et al. [3], who considered a nonlinear elasto-plastic contact law and harmonic excitation of the bit and generalizes it to impulsive loading.

Neglecting wave propagation in the system and reducing the drillstring to the drill bit itself leads to the model depicted in Figure 2. The vertical loads are of three types: (i) the static loads $F_S + Mg$, (ii) the impulsive force activation δF_T due to hammer impacts on the bit, chosen with constant impulsion and period T, and (iii) the rock reaction on the bit F_R that follows the bilinear contact law. The vertical depth is measured by the variable y.

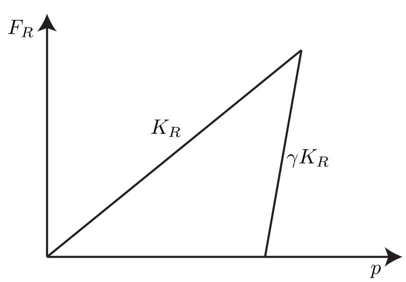

Figure 1. Bit/rock interaction law.

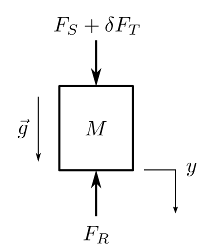

Figure 2. Drill bit free body diagram.

GOVERNING EQUATIONS

Due to the unilateral nature of the contact at the bit/rock interface, the bilinearity of the contact law and the impulsive characteristics of the percussive activation, the bit dynamics is governed by a set of piecewise-linear ordinary differential equations and the bit velocity presents a discontinuity at each force pulse. It is the hybrid nature of this linear system that renders it discontinuous and nonlinear, therefore complex.

From the piecewise nature of the contact force, we define three drilling regimes: (i) *free flight* (FF) during which the bit is off the hole bottom; (ii) *forward contact* (FC) that corresponds to contact at the bit/rock interface and downward motion; and (iii) *backward contact* (BC) where contact at the interface is established but the motion is upward.

To avoid the unbounded growth of the bit depth with respect to time, we express the equation of motion in terms of the penetration while drilling. It is defined as the advance of the bit with respect to the final contact position of the bit/rock interface during the previous drilling cycle plus the residual penetration, has the cycle not been completed.

Introducing reference scales, in time $\mathbb{T} = \sqrt{M/K_R}$ and in length $\mathbb{L}$, the latter being taken as the typical penetration per drilling cycle, we write the dimensionless governing equations in terms of the penetration θ, for each regime. They read

$$\text{FF: } \bar{\bar{\theta}} = \lambda_T, \quad \text{FC: } \bar{\bar{\theta}} + \theta = \lambda_T, \quad \text{BC: } \bar{\bar{\theta}} + \gamma\theta = \lambda_T + (\gamma - 1)\theta_p, \quad \text{IMPACT: } \bar{\theta}(\tau_i^+) = \bar{\theta}(\tau_i^-) + \Delta\bar{\theta}_i,$$

with the overhead bar referring to differentiation with respect to the dimensionless time $\tau = t/\mathbb{T}$, θ_p to the peak penetration or penetration at the transition between the forward contact and backward contact regimes, λ_T being the sum of the vertical

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dead loads and $\Delta\bar{\theta}_i$ the velocity jump at activation. The equations of motion are supplemented by transition conditions which rule the occurrence of an event, be it an impact or a change of regime. Their definition requires the introduction of two history variables: the peak and upper penetrations, θ_p and θ_u, the latter being the penetration at which contact is lost at the bit/rock interface. As examples, we give the conditions for FC to BC, BC to FF, FF to FC transitions and impact occurrence

$$h_{\mathsf{FC}\to\mathsf{BC}}(\bar{\theta}) \equiv \bar{\theta} = 0, \quad h_{\mathsf{BC}\to\mathsf{FF}}(\theta) \equiv \theta_p + \gamma(\theta - \theta_p) = 0, \quad h_{\mathsf{FF}\to\mathsf{FC}}(\bar{\theta}) \equiv \theta - \theta_u = 0, \quad h_{\Delta\bar{\theta}_i}(\tau) \equiv \tau - \tau_i = 0.$$

ANALYSIS

In order to analyze the steady-state response of this model, dedicated simulation tools have been implemented. The piecewise linear nature of the equations of motion allowing piecewise analytical solutions, a semi-analytical integration procedure has been developed and used within an arclength-parameterized continuation procedure to compute bifurcation diagrams of limit cycles by shooting. Numerical computation of Floquet multipliers is also performed by the simulation toolbox, to enable the assessment of the stability of periodic solutions.

Figure 3 shows a typical result from the continuation procedure. Limit cycles are characterized by their periodic sequence or by their motion regime giving the number of drilling cycles m per n activation period(s), m/n; for the current example (BC$\to$FF$\to$FC$\to\Delta\bar{\theta}_i\to$FC)$_\circlearrowleft$ and $1/1$. Also their stability and the corresponding average rate of penetration is evaluated for each configuration yielding the bifurcation diagram of Figure 4. As can be observed, harmonic limit cycles do experience a loss of stability under variation of the impact frequency. This is to the profit of subharmonic stable periodic orbits which are not considered in this plot. Besides this information, the diagram illustrates the existence of local optima; that is, parametric configurations that correspond to a stable steady-state harmonic periodic response with locally maximal average rate of penetration. The existence and the location of these optima depend on the bit/rock interface parameters, as highlighted by the curves for different values of the unloading interface stiffness, $\gamma = \{3, 10, 33\}$.

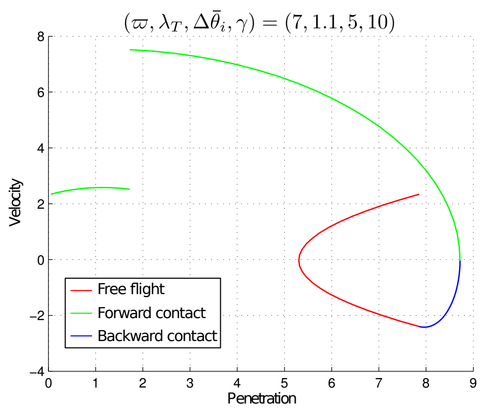

Figure 3. $1/1$ limit cycle: Phase portrait of bit trajectory.

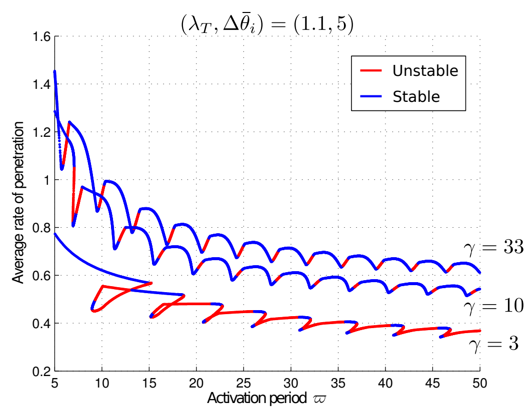

Figure 4. $m/1$ limit cycle: Average rate of penetration.

CONCLUSIONS

A physically consistent model of a down-the-hole percussively activated drill bit, that accounts for realistic bit/rock interactions and for the impulsive nature of the activation, has been proposed. By performing a bifurcation analysis using a continuation procedure, we bring to light the importance of the percussion frequency on the expected performance of the tool. Existence of optimal parametric configurations is shown as well as their dependence on the bit/rock interface parameters.

Acknowledgements

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References

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