POVERTY ALLEVIATION IN DEVELOPING COUNTRIES: THE ROLE OF INTERMEDIARIES AND TRANSPORT COSTS IN AGRICULTURE

AN APPLICATION TO THE MILK SECTOR IN SENEGAL

A thesis submitted by

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Introduction

This dissertation investigates the effects of a potential increase of production and commercialization of dairy products in Senegal, due to the emergence of intermediaries in this sector. In this country, as in most African countries, milk production takes place in an extensive pastoral or agro-pastoral system, where cattle are raised on pasture. It concerns a large part of the population, especially in rural areas: 48.12% of the Senegalese households (73.48% in rural areas) own cattle (ESPS, 2005). In general, households involved in agriculture, livestock and forest employments face poverty: 63.28% of them are considered as poor, compared to 37.82% in other employments (ESPS, 2005). In that sense, the development of the dairy sector has the potential to reduce poverty.

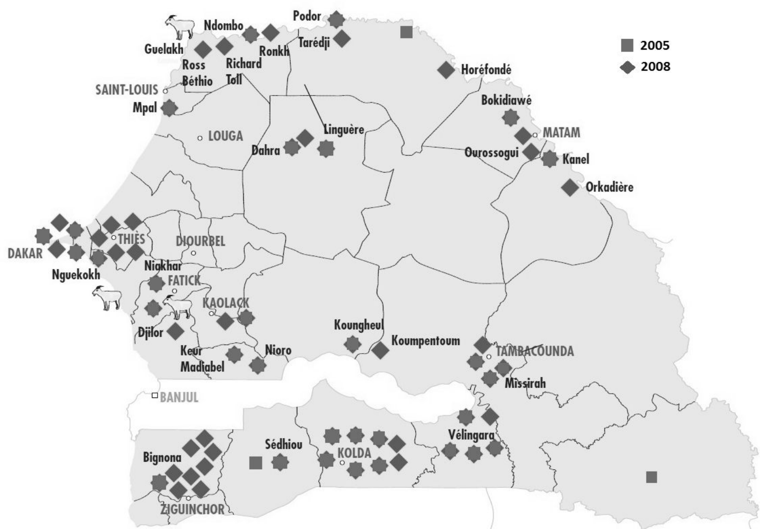
Although milk consumption in Africa is still low compared to the rest of the world, dairy products make now part of the consumption habits of most African households. In Senegal, the quantity consumed has quadrupled during the period 1961-1993. Nevertheless, despite this increased demand, the domestic milk production has risen by less than 40% during the same period (FAOSTAT, 2009).

It is often claimed that the expansion of the sector is hampered by a lack of competitiveness against imported milk powder. Indeed, currently, Senegalese demand is mainly satisfied by these imports, covering 68% of the demand (MINEFI, 2006). Producers' organizations and NGOs argue that prices remain too low due to this "unfair" international competition. This is seen as one of the reasons which keeps small producers in poverty. The famous Oxfam's campaign "Milking the CAP" was founded on the following claim: *"The EU dairy regime affects developing countries in three main ways: by depressing world market prices, by pushing developing country exporters out of third markets, and by directly undermining domestic markets in developing countries (...) The result is that domestic prices are depressed and local producers, many of whom live in poverty, are driven out of business"* (OXFAM, 2002: 16). In Belgium, Luxembourg and France, more than 30 000 people sign the petition "Lait, l'Europe est vache avec l'Afrique" based on the same assertion (CFSI, 2007).

However, other factors explain the relative inertia of the Senegalese milk sector. This is partly due to the characteristics of the livestock sector: generally, each peasant has only some cows and each one provides between 0.5 and 2 liters of milk per day. Both elements lead to small quantities of milk produced, between 2 and 10 liters per day (DUTEURTRE, 2006). The productivity per animal is determined by its breed (local cattle breeds, Zebu Gobra, Taurine N'Dama or D'jakoré are known to have low productivity) but also to the quantity of feeds available. Cattle are mainly raised on pasture where grass is only abundant during the wet season (from June to October). Feed supplements can be provided by the use of organic manure and harvest residues, notably from cotton and sesame. However, one of the main constraints for improving milk production is the difficulty for the farmers to obtain these cattle feeds (Dieye et al., 2005, Dieye, 2003).

Another factor which hampers the increase of production are high transport costs. Holloway et al. (2000) found that, in Ethiopia, each additional minute walk to the collection center reduces the marketable quantity of milk by 0.06 liters per day. In a region where milk yields per day are less than 4 liters, this is of considerable importance. High transport costs also have a negative impact on the use of feed supplement. In Kenya, Staal et al. (2002) have found that an additional 10 kilometers between the farmer and Nairobi decreases the probability of using concentrate feeding by more than 1%. The nature of the milk makes it difficult to transport on large distances. However, while production is distributed on most of the rural areas in the country, consumption is mainly concentrated in Dakar, sometimes at more than 300 kilometers from the producers. Inadequate transport infrastructure also contribute to increase the transport cost. As incurring large costs for transporting small quantities of milk may turn to be unprofitable, farmers sometimes prefer not to take part to the market, or to participate only some days, resulting in very low quantities of milk commercialized on the market.

Figure 1: Milk processing units ("mini-dairies") in Senegal (2005 and 2008)



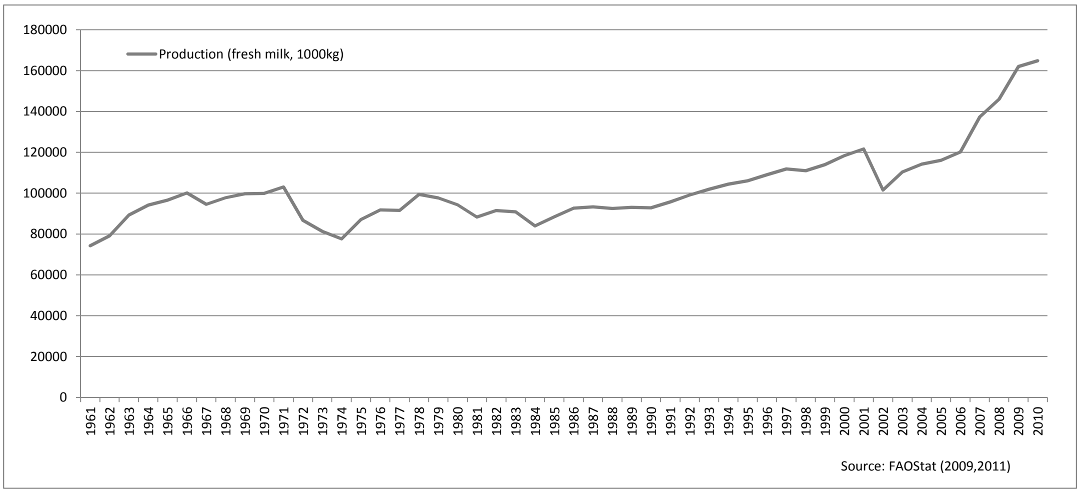
* Source: Broutin (2005) and Broutin (2008)

Since the nineties, we have seen the emergence of small-scale processing units called "mini-dairies" that play an intermediary role between the farmers and the market (Dieye et al., 2005, Corniaux et al., 2005). These intermediaries have some kind of advantage over the farmers to sell the products on the market. They use more efficient transport devices, such as trucks, they own bulk cooling tanks, such that they can stock the milk and do not have to transport it every day, etc.

These intermediaries seem to rapidly expand. Based on a survey conducted in 2002 in Kolda (Southern Senegal), Dieye et al. (2005) have reported that the quantities of milk collected by small-scale processing units in this area increased from 21250 liters in 1996 to 113600 liters in 2001 with the number of processing units increasing from 1 to 5. The quantity collected nearly doubled in the two following years (214205 liters collected in 2003) with the number of intermediaries increasing to 8 (Dieye, 2006). The same pattern is observed in the other regions. Figure 1 represents the expansion of these intermediaries between 2005 and 2008.

Potentially, these mini-dairies can play a role in the increase of local production and commercialization of dairy products in Senegal. On figure 2, one may note that domestic milk production, that have stagnated for 30 years, has begun to increase in the nineties. In this dissertation, we investigate several aspects related to these intermediaries.

Figure 2: Evolution of local milk production in Senegal (1961-2010)



Amongst the dairy products available in Senegal, milk powder represents 47% of the consumption (Duteurtre, 2006). This powder is transformed at home into liquid milk or sour milk, or directly used in the coffee for instance. Apart from the powder, dairy industry proposes transformed products that are highly appreciated by Senegalese households. Indeed, "industrial" (as opposed to home-made) sour milk accounts for 20% of the consumption (Duteurtre, 2006). Until recently, the Senegalese dairy industry produced mainly sour milk made with imported powder. However, the emergence of mini-dairies has brought a new product on the market: "industrial" sour milk made with local fresh milk.

In chapter 1, we investigate if a demand exists for this new product. In particular, we look if this product is substitutable with the powder-based sour milk, or if consumers are willing to pay a different price for a local milk based sour milk. Using data from a survey conducted on 400 households in the region of Dakar, we find evidence that consumers are willing to pay a positive premium for sour milk made with local fresh milk, based on a choice-based-conjoint analysis as well as on contingent valuation measures. Moreover, we identify some niche markets that the mini-dairies could target to sell their products at a considerably higher price than the imported ones. We also put into light that providing information to the consumers about the origin of the product is a crucial element for effectively being able to set a higher price for their products. With this respect, the creation of a certification for the local origin, as it has be considered by NGOs and producers' organizations (PROLAIT, 2009), could increase the value of local milk-based products with respect to the powder-based ones.

We observe that contracts between "mini-dairies" and farmers often involve interlinked transactions. Additionally to playing an intermediary role on the milk market, the dairy also play this role on the input market, providing the farmer the cattle feeds that are necessary for his production. In such a contract, both output and input prices are determined jointly. In the region of Kolda, DIEYE et al. (2005) report that processing units provide credit and cattle feeds to the farmers in order to increase commercial links. The two most important milk processing units in this region ("Bilaame Puul Debbo" and "Le Fermier") use three different mechanisms for linking milk purchase and feeds selling: credit for feeds purchase, direct feeds purchase for the farmer, or guarantee to the feeds seller in case of non-payment by the farmer (DIEYE, 2006). In Northern Senegal, "La Laiterie du Berger" buys large quantities of cattle feeds and resells it to the farmers at 50 percent of the market price (BATHILLY, 2007).

Chapter 2 examines the contract prices that should be optimal from the point of view of a mini-dairy using interlinked transactions. From this, we determine under which conditions the presence of such an intermediary could help to increase farmers' production and income and reduce poverty, measured by a Foster-Greer-Thorbecke indicator. We compare a benchmark spatial price discrimination case with two pricing policies that are observed in practice (uniform pricing and mill pricing) and derive some policy recommendations regarding the type of pricing that should be imposed to the intermediaries by a policy maker whose aim is to reduce poverty.

While the importance of smallholders' market participation for poverty alleviation in developing countries has been increasingly recognized (VON BRAUN and KENNEDY, 1994, HELTBERG and TARP, 2002, BARRETT, 2008 etc.), METZGER et al. (1995) report that, in Senegal, less than 25% of milk production is commercialized. According to DIEYE (2003), the presence of mini-dairies seems to improve this market participation. The survey he conducted in the peri-urban region of Kolda, where several mini-dairies are established, show that the level of milk commercialization reaches 75% in the wet season.

In chapter 3 we analyze under which condition the presence of an intermediary can improve farmers' participation to the market. We compare an interlinked contract model with a simple one, in order to derive if the interlinkage permits to improve farmers' participation. Using a Foster-Greer-Thorbecke indicator, we explore the potential poverty reduction effect of the different types of contract (interlinked or simple) and pricing policy (discrimination, uniform or mill pricing), that we decompose into two parts: on the one hand the poverty reduction due to the increase of the income of farmers who were able to take part to the market in the absence of the intermediary and, on the other hand, the poverty reduction due to the increased participation.

Finally, we want to determine under what conditions the intermediaries find it profitable to enter the market. On the one hand they have to support irreversible costly investment, and, on the other hand milk price is characterized by important volatility which leads to uncertainty and tends to discourage investment. In this context, using a net present value rule for determining the investment plant of such a firm may lead to overoptimistic results, hence, in chapter 4, we use the real option theory (Dixit and Pindyck, 1994). To better represent the reality, we take into account the possibility for the firm to further expand its collection area once the first investment is made. Taking this possibility into account drastically changes the results compared to a model where the size of the collection area is fixed. Using data on the milk sector in Senegal, we simulate these effects in a real context. We check the profitability of two existing mini-dairies and explore whether the project of transforming local milk into powder could be profitable.

Chapter 1

Willingness-to-pay for local milk-based dairy products in Senegal*†

1.1 Introduction

Senegalese (and West African) dairy sector is claimed to be under pressure due to cheap imports of milk powder coming from European Union (see for instance Oxfam, 2002 or CFSI, 2007). Indeed, currently, Senegalese demand is mainly satisfied by these imports, local production covering only 32% of the demand (MINEFI, 2006).

Amongst the dairy products available in Senegal, milk powder represents 47% of the consumption (Duteurtre, 2006). This powder is transformed at home into liquid milk or sour milk, or directly used in the coffee for instance. Apart from the powder, dairy industry proposes transformed products that are highly appreciated by Senegalese households. Indeed, "industrial" (as opposed to home-made) sour milk accounts for 20% of the consumption (Duteurtre, 2006). Until recently, the Senegalese dairy industry produced mainly sour milk made with imported powder. However, since the nineties, small-scale milk processing units, which ensure rural milk collection, seem to rapidly expand (Corniaux et al., 2005, Dieye et al., 2005). These units propose sour milk made with fresh local milk.

No significant difference seems to appear between the market prices of local milk based sour milk and imported powder based one. A possible explanation could be that the consumers are indifferent between both products. In that case, they would be perfectly substitutable, and the so-called import surge of milk powder from Europe is favorable for consumers, as it makes cheaper products available.

However, it is suggested that consumers prefer local milk based products. Broutin et al. (2006) show that 90% of households consuming local sour milk would like to increase

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†We are grateful to Tatiana Goetghebuer, Bernard Lejeune, Joe Tharakan and Vincenzo Verardi for discussion, comments and helpful suggestions, as well as to Cécile Broutin and GRET for providing the data.

their consumption but cannot do it because of the lack of availability (mentioned by more than 50% among them). Another study, from Sissokho and Sall (2001), states that 79% of the consumers consider that local milk-based dairy products have a higher quality than imported ones.

In what follows, we use data on stated preferences to confirm or infirm the assertion that local milk based products are preferred. In particular, we want to estimate if consumers are willing to pay a positive premium in order to consume local fresh milk based sour milk in opposition to a product made with imported powder. Additionally, we would like to quantify this premium.

If it turns that consumers are willing to pay a positive premium for local milk based products, the two kinds of goods are not perfectly substitutable and a difference should appear in market prices. Hence one question is why we do not observe such a difference. We suspect that part of the answer comes from the difficulty to distinguish local from imported products. Indeed, the presentation (packaging, advertising, etc.) of powder based products may sometimes induce consumers to believe that they are made with local milk (see for instance Bakhoum, 2006).

If consumers are willing to pay more for local products, it means that there exists an opportunity for local origin certification as it has be considered by NGOs and producers' organizations (Prolait, 2009). This certification would increase the value of local milk-based product with respect to the powder-based ones, giving to local producers the possibility to compete with imports, despite their higher production costs (mainly high transport costs due to the perishable nature of fresh milk and to the poor quality of road infrastructures). In this case, local milk-based products could be sold on the Senegalese market at such a price that they find a demand. If this condition is satisfied, increasing local milk production may be profitable to consumers as well as to producers. In a country where, in rural areas, seven out of ten households own cattle, this expansion would increase the income of a large share of the population.

While a reliable certification for the local origin may be difficult to implement in a developing country context, at least producers who use local raw material could implement advertising that informs consumers about the local origin of their products. With this respect, we want to identify some niche markets of particular consumers that these producers should particularly target, as they are willing to pay a higher premium for local milk based products.

From a policy perspective, better regulation could be encouraged regarding the packaging of powder-based sour milk. Currently, the only regulation imposed is to mention the quantity of milk powder used, if it is larger than 5 grams per 100 grams of milk (Broutin, and Diedhiou, 2010). However, this is not always respected (Bakhoum, 2006). Moreover, no regulation is imposed on the type of packaging, such that imported products are often presented with a local zebu cow or a Peul woman, that induce the consumer to think they are made with local milk.

Nevertheless, origin certification, advertising focused on the local origin and lobbying for better regulation are relevant only if consumers do value local products more than imported ones. This is what we analyze, using data on 400 household in the region of Dakar and two kinds of methods: choice-based-conjoint (CBC) analysis and contingent valuation (CV). Several studies have been conducted in various context to evaluate the willingness-to-pay (WTP) for the local origin, using different methods such CBC and CV but also experimental auctions or hedonic prices. However, this literature is mainly concerned by consumption choices in developed countries and, to our knowledge, evaluations of the WTP for local products have never been conducted in Africa.

In choice-based-conjoint analysis, individuals are asked to choose between alternative products defined by various attributes including the price. Comparing the choices allows to estimate the WTP for the different characteristics, notably the local origin (see for instance, Alfnes (2004) on Norwegian beef compared to Swedish and Botswana ones, Quagrainie et al. (1998) on Alberta-labeled beef or Darby et al. (2006) on labeled "Grown in Ohio" strawberries).

Contingent valuation consists in directly questioning individuals about their WTP. Conducting this type of survey on consumers from Colorado, Loureiro and Hine (2002) have found that locally grown potatoes carry a potential premium of about 10% over the initial price. Loureiro and Umberger (2003) have evaluated that respondents are willing to pay 38% more for "US Certified Steak" and 58% more for "US Certified Hamburger". In Vandermersch and Mathijs (2004)'s study, more than 50% of the respondents agree to pay 0.05 or 0.1 euros more for Belgian milk. Buchardi et al. (2005) have determined that German consumers have a higher WTP (about 0.18 euros per liter) for fresh milk from their own region compared to the same product from another region.

Both methods suffer from the so-called "hypothetical bias", the tendency for stated WTP to overestimate actual WTP (Cummings et al., 1995). It is due to the hypothetical nature of question: the transaction does not effectively occur: consumers do not really have to spend money. In the CBC case, this bias is mitigated, as consumers are asked to mimic their typical purchase choices, but not completely eliminated. Carlsson and Martinsson (2001) have shown, in the case of public goods (environmental projects) that the (hypothetical) preferences expressed in a CBC survey are not significantly different from the (actual) ones expressed when the money transfer takes place. In the case of private goods (beef steaks) however, Lusk and Schroeder (2004) have found that hypothetical responses are statistically different from the actual ones. As our analysis is focused on a pure private good, we must treat the results with caution. CBC analyses generally overestimate the WTP. In the case of contingent valuation, the hypothetical bias represents generally a major limitation. Nevertheless, in the survey we use, respondents are not directly asked to state their willingness to pay for the local origin, but instead they have to state the price they are willing to pay on one hand for local milk based sour milk and on the other hand for powder based one. Comparing these values, we expect to have an unbiased estimate of the premium they want to pay for the local origin. Contingent valuation has also the advantage of consisting in a very simple task for the respondent. In the CBC analysis however, the respondent's task is more difficult, responses may be inconsistent across questions, answers may be influenced by the complexity of the task, etc.[1]

[1]See Lusk and Hudson, (2004: 157) or Carlsson and Martinsson (2001: 180) for a more complete

This chapter is structured as follows. Next section presents an overview of observed milk prices in Senegal. In section 3, we present the methodologies we use to estimate the willingness-to-pay for local products, and describe the data. Section 4 is devoted to model specifications and hypotheses while section 5 exposes and discusses the results. Finally, section 6 concludes.

1.2 Observed prices

Table 1.1 shows the results of a linear regression of the observed prices of various sour milks on some of their characteristics, including the raw material (fresh local milk or imported powder). This analysis uses data on the prices of 41 products (7 different brands) collected in the supermarkets of Dakar in November 2005 (Duteurtre, 2006). Due to the small size of the sample, the results have to be interpreted with caution. Nevertheless, they do not show any significant difference between prices of sour milks made with powder of fresh milk, i.e. the coefficient related to the raw material is not significant.

Table 1.1: Determinants of observed prices (linear regression)

Variable	Coeff.	(s.e.)
Packaging (Bucket=1, sachet=0)	265.86***	(42.65)
Taste (Sugar=1, no sugar=0)	32.67	(42.81)
Raw Material (Fresh=1, powder=0)	0.997	(60.45)
Volume (in liters)	-79.48***	(27.09)
Constant	856.36***	(47.98)

Number of observations: 41 products. R^2=0.5318.

Dependant variable: price per liter. *** indicates significance at 1% level.

This does not necessarily implies that consumers are not willing to pay more for local products. We suspect it is partly due their inability to distinguish the raw materials. Table 1.2 reports summary results from the GRET survey[2] question *"according to you, what is the raw material of the following products (brands): powder or fresh milk?"*. The results are reported only for respondents who consume the brand. General ignorance about the raw material is noticed for the brands that are made with powder. For instance, 41.75% of the respondents consume Niiw, but only 17% among them know it is made with powder. More than 50% think it is made with fresh raw material. However, more than 75% of the respondents who consume Wayembam correctly answer that it is made with fresh milk. This seems to indicate that people consuming a product made with fresh milk do an informed choice, while people who consume sour milk made with powder might have chosen another product if they were better informed. It is even more a concern as

review of the CBC drawbacks.

[2]See the next section for a description of this survey.

85.75% of the respondents affirm that they are able to recognize fresh raw material from powder and vice-versa.

Table 1.2: Product knowledge

Brand	% of respondents who consume	% of consum. who think it is made with powder	% of consum. who think it is made with fresh milk	% of consum. who don't know
Brands made with powder				
Starlait	27.00	52.78	21.30	25.93
Sarbi	27.50	20.91	52.73	26.36
Niiw	41.75	16.77	55.69	27.54
Ma Kalait	0.50	50.00	0.00	50.00
Sen Sow	16.75	14.93	49.25	35.82
Banic	5.75	26.09	43.48	30.43
Taif Sow	7.75	25.81	41.94	32.26
Jaboot	36.25	33.79	32.41	33.79
Brand made with fresh milk				
Wayembam	16.50	10.61	77.27	12.12

It is not surprising that consumers of powder-based sour milk think it is made with fresh milk, as the advertising about these products is often ambiguous: for instance, most of the brands include Wolof words (such as "sow", which means "milk"). Even when the composition is clearly indicated, most consumers do not read it, or are not able to read it, and are more influenced by a picture of Senegalese characters or local zebu cows on the packaging.

Table 1.3: Impact of marketing on price (linear regression)

Variable	Coeff.	(s.e.)
Packaging (Bucket=1, sachet=0)	263.45***	(40.96)
Taste (Sugar=1, no sugar=0)	19.28	(45.61)
Picture (Local=1, other=0)	118.19***	(43.21)
Name language (Wolof=1, other=0)	-5.70	(51.70)
Volume (in liters)	-71.53***	(24.81)
Constant	806.61***	(61.06)

Number of observations: 38 products. R^2=0.6427.

Dependant variable: price per liter. *** indicates significance at 1% level.

A question is whether this misinformation about the composition of the product is important in determining its price. Table 1.3 gives us an indication that it has a significant impact. This table shows how those marketing characteristics affect the price. The methods and data used are the same as in table 1.1 but the explicative variable "fresh raw material" is now replaced by the characteristics that tend to persuade the consumers that the raw material is fresh milk. Subject to the same caution as before, results from table 1.3 indicate that the presence of a local image on the packaging significantly increases the price of sour milk. On average, products that have such a picture cost 118 CFA more per liter.

