A flexible two-step randomised response model for estimating the proportions of individuals with sensitive attributes

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Problem definition

Let $P$ be the proportion of individuals in a given population presenting a certain characteristic of interest (e.g., smoking, heart defect, diabetes,…).

Problem: Estimating $P$ from a sample of size $n$ drawn from the population

Classical studies: Let $r$ be the number of subjects with the characteristic in the sample, then an estimate of $P$ is given by

$$\hat{p} = \frac{r}{n}$$

and

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

IC 95%: $\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
Difficulties

When the attribute of interest is a “sensitive” one (e.g., illicit drugs consumption, psychiatric disorder,..), people tend to refuse to answer or intentionally mislead

\[ \downarrow \]

biased estimation of the true proportion \( P \) in classical method

Solution to reduce or eliminate bias linked to the wish to keep private life secret –→ Random Response Model (RRM) (Warner, 1965)
RRM approach (I)

- **Objective**: reassuring the respondent that a possible affirmative answer to sensitive questions can’t put him/her in a dangerous situation

- **Method**: - Pair the sensitive question with an unrelated one.
  - Respondent select randomly one of these two questions.
  - Interviewer records only a “Yes or No” without knowing which question has been answered.
RRM approach (II)
New approach (I)

- No unrelated question but merely the sensitive question.
- Question with categorical answers which can be combined with some flexibility to have a binary outcome.
New approach (II)

Let $f$ denote the probability of a positive answer regardless of the question,

$$f = qp + m(1 - q)$$

Then, as the probabilities of the 2 random processes $q$ and $m$ are supposed to be known (and can be fixed a priori by the investigator), the probability of a positive answer to the sensitive question is

$$p = \frac{f - m(1 - q)}{q}$$
Properties (I)

If $n_1$ and $n_{12}$ are the observed numbers of subjects responding positively according to the outcome of the Step1 and Step1+ Step2, respectively

$$\hat{f} = \frac{n_1 + n_{12}}{n}$$

Consequently, $\hat{p} = \frac{\hat{f} - m(1 - q)}{q}$ if the right hand-side is $>0$

and $\hat{p} = 0$ otherwise;
Properties (II)

Statement 1: \( \hat{p} \) is an unbiased and consistent estimator of \( p \)

- **Unbiasedness**
  \[
  E(\hat{p}) = E \left[ \frac{\hat{f} - m(1 - q)}{q} \right] \\
  = \frac{E(\hat{f}) - m(1 - q)}{q}
  \]
  As \( \hat{f} \) is an unbiased estimator of \( f \)
  \[
  \rightarrow E(\hat{p}) = \frac{f - m(1 - q)}{q} = p
  \]

- **Consistency**
  By the law of large sample, \( \hat{f} \rightarrow f \)
  Hence,
  \[
  \hat{p} = \frac{\hat{f} - m(1 - q)}{q} \rightarrow p
  \]
Statement 2: \( \text{Var}(\hat{p}) = \frac{1}{nq^2} f(1 - f) \)

\[
\text{Var}(\hat{p}) = \left( \frac{\hat{f} - m(1 - q)}{q} \right)
\]

\[
= \frac{1}{q^2} \text{Var}(\hat{f})
\]

Since \( m \) and \( q \) are constant and as \( \text{Var}(\hat{f}) = \frac{f(1 - f)}{n} \)

\[
\text{Var}(\hat{p}) = \frac{1}{nq^2} f(1 - f)
\]
Thus for large \( n \),

\[
\sqrt{\frac{1}{nq^2} \hat{f}(1 - \hat{f})} ~ N(0,1)
\]

Confidence interval

\[
\hat{p} - Q_z \left(1 - \frac{\alpha}{2}\right) \sqrt{\frac{1}{nq^2} \hat{f}(1 - \hat{f})} ; \hat{p} + Q_z \left(1 - \frac{\alpha}{2}\right) \sqrt{\frac{1}{nq^2} \hat{f}(1 - \hat{f})}
\]
One-sample power (I)