We suspect that consumers are willing to pay more for local products but are not able to do so because they are not able to recognize such products. In what follows, we test this affirmation by estimating the willingness-to-pay for the local raw material, based on hypothetical products, such that the information problem is eliminated (i.e. respondents are perfectly aware of the composition of the hypothetical product).

1.3 Data and methods

We use data from a survey realized in April 2002 in the context of the program "INCO MPE agroalimentaires" coordinated by the NGO GRET[3] (Broutin et al., 2006), on 400 households from the region of Dakar (departments of Dakar, Pikine and Rufisque).

The survey includes rating/ranking choice-based-conjoint (CBC) data about sour milk. Eight hypothetical sour milks (products A to H in table 1.4) were proposed to the respondents. These products differ by their characteristics (or attributes) and price, but are chosen to represent the reality, i.e products with the same characteristics and price might exist on the Senegalese market.[4]

Table 1.4: Hypothetical products proposed to the respondents

Product	Packaging	Taste	Raw material	Price (CFA)
A	per weight	no sugar	powder	275
B	per weight	sugar	fresh	325
C	per weight	sugar	powder	225
D	sachet	sugar	fresh	275
E	sachet	no sugar	fresh	225
F	sachet	no sugar	powder	325
G	sachet	sugar	powder	225
H	per weight	no sugar	fresh	225

All these products are liquid sour milk, made with fresh milk or with milk powder, packed individually (sachet) or sold per weight, with or without additional sugar. Note that no mention of local characteristic is made. However we use the attribute "fresh raw material" as a proxy for "local raw material". Indeed, up to now, there is not any milk powder produced in Senegal, thus the powder form of the raw material implicitly returns

[3]Groupe de recherche et d'échanges technologiques, www.gret.org.

[4]When constructing the survey, the GRET has identified four relevant attributes (packaging, taste, raw material and price) and corresponding levels using Kelly's repertory grid method (see for instance Steenkamp and Van Trijp, 1997). Combining attributes levels gave 2x2x2x3=24 possible hypothetical products, that was reduced to 8 using the SPSS Orthoplan procedure (see SPSS (2005) for more information about the procedure). This sub-set is designed to capture the main effects for each attribute level.

to its imported origin. Our own informal discussions with Senegalese consumers confirm that they consider that powder is always imported and fresh milk always local. However, we are not able, in this study, to distinguish the valuation of taste due to the freshness of the local raw material and the pure impact of the local origin.

In a first step, consumers facing the eight proposed products, were asked "which product(s) are you willing to buy now, taking into account its (their) characteristics and price?". The highest note (5) was given to this (these) product(s). In a second step, respondents were asked which product(s) they are not willing to buy, given its (their) characteristics and price. This (these) product(s) obtained the lowest note (1). In the last step, respondents had to rank the remaining products in three categories, corresponding to the notes 4, 3 and 2.

Table 1.5: CBC descriptive results

Product	Mean note	Note=1	Note=2	Note=3	Note=4	Note=5
		(least preferred)	(middle classes)			(most preferred)
A	2.59	39.75 %	12.25 %	12.00 %	21.50 %	14.50 %
B	3.17	25.75 %	12.25 %	11.00 %	21.00 %	30.00 %
C	2.77	31.75 %	15.75 %	13.00 %	23.25 %	16.25 %
D	4.10	8.50 %	3.75 %	9.25 %	21.75 %	56.75 %
E	3.94	9.25 %	5.75 %	11.25 %	29.25 %	44.50 %
F	3.20	19.50 %	16.50 %	12.75 %	27.50 %	23.7 %
G	3.84	10.00 %	10.25 %	9.50 %	26.50 %	43.75 %
H	3.22	23.00 %	11.75 %	1325 %	24.00 %	28.00 %

Number of observations: 400 households.

This scheme combines two properties that may be used for evaluate the WTP. On the one hand, people were asked to give a note (from one to five) to alternative products, this is known as rating CBC. However, the intensity of the notes may depend on unobserved individual fixed effects. Nevertheless, the particular design of the question (i.e. first giving rate 5, then rate 1, then the other rates) tends to reduce this effect. On the other hand, respondents also had to rank the alternatives from the most preferred to the least preferred one, this is a known as ranking CBC. It is commonly accepted that the first two or three ranks as well as the last two or three reflect real preferences.[5] As the GRET survey contains five ranks, we are confident that they reflect real preferences.

As we trust both rating and ranking are reliable in our setting, we will use both interpretations in our analysis. Note that tied rates/ranks are allowed, i.e. an individual may give the same rate/rank to several alternatives. Indeed, there are 8 alternatives for only 5 possible rates/ranks. Table 1.16 in Appendix illustrates the importance of tied ranks. For instance, on average, consumers give a note 5 (most preferred) to 2.6 products and a note 1 (least preferred) to 1.7 products. We will interpret tied rates/ranks as follows: when a consumer gives the same note for two products, we consider he is

[5] See for instance Wilson and Corlett (1995: 77).

indifferent between them. But it could also be considered that a ranking for these goods exists, but is unknown.

Table 1.5 gives some descriptive results from the CBC data. The hypothetical product that receives the highest average note (4.10) is the product D that costs 275 CFA and has the following characteristics: individually packed (sachet), with sugar and made with fresh milk. 56.75% of the interviewed consumers gave a note 5 (the highest note) to this product. The product that receives the lowest average note (2.59) is product A. 39.75% of the respondents gave it a note 1 (the lowest note).

Table 1.6: Contingent Valuation descriptive results

Question:	Mean	Std. Dev.	Min	Max
"What is a **reasonable** price for a sachet of 1/2 l of sour milk made with powder?"	312.72	120.62	125	1500
"At what price do you think a sachet of 1/2 l of sour milk made with powder is **expensive** but you still buy it?"	406.45	154.19	200	2000
"At what price do you think a sachet of 1/2 l of sour milk made with powder is **so expensive** that you do not buy it?"	563.41	262.36	250	3000
"At what price do you think a sachet of 1/2 l of sour milk made with powder is **cheap** but you still buy it?"	240.12	90.66	125	1000
"At what price do you think a sachet of 1/2 l of sour milk made with powder is **so cheap** that you doubt about its quality and you do not buy it?"	178.38	48.41	125	500

Number of observations: 399 households.

Question:	Mean	Std. Dev.	Min	Max
"What is a **reasonable** price for a sachet of 1/2 l of sour milk made with fresh milk?"	339.56	121.02	100	1000
"At what price do you think a sachet of 1/2 l of sour milk made with fresh milk is **expensive** but you still buy it?"	438.56	166.80	150	1500
"At what price do you think a sachet of 1/2 l of sour milk made with fresh milk is **so expensive** that you do not buy it?"	602	234.64	200	1800
"At what price do you think a sachet of 1/2 l of sour milk made with fresh milk is **cheap** but you still buy it?"	247.94	101.88	75	800
"At what price do you think a sachet of 1/2 l of sour milk made with fresh milk is **so cheap** that you doubt about its quality and you do not buy it?"	150.5	62.96	25	500

Number of observations: 400 households.

In addition to the CBC data, the GRET survey contains information about contingent valuation. Indeed, consumers were asked to answer to various questions about the price they find reasonable for sour milk made with powder and made with fresh raw material (see table 1.6 for descriptive results). Figure 1.1 is based on the cumulative density for the questions "what is a reasonable price for ...?" and "at what price do you think ... is expensive but you still buy it?". The curves for fresh raw material are above the corresponding one for powder, indicating that, for any given price p, a higher proportion of the consumers find p reasonable (resp. expensive) for sour milk made with fresh milk than for sour milk made with powder. For any proportion of the consumers, the price that is found reasonable (resp. expensive) for fresh raw material is higher that the reasonable (resp. expensive) price for powder.

Figure 1.1: Contingent valuation of reasonable and expensive prices

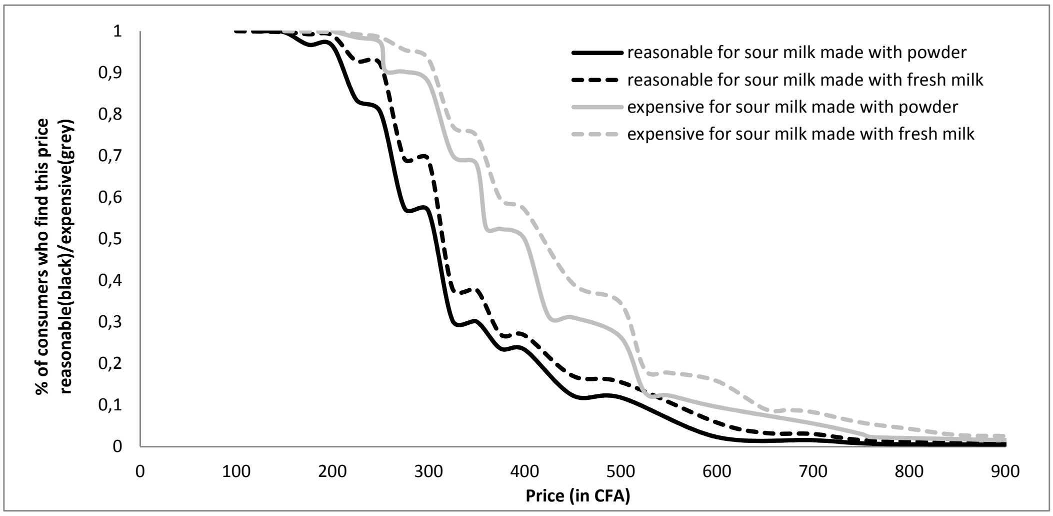

Table 1.7: Definition of contingent valuation measures

Variable	Definition
$Premium_R$	Reasonable price for sour milk with fresh raw material - Reasonable price for sour milk with powder
$Premium_E$	Expensive price for sour milk with fresh raw material - Expensive price for sour milk with powder
$Premium_{\%}$	(Reasonable price for sour milk with fresh raw material - Reasonable price for sour milk with powder) /Reasonable price for sour milk with powder

As already explained, CBC analysis as well as contingent valuation tend to overestimate the WTP due to the presence of an hypothetical bias. As individuals are not in a real situation of purchase, they tend to report higher stated WTP than the actual one. As CBC mimics consumers' behavior, it is assumed to reduce the bias (while not eliminating it, especially in the evaluation of WTP for private goods). For that reason CBC is generally preferred to contingent valuation. However, contingent valuation measures in the GRET survey provide reliable estimates of the WTP for the fresh raw material. Indeed, individual were asked, separately, to determine a reasonable price for sour milk made with powder and then made with fresh raw material. It can be reasonably assumed that the hypothetical bias acts the same way on both answers. Using the difference between them as a measure of the WTP for fresh raw material eliminates the bias, assuming it is additive. We used various measures, based on that difference, that are summarized in table 1.7. Histograms of the frequency distribution are presented for two of them on figure 1.2.

Figure 1.2: Contingent valuation measures (histograms)

(a) $Premium_R$ (b) $Premium_E$

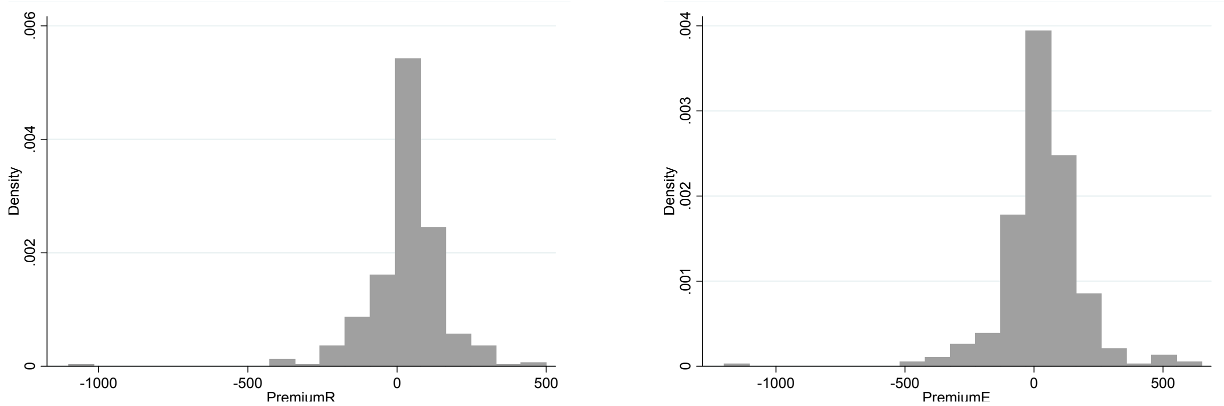

Table 1.8: Definition of socio-economic and demographic variables

Variable	Definition	
Department	Department	Dakar=0
		Pikine=1
		Rufisque=2
Ethnicity	Respondent's ethnicity	Wolof=0
		Peul/Toucouleur=1
		Other(ethn. minority)=2
Size	Number of members	Small: less than 5 members
	in the household	Big: more than 10 members
High education	Respondent's education	Secondary or more=1
		Others=0
Food expenses	Household's food expenses	Low: ≤75000 CFA
	per month	High: >150000 CFA
Housing	Housing type	Regular (with or without floor)=0
		High standing=1
		Social or provisional=2
TV	Color TV ownership	No=0 Yes=1

The survey also includes data about respondents and households' socio-economic and demographic characteristics such as department, ethnicity, education, size of the household, food expenses, etc. The definitions of the variables we use are presented in table 1.8. Some descriptive statistics are given in table 1.9. Households from Dakar department as well as medium size households are slightly overrepresented in the sample. It has to be noted that only households who consume sour milk were surveyed.

Table 1.9: Descriptive statistics

		Population[a] (Dakar Region)	Sample[b]
Department	Dakar	42%[d]	48.5%
	Pikine[c]	45.4%[d]	40.25%
	Rufisque	12.6%[d]	11.25%
Age (chief of household)	15 to 24	0.94%	0.75%
	25 to 34	10.45%	5.75%
	35 to 44	22.28%	19.5%
	45 to 54	28.09%	28.25%
	55 to 64	19.9%	21.75%
	65 and more	16.78%	15%
	Don't know/answer	1.56%	9%
Household size	Less than 5	23.15%	10.5%
	5 to 10	43.06%	62.5%
	More than 10	33.79%	27%
Housing ownership	Owner	62.39%	65.75%
	Tenant	33.85%	30%
	Free housing	3.32%	4.25%
	Others	0.44%	
Mean annual food expenses	(CFA)	1 291 085	1 220 022

[a]ESPS (2005), 1598 households in the Region of Dakar.
[b]GRET (2002), 400 households in the Region of Dakar.
[c]Since 2002, the department of Pikine has been divided into department of Guédiawaye and the new department of Pikine. Pikine population data for 2006 are calculated as the sum of the population of both new departments.
[d]ANSD (2006).

In spite of this, we trust there is no selection bias. Firstly, when doing inference, the population we are interested in is the population of sour milk consumers. Indeed, we would like to assess the additional price that those consumers are willing to pay to consume a local product rather than an imported one. We can reasonably believe that individuals who currently do not consume any kind of sour milk are not willing to consume local milk-based sour milk, and a fortiori, to pay an additional premium for it. Secondly, even if we do not know how non-consumers value the various kinds of sour milk, this only has a minor impact on the entire population behavior, as they represent a very small part of this population. Indeed, virtually all households do consume sour milk. For instance, in a survey of 82 households from Dakar, Duteurtre and Broutin

(2006)[6] have observed that all of them consume sour milk during the month following Ramadan.

1.4 Model specifications and hypotheses

1.4.1 Choice-based-conjoint analysis

Respondents' choices to the CBC questionnaire are modeled according to McFadden's Random Utility Model (RUM) (see for instance Anderson et al., 1992 or Louviere et al., 2000). We assume that, given a set of alternatives, the consumers choose the alternative that maximizes their utility. The utility U_{ij} that individual i gets by choosing alternative j is unobservable (latent variable) but can be defined by a deterministic component (V_{ij}) which is observable and a stochastic error term (ϵ_{ij}) which is not observable:

$$U_{ij} = V_{ij} + \epsilon_{ij} \tag{1.4.1}$$

We assume V_{ij} can be represented by the following additive linear function:

$$V_{ij} = \gamma Z_j + \theta p_j \tag{1.4.2}$$

where Z_j is a vector of attributes of the product j, p_j is the price of the product j, γ is a vector of coefficients to be estimated, θ is a coefficient to be estimated (expected to be negative).[7] This simple utility function (1.4.2) provides the main effects of the model. It indicates how each attribute affects the level of utility, when isolated from the other attributes. Indeed γ_k (element k of vector γ) represents how the attribute z_k (element k in each vector Z_j) contributes to the individual's utility.

From this expression, one can easily define the (deterministic) willingness-to-pay for an attribute (Champ et al., 2003: 189). Indeed, by differentiating equation (1.4.2), we see that the coefficient γ_k is nothing else that the marginal utility provided by the attribute z_k (i.e. $\partial V_{ij}/\partial z_k$). θ may be interpreted in a same way as the marginal utility of money ($\partial V_{ij}/\partial p_j$), such that the ratio $-\gamma_k/\theta = -(\partial V_{ij}/\partial z_k)/(\partial V_{ij}/\partial p_j)$ represents the marginal rate of substitution between the attribute z_k and money.[8] Facing any change in attribute z_k which would increase the utility V_{ij}, the individual is willing to pay the premium $-\gamma_k/\theta$ that keeps utility constant. Alternatively, he has to be paid $-\gamma_k/\theta$ to accept a change in attribute z_k that would decrease his utility.

[6] Referenced by Dia et al. (2008: 39).

[7] Note that a product-specific intercept (to be estimated) would have been included. Such an intercept α_j would represent the effect of non included (maybe non observable, such as quality) attributes of product j. As in the data, products are precisely defined by their four attributes, we assume $\alpha_j = 0$. An intercept to be estimated may be useful when alternatives are, for example, various brands of products, which implicitly represent their attributes.

[8] We expect that $-\gamma_k/\theta$ has the sign of γ_k, as θ is expected to be negative.

In particular, we estimate the following empirical specification:

$$V_{ij} = \gamma_1 Package_j + \gamma_2 Taste_j + \gamma_3 RawMaterial_j + \theta p_j \tag{1.4.3}$$

in order to evaluate, among others, the WTP for fresh raw material $-\gamma_3/\theta$.

To control for heterogeneity among consumers, we include socio-economic and demographic variables in the specification:

$$V_{ij} = \gamma Z_j + \theta p_j + \delta X_i \tag{1.4.4}$$

where X_i is a vector of individual i's characteristics and δ is a vector of coefficients to be estimated. In that model, the utility is not only affected by the attributes of the product but also by the individual's own characteristics.

Consumers' characteristics may not only affect their utility but also their preferences for the attributes of the products. To treat this, we include interactions effects:

$$V_{ij} = \gamma Z_j + \theta p_j + \delta X_i + \beta(X_i Z_j) \tag{1.4.5}$$

where β is a vector of coefficients to be estimated.

The WTP for an attribute z_k can still be defined as the marginal rate of substitution between attribute z_k and money. That is:

$$-\frac{\partial V_{ij}/\partial z_k}{\partial V_{ij}/\partial p_j} = -\frac{\beta X_i + \gamma_k}{\theta} \tag{1.4.6}$$

Here, the WTP for an attribute depends on socio-economic variables and differs thus among individuals.

Precisely, we are interested in measuring the effect of socio-economic variables such as income, education and household's size on the WTP for fresh raw material rather than powder. This has two main implications. Firstly, it will allow to identify niche markets of consumers that are willing to pay relatively more than others to consume fresh milk. Local producers should specially target these consumers to sell their differentiated product at a higher price. Secondly, as it is generally admitted[9] that richer individuals have a preference for higher quality goods, wealthier households' preferences provide interesting information about the perception of the products. If they preferred fresh milk even more than poorer households, this would be a strong indication that fresh milk has a higher *perceived* quality. It is not clear, a priori, which raw material, from the powder or the fresh milk, is perceived to have the highest quality. Indeed, fresh milk may be collected in poor sanitary conditions, but comes from local cows, and corresponds more to Senegalese rural habits, while powder production is assumed to be more controlled but consumers may think that nutritive properties or taste have been altered.

In the particular model

$$V_{ij} = \gamma_1 Package_j + \gamma_2 Taste_j + \gamma_3 RawMaterial_j + \theta p_j + \delta X_i + \beta(Wealth_i * RawMaterial_j) \tag{1.4.7}$$

[9] See for instance Bils and Klenow (2001) or Manig and Moneta (2009).

(where Wealth=1 if the household is in the wealthier category), we expect γ_3 to be positive (i.e. consumers are willing to pay more for fresh raw material). If γ_3 was not significantly different from zero, consumers would just be indifferent between powder or fresh raw material. However, we have no particular expectation on the effect of wealth β. If β is positive, fresh raw material can be assimilated to high quality product, and wealthier individuals are willing to pay even more than other individuals for this attribute. If β is negative, then powder represents quality and wealthier individuals, who have a higher preference for quality, are willing to pay less than other individuals for fresh raw material.

For other major socio-economic characteristics, we expect the following results. Education should have a positive effect on the WTP for fresh raw material as more educated individuals may be more informed of the social and nutritional implications of consuming fresh milk. Being Peul, as opposed to other ethnicities, may also affect positively this WTP, as Peuls, traditionally involved in the livestock sector, should be more concerned by local producers' difficulties. Finally, we expect small and big households to have a different WTP for local raw material as preference for feeding the children may be different from adults' taste.

Ordered Logit and Probit Models (Random Utility Models) are suitable to evaluate the WTP.[10] However, Ordered Logit requires that the assumption of independence of irrelevant alternatives (IIA) holds. The relative probability of choosing alternative j versus alternative l has to be independent of which other alternatives are available as well as of which alternatives have been already chosen (Long and Freese, 2006: 341). Using a Hausman test and comparing the full model with a reduced model on a subset of alternatives, we can show that IIA assumption does not hold. For example comparing the full model with a model excluding profile G, Hausman test (not reported) rejects the null hypothesis of IIA ($\chi^2_4 = 13.65,\ p < 0.01$). We choose to use an Ordered Probit Model as it does not rely on the IIA assumption. Nevertheless, using an Ordered Logit Model doesn't change much the results (not reported).

The dependent variable we focus on is the note m given by the individual i to the hypothetical product j.[11] Ordered Probit Model assumes that the alternative j receives a note m if the utility from this product crosses an unknown threshold:

$$note(j) = m \quad \text{if} \quad \alpha_{m-1} < U_{ij} \leq \alpha_m$$

As U_{ij} crosses increasing thresholds (from $\alpha_0 = -\infty$ to $\alpha_M = \infty$), the note attributed to j moves up. The probability that individual i gives a note m (=1,...,5) to the product j is given by:

$$P_{ijm} = Prob[\alpha_{m-1} < V_{ij} + \epsilon_{ij} \leq \alpha_m] = Prob[\alpha_{m-1} - V_{ij} < \epsilon_{ij} \leq \alpha_m - V_{ij}]$$

[10] The rating/ranking nature of the data allows us to use both Ordered and Rank-Ordered Models. We have compared both types in the Logit case. As they provide similar results (not reported), we use the simplest one, that is, the Ordered Model.

[11] The database contains 3200 observations (400 households i* 8 alternatives j to be rated) for that dependent variable.

Using 1.4.5,

$$P_{ijm} = \Phi(\alpha_m - \beta(X_i Z_j) - \gamma Z_j - \theta p_j) - \Phi(\alpha_{m-1} - \beta(X_i Z_j) - \gamma Z_j - \theta p_j) \qquad (1.4.8)$$

where $\Phi(.)$ is the cumulative density function for standard normal distributed errors.

1.4.2 Contingent valuation

Based on various contingent valuation measures from the survey (see table 1.7), we estimate the stated WTP for fresh raw material depending on socio-economic and demographic characteristics, using the following linear regression:

$$Premium_i = a + bX_i + e_i \qquad (1.4.9)$$

where $Premium_i$ is the measure of the additional amount that individual i is willing to pay to consume sour milk made with fresh raw material rather than with powder, X_i is a vector of socio-economic and demographic variables, b is a vector of coefficients to be estimated, a is a constant to be estimated and e_i is the error term.

1.5 Results

1.5.1 Choice-based-conjoint analysis

Table 1.10: Ordered Probit Model

Variable		Coefficient	(Std. Err.)	WTP[a]
Package (per weight=1)	γ_1	-0.630***	(0.050)	-357.8
Taste (Sugar=1)	γ_2	0.205***	(0.045)	116.5
Raw material (Fresh=1)	γ_3	0.402***	(0.049)	228.3
Price	θ	-0.002***	(0.000)	
	α_1	-1.345***	(0.114)	
	α_2	-0.979***	(0.114)	
	α_3	-0.651***	(0.115)	
	α_4	0.024	(0.114)	

Log-Likelihood: -4690.959. Number of observations: 3200 (400 groups).
Standard errors are clustered. *** indicates significance at 1% level.
[a] WTP estimates are given by $-\gamma_k/\theta$.

Table 1.10 reports the results from the Ordered Probit Model with specification (1.4.3). All the coefficients are statistically significant at 1% level. As expected, individuals seem to prefer a sour milk with the following characteristics: individually packed (sachet), with sugar and made with fresh raw material.

The packaging has the most crucial importance ($|\gamma_1| = 0.63$). Preference for fresh milk is also major: keeping other attributes (package and taste) unchanged, the marginal

WTP for fresh raw material $-\gamma_3/\theta$ is around 228 CFA. It means that, all other things being equal, the representative household is willing to pay 228 CFA more to consume a product made with fresh milk rather than a product made with powder.

Table 1.11: Ordered Probit Model (heterogeneity among consumers)

Variable		Coefficient	(Std. Err.)	WTP[a]	dy/dx[b]	(Std. Err.)
Package (per weight=1)	γ_1	-0.633***	(0.051)	-355.2	-0.209***	(0.015)
Taste (Sugar=1)	γ_2	0.206***	(0.045)	115.6	0.068***	(0.015)
Raw material (Fresh=1)	γ_3	0.405***	(0.049)	227.5	0.134***	(0.016)
Price	θ	-0.002***	(0.000)		-0.001***	(0.000)
Pikine	δ_1	0.157***	(0.048)		0.052***	(0.016)
Rufisque	δ_2	0.231**	(0.098)		0.076**	(0.032)
Ethn. minority	δ_3	0.020	(0.048)		0.007	(0.016)
Peul	δ_4	0.065	(0.064)		0.021	(0.021)
Small household	δ_5	-0.096	(0.066)		-0.032	(0.022)
Big household	δ_6	-0.048	(0.050)		-0.016	(0.016)
High education	δ_7	-0.054	(0.045)		-0.0178	(0.015)
Low expenses	δ_8	0.024	(0.053)		0.008	(0.017)
High expenses	δ_9	0.032	(0.057)		0.011	(0.019)
	α_1	-1.277***	(0.117)			
	α_2	-0.910***	(0.117)			
	α_3	-0.580***	(0.119)			
	α_4	0.101	(0.118)			

Log-Likelihood: -4676.2297. Number of observations: 3200 (400 groups). Std. err. are clustered.

*** and ** indicate significance at 1% and 5% level.

[a] WTP estimates are given by $-\gamma_k/\theta$.

[b] Average marginal response of the probability of giving a note 5 to the product when a regressor changes and the others are unchanged. Average probability of note 5 is 0.3217.

Controlling for individuals' characteristics does not change much the results (table 1.11). With specification (1.4.4), the marginal WTP for fresh raw material $-\gamma_3/\theta$ is around 227 CFA.

The average marginal effects from the Ordered Probit Model are also illustrated in table 1.11. The average probability that a respondent gives a note 5 to the proposed hypothetical product increases by 13 points if the product is made with fresh raw material. Adding sugar increases the probability of a note 5 by 6.8 points and going to an individual packaging increases it by 21 points, all other things equal.

The effects reported in table 1.11 are the marginal effects averaged for all individuals. They have to be distinguished from the marginal effects for an average individual (not reported here). Indeed an "average" individual (that is, with the following characteristics: from Dakar, Wolof, medium size household, low education and medium food expenses) has a probability of 52.6% of giving a note 5 to the product that has the following attributes: sachet, sugar, fresh raw material, i.e. the product with all the most preferred attributes when its price is 250 CFA (a common market price). At the same price, the product with all the least preferred attributes (per weight, without sugar, made with powder) receives a note 5 with a probability of 11.9%. If the "most preferred" product was free (price was zero), the probability of receiving a rate 5 would be 69.5%.

Table 1.12: Ordered Probit Model (with interactions)

		Model a		Model b		Model c	
Variable		Coeff.	(s.e.)	Coeff.	(s.e.)	Coeff.	(s.e.)
Package (per weight=1)	γ_1	-0.634***	(0.051)	-0.636***	(0.051)	-0.634***	(0.051)
Taste (Sugar=1)	γ_2	0.206***	(0.045)	0.206***	(0.045)	0.206***	(0.045)
Raw material (Fresh=1)	γ_3	0.374***	(0.076)	0.489***	(0.062)	0.340***	(0.060)
Price	θ	-0.002***	(0.000)	-0.002***	(0.000)	-0.002***	(0.000)
Pikine	δ_1	0.157***	(0.048)	0.157***	(0.048)	0.157***	(0.048)
Rufisque	δ_2	0.231**	(0.098)	0.231**	(0.098)	0.232**	(0.098)
Ethn. minority	δ_3	0.020	(0.048)	0.021	(0.048)	0.020	(0.048)
Peul	δ_4	0.064	(0.065)	0.065	(0.065)	0.064	(0.064)
Small household	δ_5	-0.096	(0.066)	-0.191*	(0.101)	-0.096	(0.066)
Big household	δ_6	-0.049	(0.050)	0.138*	(0.072)	-0.048	(0.050)
High education	δ_7	-0.053	(0.045)	-0.054	(0.045)	-0.136**	(0.067)
Low expenses	δ_8	0.042	(0.075)	0.024	(0.053)	0.024	(0.053)
High expenses	δ_9	-0.082	(0.086)	0.032	(0.057)	0.032	(0.057)
Low exp.*Raw material	β_1	-0.037	(0.107)				
High exp.*Raw material	β_2	0.234*	(0.132)				
Small hh*Raw material	β_3			0.194	(0.159)		
Big hh*Raw material	β_4			-0.375***	(0.108)		
High educ.*Raw material	β_5					0.168*	(0.101)
	α_1	-1.295***	(0.120)	-1.240***	(0.117)	-1.310***	(0.118)
	α_2	-0.927***	(0.120)	-0.872***	(0.117)	-0.942***	(0.119)
	α_3	-0.596***	(0.122)	-0.540***	(0.119)	-0.611***	(0.120)
	α_4	0.085	(0.121)	0.144	(0.119)	0.070	(0.119)

Log-Likelihood: model a: -4672.7878, model b: -4664.3479, model c: -4673.9821.
Number of observations: 3200 (400 groups). Standard errors are clustered.
***, ** and * indicate significance at 1%, 5% and 10% level.

Table 1.12 presents the results obtained from the Ordered Probit Model that includes interaction effects (specification (1.4.5)). Model *a* corresponds to the particular specification (1.4.7). The WTP for fresh raw material, for the base category household (that is with monthly food expenses between 75 000 and 150 000 CFA) is 210 CFA ($-\gamma_3/\theta$).

The interaction between food expenses and raw material is quite interesting. The WTP for fresh raw material, for a family with a low level of food expenses (less than 75 000 CFA/month) is not significantly different from the reference household's one. However, wealthier households (with food expenses higher than 150 000 CFA/month) have a WTP for this attribute of 341 CFA ($-(\gamma_3 + \beta_2)/\theta$). Subject to the assumption we have adopted, this seems to indicate that sour milk made with fresh raw material is considered to have a higher perceived quality than sour milk made with powder. One may criticize the use of food expenses as a measure of wealth. Nevertheless, using another usual wealth indicator (the ownership of a color TV) does not affect the results (see table 1.18 (b) in Appendix), indicating their robustness. Comparing the wealthiest households with the poorest (instead of the one who has medium expenses) even increases a bit the importance and significance of the effect (table 1.18 (c) in Appendix).

Model *b* in table 1.12 shows that medium size households have a WTP for fresh raw material of 275 CFA ($-\gamma_3/\theta$). Smaller families (less than 5 members) are not different from them. Bigger households, however, have a quite smaller WTP for fresh raw material: 64 CFA ($-(\gamma_3 + \beta_4)/\theta$). This may be partially explained by an income effect as, ceteris paribus, bigger households have a lower income per capita and the control variable *Food expenses* only represents total income. With lower income per capita, bigger households are willing to pay less for fresh raw material. This intuitive interpretation is similar to the previous one about poorer versus wealthier households. Income effect is only part of the story however. Using a proxy[12] of the income per capita as control variable instead of *Food expenses*, β_4 is still significantly negative, indicating that bigger households are ready to pay less for fresh raw material, certainly due to differences in taste between the members of big and small families.

Model *c* in table 1.12 indicates that consumers with a high education (superior to secondary school) are willing to pay more for fresh raw material (β_5 is significantly positive) than less educated ones. They have a marginal WTP of 285 CFA for this attribute ($-(\gamma_3+\beta_5)/\theta$), while less educated consumers have a WTP of 191 CFA ($-\gamma_3/\theta$).

We see that the WTP for fresh raw material greatly depends on the characteristics of the households. There clearly exist some niche markets (i.e. wealthier and educated consumers), that milk producers may target to sell the local milk-based dairy products.

The interaction effect of being Peul on the preference for raw material is not significant (β_6 in table 1.17 in Appendix) indicating that Peuls do not seem to be willing to pay more for fresh raw material. This may be an indication that the choice of the preferred raw material is dictated by taste and quality considerations more than by a wish to support local producers.

We suspect that the rating/ranking CBC data overestimate the willingness-to-pay because individuals are not in a real situation of purchase (they do not have to spend money), or because of the difficulty of the ranking task. Indeed, saying that individuals are willing to pay 228 CFA more for a product that already costs 250 CFA, that is, saying that they are ready to pay almost the double of the current price, seems unrealistic. However, the results show that individuals are willing to pay a significantly positive premium for fresh raw material. We can use the lower bound of a 95% confidence interval as the lower limit for the WTP, interpreting that the true value of the WTP has a probability 0.975 to be above this limit.

Confidence intervals for the main estimates of the WTP for fresh raw materials are reported in table 1.13. They are calculated using the delta method, assuming that the WTP is normally distributed. Indeed, it is reasonable to suppose that the coefficients of

[12] Food expenses/(number of children $+2$).

Table 1.13: WTP for fresh raw material: estimates and confidence intervals

Model	WTP estimate	Lower bound of CI[a] at 95%	Upper bound of CI[a] at 95%
Ord. Probit on (1.4.3) (table 1.10)	228.32	113.82	342.82
Ord. Probit on (1.4.4) (heterog., table 1.11)	227.48	114.33	340.64
Ord. Probit on (1.4.5) (interact., table 1.12):			
Model a (base category household)	209.63	86.08	333.17
Model b (base category household)	274.61	140.78	408.44
Model c (base category household)	190.96	84.22	297.69

[a]Confidence intervals at 95% level calculated with delta method.

an Ordered Probit Model are normally distributed when the sample is large. As the WTP is a ratio of two normally distributed variables, its distribution is approximately normal when the coefficient of variation of the denominator is small[13](Hole, 2006). Confidence intervals are quite large, indicating that the estimation of mean WTP is imprecise.

While we may easily trust that products receiving note 5 are the most preferred and that products receiving note 1 are the least preferred, it may be argued that consumers may not be able to rank intermediate products in accordance with their real preferences. To test for the robustness regarding this point we use two alternative specifications. First, we gather middle classes (notes 2, 3 and 4) and use an Ordered Probit Model with only three categories instead of five. Table 1.19 in Appendix indicates that main results, in terms of significance and sign, are not affected. Second, we use a Binary Probit Model where the product is considered to be chosen (choice=1) if it receives the note 5 and not chosen (choice=0) if it receives a note lower than 5 (i.e. 1, 2, 3 or 4). Table 1.20 in Appendix indicates also that main results are not altered, neither in terms of significance or sign, except for the interaction effect between education and raw material.

Table 1.14 reports average marginal effects from the Ordered Probit Model with interactions. Going from a powder raw material to a fresh one increases the probability of receiving a note 5 by 11 to 16 points of probability, depending on the specification.

Interaction effects must be interpreted with caution as, in non-linear models, a rigorous test for those effects must be based on the estimated cross-partial derivative[14], which is not the case in table 1.14. To test for the robustness of the results concerning these effects, we have checked their significance using the method proposed by Norton et al. (2004). Results from the Binary Probit Model in table 1.21 in Appendix indicate that, for models a and b, significance is not affected. Estimated interaction effects are even bigger with this method. The interaction effect between high education and raw material (model c) is no longer significant.

[13]Precisely, it has to be less than 0.39 (Hayya et al., 1975). In our case, for instance in the simple model presented in table 1.10, $s.e.(\theta)/\theta = 0.262 < 0.39$.

[14]For the same reason, only one interaction effect is included in each model in table 1.12. Nevertheless, including the three interactions terms in the same model does not change much the results, expect for the education effect (see table 1.22 in Appendix).

Wealthiest households' probability of choosing a product is increased by 9.5 points if the product is made with fresh raw material instead of powder. This effect is even stronger for products whose predicted probability of being chosen is high (see figure 1.3 in Appendix). For big households, the probability of choosing a product decreases by 17 points when it is made with fresh raw material and this negative effect is even stronger for products that have higher predicted probability of being chosen (see figure 1.4 in Appendix).

Table 1.14: Marginal effects from Ordered Probit Model (heterogeneity among consumers)

	Model a		Model b		Model c	
Variable	dy/dx[a]	(s.e.)	dy/dx[a]	(s.e.)	dy/dx[a]	(s.e.)
Package[b] (per weight=1)	-0.210***	(0.015)	-0.209***	(0.015)	-0.209***	(0.015)
Taste[b] (Sugar=1)	0.068***	(0.015)	0.068***	(0.015)	0.068***	(0.015)
Raw material[b] (Fresh=1)	0.124***	(0.025)	0.161***	(0.020)	0.112***	(0.020)
Price	-0.001***	(0.000)	-0.001***	(0.000)	-0.001***	(0.000)
Pikine[b]	0.052***	(0.016)	0.052***	(0.016)	0.052***	(0.016)
Rufisque[b]	0.076**	(0.032)	0.076**	(0.032)	0.076**	(0.032)
Ethn. minority[b]	0.007	(0.016)	0.007	(0.016)	0.007	(0.016)
Peul[b]	0.021	(0.021)	0.021	(0.021)	0.021	(0.021)
Small household[b]	-0.032	(0.022)	-0.062*	(0.033)	-0.032	(0.022)
Big household[b]	-0.016	(0.0164)	0.045*	(0.024)	-0.016	(0.016)
High education[b]	-0.017	(0.015)	-0.018	(0.015)	-0.045**	(0.022)
Low expenses[b]	0.014	(0.0246)	0.008	(0.017)	0.008	(0.017)
High expenses[b]	-0.027	(0.028)	0.011	(0.019)	0.011	(0.019)
Low exp.*Raw material[b]	-0.012	(0.035)				
High exp.*Raw material[b]	0.077*	(0.044)				
Small hh*Raw material[b]			0.064	(0.052)		
Big hh*Raw material[b]			-0.123***	(0.035)		
High educ.*Raw material[b]					0.055*	(0.033)

Number of observations: 3200 (400 groups). Standard errors are clustered.

***, ** and * indicate significance at 1%, 5% and 10% level.

[a] Average marginal response of the probability of giving a note 5 to the product when a regressor changes and the others are unchanged.

[b] dy/dx is for discrete change of dummy variable from 0 to 1.

1.5.2 Contingent valuation

The results from the linear regression (1.4.9), using various measures of *Premium* (see table 1.7), are presented in table 1.15. Some results are consistent with the CBC analysis, particularly, wealthier households have a higher willingness-to-pay for fresh raw material ($b_9 > 0$ and significant in all the three models). Consumers from Pikine are ready to pay much less for this attribute than consumers from Dakar (b_1 negative and highly significant) which did not appear in the CBC results (see table 1.17 (b) in Appendix where β_7 is not significant). Households in Pikine are, on average, bigger and poorer than other households. However, as we control for households' size and wealth, this does not explain the important impact of the regional dummy.

Table 1.15: Contingent valuation : linear regressions

		(a) $Premium_R$		(b) $Premium_E$		(c) $Premium_{\%}$	
Variable		Coeff.	(s.e.)	Coeff.	(s.e.)	Coeff.	(s.e.)
Pikine	b_1	-50.491***	(14.072)	-63.180***	(17.240)	-0.161***	(0.040)
Rufisque	b_2	-4.095	(21.392)	-2.362	(26.209)	-0.002	(0.060)
Ethn. minority	b_3	-12.257	(14.834)	0.103	(18.174)	0.020	(0.042)
Peul	b_4	8.736	(17.110)	4.841	(20.964)	0.014	(0.048)
Small household	b_5	6.592	(21.241)	18.261	(26.025)	0.035	(0.060)
Big household	b_6	-7.669	(14.514)	-6.931	(17.783)	-0.043	(0.041)
High education	b_7	10.119	(13.578)	11.177	(16.635)	0.034	(0.038)
Low expenses	b_8	28.081*	(14.814)	33.221*	(18.150)	0.035	(0.042)
High expenses	b_9	30.024*	(17.395)	49.217**	(21.313)	0.102**	(0.049)
Constant	a	30.043**	(14.888)	29.397	(18.240)	0.164***	(0.042)

Number of observations: 400 households.

***, ** and * indicate significance at 1%, 5% and 10% level.

Based on model (a), consumers from the base category are, on average, willing to pay a premium of 30 CFA for fresh raw material. For wealthier households, this premium is around 60 CFA. Results from the other models are similar. Based on model (c), reference households are willing to pay a premium of 16.4% above the initial price. For instance, if sour milk made with powder costs 250 CFA, they are willing to pay 291 CFA for a product made with fresh milk, that is 41 CFA more. For wealthier individuals, this premium is 26.6%, or 66.5 CFA if the initial price is 250 CFA. One may think that the WTP is better represented by the threshold price above which the consumers stops to buy the product. Linear regressions from premiums based on this threshold are represented in table 1.23 in Appendix and present results similar to table 1.15.

Those results confirm our previous observations. Firstly, CBC results are upward biased, certainly due to the difficulty of the ranking task as well as to the hypothetical nature of the questions. While contingent valuation analyses are generally more biased than the CBC ones, the transformation we use here permits to give reliably unbiased estimates with the contingent valuation method. Secondly, we may be confident that consumers are willing to pay a positive premium for fresh raw material, even if we can not unequivocally quantify this premium. Thirdly, wealthier individuals are willing to pay even more than other consumers to get a product made with fresh milk rather than powder.

1.5.3 Discussion

The previous analysis seems to assess that consumers are willing to pay a positive premium for local milk based products. We suspect that this preference is not transfered to market prices because consumers are not able to distinguish both kinds of products. As it has been suggested by our analysis of observed prices, it means that the consumers agree to pay a positive premium for products they think are local but that are actually made with imported powder.

We check that this misinformation has no impact on the WTP, that is, that better informed consumers are not significantly different from other consumers regarding the way they value the local origin. We do this by including the following indicator of knowledge as control variable in the various model specifications we used:

$$K_i = \frac{\#\text{ of (powder-based) brands consumed and correctly known by individual } i}{\#\text{ of (powder-based) brands consumed by individual } i}$$

It turns out that this indicator is not significant neither when included in the Ordered Probit Model, with and without interactions (CBC analysis), neither when included in the linear regression of the contingent valuation analysis (results are not reported here). The same applies for a dummy variable indicating that the score K_i (between 0 and 1) is higher than a threshold value, say for instance 0.5.

As a better knowledge does not seem to influence the WTP, improving this knowledge would permit that consumers agree to pay more for products that are *actually* made with fresh milk. A clear implication of this analysis is that any policy that leads to a better information could allow local producers to sell their products on the market at a higher price, while still finding a demand. Such policies include improving local products advertising, creating certification for the local origin or improving and enforcing regulations about the packaging of powder based products.

1.6 Conclusions

In this chapter, we estimated the Senegalese consumers' willingness-to-pay for a fresh (or local) raw material in the composition of sour milk. Using choice-based-conjoint data, we found that consumers are, on average, willing to pay a premium around 220 CFA, depending on the specification. An Ordered Probit Model that controls for consumers heterogeneity, estimates this WTP at 227 CFA with a large confidence interval (from 104 to 351 CFA at 95% level). It means that, on average, a household from the base category is ready to pay 227 CFA more to obtain sour milk made with fresh milk rather than with powder.

This estimation is suspected to be upward biased due to the hypothetical nature of the question. It is generally assumed that contingent valuation analyses lead to an over-estimation of the WTP even larger that the CBC does. However, using a transformation of two direct contingent valuation questions, we obtain an indirect contingent valuation measure that seems to give unbiased (or less biased) estimation of the WTP. With this method, the willingness-to-pay for the fresh raw material is estimated to 30 CFA, or 16% above the initial price, depending on the specification.

The willingness-to-pay greatly depends on the characteristics of the households and there clearly are some niche markets that milk producers may target to sell the local milk-based dairy products. Wealthier households are willing to pay more than the other households, indicating that fresh raw material may be assimilated to superior *perceived* quality. This higher willingness-to-pay from the wealthier households is confirmed by both CBC and contingent valuation analyses. Big households are ready to pay much less than the base category ones, certainly due to difference in taste between children and adults. Highly educated respondents have a higher WTP than less educated ones. Surprisingly, being Peul does not affect the WTP for fresh raw material in spite of Peuls' traditional implication in the livestock sector.

It has been shown that consumers are not currently able to distinguish powder-based products from those made with fresh milk. This implies that market prices are not differentiated despite the fact that consumers are willing to pay more for the second ones. A better regulation for dairy products made with powder, coupled with a good marketing of local products, targeted to the niche markets we defined, might allow local producers to sell their product at a higher price, which could compensate the higher transport costs they face and hence improve their incomes and livelihood.

We are aware of the weaknesses of the present analysis, that may be improved in future researches, mainly by constructing new databases that better fit our objectives. First, GRET database only contains information about fresh raw material which is a proxy for local raw material. Hence we are not able to distinguish the valuation of taste due to freshness from the pure effect of locality. Second, we suspect that the rating/ranking CBC data overestimate the willingness-to-pay because individuals are not in a real situation of purchase (they do not have to spend money), or because of the difficulty of the ranking task. The present analysis gives us an indication that consumers are willing to pay a positive premium for local products. The existence of this significantly positive premium is confirmed by contingent valuation measures that are assumed to be unbiased. However, we should not trust the CBC evaluation of the magnitude of the premium. Reliable estimation of the WTP should be obtained by observing individuals in a real environment, such as in an experimental framework or by observing real purchase behavior on the market. In spite of these restrictions, this chapter gives a first insight of consumers' preferences for local milk-based dairy products and encouraging results for future researches.