Determination of sample size $n$, so that $|\hat{p} - p| \leq \Delta$

at the $\alpha\%$ significance level

$$\Delta = Q_Z \left(1 - \frac{\alpha}{2}\right) \sqrt{\frac{1}{nq^2} f(1-f)} \quad \Rightarrow \quad n = \frac{Q_Z^2 \left(1 - \frac{\alpha}{2}\right) f(1-f)}{q^2 \Delta}$$

As $f = qp + m(1-q)$,

$$n = \frac{Q_Z^2 \left(1 - \frac{\alpha}{2}\right) [qp + m(1-q)][1 - qp - m(1-q)]}{q^2 \Delta}$$
One-sample problem (II)

Example based on the prevalence of the use of cannabis.
Hypothesis: \( \Delta = 2\% \)
\[ \alpha = 5\% \]
\[ m = \frac{5}{6} \quad \text{and} \quad q = 0.5 \]
\[ p = 25\% \]

Plugging these values in the previous formula, provides

\[ \rightarrow n \approx 191 \]
Application : Question

During all life, how many times have you used illicit drug?

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<table>
<thead>
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<tbody>
<tr>
<td>1</td>
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<td>3</td>
<td>=</td>
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<td>4</td>
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<tr>
<td>5</td>
<td>=</td>
</tr>
<tr>
<td>6</td>
<td>=</td>
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Flip the coin (RP1), if the result is tail answer to question «a». If the result is head, go to «b».

a) During all life, how many times have you used illicit drug?

1 = Never
2 = 1-2 times
3 = 3-5 times
Response □
4 = 6-9 times
5 = 10-19 times
6 = 20 times or more

b) Roll the dice once (RP2). What is the result?
Undergraduate students registered at the University of Liège during the academic year 2003-2004

<table>
<thead>
<tr>
<th>Characteristics of the students (n=435)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (year)</td>
</tr>
<tr>
<td>Gender</td>
</tr>
<tr>
<td>Female (%)</td>
</tr>
<tr>
<td>Male (%)</td>
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</tbody>
</table>
Application: Practical aspect

- During supervised practical works
  → more receptive and concentrated

- Explanation

- Closed box
Application: Results

Sensitive Question
“Yes” answer : 2-6 (1 or more)
“No” answer : 1 (Never)

→ $m = \frac{5}{6}$

<table>
<thead>
<tr>
<th>Sensitive question</th>
<th>Prevalence (± SE)</th>
<th>95% CI</th>
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<tr>
<td>Illicit drug</td>
<td>40.9 (± 4.7)</td>
<td>31.7 - 50.0</td>
</tr>
</tbody>
</table>
Application: Flexibility (I)

Sensitive Question
“Yes” answer : 4-6 (6 or more)
“No” answer : 1-3 (Never - 5 times)

\[ m = \frac{1}{2} \]

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<th>95% CI</th>
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<tr>
<td>Illicit drug</td>
<td>26.7 (± 4.7)</td>
<td>17.6 – 35.9</td>
</tr>
</tbody>
</table>
Sensitive Question
“Yes” answer: 6 (20 or more)
“No” answer: 1-5 (Never - 19 times)

\[ m = 1/6 \]

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<th>Sensitive question</th>
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<th>95% CI</th>
</tr>
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<tbody>
<tr>
<td>Illicit drug</td>
<td>16.4 (± 3.6)</td>
<td>9.3 – 23.8</td>
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</table>
A classical anonymous questionnaire was distributed to 
n = 462 undergraduate students also registered at the University of 
Liège during the academic year 2003-2004.

<table>
<thead>
<tr>
<th>Illicit drug</th>
<th>Prevalence (± SE)</th>
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<tr>
<td>Classical (n=462)</td>
<td>24.2 (± 2.0)</td>
</tr>
<tr>
<td>RRM (n=435)</td>
<td>16.4 (± 3.6)</td>
</tr>
</tbody>
</table>
Conclusion

- The problem of estimating the prevalence of sensitive attributes is quite common (public health, social life, economy, …)

- New approach of RRM by a two-step (RP1 – RP2) rather than by a one-step (RP1) random process response

- Flexibility with RP2 (question with multiple answers, modify)

- Can the two-step RRM be a substitute for the classical approach in general?

- Sensitivity analyses should be carried on.