Appendices

Table 1.16: Tied ranks

	note 1	note 2	note 3	note 4	note 5
% who gave the note to 0 product	16.50	41.50	37.25	9.00	1.50
% who gave the note to 1 product	33.75	36.25	37.75	32.75	19.75
% who gave the note to > 1 product	49.75	22.25	25.00	58.25	78.75
Average number of products	1.675	0.8825	0.920	1.9475	2.575

Table 1.17: Ordered Probit Model (with interactions)

		Model a		Model b	
Variable		Coeff.	(s.e.)	Coeff.	(s.e.)
Package (per weight=1)	γ_1	-0.633***	(0.051)	-0.633***	(0.040)
Taste (Sugar=1)	γ_2	0.206***	(0.045)	0.206***	(0.039)
Raw material (Fresh=1)	γ_3	0.404***	(0.055)	0.417***	(0.055)
Price	θ	-0.002***	(0.000)	-0.002***	(0.000)
Pikine	δ_1	0.157***	(0.048)	0.156***	(0.060)
Rufisque	δ_2	0.231**	(0.098)	0.286***	(0.092)
Ethn. minority	δ_3	0.020	(0.048)	0.020	(0.046)
Peul	δ_4	0.061	(0.085)	0.065	(0.053)
Small household	δ_5	-0.096	(0.066)	-0.096	(0.066)
Big household	δ_6	-0.048	(0.050)	-0.049	(0.045)
High education	δ_7	-0.054	(0.045)	-0.054	(0.042)
Low expenses	δ_8	0.024	(0.053)	0.0240	(0.046)
High expenses	δ_9	0.032	(0.057)	0.032	(0.054)
Peul*Raw material	β_6	0.007	(0.120)		
Pikine*Raw material	β_7			0.003	(0.082)
Rufisque*Raw material	β_8			-0.112	(0.128)
	α_1	-1.278***	(0.118)	-1.272***	(0.137)
	α_2	-0.911***	(0.118)	-0.905***	(0.137)
	α_3	-0.580***	(0.119)	-0.574***	(0.136)
	α_4	0.100	(0.119)	0.107	(0.136)

Log-Likelihood: model a: -4676.2272, model b: -4675.8071. Standard errors are clustered.
Nb of observations: 3200 (400 groups). *** and ** indicate significance at 1% and 5% level.

Table 1.18: Robustness: other income related variables (ordered probit)

		Model a		Model b		Model c	
Variable		Coeff.	(s.e.)	Coeff.	(s.e.)	Coeff.	(s.e.)
Package (per weight=1)	γ_1	-0.634***	(0.051)	-0.634***	(0.051)	-0.634***	(0.039)
Taste (Sugar=1)	γ_2	0.206***	(0.045)	0.206***	(0.045)	0.206***	(0.039)
Raw material (Fresh=1)	γ_3	0.370***	(0.062)	0.219**	(0.092)	0.337***	(0.061)
Price	θ	-0.002***	(0.000)	-0.002***	(0.000)	-0.002***	(0.000)
Pikine	δ_1	0.171***	(0.045)	0.175***	(0.047)	0.157***	(0.044)
Rufisque	δ_2	0.242**	(0.098)	0.231**	(0.098)	0.231***	(0.067)
Ethn. minority	δ_3	0.021	(0.048)	0.025	(0.048)	0.0202	(0.046)
Peul	δ_4	0.068	(0.064)	0.073	(0.064)	0.064	(0.053)
Small household	δ_5	-0.094	(0.065)	-0.093	(0.067)	-0.096	(0.066)
Big household	δ_6	-0.046	(0.050)	-0.049	(0.050)	-0.049	(0.045)
High education	δ_7	-0.055	(0.046)	-0.067	(0.046)	-0.053	(0.042)
Low type housing		0.030	(0.069)				
High type housing		-0.200	(0.124)				
TV				-0.059	(0.066)		
Medium expenses						-0.042	(0.063)
Large expenses						-0.124	(0.076)
Low housing*Raw mat.		0.039	(0.102)				
High housing*Raw mat.		0.418	(0.262)				
TV*Raw material				0.246**	(0.107)		
Medium exp.*Raw mat.						0.037	(0.086)
High exp.*Raw mat.						0.270**	(0.107)
	α_1	-1.285***	(0.117)	-1.335***	(0.129)	-1.336***	(0.140)
	α_2	-0.918***	(0.117)	-0.967***	(0.129)	-0.969***	(0.139)
	α_3	-0.587***	(0.118)	-0.636***	(0.130)	-0.638***	(0.139)
	α_4	0.094	(0.118)	0.046	(0.131)	0.043	(0.138)

Log-Likelihood: model a: -4673.1, model b: -4671.9676, model c: -4672.7878.

Number of observations: 3200 (400 groups). Standard errors are clustered.

*** and ** indicate significance at 1% and 5% level.

Table 1.19: Robustness: grouped middle classes (ordered probit)

		Model a		Model b		Model c		Model d		Model e	
Variable		Coeff.	(s.e.)	Coeff.	(s.e.)	Coeff.	(s.e.)	Coeff.	(s.e.)	Coeff.	(s.e.)
Package (per weight=1)	γ_1	-0.626***	(0.051)	-0.628***	(0.051)	-0.630***	(0.051)	-0.631***	(0.051)	-0.629***	(0.051)
Taste (Sugar=1)	γ_2	0.216***	(0.046)	0.216***	(0.046)	0.216***	(0.046)	0.217***	(0.047)	0.216***	(0.046)
Raw material (Fresh=1)	γ_3	0.394***	(0.051)	0.396***	(0.051)	0.343***	(0.077)	0.485***	(0.064)	0.327***	(0.062)
Price	θ	-0.002***	(0.000)	-0.002***	(0.000)	-0.002***	(0.000)	-0.002***	(0.000)	-0.002***	(0.000)
Pikine				0.119**	(0.047)	0.119**	(0.047)	0.120**	(0.047)	0.119**	(0.047)
Rufisque				0.158	(0.101)	0.158	(0.101)	0.158	(0.102)	0.159	(0.101)
Ethn. minority				0.012	(0.047)	0.012	(0.047)	0.012	(0.047)	0.012	(0.047)
Peul				0.066	(0.063)	0.066	(0.063)	0.066	(0.063)	0.066	(0.063)
Small household				-0.096	(0.065)	-0.096	(0.065)	-0.176*	(0.104)	-0.096	(0.065)
Big household				-0.049	(0.049)	-0.049	(0.049)	0.141*	(0.073)	-0.049	(0.049)
High education				-0.044	(0.044)	-0.044	(0.044)	-0.044	(0.044)	-0.132*	(0.068)
Low expenses				0.026	(0.053)	0.031	(0.076)	0.027	(0.053)	0.026	(0.053)
High expenses				-0.011	(0.057)	-0.155*	(0.086)	-0.011	(0.057)	-0.011	(0.057)
Low exp.*Raw material	β_1					-0.010	(0.112)				
High exp.*Raw material	β_2					0.293**	(0.136)				
Small hh*Raw material	β_3							0.161	(0.169)		
Big hh*Raw material	β_4							-0.382***	(0.113)		
High educ.*Raw material	β_5									0.178*	(0.105)
	α_1	-1.305***	(0.121)	-1.262***	(0.123)	-1.291***	(0.126)	-1.223***	(0.123)	-1.297***	(0.125)
	α_2	0.062	(0.122)	0.110	(0.125)	0.083	(0.127)	0.154	(0.125)	0.076	(0.126)

Log-Likelihood: model a: -3167.9884, model b: -3158.6391, model c: 3154.2467, model d: 3147.7665, model e: 3156.2902.

Number of observations: 3200 (400 groups). Standard errors are clustered.

***, ** and * indicate significance at 1%, 5% and 10% level.

Table 1.20: Probit Model

		Model a		Model b		Model c	
Variable		Coeff.	(s.e.)	Coeff.	(s.e.)	Coeff.	(s.e.)
Package (per weight=1)	γ_1	-0.584***	(0.056)	-0.587***	(0.056)	-0.584***	(0.055)
Taste (Sugar=1)	γ_2	0.273***	(0.054)	0.274***	(0.055)	0.273***	(0.054)
Raw material (Fresh=1)	γ_3	0.400***	(0.066)	0.594***	(0.071)	0.413***	(0.075)
Price	θ	-0.001***	(0.001)	-0.001***	(0.001)	-0.001***	(0.001)
Pikine		0.222***	(0.052)	0.229***	(0.055)	0.224***	(0.056)
Rufisque		0.225*	(0.132)	0.226*	(0.132)	0.226*	(0.131)
Ethn. minority		0.054	(0.059)	0.054	(0.060)	0.055	(0.060)
Peul		0.077	(0.072)	0.077	(0.073)	0.079	(0.073)
Small household		-0.059	(0.082)			-0.060	(0.082)
Big household		-0.086	(0.062)	0.189**	(0.090)	-0.087	(0.061)
High education		-0.003	(0.056)	-0.009	(0.056)	-0.065	(0.091)
Low expenses				-0.010	(0.066)	-0.007	(0.066)
High expenses		-0.140	(0.104)	0.015	(0.071)	0.015	(0.070)
High exp.*Raw material	β_2	0.290*	(0.148)				
Big hh*Raw material	β_4			-0.509***	(0.126)		
High educ.*Raw material	β_5					0.114	(0.123)
	α	-0.281*	(0.161)	-0.383**	(0.163)	-0.284*	(0.161)

Log-Likelihood: model a: -1856.9146, model b: -1848.8712, model c: -1859.1067.

Number of observations: 3200 (400 groups). Standard errors are clustered.

***, ** and * indicate significance at 1%, 5% and 10% level.

Table 1.21: Norton et al. (2004)'s method for interaction effects

	Model a		Model b		Model c	
Variable	Int. effect	(s.e.)	Int. effect	(s.e.)	Int. effect	(s.e.)
High exp.*Raw material	0.095**	(0.048)				
Big hh*Raw material			-0.171***	(0.042)		
High educ.*Raw material					0.037	(0.040)

Number of observations: 3200 (400 groups).

*** and ** indicate significance at 1% and 5% level.

Figure 1.3: Interaction effect High exp.*Raw material from Probit Model

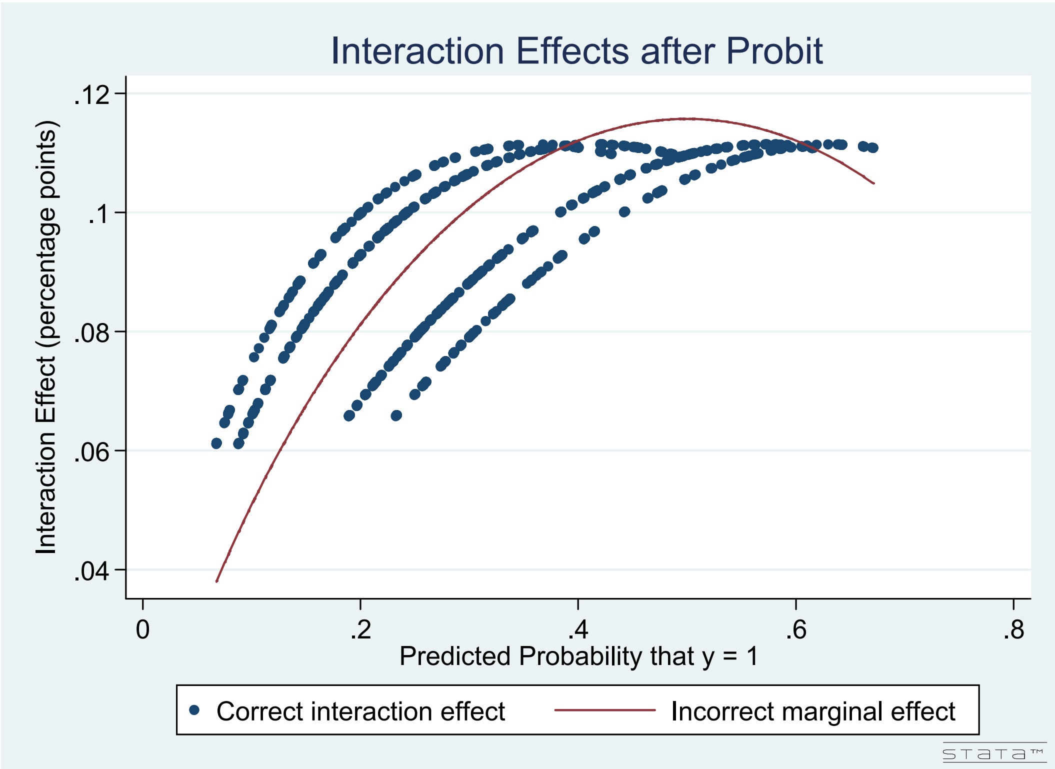

Figure 1.4: Interaction effect Big hh*Raw material from Probit Model

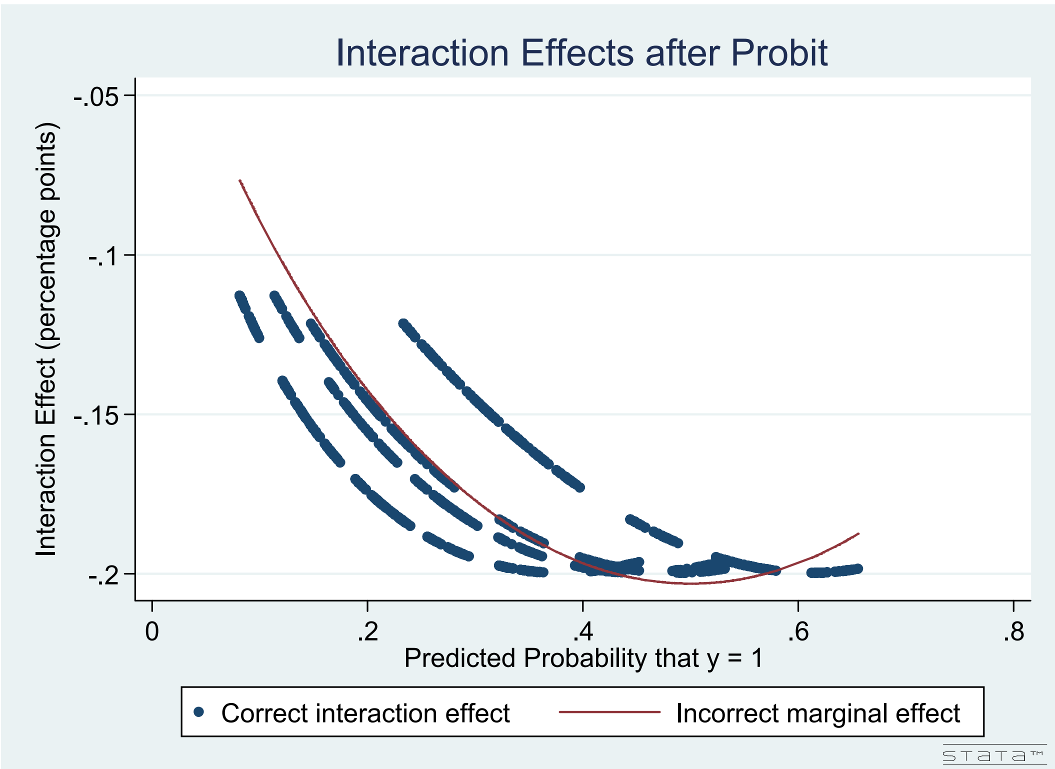

Table 1.22: Ordered Probit Model with three interactions

Variable		Coeff.	(s.e.)
Package (per weight=1)	γ_1	-0.637***	(0.039)
Taste (Sugar=1)	γ_2	0.207***	(0.039)
Raw material (Fresh=1)	γ_3	0.425***	(0.078)
Price	θ	-0.002***	(0.000)
Pikine	δ_1	0.157***	(0.044)
Rufisque	δ_2	0.232***	(0.067)
Ethn. minority	δ_3	0.021	(0.046)
Peul	δ_4	0.064	(0.053)
Small household	δ_5	-0.193**	(0.092)
Big household	δ_6	0.137**	(0.063)
High education	δ_7	-0.081	(0.059)
Low expenses	δ_8	0.036	(0.063)
High expenses	δ_9	-0.093	(0.075)
Low expenses*Raw mat.	β_1	-0.024	(0.088)
High expenses*Raw mat.	β_2	0.257**	(0.107)
Small hh*Raw mat.	β_3	0.197	(0.132)
Big hh*Raw mat.	β_4	-0.374***	(0.090)
High educ.*Raw mat.	β_5	0.057	(0.084)
	α_1	-1.273***	(0.140)
	α_2	-0.905***	(0.139)
	α_3	-0.572***	(0.139)
	α_4	0.113	(0.139)

Log-Likelihood: -4659.95.

Number of observations: 3200 (400 groups). Standard errors are clustered.

*** and ** indicate significance at 1% and 5% level.

Table 1.23: Contingent valuation : linear regressions

		(a) $Premium_{SE}$		(b) $Premium_{\%E}$		(c) $Premium_{\%SE}$	
Variable		Coeff.	(s.e.)	Coeff.	(s.e.)	Coeff.	(s.e.)
Pikine	b_1	-39.802	(28.298)	-0.159***	(0.037)	-0.110***	(0.040)
Rufisque	b_2	40.015	(43.019)	0.016	(0.056)	0.061	(0.061)
Ethn. minority	b_3	-19.425	(29.830)	0.0488	(0.039)	0.004	(0.042)
Peul	b_4	-18.351	(34.409)	0.006	(0.045)	-0.010	(0.049)
Small household	b_5	8.113	(42.717)	0.055	(0.056)	0.056	(0.061)
Big household	b_6	8.834	(29.188)	-0.019	(0.038)	0.014	(0.042)
Low expenses	b_8	7.032	(29.791)	0.043	(0.039)	0.014	(0.042)
High expenses	b_9	48.728	(34.982)	0.134***	(0.046)	0.091	(0.050)
High education	b_7	12.318	(27.305)	0.025	(0.036)	0.027	(0.039)
Constant	a	38.209	(29.939)	0.116***	(0.039)	0.124***	(0.043)

$Premium_{SE}$ = Price so expensive that the consumer does not buy sour milk with fresh raw material - Price so expensive that the consumer does not buy sour milk with powder
$Premium_{\%E}$ = (Expensive price for sour milk with fresh raw material - Expensive price for sour milk with powder)/Expensive price for sour milk with powder
$Premium_{\%SE}$ = (Price so expensive that the consumer does not buy sour milk with fresh raw material - Price so expensive that the consumer does not buy sour milk with powder)/ Price so expensive that the consumer does not buy sour milk with powder
Number of observations: 400 households.
***, ** and * indicate significance at 1%, 5% and 10% level.

Chapter 2

Intermediaries, transport costs and interlinked transactions*†

2.1 Introduction

This chapter considers the potential poverty alleviation effect from an increased production and commercialization of dairy products in Senegal, due to the emergence of intermediaries in this sector. As in most African countries, increased domestic dairy production could generate income for a large part of the population (Staal et al., 1997, Delgado et al., 1999). Indeed, in Senegal 48.12% of the population (73.48% in rural areas) own cattle (ESPS, 2005), most of them being poor: 63.28% of the households involved in agriculture, livestock and forest employments face poverty compared to 37.82% in other employments. In that sense, the development of the dairy sector has the potential to reduce poverty.

Although milk consumption in Africa is still low compared to the rest of the world, dairy products make now part of the consumption habits of most African households. In Senegal, the quantity consumed has quadrupled during the period 1961-1993. Nevertheless, despite this increased demand, the domestic milk production has risen by less than 40% during the same period, most of the demand being satisfied by the increased imports (FAOstat, 2009).

This stagnation of the domestic milk production is partly due to the characteristics of the livestock sector: generally, each peasant has only some cows and each one provides between 0.5 and 2 liters of milk per day. Both elements lead to small quantities of milk produced, between 2 and 10 liters per day (Duteurtre, 2006). The productivity per animal is determined by its breed (local cattle breeds, Zebu Gobra, Taurine N'Dama or D'jakoré are known to have low productivity) but also to the quantity of feeds available.

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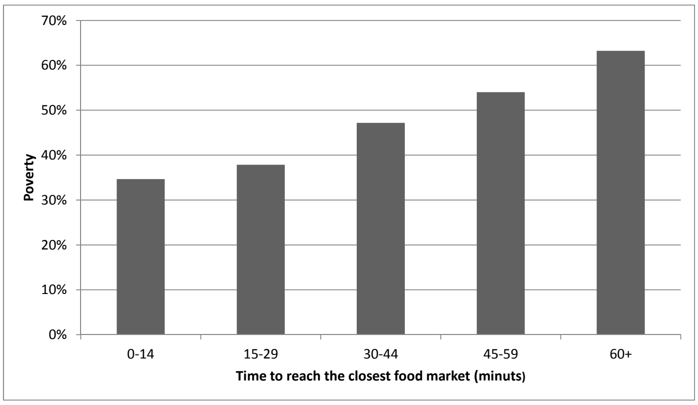
Around 70% of the Senegalese livestock sector take place in an agro-pastoral system where cattle are raised on pasture but feed supplements are provided by the use of organic manure and harvest residues, notably from cotton and sesame. One of the main constraints for improving milk production is the difficulty for the farmers to obtain these cattle feeds (Dieye et al., 2005, Dieye, 2003).

Another factor which hampers the increase of production are high transport costs. The nature of the milk makes it difficult to transport on large distances. However, while production is distributed on most of the rural areas in the country, consumption is mainly concentrated in Dakar, sometimes at more than 300 kilometers from the producers. Inadequate transport infrastructure also contribute to increase the transport costs.

As incurring large costs for transporting small quantities of milk may turn to be unprofitable, farmers often prefer not to take part to the market, or to participate only sometimes, resulting in very low quantities of milk commercialized on the market. Holloway et al. (2000) found that, in Ethiopia, each additional minute walk to the collection center reduces the marketable quantity of milk by 0.06 liters per day. In a region where milk yields per day are less than 4 liters, this is of considerable importance. High transport costs also have a negative impact on the use of feed supplements. In Kenya, Staal et al. (2002) have found that an additional 10 kilometers between the farmer and Nairobi decreases the probability of using concentrate feeding by more than 1%. More isolated farmers are also poorer. Figure 2.1 shows how poverty increases with the time necessary to reach the market.

Figure 2.1: Senegal: poverty increases with distance



* Source: ESPS (2005)

Since the nineties, we have seen the emergence of small-scale processing units called "mini-dairies" (see figure 1, page 8) that play an intermediary role between the farmers and the market (Dieye et al., 2005, Corniaux et al., 2005). These intermediaries have some kind of advantage over the farmers to sell the products on the market. They use more efficient transport devices, such as trucks, they own bulk cooling tanks, such that they can stock the milk and do not have to transport it every day, etc. This cost advantage requires a fixed cost, which for isolated farmers with a low income (of which a large part is used to buy food) is important and cannot be borne by each farmer on his own.

In this chapter we investigate the impact of the presence of such intermediaries on production, farmers' income and poverty. On figure 2 (page 9), one may note that the domestic milk production, that have stagnated for 30 years, has begun to increase in the nineties. One possible explanation for this evolution lies in the emergence of the so-called "mini-dairies". In what follows, we design a model that determine how the presence of intermediaries could help to increase small farmers' production, and examine under which conditions poor households could benefit from it.

Contracts between intermediaries and farmers often involve interlinked transactions. In the region of Kolda, Dieye et al. (2005) report that processing units provide credit and cattle feeds to the farmers in order to increase commercial links. The two most important milk processing units in this region ("Bilaame Puul Debbo" and "Le Fermier") use three different mechanisms for linking milk purchase and feeds selling: credit for feeds purchase, direct feeds purchase for the farmer, or guarantee to the feeds seller in case of non-payment by the farmer (Dieye, 2006). In the North, "La Laiterie du Berger" buys large quantities of cattle feeds and resells it to the farmers at 50 percent of the market price (Bathilly, 2007).

While these interlinked contracts have been shown to be efficient, it has also been shown that any efficiency gain is completely appropriated by the intermediary. This means that interlinked contracts do not allow the farmer to benefit from the intermediary's cost advantage. For instance, Gangopadhyay and Sengupta (1987) analyze how the trader's cost advantage on the input market induces him to provide inputs at a low price, and to extract all the surplus from the producer by setting a low price for the output, such that the producer finally obtains his reservation income. Extrapolating this result to our setting, this would imply that the presence of intermediaries has little effect on the reduction of poverty amongst farmers in rural areas. However, these results from the interlinked contracts literature have been obtained under the assumptions that the intermediary is a profit maximizer and sets a different contract for each farmer.

In practice, we observe that there is a lot of diversity regarding the nature of the mini-dairies. Intermediaries are not necessarily profit maximizers and we observe that local producers' associations, NGOs, cooperatives or even public organizations set up trading structures whose aim is to improve farmers' living conditions and income (Chau et al., 2009). In addition, a recent report from the United Nations encourages farmers to develop cooperative structures, in order to realize economies of scale (De Schutter, 2010). The Directory of Women in Livestock DINFEL ("Directoire National des Femmes en Elevage") is an example of such a non-profit mini-dairy. It collects around 400 liters of milk per day in the region of Dahra, transforms it and sells it in the region of Dakar. Even for-profit processing units seem to have non-profit secondary objectives. For instance "La Laiterie du Berger" claims that one of its objectives is to increase farmers' income (own interview, 2009).

Interlinked contracts have been extensively analyzed in a non-spatial context. However, the spatial dimension plays a key role in agricultural sectors in developing countries. In Senegal, areas of milk production are located far from the capital city (360 km for Richard-Toll where "La Laiterie du Berger" operates, 250 km for Dahra where is the DINFEL collection area), while most of the consumers are located in Dakar. On average, households' expense for milk consumption is 218 CFA per day in Dakar whereas it is 107.5 CFA in other regions (ESPS, 2005). Even at the rural level, transport cost is important compared to the price received by the farmers. In Kolda, where the price received by the producers ranges between 75 and 150 CFA, transport by bicycle costs between 20 and 25 CFA per liter (Dia, 2002). Motorized transport is even more costly, according to one of the managers of "La Laiterie du Berger" (personal interview, 2009), average transport cost on its collection area is 100 CFA per liter, while farmers receive 200 CFA per liter.

The interlinked contracts literature assumes that the firm is able to offer different contracts to different agents (Gangopadhyay and Sengupta, 1987). In a spatial context, this would correspond to assuming that the intermediary perfectly price discriminates between spatially dispersed farmers. Spatial price discrimination is only one possible pricing policy. It implies that the intermediary collects the product himself from the farmers. There are, however, other modes of collection and hence other pricing policies that the intermediaries may choose. For instance, "La Laiterie du Berger" organizes milk collection and pays all the farmers the same price, independent of the distance. This corresponds to uniform pricing. In "Le Fermier" however, farmers are responsible for transport, such that the ones who are located far from the processing unit receive a considerably lower net price than the closer ones. This corresponds to mill pricing.

For a farmer at a given location, the choice of a particular pricing policy may be important. Mill pricing, where farmers have to support transport cost, is disadvantageous for those located far away. Uniform pricing may seem fairer, as all the producers receive the same price. However, the closest ones may receive a lower net price than if they were themselves responsible for the transport.

In this chapter, we develop an interlinked contract model in which the intermediary has a transport cost advantage. Regarding interlinked transactions literature, our contribution is threefold. First, we develop a contract model where interlinkage is motivated by the trader's advantage in transport costs. While various rationales for the existence of such transactions have been analyzed[1], to our knowledge, difference in transport costs has not be considered.

Second, when heterogeneity amongst farmers is taken into account, it is generally done by assuming that the reservation incomes are different for each farmer with these reservation incomes being exogenously determined (for instance, Gangopadhyay and Sengupta, 1987). In our model, the heterogeneity amongst farmers is due to their spatial dispersion. It means that the location affects the farmer's income both in and outside the contract. It implies that the agents' rents (what they obtain above their reservation utility) may fail to be monotonic and that nonmonotonic rents could emerge (Jullien, 2000). Since standard contract theory relies heavily on the monotonicity of the rent, we have to use alternative methods to characterize the optimal contract.

[1]From rationed or imperfect rural credit (Gangopadhyay and Sengupta, 1987; Chakrabarty and Chaudhuri, 2001), output market price uncertainty (Chaudhuri and Gupta, 1995), risk aversion (Basu, 1983; Basu, Bell and Bose, 2000), unobservable tenant effort (Braverman and Stiglitz, 1982; Mintra, 1983) to the inability to collude (Motiram and Robinson, 2010).

Third, as opposed to the literature that only looks at situations in which the trader is able to perfectly price discriminate among heterogeneous farmers, we explore the effects of other spatial price policies, namely uniform and mill pricing. In this context, we show that one of the main results of this literature still holds: the intermediary has an interest in providing the input at a price under the market price, in order to extract as much as possible the generated surplus from the farmer. However, when intermediaries are not allowed to discriminate perfectly, the farmers may gain from the contract, such that the presence of an intermediary may help to reduce their poverty.

We compare the outcomes of the different policies (discriminatory, mill and uniform pricing) in terms of income, poverty and regional disparities in order to arrive at some policy recommendations as to the type of spatial pricing policy that should be used. In particular, we look at what a benevolent policy maker who wants to decrease poverty but is unable to impose a complex tax and subsidy scheme should impose as a spatial pricing policy to be used by intermediaries. Alternatively, our results regarding the recommended pricing policy can be seen as the pricing policy that an external donor which helps set up agricultural intermediaries with a view of reducing rural poverty should impose as a condition to those intermediaries.

The chapter is structured as follows. The next section describes the model and its assumptions. Sections 3, 4 and 5 develop the interlinked transaction model for a for-profit intermediary in the cases of spatial price discrimination, uniform pricing, and mill pricing. Section 6 extends the model to the case of a non-profit organization. Section 7 discusses the implications of pricing policy choice for profit, farmers' income, level of production, regional differences among farmers, poverty and so on. Finally, section 8 concludes.

2.2 Model

We analyze the impact of transport costs and interlinked transactions on poverty in the following theoretical framework. Geographical locations are represented along a linear segment of size $r + R$. A final good market is located at the origin 0. We consider one agricultural good whose price p is set on this market. We assume that the different agents in our model do not have an impact on this price.[2] This good is consumed at location 0 which can be assumed to be an urban center. At a distance r from this urban center, there is a rural area which has a geographical extend R. Farmers are uniformly distributed over this area. Each farmer produces the agricultural good according to the same production function $f(k)$, where k is the quantity of input he uses. This input is sold at price i on the market at location 0. The production function has the usual properties: $f(.)$ is continuously differentiable, $f(0) = 0$, $f_k = \frac{df}{dk} > 0$, $\lim_{k\to 0} f_k = \infty$, $\lim_{k\to\infty} f_k = 0$ and $\frac{d^2f}{dk^2} < 0$. Farmers are assumed to be profit maximizers. A farmer located at x facing prices $p_F(x)$ and $i_F(x)$ maximizes his income $y(x, p_F(x), i_F(x))$ by

[2] This can be the case for example because we are in a small open economy and the price of this good is determined on world markets.

using the optimal quantity $k(x, p_F(x), i_F(x))$ (for simplicity, as long as it does not cause any confusion, shortcut notations $y(x)$ and $k(x)$ will be used):

$$\max_{k(x)} \; y(x) = p_F(x) f(k(x)) - i_F(x) k(x) \tag{2.2.1}$$

The existence of an interior solution to this problem is guaranteed by the above assumptions regarding the production function. The choice of input quantity satisfies the following necessary condition:

$$\frac{df}{dk} = \frac{i_F(x)}{p_F(x)} \tag{2.2.2}$$

To transport the agricultural good to the market, farmers face high transport costs. These costs are assumed to be linear in distance for the output. To simplify the analysis we assume that transport costs are negligible for the input and set them equal to zero. A farmer located at a distance x from the market faces a transport cost τx and hence this farmer can obtain a net price $p_F(x) = p - \tau x$ for the good he produces. All farmers are assumed to be able to sell profitably on the market which implies the following restriction, $p > \underline{p} \equiv \tau r + \tau R$.

An intermediary is located at r.[3] This trader offers interlinked contracts to the geographically dispersed producers. There is an input-output interlinked relationship between them: on the one hand he buys the output from the farmers and, on the other hand, sells them an input necessary for their production. Prices for both input and output are simultaneously fixed in the contract between the trader and the farmer. The trader sells the agricultural output from the farmers and buys input for them on the market located in 0, at market price p and i. The intermediary is assumed to have a cost advantage. Here, we assume that the trader has an advantage to transport the good between r and 0. Transport costs for the trader are given by $t(x) = \theta r + \tau(x - r)$ per unit of output transported with $\theta < \tau$.

The sequence is the following. In a first step, the trader proposes a contract $(p_C(x), i_C(x))$ to each farmer located on the segment $[r, r + R]$.[4] Very often, the quantities produced by each individual farmer are small. We assume that contract prices do not depend on the quantity sold. The farmer located at x receives $p_C(x)$ per unit of output

[3]In developing countries, poor infrastructures in rural area reduce the incentives for firms to locate within this area. By locating just outside of the rural area, the trader has a better access to roads, electricity, water, etc. Because of the limited number of farmers involved and the potentially large investment costs, the intermediary is assumed to have monopoly/monopsony power when he trades with the farmers. On the final market, however, the intermediary is price-taker.

[4]The limit $r + R$ can be seen has a physical limit for the production area. It can be due to the existence of a national border, to the absence of farmers beyond a certain distance, or to technical limits for transporting perishable goods over long distances. For social reasons, the trader may not be able to contract only with some farmers of a local community. Hence we impose that the trader contracts with all farmers located before $r + R$. There are parameters values for which it is profitable for the trader to do so. A sufficient condition for this to be the case is that $\tau r - \theta r > 3\tau R/2$. Indeed, we will see in chapter 3 that the optimal size of the production area, should the intermediary be able to choose it, is given by (3.4.9), (3.4.16) or (3.4.29). This is larger than R when $\tau r - \theta r > 3\tau R/2$ is satisfied.

and pays $i_C(x)$ per unit of input. Each farmer can individually accept or reject the contract. In a second step, the farmer chooses his optimal quantity of input, which determines his level of production. If he has accepted the contract, he faces prices $(p_C(x), i_C(x))$ and chooses optimal input use $k^*(x) = k(x, p_C(x), i_C(x))$. If he rejects the contract, he sells his production directly to the final market. The same applies to the purchase of inputs. In this case, he chooses the optimal amount of inputs k^0 as a function of market prices (p, i) as well as of the transport cost he has to support, that is $k^0(x) = k(x, p - \tau x, i)$. In a last step, output is produced and is sold on the market, directly by the farmer (if he has rejected the contract) or via the trader (if the farmer has accepted the contract).

This means that the trader's problem can be characterized as follows:

$$\max_{p_C(x), i_C(x)} \Pi = \int_r^{r+R} (p - \theta r - \tau(x - r) - p_C(x)) f(k^*(x)) + (i_C(x) - i)k^*(x)dx - F \quad (2.2.3)$$

where F is the fixed cost necessary to obtain the transport cost advantage[5], subject to the demand for input (2.2.2) and the following participation constraint:

$$y(x) \equiv p_C(x) f(k^*(x)) - i_C(x)k^*(x) \geq y^0(x) \equiv (p - \tau x) f(k^0(x)) - ik^0(x) \quad (2.2.4)$$

One of the questions we will be looking at in the following sections is whether, without state intervention, the different outcomes are socially optimal. Due to the farmer's cost disadvantage compared to the trader, his stand-alone production is not socially efficient. Indeed, the efficient input use $k^{\#}(x)$ maximizes the sum of trader's profit and farmer's incomes $\int_r^{r+R} (p - \theta r - \tau(x - r)) f(k(x)) - ik(x)dx$ and satisfies

$$\frac{df}{dk} = \frac{i}{p - \theta r - \tau(x - r)} \quad (2.2.5)$$

Given $\theta < \tau$ and the concavity of production function, this implies that $k^0(x) < k^{\#}(x)\ \forall x$.

In the following sections we will look at different ways in which the trader can set contracts with farmers who are geographically dispersed.

2.3 Spatial price discrimination

The trader proposes a contract $(p_D(x), i_D(x))$ to the farmer located in x. This contract can be different, depending on the location of the farmer and the difference in two farmers' contracts does not necessarily represent the difference in transport costs between them. Each farmer can individually accept or refuse the contract proposed. Hence, to maximize his total profit, the trader chooses a contract which maximizes the profit he makes at each location.

[5] Hereafter, we omit this cost F as it has no influence on the optimization result. We assume that F is not too high with respect to the profit that can be made by the intermediary.

From equations (2.2.3) and (2.2.4), the trader's problem may be written as:

$$\max_{p_D(x), i_D(x)} \quad \pi(x) = (p - \theta r - \tau(x-r))f(k^*(x)) - ik^*(x) - (p_D(x)f(k^*(x)) - i_D(x)k^*(x)) \tag{2.3.1}$$

$$\text{s.t. } g(x) \equiv p_D(x)f(k^*(x)) - i_D(x)k^*(x) - y^0(x) \geq 0 \tag{2.3.2}$$

The Lagrangian is given by:

$$\mathcal{L} = (p-\theta r-\tau(x-r))f(k^*(x)) - ik^*(x) + (\lambda(x)-1)(p_D(x)f(k^*(x)) - i_D(x)k^*(x)) - \lambda(x)y^0(x) \tag{2.3.3}$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_D(x)}{p_D(x)}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_D(x)} = \left((p - \theta r - \tau(x-r))\frac{i_D(x)}{p_D(x)} - i\right)\frac{\partial k^*(x)}{\partial p_D(x)} + (\lambda(x)-1)f(k^*(x)) = 0 \tag{2.3.4}$$

$$\frac{\partial \mathcal{L}}{\partial i_D(x)} = \left((p - \theta r - \tau(x-r))\frac{i_D(x)}{p_D(x)} - i\right)\frac{\partial k^*(x)}{\partial i_D(x)} + (\lambda(x)-1)(-k^*(x)) = 0 \tag{2.3.5}$$

$$\lambda(x) \geq 0, \;\; g(x) \geq 0, \;\; \lambda(x)g(x) = 0 \tag{2.3.6}$$

From (2.3.5),

$$\lambda(x) - 1 = \left((p - \theta r - \tau(x-r))\frac{i_D(x)}{p_D(x)} - i\right)\frac{\partial k^*(x)}{\partial i_D(x)}\frac{1}{k^*(x)} \tag{2.3.7}$$

Substituting (2.3.7) in (2.3.4) we have:

$$\left((p - \theta r - \tau(x-r))\frac{i_D(x)}{p_D(x)} - i\right)\left(\frac{\partial k^*(x)}{\partial p_D(x)} + \frac{\partial k^*(x)}{\partial i_D(x)}\frac{f(k^*(x))}{k^*(x)}\right) = 0 \tag{2.3.8}$$

If the second term was equal to zero, it can be shown that $y(x) = 0$ so that $g(x) < 0$, which contradicts (2.3.6).[6] Thus, the first term has to be equal to zero, that is:

$$\frac{i_D(x)}{p_D(x)} = \frac{i}{p - \theta r - \tau(x-r)} \tag{2.3.9}$$

Equation (2.3.9) characterizes the optimal contract $(p_D(x), i_D(x))$. This contract induces the farmer to increase his level of input (as well as his level of output) with respect to the levels he would have chosen in the stand-alone case, even though he receives the same income, as it is stated in the following proposition.

[6]Indeed, implicit function theorem applied to equation (2.2.2) gives (omitting the argument (x)): $\frac{dk}{dp_F} = -\frac{\frac{df}{dk}}{p_F\frac{d^2f}{dk^2}}$ and $\frac{dk}{di_F} = -\frac{-1}{p_F\frac{d^2f}{dk^2}}$. By the envelop theorem, we know that $\frac{dk}{dp_F} = \left.\frac{\partial k}{\partial p_F}\right|_{k=k^*}$ and $\frac{dk}{di_F} = \left.\frac{\partial k}{\partial i_F}\right|_{k=k^*}$ that we call $\frac{\partial k^*}{\partial p_F}$ and $\frac{\partial k^*}{\partial i_F}$. Thus, $\frac{\partial k^*}{\partial p_F} = -\frac{\frac{df}{dk}}{p_F\frac{d^2f}{dk^2}}$ and $\frac{\partial k^*}{\partial i_F} = -\frac{-1}{p_F\frac{d^2f}{dk^2}}$, which implies $-\frac{\partial k^*/\partial p_F}{\partial k^*/\partial i_F} = \frac{df}{dk}$. By (2.2.2), this is equal to $\frac{i_F}{p_F}$. If the second term in (2.3.8) was equal to zero, we would have $-\frac{\partial k^*/\partial p_F}{\partial k^*/\partial i_F} = \frac{f(k^*)}{k^*}$, thus $\frac{f(k^*)}{k^*} = \frac{i_F}{p_F} \Leftrightarrow p_F = \frac{i_F k^*}{f(k^*)}$. Substituting in the farmer's income would give $y(x) = p_F f(k^*) - i_F k^* = 0$.

Proposition 1. ***Under spatial price discrimination, when the trader is profit maximizer, each farmer's income is pushed down to his reservation level ($y(x) = y^0(x)\ \forall x$), while each farmer uses the efficient quantity of inputs, which is larger than in his stand-alone situation:*** $k^*(x) = k^\#(x) > k^0(x)$.

Proof of proposition 1: As the ratio of input price to output price is given by (2.3.9), this tells us, by using (2.2.2) and comparing it to (2.2.5), that the farmer will choose the efficient level of input: $k^*(x) = k^\#(x)$. Given that $\tau > \theta$ and that $f(k)$ is strictly concave and using (2.2.2) with respectively $(p_F(x), i_F(x)) = (p_D(x), i_D(x))$ and $(p_F(x), i_F(x)) = (p - \tau x, i)$, we have that $k^*(x) > k^0(x)$. Substituting (2.3.9) in (2.3.7) gives $\lambda(x) = 1$. From (2.3.6), this implies that the individual rationality constraint is binding: $g(x) \equiv p_D(x) f(k^*(x)) - i_D(x) k^*(x) - y^0(x) = 0$. $\square$

Substituting (2.3.9) in the binding participation constraint $g(x) = 0$ gives:

$$p_D(x) = (p - \theta r - \tau(x - r)) \underbrace{\frac{(p - \tau x) f(k^0(x)) - i k^0(x)}{(p - \theta r - \tau(x - r))\, f(k^*(x)) - i k^*(x)}}_{\eta_D(x)} \tag{2.3.10}$$

$$i_D(x) = i \underbrace{\frac{(p - \tau x) f(k^0(x)) - i k^0(x)}{(p - \theta r - \tau(x - r))\, f(k^*(x)) - i k^*(x)}}_{\delta_D(x)} \tag{2.3.11}$$

Proposition 2. ***Under spatial price discrimination, the profit maximizing contract is characterized by*** $\eta_D(x) = \delta_D(x) < 1$ ***which implies*** $i_D(x) < i$ ***and*** $p_D(x) < p - \theta r - \tau(x - r)$***: the trader "loses" on the input trading and "gains" on the output trading.***

Proof of proposition 2: As $k^*(x) = k^\#(x)$ (proposition 1), $\eta_D(x) = \delta_D(x)$ may be written as:

$$\eta_D(x) = \delta_D(x) = \frac{\max_k\ (p - \tau x) f(k) - ik}{\max_k\ (p - \theta r - \tau(x - r)) f(k) - ik}$$

Using the envelop theorem and since by assumption $\theta < \tau$, this implies that $\eta_D(x) = \delta_D(x) < 1$. Using this result with (2.3.10) and (2.3.11) this implies that $p_D(x) < p - \theta r - \tau(x - r)$ and $i_D(x) < i$. $\square$

Gangopadhyay and Sengupta (1987) obtain similar results. They analyzed interlinked contracts when input market is characterized by an imperfection, such that the farmer faces a higher input price than the firm. They show that the trader has an interest to "subsidize" the input and "tax" the output, and that this type of contract allows him to appropriate himself all the efficiency gain (i.e. farmers' incomes are pushed down to their reservation income). In our context, the difference between the trader and the farmer lies in the (output) transport costs, and the previous analysis shows that their results remain valid in this context. If the trader did not propose an interlinked contract but only proposed a contract regarding the output price, he would not have been able to push all the farmers' incomes down to their reservation level.[7] Both instruments, output and input prices, are necessary for the trader to capture completely the efficiency gain. The strategy of "La Laiterie du Berger" that sells cattle feed to farmers at 50% of the market price (personal interview, 2009) is thus constistent with our analysis. In other contexts also, evidence suggests that in interlinked contracts the input is sold at a discount.[8]

It can be easily seen, as it is done in GANGOPADHYAY and SENGUPTA (1987), that, if there was no difference between the trader and the farmer (i.e. $\tau = \theta$), the optimal contract would be characterized by $\eta_D(x) = \delta_D(x) = 1$, and the role of the trader would be irrelevant. If he has no cost advantage, the trader is not able to organize the production in a more efficient way than farmers do.

Proposition 3. ***Under spatial price discrimination,*** $i_D(x) < i$ ***and*** $p_D(x) < p - \tau x$***: each farmer "loses" on the output trading and "gains" on the input trading.***

Proof of proposition 3: From (2.3.10), $p_D(x) < p - \tau x$ if

$$f(k^0(x)) - \frac{i}{p - \tau x} k^0(x) < f(k^*(x)) - \frac{i}{p - \theta r - \tau(x - r)} k^*(x)$$

From (2.2.2), (2.2.5) and proposition 1, this is equivalent to

$$f(k^0(x)) - \left.\frac{df}{dk}\right|_{k(x)=k^0(x)} k^0(x) < f(k^{\#}(x)) - \left.\frac{df}{dk}\right|_{k(x)=k^{\#}(x)} k^{\#}(x)$$

This is true provided that the production elasticity $\frac{df}{dk}\frac{k}{f(k)}$ is constant or decreasing in k. The result $i_D(x) < i$ follows from proposition 2. □

When involved in the interlinked transaction, each farmer receives a price for the output which is lower than the net price he would have received in the stand-alone situation. This "loss" on the output trading is compensated by a "gain" on the input trading, such that, as proposition 1 states, each farmer gets an income $y(x)$ from the contract which is exactly equal to his reservation income $y^0(x)$.

The results show that farmers are treated differently depending on their location. On the one hand, farmers located far from the market receive a lower price for their output, but on the other hand they also pay a lower price for input. Moreover, those farmers receive a smaller share of the net price received by the trader on the market for the output and pay a lower part of the input price. Indeed, from (2.3.10) and (2.3.11),

[7] In particular, with $f(k) = 2\sqrt{k}$, it can be shown that only the farmers located before $\frac{p+\theta r}{\tau} - r$ receive their reservation income. Farmers further away get a positive surplus from such a contract.

[8] In Kenya, British American Tobacco Ltd delivers input to farmers at prices that are "*in most cases lower than the Nairobi wholesale prices for similar products*", while Kenya Tea Development Agency Ltd supplies bags of fertilizer at a price "*significantly lower than the wholesale price in Nairobi and much lower than the retail price offered to the smallholders by the village-level stockists*" (IFAD, 2003). Sometimes, input is even given for free (KOO, 2011, IFAD, 2003).

it can be shown[9] that $p_D(x)$, $i_D(x)$, and $\eta_D(x) = \delta_D(x)$ are decreasing in x. Contract prices $p_D(x)$ and $i_D(x)$ are increasing with the output market price p.[10] We also have that $\eta_D(x)$ $(= \delta_D(x))$ increase with p which means that trader's mark-up on the output and discount on the input are lower when p is higher.

2.4 Uniform pricing

Under uniform pricing policy, the trader is constrained to propose the same contract (p_U, i_U) to all farmers (where p_U and i_U are independent of x). Each farmer can individually accept or refuse the contract proposed.

The trader's problem can be written as:

$$\max_{p_U, i_U} \Pi = \int_r^{r+R} (p - \theta r - \tau(x - r)) f(k^*) - ik^* - (p_U f(k^*) - i_U k^*) dx$$

$$\text{s.t. } g(x) \equiv p_U f(k^*) - i_U k^* - y^0(x) \geq 0 \ \forall x$$

Note that k^* is the same for all farmers, independent of their location (see (2.2.2) where $p_F(x) = p_U$ and $i_F(x) = i_U$ are independent of x). As farmers are distributed on the interval $[r, r+R]$, there is a continuum of participation constraints $g(x)$ with $x \in [r, r+R]$. The satisfaction of the constraint for the first farmer (located at r) is sufficient to ensure that it is satisfied for all farmers located further (in $x \in]r, r+R]$). Indeed, as k^* is constant for all x and $y^0(x)$ is strictly decreasing in x, $g(x)$ is strictly increasing in x.

Thus, we can replace the continuum of constraints $g(x) \geq 0$ by the unique constraint $g(r) \geq 0$ (see for instance Bolton and Dewatripont, 2005: 82). The problem is now the following:

$$\max_{p_U, i_U} \Pi = R\left(\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^* - (p_U f(k^*) - i_U k^*)\right)$$

$$\text{s.t. } g(r) \equiv p_U f(k^*) - i_U k^* - y^0(r) \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = R\left(\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^*\right) + (\lambda - R)(p_U f(k^*) - i_U k^*) - \lambda y^0(r) \qquad (2.4.1)$$

---

[9] First derivative of $\eta_D(x)$ with respect to x is negative if $f(k^*(x))[(p - \tau x) f(k^0(x)) - ik^0(x)] < f(k^0(x))[(p - \theta r - \tau(x - r)) f(k^*(x)) - ik^*(x)]$. As $\theta < \tau$, a sufficient condition is $k^0(x)/f(k^0(x)) > k^*(x)/f(k^*(x))$ which is ensured by the concavity of the production function and the fact that $k^*(x) > k^0(x)$ from proposition 1. As $\eta_D(x)$ is decreasing in x, it follows that $p_D(x)$ and $i_D(x)$ are also decreasing in x since $\frac{\partial p_D(x)}{\partial x} = \frac{\partial \eta_D(x)}{\partial x}(p - \theta r - \tau(x - r)) - \tau\eta_D(x) < 0$ and $\frac{\partial i_D(x)}{\partial x} = \frac{\partial \eta_D(x)}{\partial x} i < 0$.

[10] For instance, Strohm and Hoeffler (2006) have reported that Deepa Industries in Kenya paid a higher price to potatoes producers than originally agreed because the market price had risen.

Noting that at equilibrium $\frac{df}{dk} = \frac{i_U}{p_U}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_U} = R\left(\left(p - \theta r - \tau\frac{R}{2}\right)\frac{i_U}{p_U} - i\right)\frac{\partial k^*}{\partial p_U} + (\lambda - R)\, f(k^*) = 0 \tag{2.4.2}$$

$$\frac{\partial \mathcal{L}}{\partial i_U} = R\left(\left(p - \theta r - \tau\frac{R}{2}\right)\frac{i_U}{p_U} - i\right)\frac{\partial k^*}{\partial i_U} + (\lambda - R)\,(-k^*) = 0 \tag{2.4.3}$$

$$\lambda \geq 0,\;\; g(r) \geq 0,\;\; \lambda g(r) = 0 \tag{2.4.4}$$

From (2.4.3),

$$\lambda - R = R\left(\left(p - \theta r - \tau\frac{R}{2}\right)\frac{i_U}{p_U} - i\right)\frac{\partial k^*}{\partial i_U}\frac{1}{k^*} \tag{2.4.5}$$

Substituting (2.4.5) in (2.4.2) we have:

$$R\left(\left(p - \theta r - \tau\frac{R}{2}\right)\frac{i_U}{p_U} - i\right)\left(\frac{\partial k^*}{\partial p_U} + \frac{\partial k^*}{\partial i_U}\frac{f(k^*)}{k^*}\right) = 0 \tag{2.4.6}$$

As R is strictly positive, the first and/or second term between brackets has to be equal to zero. If the last term was equal to zero, it can be shown[11] that $y(x) = 0\ \forall x$ so that $g(x) < 0\ \forall x$, which contradicts (2.4.4). Thus, the first term has to be equal to zero, that is:

$$\frac{i_U}{p_U} = \frac{i}{p - \theta r - \tau\frac{R}{2}} \tag{2.4.7}$$

Equation (2.4.7) characterizes the optimal contract (p_U, i_U). This contract implies that each farmer receives the same income from the contract as the stand-alone income of the first farmer.

Proposition 4. ***Under uniform pricing, when the trader is profit maximizer, if the trader's cost advantage is large enough ($\tau r - \theta r > \tau(R/2)$) the closest farmer's income is pushed down to his reservation level ($y(r) = y^0(r)$) while the others obtain a positive surplus from the contract. Each farmer increases his quantity of inputs with respect to the stand-alone situation ($k^*(x) > k^0(x)$). If his cost advantage is too small ($\tau r - \theta r \leq \tau(R/2)$), the trader is not able to make a positive profit.***[12]

Proof of proposition 4: Substituting (2.4.7) in (2.4.5) gives $\lambda = R$. From (2.4.4), this implies that the individual rationality constraint is binding: $g(r) \equiv p_U f(k^*) - i_U k^* - y^0(r) = 0$. If $\tau r - \theta r > \tau(R/2)$, given that that $f(k)$ is strictly concave and using (2.2.2) with respectively $(p_F(x), i_F(x)) = (p_U, i_U)$ and $(p_F(x), i_F(x)) = (p - \tau x, i)$, we

[11] Indeed, we have shown (see footnote 6) that $-\frac{\partial k^*(x)/\partial p_F}{\partial k^*(x)/\partial i_F} = \frac{f(k^*(x))}{k^*(x)}$, implies $y(x) = 0$.

[12] If the intermediary had the possibility to collect the good only on a part of collection area R, he would do so, this strategy leading to a positive profit. Here, we assume this is not possible. For a discussion about the optimal size of the collection area, see chapter 3.

have that $k^* > k^0(x)$. If $\tau r - \theta r \leq \tau(R/2)$, we have that $k^* \leq k^0(x)$. Given that $g(r) = 0$, the profit is $\Pi = R\left(\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^* - [(p - \tau r)f(k^0(r)) - ik^0(r)]\right)$. From $k^* \leq k^0(r)$ and $\tau r \leq \theta r + \tau(R/2)$, we have that $\Pi \leq 0$. $\square$

Contrary to the spatial price discrimination case, when the trader is able to operate profitably under uniform pricing, all the farmers except the first one see an increase in their income with respect to their stand-alone situation. Using this policy, " La Laiterie du Berger" claims that its presence has allowed to triple the income of the farmers involved (PhiTrust, 2011).[13]

Substituting (2.4.7) in the binding participation constraint $g(r) = 0$ gives:

$$p_U = \left(p - \theta r - \tau\frac{R}{2}\right) \underbrace{\frac{(p - \tau r)f(k^0(r)) - ik^0(r)}{\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^*}}_{\eta_U} \tag{2.4.8}$$

$$i_U = i \underbrace{\frac{(p - \tau r)f(k^0(r)) - ik^0(r)}{\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^*}}_{\delta_U} \tag{2.4.9}$$

Proposition 5. ***Under uniform pricing, when*** $\tau r - \theta r > \tau(R/2)$***, the profit maximizing contract is characterized by*** $\eta_U = \delta_U < 1$ ***which implies*** $i_U < i$ ***and*** $p_U < p - \theta r - \tau(R/2)$***: the trader "loses" on the input trading and "gains" on average on the output trading.***

Proof of proposition 5: Note, from (2.4.7) and (2.2.5), that $k^* = k^{\#}(r+(R/2))$. Thus, $\eta_U = \delta_U$ may be written as:

$$\eta_U = \delta_U = \frac{\max_k \, (p - \tau r)f(k) - ik}{\max_k \, (p - \theta r - \tau(R/2))f(k) - ik}$$

Using the envelop theorem, $\tau r - \theta r > \tau(R/2)$ implies that $\eta_U = \delta_U < 1$. Using this result with (2.4.8) and (2.4.9) implies that $p_U < p - \theta r - \tau(R/2)$ and $i_U < i$. $\square$

Propositions 4 and 5 imply that, only if there exists a *sufficient* difference in transport costs ($\tau r - \theta r > \tau(R/2)$), the trader is able to make a positive profit. In this case, he "loses" on the input trading and "gains" on the output trading, as the average net price he receives on the market is higher than the price he pays to each farmer, similarly to what happens in the spatial price discrimination case. However, if his cost advantage is too

[13] Higher income due to the contract is also consistent with empirical evidence in other contexts. Indeed, Warning and Key (2002) have estimated an increase in gross agricultural income of 207000 CFA for Senegalese peanut producers that have accepted a contract with "arachide de bouche". Similarly, Simons et al. (2005) have found that the contracts for seed corn in East Java and for broilers in Lombok made significant contributions to farmers' capital returns.

small, he is not able to profitably induce farmers to organize production in a more efficient way. This result is in contrast with the result obtained under price discrimination, where the trader is able to exploit his cost advantage, even if the advantage is very small.

As it was the case with spatial price discrimination, when the trader's cost advantage is large enough, contract prices under uniform pricing p_U and i_U are increasing with the output market price p. The same applies for $\eta_U = \delta_U$, which means that farmers receive a higher share of trader's gain on the output transaction, but pay a higher share of the input price, when p is higher.

2.5 Mill pricing

Under a mill pricing policy, the trader pays the same mill price to all farmers. He has to propose the same contract (p_M, i_M) to all farmers (where p_M and i_M are independent of x) but farmers have to support the transport costs. Thus, the net price received by the farmer for the output is $p_F(x) = p_M - \tau(x - r)$.

From equations (2.2.3) and (2.2.4), the trader's problem may be written as:

$$\max_{p_M;i_M} \Pi = \int_r^{r+R} (p - \theta r - p_M) f(k^*(x)) + (i_M - i)k^*(x)dx \tag{2.5.1}$$

$$\text{s.t. } g(x) \equiv (p_M - \tau(x-r))f(k^*(x)) - i_M k^*(x) - y^0(x) \geq 0 \ \forall x$$

As farmers are distributed on the interval $[r, r+R]$, there is a continuum of participation constraints $g(x)$ with $x \in [r, r+R]$. Contrary to the uniform pricing case, which constraint(s) will be binding at the optimum is not obvious. Indeed, one cannot determine a priori if the contract income decreases at a faster rate with distance than the reservation income. As Jullien (2000) shows, when both reservation and contract utility depend on the agent's type (in our case, his location), it may be the case that the constraint is binding at one end of the interval of agent's type, but it may also be the case that one or several interior agents face binding participation constraints while extreme agents do not. In the proof of lemma 1 (appendix 2.A), we show that, if the production function is homogeneous, the latter does not occur. Indeed, we show that the outcome may only be characterized by one of the four following cases: (1) the last participation constraint is binding and only the most distant farmer's income is pushed down to the reservation level. Other farmers get a positive surplus. This happens if contract prices p_M and i_M are such that the income from the contract decreases less rapidly with distance than the reservation income. (2) the first participation constraint is binding and only the first farmer's income is pushed down to the reservation level while other farmers get a positive surplus. This is possible if contract prices p_M and i_M are such that the income from the contract decreases more rapidly with distance than the reservation income. (3) all constraints are binding and all the farmers are pushed down to their reservation income. This is the case if the trader decides to set $p_M = p - \tau r$ and $i_M = i$. (4) no constraint is binding.

Lemma 1. ***Under mill pricing, if the production function is homogeneous of degree*** $h < 1$***,*** $g(r) \geq 0$ ***and*** $g(r+R) \geq 0$ ***are sufficient to ensure that*** $g(x) \geq 0 \ \forall x$***.***

Proof of lemma 1: See appendix 2.A.

Using lemma 1, the problem can be written as:

$$\max_{p_M, i_M} \Pi = \int_r^{r+R} (p - \theta r - p_M) f(k^*(x)) + (i_M - i) k^*(x) dx$$

$$\text{s.t. } g(r) \equiv p_M f(k^*(r)) - i_M k^*(r) - y^0(r) \geq 0$$

$$\text{and } g(r+R) \equiv (p_M - \tau R) f(k^*(r+R)) - i_M k^*(r+R) - y^0(r+R) \geq 0$$

In lemma 2, we prove that the case where contract prices are such that only the first farmer's income is pushed down to the reservation level (case (2) above) is dominated by the replication of the stand-alone situation (case (3)). Indeed, in the first case, the trader induces all the farmers to decrease their production, compared to their stand-alone level, which is not optimal from the trader's point of view. Hence, if the first farmer's participation constraint is binding at the optimum this implies that all the participation constraints are binding at the optimum and that $i_M = i$ and $p_M = p - \tau r$.

Lemma 2. ***Under mill pricing, if the production function is homogeneous of degree*** $h < 1$ ***and*** $g(r) = 0$ ***at the optimum, this implies that*** $g(x) = 0$ ***at the optimum for all*** x***.***

Proof of lemma 2: See appendix 2.B.

Figure 2.2 illustrates the results from lemmas 1 and 2, when $f(k) = 2\sqrt{k}$. Binding constraints $g(x) = 0$ are represented by curves with the form $i_M = \frac{(p_M - \tau(x-r))^2}{(p-\tau x)^2} i$. They cross at the point $(p - \tau r, i)$. When $p_M > p - \tau r$ the curve corresponding to the constraint of the farmer located in r is below all the other curves, while when $p_M < p - \tau r$ the curve corresponding to the constraint of the farmer located in $r + R$ is below all the other curves. Lemma 1 implies that the couple (p_M, i_M) resulting from the maximization (2.5.1) is located either within the colored area, either on one of its borders. Lemma 2 implies that this outcome can not be located on the dashed curve.

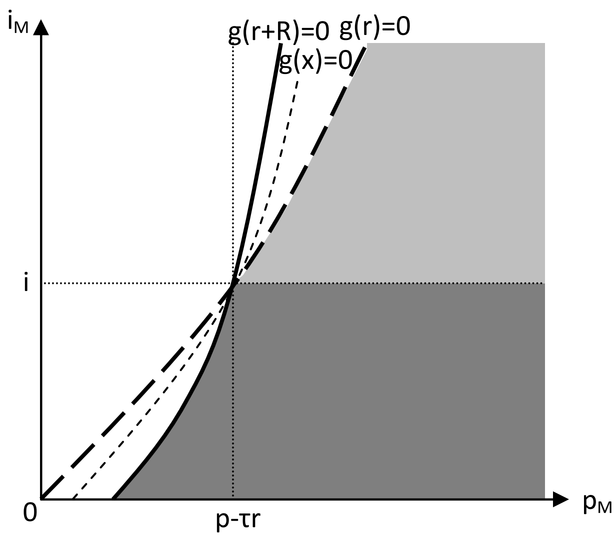
2.5.1 Model with a specific production function

In what follows, we use a particular production function to derive some characteristics of the equilibrium.

Assumption 1. $f(k) = 2\sqrt{k}$***.***

If the participation constraint of the most remote farmer is binding, this implies that $i_M < i$. If it was not the case, the binding participation constraint would imply that $p_M > p - \tau r$, and this, in turn, would not respect the participation constraint for the other farmers. However, the unconstrained equilibrium could be such that $i_M > i$ and $p_M > p - \tau r$. Indeed, a priori one could think that it would be possible to find a contract such that each farmer losses on the input but gains on the output, while no participation

Figure 2.2: Continuum of constraints



constraint is binding (this corresponds to the dark gray area on figure 2.2). In what follows, we show that the trader has no interest to do so, such that, at the optimum, $i_M \leq i$ always holds. On figure 2.2, proposition 6 implies that the couple (p_M, i_M) resulting from the maximization (2.5.1) is located within the light gray area, or on one of its borders.

Proposition 6. ***Under mill pricing and assumption 1, the profit maximizing contract is characterized by $i_M \leq i$. The trader "loses" on the input trading.***

Proof of proposition 6: See appendix 2.D.

Corollary 1. ***Under mill pricing and assumption 1, the profit maximizing interlinked contract implies that each farmer increases the quantity of inputs he uses, and hence increases his production, compared to his stand-alone alternative.***

Proof of corollary 1: The participation constraint has to be satisfied for all x. As the production function is homogeneous, that means $i_M k^*(x) - ik^0(x) \geq 0$ (see also appendix 2.A). From proposition 6, $i_M \leq i$, which implies $k^*(x) \geq k^0(x)$ for the participation constraints to be satisfied. □

The result of farmers increasing their output (also obtained under discriminatory and uniform pricing) is consistent with what is observed in the milk sector in Senegal. In particular, "La Laiterie du Berger" claims that the feed supplements it provides to the farmers have helped them to increase their production, especially during the dry season (own interview, 2009). This is also observed in other sectors using interlinked contracts.[14]

Using lemma 2 and proposition 6, the problem can be written as:

$$\max_{p_M, i_M} \Pi = \int_r^{r+R} (p - \theta r - p_M) f(k^*(x)) + (i_M - i) k^*(x) dx$$

$$\text{s.t. } g(r+R) \equiv (p_M - \tau R) f(k^*(r+R)) - i_M k^*(r+R) - y^0(r+R) \geq 0$$

$$\text{and } i - i_M \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = \int_r^{r+R} (p - \theta r - p_M) f(k^*(x)) + (i_M - i) k^*(x) dx$$

$$+ \lambda \big((p_M - \tau R) f(k^*(r+R)) - i_M k^*(r+R) - y^0(r+R) \big) + \mu (i - i_M) \tag{2.5.2}$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_M}{p_M - \tau(x-r)}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_M} = \lambda f(k^*(r+R)) + \int_r^{r+R} \left((p - \theta r - p_M) \frac{i_M}{p_M - \tau(x-r)} + i_M - i \right) \frac{\partial k^*}{\partial p_M} - f(k^*(x)) dx = 0 \tag{2.5.3}$$

$$\frac{\partial \mathcal{L}}{\partial i_M} = -\lambda k^*(r+R) - \mu + \int_r^{r+R} \left((p - \theta r - p_M) \frac{i_M}{p_M - \tau(x-r)} + i_M - i \right) \frac{\partial k^*}{\partial i_M} + k^*(x) dx = 0 \tag{2.5.4}$$

$$\lambda \geq 0, \; g(r+R) \geq 0, \; \lambda g(r+R) = 0 \tag{2.5.5}$$

$$\mu \geq 0, \; i - i_M \geq 0, \; \mu(i - i_M) = 0 \tag{2.5.6}$$

Contrary to uniform pricing and spatial price discrimination, under mill pricing the optimum is not always constrained. Whether the optimum is constrained or unconstrained depends on the output price p as well as on the trader's cost advantage $\tau - \theta$.

Proposition 7. ***Under mill pricing and assumption 1:***

- ***If the trader has a large cost advantage ($\tau r - \theta r > \tau R$), if the output price is large ($p > \bar{p}$ with $\bar{p}$ unique) the most distant farmer's income is pushed down to his reservation level ($g(r+R) = 0$) while other farmers obtain a positive surplus from the contract. For a lower output price ($p \in [\underline{p}, \bar{p}]$), all the farmers, including the last one, obtain a positive surplus from the contract ($g(x) > 0 \; \forall x$).***

[14] In the Indian poultry sector, Ramaswami et al. (2006) have found that contract production is more efficient than noncontract one and that the efficiency surplus is largely appropriated by the processor. In Ethiopia, Tadesse and Guttormsen (2009) have estimated that producers of haricot bean who are in relational (interlinked) contract supply about 27% more than farmers in spot markets.

- ***If $\tau R/2 < \tau r - \theta r \leq \tau R$, then the most distant farmer's income is pushed down to his reservation level ($g(r+R) = 0$) while other farmers obtain a positive surplus from the contract $\forall\ p > \underline{p}$.***

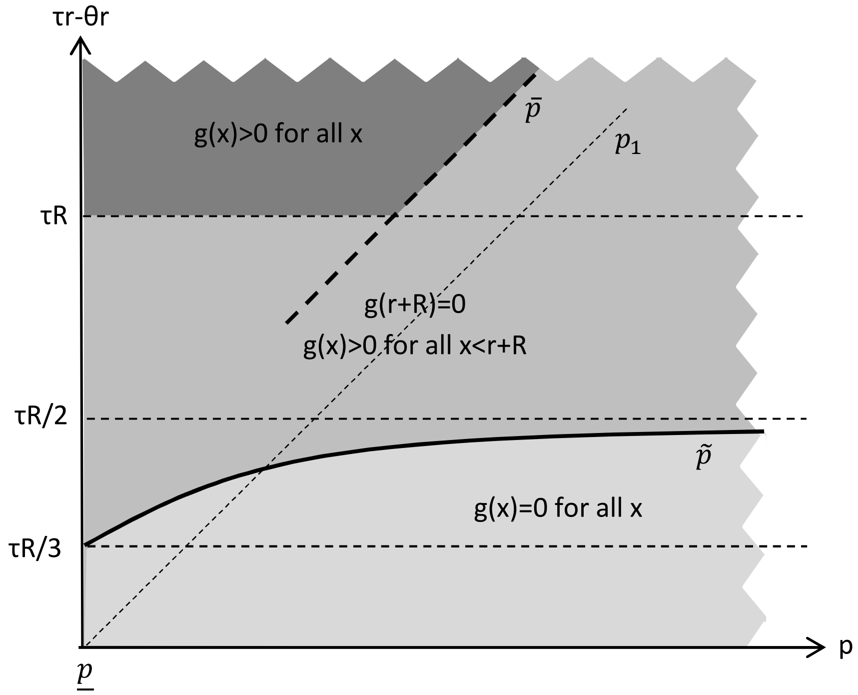
- ***If $\tau R/3 < \tau r - \theta r \leq \tau R/2$, if the output price is large ($p > \tilde{p}$ with $\tilde{p}$ unique) then all the farmers' incomes (including the last one's) are pushed down to their reservation level ($g(x) = 0\ \forall x$) and the stand-alone situation is simply replicated. For a lower output price ($p \in [\underline{p}, \tilde{p}]$), only the most distant farmer's income is pushed down to his reservation level ($g(r+R) = 0$) while other farmers obtain a positive surplus from the contract.***

- ***If the trader has a small cost advantage ($\tau r - \theta r \leq \tau R/3$), then all the farmers' incomes (including the last one's) are pushed down to their reservation level ($g(x) = 0\ \forall x$) and the stand-alone situation is simply replicated, $\forall\ p > \underline{p}$.***

Proof of proposition 7: See appendix 2.E.

These results (illustrated by the stylized figure 2.3) show that under mill pricing the optimal pricing by the trader is not always to simply charge farmers the prices they face in a stand-alone situation and to make a profit from the transport cost advantage he has. In particular, if his cost advantage is large enough, the trader uses it to introduce a "distortion" in the prices in order to induce farmers to produce more and hence increase his profit even more.

Figure 2.3: Proposition 7



Note that for some values of the parameters ($\tau r - \theta r > \tau R$ and $p < \bar{p}$) all the farmers can benefit from contracting with a trader. In a context where agricultural output prices are often driven down by international competition, this result is particularly interesting. In contrast, under the two other pricing policies (discrimination or uniform pricing), there is always at least one farmer who is pushed down to his reservation income, for any p.

If the trader's transport cost advantage is large, but the output price is low, the optimum is unconstrained, meaning that the contract which is optimal from the trader's point of view leads to incomes for the farmers that are higher than their stand-alone incomes. This is due to the low level of the stand-alone income which is a consequence of both low output price and high farmer's transport cost. When the output price is larger, this is no longer possible. Indeed, as θ is bounded at 0, the trader's cost advantage cannot be larger than τ, and cannot compensate the increase in the reservation income due to a higher output price.

On the contrary, if the trader's transport cost advantage is low, if the output price is large, the contract which is optimal from the trader's point of view leads to incomes for the farmers that are lower than their stand-alone incomes. Indeed, the high output price lead to large reservation incomes that cannot be compensated by the trader's cost advantage, as it is too small. In this case, the best the trader is able to do in order that the farmers accept the contract, is to replicate their stand-alone situation.

2.6 The case of a non-profit trader

Most of the literature about spatial price policies and/or about interlinked transactions assumes that the trader is a profit maximizing firm. However in agriculture and livestock sector in developing countries, several local producers' associations and NGOs try to increase the income and welfare of producers. In order to take this into account, we look at the case in which the trader is a non-profit organization. When the trader defends the interest of farmers, the objective function can take different forms. Following the literature, we look at two different cases: a first case where the trader maximizes the sum of farmers's incomes and a second case where the trader maximizes total earnings (i.e. the sum of profit and farmers' income) assuming that the profit can be distributed to members (see for instance Royer, 2001).

2.6.1 Spatial price discrimination

When the trader maximizes total farmers' income, subject to a non-negative profit and participation of all farmers, his problem is the following:

$$\max_{p_D(x), i_D(x)} \; y(x) = p_D(x) f(k^*(x)) - i_D(x) k^*(x)$$

$$\text{s.t. } g(x) \equiv p_D(x) f(k^*(x)) - i_D(x) k^*(x) - y^0(x) \geq 0$$

$$\text{and } \pi(x) \equiv (p - \theta r - \tau(x-r)) f(k^*(x)) - ik^*(x) - (p_D(x) f(k^*(x)) - i_D(x) k^*(x)) \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = \mu(x)\left((p - \theta r - \tau(x-r)) f(k^*(x)) - ik^*(x)\right) + (1 - \mu(x) + \lambda(x))(p_D(x) f(k^*(x)) - i_D(x) k^*(x))$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_D(x)}{p_D(x)}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_D(x)} = \mu(x)\left((p-\theta r-\tau(x-r))\frac{i_D(x)}{p_D(x)} - i\right)\frac{\partial k^*(x)}{\partial p_D(x)} + (1-\mu(x)+\lambda(x))f(k^*(x)) = 0 \tag{2.6.1}$$

$$\frac{\partial \mathcal{L}}{\partial i_D(x)} = \mu(x)\left((p-\theta r-\tau(x-r))\frac{i_D(x)}{p_D(x)} - i\right)\frac{\partial k^*(x)}{\partial i_D(x)} + (1-\mu(x)+\lambda(x))\left(-k^*(x)\right) = 0 \tag{2.6.2}$$

$$\lambda(x) \geq 0,\ g(x) \geq 0,\ \lambda(x)g(x) = 0 \tag{2.6.3}$$

$$\mu(x) \geq 0,\ \pi(x) \geq 0,\ \mu(x)\pi(x) = 0 \tag{2.6.4}$$

Optimal contract is characterized by $\mu(x) = 1$, $\lambda(x) = 0$ and:

$$\frac{i_D(x)}{p_D(x)} = \frac{i}{p-\theta r-\tau(x-r)} \tag{2.6.5}$$

This price ratio is the same as for the profit maximizing trader (see (2.3.9)). This implies that the optimal level of input chosen by each farmer $k^*(x)$ is the same, whether the trader is profit maximizer or not. The same applies for the output production level $f(k^*(x))$. Under this pricing policy, both non-profit and for-profit traders induce farmers to choose the efficient level of inputs. This means that maximizing farmers' income does not lead to any efficiency loss compared to maximizing profit.

From (2.6.4), and $\mu(x) = 1$, we know that, at the optimum, the profit is equal to zero (the constraint on profit is binding). Substituting (2.6.5) in $\pi(x) = 0$ gives:

$$p_D(x) = p - \theta r - \tau(x-r), \quad i_D(x) = i$$

Comparing these expressions to the for-profit case ((2.3.10) and (2.3.11)), we see that when the trader is an income maximizer, he does not make any gain nor loss neither on the input nor on the output. The trader transfers the net prices he faces to the farmers.

It has to be noted that when the trader maximizes farmers' income under price discrimination, while he has the possibility of setting for different farmers different contracts which do not reflect the difference in transport costs, it is not optimal for him to do so. The optimal contracts for different farmers will reflect the differences in transport costs and hence are the same as the optimal contracts under mill pricing.

The price ratio (2.6.5) is the result of profit maximization or farmers' income maximization or even the maximization of the sum of profit and farmers' income. Indeed, assume a cooperative maximizes such a function. The problem is then the following:

$$\max_{p_D(x), i_D(x)} (p-\theta r-\tau(x-r))f(k^*(x)) - ik^*(x)$$

$$\text{s.t. } g(x) \equiv p_D(x)f(k^*(x)) - i_D(x)k^*(x) - y^0(x) \geq 0$$

$$\text{and } \pi(x) \equiv (p-\theta r-\tau(x-r))f(k^*(x)) - ik^*(x) - (p_D(x)f(k^*(x)) - i_D(x)k^*(x)) \geq 0$$

The Lagrangian is given by (for ease of notation, we drop the argument (x)):

$$\mathcal{L} = (1+\mu)\left((p-\theta r-\tau(x-r))f(k^*) - ik^*\right) + (\lambda-\mu)(p_D f(k^*) - i_D k^*) - \lambda y^0$$

The Kuhn-Tucker first-order conditions imply that $\lambda(x) = \mu(x) = 0$ and that the optimal contract is characterized by (2.6.5). As p_D and i_D enter the objective function only through k^* and not separately, there exists a continuum of p_D and i_D (which satisfy (2.6.5)) which maximizes this objective function. This leads us to the following proposition:

Proposition 8. ***Under spatial price discrimination, when total earnings are maximized, the optimal contract is defined by*** $(p_D(x), i_D(x)) = (\psi_D(x)(p - \theta r - \tau(x-r)), \psi_D(x)i)$ ***where*** $\psi_D(x) \in [\eta_D(x), 1]$, $\eta_D(x)$ ***is defined in*** (2.3.10)***. If*** $\psi_D(x) = \eta_D(x)$***, then*** $g(x) = 0$ ***and the profit is maximized. If*** $\psi_D(x) = 1$***, then*** $\pi(x) = 0$ ***and farmer's income is maximized.***

Whether the trader maximizes profit, farmers' income or total earnings, the optimal contract is always defined by the same price ratio, implying that the efficient outcome is reached. Only the distribution of the efficiency gain between agents is different. Profit maximization and income maximization can be seen as two particular cases of the total earnings maximization. If profit is maximized, the trader acquires all the efficiency gain, while if income is maximized, it is acquired by the farmers.

2.6.2 Uniform pricing

Under uniform pricing, as k^* is independent of x, the trader's problem, when he maximizes total farmers' income subject to non-negative profit and participation of all farmers, is the following:

$$\max_{p_U, i_U} R(p_U f(k^*) - i_U k^*)$$

$$\text{s.t } g(r) \equiv p_U f(k^*) - i_U k^* - y^0(r) \geq 0$$

$$\text{and } \Pi = R\left(\left(p - \theta r - \tau \frac{R}{2}\right) f(k^*) - ik^* - (p_U f(k^*) - i_U k^*)\right) \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = \mu R\left(\left(p - \theta r - \tau \frac{R}{2}\right) f(k^*) - ik^*\right) + (R - \mu R + \lambda)(p_U f(k^*) - i_U k^*)$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_U}{p_U}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_U} = \mu R\left(\left(p - \theta r - \tau \frac{R}{2}\right) \frac{i_U}{p_U} - i\right) \frac{\partial k^*}{\partial p_U} + (R - \mu R + \lambda) f(k^*) = 0 \tag{2.6.6}$$

$$\frac{\partial \mathcal{L}}{\partial i_U} = \mu R\left(\left(p - \theta r - \tau \frac{R}{2}\right) \frac{i_U}{p_U} - i\right) \frac{\partial k^*}{\partial i_U} + (R - \mu R + \lambda)(-k^*) = 0 \tag{2.6.7}$$

$$\lambda \geq 0, \;\; g(r) \geq 0, \;\; \lambda g(r) = 0 \tag{2.6.8}$$

$$\mu \geq 0, \;\; \Pi \geq 0, \;\; \mu \Pi = 0 \tag{2.6.9}$$

The optimal contract is characterized by $R - \mu R + \lambda = 0$ and:

$$\frac{i_U}{p_U} = \frac{i}{p - \theta r - \tau \frac{R}{2}} \tag{2.6.10}$$

This price ratio is the same as for the profit maximizing trader (see (2.4.7)). This implies that the optimal level of input chosen by farmers, k^*, is the same, whether the trader is a profit maximizer or an income maximizer. Hence, as it was the case for the for-profit, if he has a sufficient cost advantage, the income maximizer trader is able to induce each farmer to increase his production with respect to his stand-alone alternative.

From (2.6.8), (2.6.9) and $R - \mu R + \lambda = 0$, we know that, an optimum exists only if $\tau r - \theta r \geq \tau(R/2)$. At the optimum, the profit is equal to zero (the constraint on profit is binding). Substituting (2.6.10) in $\Pi = 0$ gives:

$$p_U = p - \theta r - \tau(R/2), \quad i_U = i$$

Comparing these expressions to the for-profit case ((2.4.8) and (2.4.9)), we see that when the trader is an income maximizer, he does not make any gain nor loss neither on the input nor on the output.

It has to be noted that with those prices, when $\tau r - \theta r < \tau(R/2)$ the trader is not able to induce participation of the first farmers. Only if the difference in transport cost is large enough all farmers accept the contract proposed by the income-maximizer trader.

Here again, the price ratio (2.6.10) also corresponds to total earnings maximization. Indeed, assuming a cooperative maximizes the sum of profit and farmers' income, the problem is the following:

$$\max_{p_U, i_U} R\left(\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^*\right)$$

$$\text{s.t } g(r) \equiv p_U f(k^*) - i_U k^* - y^0(r) \geq 0$$

$$\text{and } \Pi \equiv R\left(\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^* - (p_U f(k^*) - i_U k^*)\right) \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = (1+\mu)R\left(\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^*\right) + (\lambda - \mu)(p_U f(k^*) - i_U k^*) - \lambda y^0(r)$$

The Kuhn-Tucker first-order conditions imply that $\lambda = \mu$ and that the optimal contract is characterized by (2.6.10). As p_U and i_U enter the objective function only through k^* and not separately, there exists a continuum of p_U and i_U (which satisfy (2.6.10)) which maximizes this objective function. This leads us to the following proposition:

Proposition 9. ***Under uniform pricing and*** $\tau r - \theta r > R/2$***, when total earnings are maximized, the optimal contract is defined by*** $(p_U, i_U) = (\psi_U(p - \theta r - \tau(R/2)), \psi_U i)$ ***where*** $\psi_U \in [\eta_U, 1]$***,*** η_U ***is defined in*** (2.4.8)***. If*** $\psi_U = \eta_U$***, then*** $g(r) = 0$ ***and the profit is maximized. If*** $\psi_U = 1$***, then*** $\Pi = 0$ ***and total farmers' income is maximized.***

2.6.3 Mill pricing

When the trader maximizes total farmers' income subject to non-negative profit and participation of all farmers, his problem is the following[15]:

$$\max_{p_M, i_M} \int_r^{r+R} ((p_M - \tau(x-r))f(k^*(x)) - i_M k^*(x))dx$$

$$\text{s.t. } \Pi \equiv \int_r^{r+R} (p-\theta r-\tau(x-r))f(k^*(x))-ik^*(x)-((p_M-\tau(x-r))f(k^*(x))-i_M k^*(x))dx \geq 0$$

$$\text{and } g(r) \equiv p_M f(k^*(r)) - i_M k^*(r) - y^0(r) \geq 0$$

$$\text{and } g(r+R) \equiv (p_M - \tau R)f(k^*(r+R)) - i_M k^*(r+R) - y^0(r+R) \geq 0$$

As it was shown that under spatial price discrimination the optimal pricing policy is a mill pricing policy, the result regarding the optimal mill pricing one is immediate:

$$p_D(x) = p - \theta r - \tau(x-r) = p_F(x) = p_M - \tau(x-r) \Leftrightarrow p_M = p - \theta r$$

$$i_D(x) = i = i_F(x) = i_M \Leftrightarrow i_M = i$$

$$\Pi = 0$$

Proposition 10. ***Under mill pricing, when farmers' income is maximized, the optimal contract is characterized by*** $p_M = p - \theta r$ ***and*** $i_M = i$***. This equilibrium is equivalent to the one obtained from spatial price discrimination, in terms of farmer's income as well as input choice and output production.***

Contrary to the two other pricing policies, under mill pricing, the optimal contract will be different depending on whether the trader maximizes farmers' income or profit. Only income maximization corresponds to total earnings maximization. Indeed, assuming a cooperative maximizes the sum of profit and farmers' income, the problem is the following[16]:

$$\max_{p_M, i_M} \int_r^{r+R} (p - \theta r - \tau(x-r))f(k^*(x)) - ik^*(x)dx$$

$$\text{s.t. } \Pi \equiv \int_r^{r+R} (p-\theta r-\tau(x-r))f(k^*(x))-ik^*(x)-((p_M-\tau(x-r))f(k^*(x))-i_M k^*(x))dx \geq 0$$

$$\text{and } g(r) \equiv p_M f(k^*(r)) - i_M k^*(r) - y^0(r) \geq 0$$

$$\text{and } g(r+R) \equiv (p_M - \tau R)f(k^*(r+R)) - i_M k^*(r+R) - y^0(r+R) \geq 0$$

[15] If the production function is homogeneous of degree $h < 1$ (see lemma 2).

[16] If the production function is homogeneous of degree $h < 1$ (see lemma 2).

The Lagrangian is given by:

$$
\begin{aligned}
\mathcal{L} &= (1+\mu)\int_r^{r+R} ((p-\theta r-\tau(x-r))f(k^*(x))-ik^*(x))\,dx \\
&-\mu\int_r^{r+R} ((p_M-\tau(x-r))f(k^*(x))-i_M k^*(x))dx \\
&+\lambda_1(p_M f(k^*(r))-i_M k^*(r)-y^0(r))+\lambda_2((p_M-\tau R)f(k^*(r+R))-i_M k^*(r+R)-y^0(r+R))
\end{aligned}
$$

From the Kuhn-Tucker first-order conditions, either $\Pi = 0$ or $\mu = 0$. In the first case, the problem is similar to the total farmers' income maximization case, while in the second, it is similar to profit maximization. As the first case is similar to non-profit spatial price discrimination, which has been shown to provide the highest sum of profit and farmers' income, the first case is preferred to the latter one. Contrary to other policies, under mill pricing, only one contract leads to maximized total earnings and this contract corresponds to farmers' income maximization.

While imposing a mill pricing policy to a for-profit trader leads to a loss of efficiency, this is not the case for a non-profit trader. Indeed, the latter induces farmers to produce efficiently, and all the efficiency gain (compared to the stand-alone farmers' situation) is acquired by the farmers.

2.7 Poverty and policy implications

As explained before, Senegalese milk production is characterized by a low production and a low quantity of input (cattle feeds) used. Milk producers have low income and most of them can be considered as poor. Empirical literature on various agricultural sectors in developing countries shows that these elements are worsening with distance, remote farmers using less inputs (Staal et al., 2002), producing or selling less (Holloway et al., 2000, Stifel and Minten, 2008) and having lower income (Jacoby, 2000) than the less isolated ones. Improving their living conditions may contribute to reduce rural poverty and boost socio-economic development in rural areas. In this context, we look at the measures that can be adopted by policy makers to increase farmers' production, input use and income.

It has been shown that, whatever the pricing policy chosen, the use of an interlinked contract by an intermediary who has a sufficient transport cost advantage incites each farmer to increase the level of input he uses, compared to what he used in the stand-alone case and hence to increase his production. However, farmers are not always able to improve their income, as all the efficiency gain can be acquired by the trader. A policy maker or an external donor may use appropriate regulations on the spatial pricing policy to be used in the interlinked contract in order to improve farmers' livelihoods.

In particular, we look at what a benevolent policy maker who wants to decrease poverty but is unable to impose a complex tax and subsidy scheme should impose as a spatial pricing policy to be used by intermediaries. Similarly, we look at the conditions an external donor should impose to the intermediaries he promotes, when his aim is reducing rural poverty.

Following FOSTER et al. (1984), we adopt the following poverty indicator:

$$Pov_\alpha = \frac{1}{R}\int_{r+q}^{r+R}\left(\frac{z-y(x)}{z}\right)^\alpha dx \tag{2.7.1}$$

where $z > 0$ is poverty line ($z - y(x)$ is the income shortfall of the farmer located in x), q is the number of poor farmers (having income no greater than z) and α can be seen as a measure of poverty aversion, a larger α giving greater emphasis to the poorest farmers. Larger is Pov_α, higher is the poverty. We will compare the outcomes of the different pricing policies in terms of this indicator, in order to derive which policy used by the intermediary performs better in reducing poverty with respect the stand-alone situation.

We also use the squared coefficient of variation as a measure of the inequality amongst the poor (Foster et al., 1984):

$$Inequality = \frac{1}{(R-q)}\int_{r+q}^{r+R}\left(\frac{\bar{y}-y(x)}{\bar{y}}\right)^2 dx \tag{2.7.2}$$

where $\bar{y} = \frac{1}{(R-q)}\int_{r+q}^{r+R} y(x)dx$ is the average income for the poor farmers. This measure of the inequality is associated with Pov_2 in the sense that it is obtained when $R-q$ and $\bar{y}$ are substituted for R and z in the definition (2.7.1) with $\alpha = 2$. The indicator defined in (2.7.2) ranges between 0 and 1, being equal to 0 when perfect equality is satisfied.

Particular attention has to be given to the observed diversity regarding the nature of the intermediaries. As policy recommendations are different for non-profit traders and for-profit traders, we analyze them separately.

2.7.1 For-profit trader

If discrimination is possible and costless, in a laissez-faire situation, the for-profit trader will choose to discriminate, as it leads to the highest profit. In this situation, the efficient optimum is reached. However, no farmer's poverty is reduced, as they all get the same income as in their stand-alone initial situation. While the presence of a trader who has a transport cost advantage is beneficial from an efficiency point of view, it is not from a poverty reduction one.

A policy maker whose aim is to increase farmers' incomes may want to tax trader's profit in order to distribute it among farmers. However, it is possible that the public authorities in developing countries do not have the power of doing so. In what follows, we assume the policy maker is only able to impose a pricing policy.

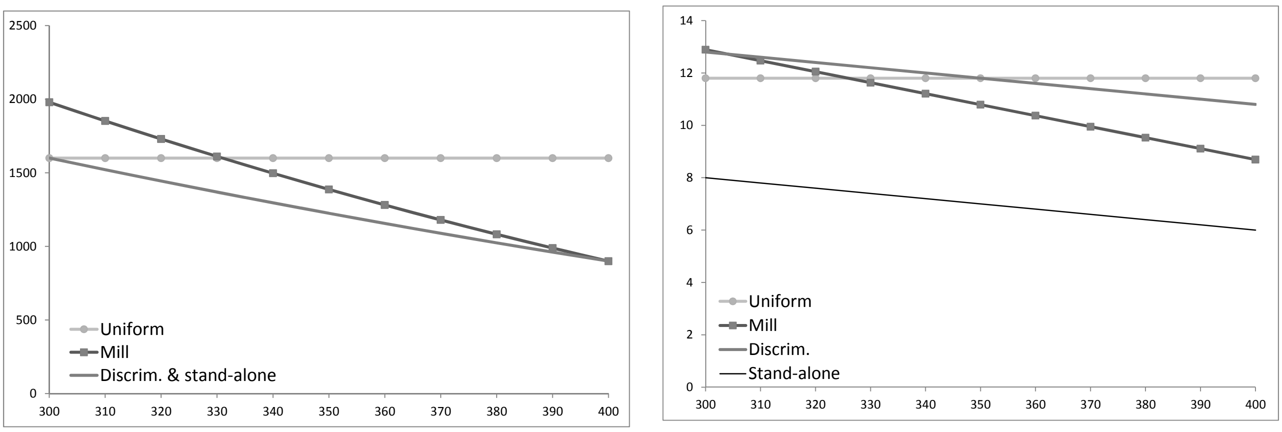
If the trader's transport cost advantage is large enough, imposing uniform pricing leads to an increased income for the poorest farmers, while richer ones are not worse off. Indeed, under this policy, only the farmer the closest to the market, that is, the one who has the highest initial income, is not able to increase his revenue. All the others are able to obtain a positive surplus from the contract, and hence to increase their income. Equality among farmers is ensured, as they all receive the same income and produce the same quantity. However, if the difference in transport cost between the trader and the farmers is small, imposing uniform pricing does not allow the trader to make a positive profit and to exploit his cost advantage to increase production.

If the trader has a sufficiently large cost advantage, imposing him to use mill pricing also increases the revenue of most of the farmers. But, contrary to the uniform pricing, farmers far from the trader, who were already poor, gain less than the one close to the trader. Mill pricing increases inequality among farmers, with respect to their stand-alone situation, but also with respect to a situation where the trader is allowed to spatially discriminate.

The previous discussion is illustrated by figure 2.4, which represents farmers' income and output as a function of the distance, under the three pricing policies when $\tau r - \theta r > \tau R$ and $p > \bar{p}$. Both uniform and mill pricing policies have positive effects on the income of most of farmers. Hence, if the policy maker is concerned only by farmers' revenue, spatial price discrimination should be prohibited. The choice of the profit-maximizing trader among the two remaining policies is not obvious. Numerical simulations show that the trader tends to prefer mill pricing when the output price p is large. However, when p is small, situations may occur where trader's profit is higher under uniform pricing. This is particularly true when r is large or $\tau - \theta$ is large, that is, when the trader has an important cost advantage compared to farmers.

Figure 2.4: Comparison of spatial pricing policies

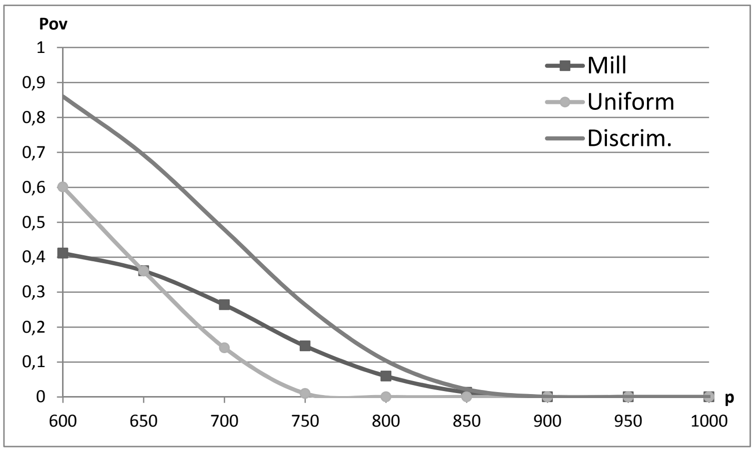
(a) Farmer's income $y(x)$ (b) Output produced $f(k(x))$



* Choice of the parameters: $r = 300$, $R = 100$, $p = 700$, $\tau = 1$, $i = 100$ and $\theta = 0.2$. Note that parameters are such that the uniform pricing contract is profitable for the trader and such that the mill pricing contract is constrained for the last farmer.

Producers' organizations in developing countries and NGOs argue that prices for agricultural goods are too low and claim that they remain low due to "unfair" international competition caused by subsidized exports from industrialized countries. This is seen as one of the reasons which keeps small producers in poverty (see for instance Oxfam (2002) or CFSI (2007) on the milk sector). In a context in which p is very low, imposing mill pricing to a trader who has a large cost advantage may result in increasing all farmers' income, including the most distant one. Numerical simulations also show that, when p is small, mill pricing may be preferred to uniform pricing by a majority of farmers [17] and that the sum of all farmer's incomes may be higher under mill pricing. If the policy-maker's objective is to choose a policy that increases farmers' total income and/or is preferred by the majority of them, then imposing mill pricing in a context of low output price is relevant. When the output price is large, however, uniform pricing is preferred by a majority of farmers and leads to a higher total farmers' income, even if the first farmer's income is always pushed down to his reservation level.

Figure 2.5: Comparison of spatial pricing policies: poverty



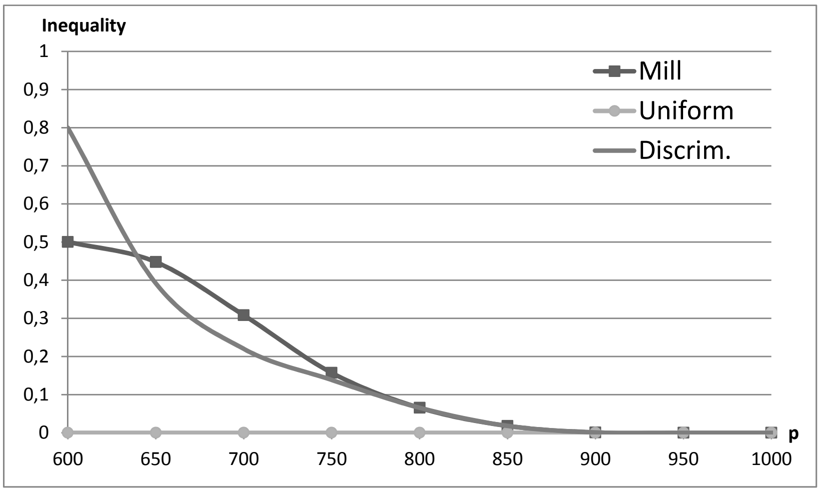
* Stand-alone situation corresponds to spatial price discrimination. Choice of the parameters: $z = 1000$, $r = 300$, $R = 100$, $\tau = 1.5$, $i = 100$ and $\theta = 0.2$. Note that parameters are such that the uniform pricing contract is profitable for the trader.

Regarding poverty, measured by the indicator defined in (2.7.1), spatial price discrimination does not contribute to poverty reduction, as it does not permit to increase farmers' income. Numerical simulations (see figure 2.5) show that mill pricing tends to perform better in reducing poverty for low values of p while uniform pricing dominates when the output price is larger. Note that, when poverty aversion is large, that is α is large (not represented here), uniform pricing dominated mill pricing in terms of poverty reduction, as more emphasis is given to the poorest (the most distant farmers) who have a larger income under uniform pricing. With very large α, Pov_α approaches a Rawlsian measure which considers only the income of the poorest farmer. If the policy maker has a Rawlsian objective, the uniform pricing policy should always be encouraged.

The effect of the pricing policies on the inequality amongst the poor is illustrated in figure 2.6. It can be seen that uniform leads to perfect equality, as all the farmers get the

[17] That is, the median farmer located in $r + R/2$ has a higher income under mill than under uniform pricing.

Figure 2.6: Comparison of spatial pricing policies: inequality



* Stand-alone situation corresponds to spatial price discrimination. Choice of the parameters: $r = 300$, $R = 100$, $\tau = 1.5$, $i = 100$ and $\theta = 0.2$. Note that parameters are such that the uniform pricing contract is profitable for the trader.

same income, while mill pricing may lead to the highest level of inequality, the closest farmers being favored with respect to the most distant ones.

Finally, if the trader's cost advantage is small, imposing uniform pricing implies that he will not be able to play a role in poverty reduction given that in that case the profit that he would obtain is negative. Under mill pricing however, the trader is able to contract profitably with the farmers. Not only can he replicate the stand-alone situation but when the output price is not too large, he can even propose a contract in which all the farmers except the last one increase their income.

2.7.2 Non-profit trader

Whatever the pricing policy used by the non-profit trader, he uses his cost advantage to improve all the farmer's income, compared to their stand-alone initial income. As in the for-profit case, the non-profit intermediary who has a sufficiently large cost advantage incites all the farmers to increase the quantity of input they use as well as their level of production. If the difference in transport costs between the trader and the farmers is small, imposing an uniform pricing policy hampers the non-profit trader to use his advantage to increase farmers' income. Under mill pricing policy however, even if the trader has a small advantage, he is able to promote increased production and input use.

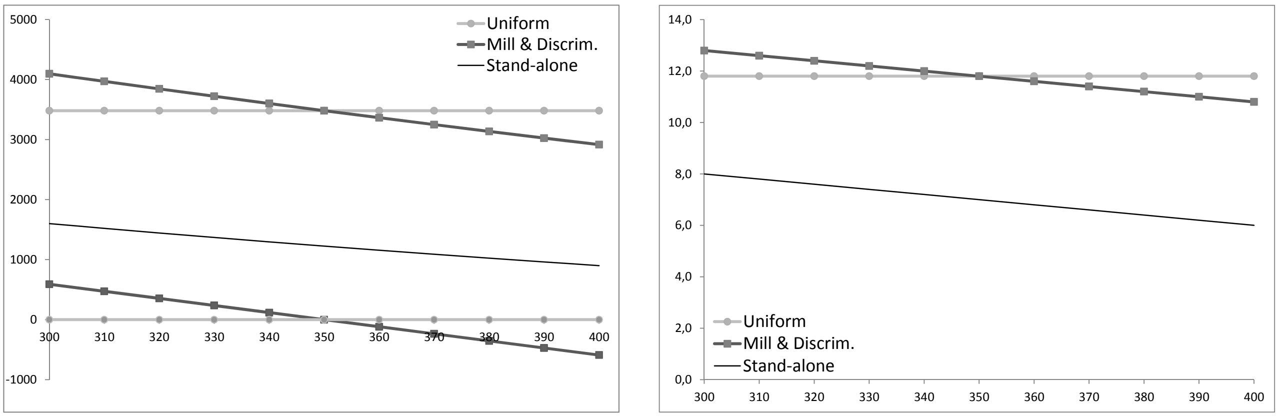
In a laissez-faire situation, it is optimal for the non-profit trader to use mill pricing. Even if the trader is allowed to price discriminate he will choose to charge prices which reflect the difference in transport costs. Hence, in this case mill pricing and discrimination pricing are equivalent. If the trader's advantage is large enough, imposing uniform pricing would serve the equality objective as in this case all farmers get the same income.

Moreover, this policy allows to increase relatively more the income of the poorest (i.e. the most distant) farmers, compared to the less remote ones. Compared to uniform pricing, with a mill pricing policy, the most distant farmers receive a lower price for their output, hence get a lower income. However, the difference in the price received exactly reflects the difference in transport cost, and each farmer receives the same additional income from the contract, compared to his initial income. This discussion is illustrated by figure 2.7.

Figure 2.7: Farmers' income maximization

(a) Farmer's income $y(x)$ (up) and trader's profit $\pi(x)$ (down)

(b) Output produced $f(k(x))$



* Choice of the parameters: $r = 300$, $R = 100$, $p = 700$, $\tau = 1$, $i = 100$ and $\theta = 0.2$.

From the efficiency perspective, when the trader is non-profit mill pricing is preferred to uniform pricing since this generates the highest possible surplus. However, from a poverty point of view, numerical simulations show that uniform pricing performs better in reducing poverty than mill pricing. [18]

2.8 Conclusions

In this chapter, we develop a model of input-output interlinked contracts between a trader and geographically dispersed farmers, and analyze the implications of different spatial pricing policies used by this trader. We look at three different spatial price policies, namely spatial price discrimination, uniform pricing and mill pricing.

We assume an agricultural output market that is characterized by large transport costs. The intermediary has a (transport) cost advantage over the farmers from whom it buys their production. This cost difference leads to an input-output interlinked contract

[18]In very extreme cases (very large poverty line and very low poverty aversion), mill pricing could dominate for small p.

between the intermediary and the farmer. A first result is that the use of an interlinked contract by a trader who has a sufficient transport cost advantage leads to an increase of the farmer' production, independently of the type of pricing policy used by the intermediary.

If the for-profit intermediary is able to perfectly discriminate contracts between farmers, this would be his preferred option. This allows him to push all the farmers' incomes down to their stand-alone initial income and hence appropriate all the efficiency gain generated by the contract. If this is the case, the presence of the intermediary, while improving agricultural efficiency, does not directly help to reduce rural farmers poverty. In practice discriminatory pricing might not be feasible and other pricing policies exist, such as uniform pricing, where the trader bears the transport costs and concludes the same contract with all the farmers, or mill pricing, where farmers are in charge of transport, and receive the same price at the mill. If the trader's cost advantage is large enough, we show that in both cases, most of the farmers get a positive surplus from the contract, while the trader is still able to make a profit. In the mill pricing case, under some conditions we can have a situation in which all the farmers, including those located the furthest from the market, see an increase in their income.

We show that imposing a uniform pricing policy to the trader who has a sufficiently large cost advantage leads to an increase of isolated farmers' income. Providing the same income to all farmers, uniform pricing favors relatively more isolated farmers, since they are the ones who initially receive a lower income. Moreover, when the output market price is large enough, uniform pricing also leads to a reduction of farmers' poverty, that we define by a Foster-Greer-Thorbecke indicator. In this case, it is also preferred to mill pricing by a majority of farmers, and it leads to higher total farmers' income.

In developing countries, agricultural market prices are often driven down by international competition. If output market prices are very low, imposing mill pricing may be relevant. Indeed, it may increase all farmers' income, including the closest and the most distant one. This is not possible under uniform pricing, whatever the output market price. When the output market price is low, mill pricing performs better in reducing poverty than uniform pricing does. Moreover, there may be cases in which both total farmers' income and median farmer's income are higher under mill than under uniform pricing. Additionally, if the trader only has a small cost advantage, under mill pricing he still may be able to increase most of the farmers' income, while under uniform pricing he cannot profitably contract with the farmers.

Since, in developing countries, there are several examples of NGOs and farmers' associations setting up intermediaries, we also look at a situation in which the trader maximizes total farmers' income. For a given pricing rule, such a trader is able to generate at least the same level of efficiency surplus as the for-profit does. We also show that it is optimal, from the non-profit firm's point of view, to use a mill pricing scheme, also implying that, in this case, the efficient outcome is reached under mill pricing. From a poverty perspective however, uniform pricing seems to performs better.

Under mill pricing, maximizing the total earnings (the sum of trader's profit and farmers' income) implies that the profit of the intermediary will be nil, while under the other pricing policies, its profit can be strictly positive. Under mill pricing, only one vector of prices corresponds to total earnings maximization. Under discriminatory and uniform pricing, there is a continuum of prices which will maximize the total earnings. Within this range, some prices favor more the farmers while others favor more the trader.

We also generalize the result found in Gangopadhyay and Sengupta (1987) that the trader has an interest in giving a discount to the farmer on the input price. If the trader's cost advantage is sufficiently large, this is true for all three pricing policies considered.

The model developed here gives potential avenues for future research. First, in certain cases, the choice of the size of the collection area may be important to the trader. In that case, rather than considering the number of farmers as being fixed, the number of participants may constitute a choice variable for the trader. A possible extension of our model would consider how the number of suppliers is endogenously chosen. This would also allow to analyze the impact of pricing policy choice on the inclusion of isolated farmers in a collection area. Secondly, it may be interesting to see whether besides the pricing policies considered in this chapter, other spatial pricing policies such as two part pricing would permit poverty reduction.

Appendices

2.A Proof of lemma 1

Using the envelop theorem, we have for a participation constraint at location x

$$\frac{\partial g(x,p_M,i_M)}{\partial x} = -\tau\left(f(k^*(x)) - f(k^0(x))\right) \lesseqgtr 0 \Leftrightarrow k^*(x) \gtreqless k^0(x) \tag{2.8.1}$$

If $f(k)$ is homogeneous of degree h, then, using Euler's theorem, the farmer's income is given by $y(x) = i_M k^*(x)\left(\frac{1}{h}-1\right)$ while his reservation income is given by $y^0(x) = ik^0(x)\left(\frac{1}{h}-1\right)$. Thus $g(x) = y(x) - y^0(x) = \left(i_M k^*(x) - ik^0(x)\right)\left(\frac{1}{h}-1\right)$.

Define $\tilde{x}$ as a location where the participation constraint is binding for a couple (p_M, i_M). We have $g(\tilde{x}, p_M, i_M) = 0 \Leftrightarrow i_M k^*(\tilde{x}) - ik^0(\tilde{x}) = 0$, or equivalently,

$$k^*(\tilde{x}) = \frac{i}{i_M} k^0(\tilde{x}) \tag{2.8.2}$$

Substituting (2.8.2) in (2.8.1), we have that, if $\tilde{x}$ exists in $[r, r+R]$:

$$\left.\frac{\partial g(x,p_M,i_M)}{\partial x}\right|_{x=\tilde{x}} \lesseqgtr 0 \;\Leftrightarrow\; i_M \lesseqgtr i \tag{2.8.3}$$

(2.8.3) implies that, if $i_M < i$, the only possible value for $\tilde{x}$ is $\tilde{x} = r+R$. If $i_M > i$, then the only possible value for $\tilde{x}$ is $\tilde{x} = r$. Finally, if $i_M = i$, if the participation constraint is binding somewhere, it has to bind everywhere: if $\tilde{x}$ exists, we have $\tilde{x} = x \; \forall x \in [r, r+R]$.

The optimum is thus characterized by one of the following cases: (1) $\tilde{x} = r+R \Rightarrow g(r+R) = 0$ and $g(x) > 0 \; \forall x \in [r, r+R[$, (2) $\tilde{x} = r \Rightarrow g(r) = 0$ and $g(x) > 0 \; \forall x \in]r, r+R]$, (3) $\tilde{x} = x \; \forall x \in [r, r+R] \Rightarrow g(x) = 0 \; \forall x \in [r, r+R]$ and (4) $\tilde{x}$ does not exist in $[r, r+R] \Rightarrow g(x) > 0 \; \forall x \in [r, r+R]$. Hence $g(r) \geq 0$ and $g(r+R) \geq 0$ are sufficient to ensure that $g(x) \geq 0 \; \forall x \in [r, r+R]$. □

2.B Proof of lemma 2

We show that it is always possible to find prices (p_M, i_M) such that the profit is higher than in case (2), which is characterized by $p_M > p - \tau r$, $i_M > i$ and $g(r) = 0$ at the optimum. For the other constraints to be satisfied but not binding, we need that $\left.\frac{\partial g(x,p_M,i_M)}{\partial x}\right|_{x=r} > 0$ and, from (2.8.3), $i_M > i$. From (2.8.1), this would imply $k^*(r) < k^0(r)$. As production function is concave, using (2.2.2), it would imply $\frac{p_M}{i_M} < \frac{p-\tau r}{i}$. Subtracting $\frac{\tau(x-r)}{i}$ on both sides and given that $i_M > i$, this would give $\frac{p_M - \tau(x-r)}{i_M} < \frac{p-\tau x}{i}$, thus $k^*(x) < k^0(x) \; \forall x$. Trader's profit would be given by $\Pi = \int_r^{r+R} (p - \theta r - \tau(x-r)) f(k^*(x)) - ik^*(x) - y(x) dx$. The trader could always increase his profit by replicating farmers' stand-alone situations (that is, proposing a contract where $p_M = p - \tau r$ and $i_M = i$, each farmer using exactly $k^0(x)$ and getting his reservation income $y^0(x)$). In this case the profit is given by $\Pi' = \int_r^{r+R} (p -$

$\theta r - \tau(x-r))f(k^0(x)) - ik^0(x) - y^0(x)dx$. This is always higher than Π. Indeed, from participation constraints, $y(x) \geq y^0(x)$, and, given our assumptions on $f(k)$, the function $(p - \theta r - \tau(x-r))f(k(x)) - ik(x)$ is concave in $k(x)$ and maximized in $k^{\#}(x)$ defined by (2.2.5). Comparing with (2.2.2) we see that $k^{\#}(x) > k^0(x)$. Thus, $k^{\#}(x) > k^0(x) > k^*(x)$, implying that $k^0(x)$ and $k^*(x)$ lie in the increasing part of the function, thus $(p-\theta r-\tau(x-r)f(k^0(x)))-ik^0(x) > (p-\theta r-\tau(x-r)f(k^*(x)))-ik^*(x) \quad \forall x$. As trader's profit could always be increased, the case (2) cannot characterize the optimum. Eliminating case (2) from the possible outcomes, the first farmer's participation constraint can never be the only one to be binding at the equilibrium. □

2.C Mill pricing: unconstrained outcome

The unconstrained outcome is the solution to the maximization problem when $\lambda = 0$ and $\mu = 0$. Replacing in (2.5.3) and (2.5.4) when $f(k) = 2\sqrt{k}$ and simplifying gives:

$$(p - \theta r - p_M) - \frac{i}{i_M}\left(p_M - \tau\frac{R}{2}\right) = 0 \tag{2.8.4}$$

$$(p - \theta r - p_M)\left(p_M - \tau\frac{R}{2}\right) + \left(\frac{1}{2} - \frac{i}{i_M}\right)\left(p_M^2 - p_M\tau R + \frac{\tau^2R^2}{3}\right) = 0 \tag{2.8.5}$$

Characteristics of the unconstrained equilibrium

The two focs can be combined as

$$H\left(p_M\right) \equiv$$

$$\left(p_M - \frac{\tau R}{2}\right)\left[\left(p_M - \frac{\tau R}{2}\right)^2 - \frac{\tau^2R^2}{12}\right] + \frac{2\tau^2R^2}{12}\left[2\left(p_M - \frac{\tau R}{2}\right) - \left(p - \theta r - \frac{\tau R}{2}\right)\right] = 0$$

We have $H\left(\frac{\tau R}{2}\right) < 0$ and $H\left(p - \theta r\right) > 0$. In addition, we have $H'\left(p_M\right) > 0$ which means that there is a unique value for p_M between $\frac{\tau R}{2}$ and $p - \theta r$ such that $H\left(p_M\right) = 0$. If there is a solution such that $p_M > \tau R$, then $i_M < i$. To see this, note that whenever $p_M > \tau R$ the term between the first square brackets is positive which implies that the term between the second square brackets has to be negative. Plugging this in the equation (2.8.4) implies that $i_M < i$.

To establish under what conditions $p_M = p - \tau r$ we evaluate $H\left(p_M\right)$ at $p_M = p - \tau r$ which yields

$$n\left(p\right) \equiv H\left(p - \tau r\right) \equiv \left(p - \tau r - \frac{\tau R}{2}\right)^3 + \frac{\tau^2R^2}{4}\left(p - \tau r - \frac{\tau R}{2}\right) - \frac{\tau^2R^2}{6}\left(p - \theta r - \frac{\tau R}{2}\right)$$

We have

$$\begin{aligned}
\frac{\partial n(p)}{\partial p} &= 3\left(p-\tau r-\frac{\tau R}{2}\right)^2+\frac{\tau^2R^2}{4}-\frac{\tau^2R^2}{6}>0\\
n(0) &= \left(-\tau r-\frac{\tau R}{2}\right)^3+\frac{\tau^2R^2}{4}\left(-\tau r-\frac{\tau R}{2}\right)-\frac{\tau^2R^2}{6}\left(-\theta r-\frac{\tau R}{2}\right)\\
&= -\left(\tau r+\frac{\tau R}{2}\right)^3+\frac{\tau^2R^2}{2}\left[\frac{1}{2}\left(-\tau r-\frac{\tau R}{2}\right)-\frac{1}{3}\left(-\theta r-\frac{\tau R}{2}\right)\right]\\
&= -\left(\tau r+\frac{\tau R}{2}\right)^3+\frac{\tau^2R^2}{2}\left[\frac{1}{2}\left(-\tau r-\frac{\tau R}{2}\right)-\frac{1}{3}\left(-\theta r-\frac{\tau R}{2}\right)\right]\\
&= -\left(\tau r+\frac{\tau R}{2}\right)^3+\frac{\tau^2R^2}{2}\left[-\frac{\tau R}{12}-\frac{1}{3}\left(\frac{3}{2}\tau r-\theta r\right)\right]<0\\
n(p_1) &= \left(\tau r-\theta r+\frac{\tau R}{2}\right)^3+\frac{\tau^2R^2}{4}\left(\tau r-\theta r+\frac{\tau R}{2}\right)-\frac{\tau^2R^2}{6}\left(2\tau r-2\theta r+\frac{\tau R}{2}\right)\\
&= \left(\tau r-\theta r+\frac{\tau R}{2}\right)^3+\frac{\tau^2R^2}{4}\left(\tau r-\theta r+\frac{\tau R}{2}\right)-\frac{\tau^2R^2}{3}\left(\tau r-\theta r+\frac{\tau R}{4}\right)>0
\end{aligned}$$

where $p_1 = 2\tau r-\theta r+\tau R$.

These three elements together imply that there is a unique $p_0 \in [0, p_1]$ such that $n(p_0) = 0$ and $p_M = p_0 - \tau r$.

Proof of $0 < dp_M/dp < 1$ if the optimum is unconstrained

Taking total derivatives of (2.8.4) and (2.8.5), equalizing them to zero and rearranging:

$$-\left(1+\frac{i}{i_M}\right)\frac{dp_M}{dp}+\frac{i}{i_M^2}\left(p_M-\frac{\tau R}{2}\right)\frac{di_M}{dp}=-1 \tag{2.8.6}$$

$$\left(\frac{p-\theta r-p_M}{p_M-\frac{\tau R}{2}}-2\frac{i}{i_M}\right)\frac{dp_M}{dp}+\frac{\frac{i}{i_M^2}\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)}{p_M-\frac{\tau R}{2}}\frac{di_M}{dp}=-1 \tag{2.8.7}$$

Using Cramer's rule on this two-equations system, we can calculate dp_M/dp as:

$$\frac{dp_M}{dp}=\frac{-\frac{\frac{i}{i_M^2}\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)}{p_M-\frac{\tau R}{2}}+\frac{i}{i_M^2}\left(p_M-\frac{\tau R}{2}\right)}{-\left(1+\frac{i}{i_M}\right)\frac{\frac{i}{i_M^2}\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)}{p_M-\frac{\tau R}{2}}-\frac{i}{i_M^2}\left(p_M-\frac{\tau R}{2}\right)\left(\frac{p-\theta r-p_M}{p_M-\frac{\tau R}{2}}-2\frac{i}{i_M}\right)}$$

$$\Leftrightarrow \frac{dp_M}{dp}=\frac{\frac{\tau^2R^2}{12}}{\left(1+\frac{i}{i_M}\right)\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)+\left(p_M-\frac{\tau R}{2}\right)^2\left(\frac{p-\theta r-p_M}{p_M-\frac{\tau R}{2}}-2\frac{i}{i_M}\right)}$$

From (2.8.4), $i/i_M = (p-\theta r-p_M)/(p_M-\tau R/2)$, thus:

$$\Leftrightarrow \frac{dp_M}{dp}=\frac{\frac{\tau^2R^2}{12}}{\left(1+\frac{i}{i_M}\right)\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)+\left(p_M-\frac{\tau R}{2}\right)^2\left(-\frac{i}{i_M}\right)}$$

$$\Leftrightarrow \frac{dp_M}{dp} = \frac{1}{\frac{12}{\tau^2 R^2}\left(p_M - \frac{\tau R}{2}\right)^2 + 1 + \frac{i}{i_M}} \tag{2.8.8}$$

From this expression, $0 < \frac{dp_M}{dp} < 1$. □

Proof of $di_M/dp < 0$ if the optimum is unconstrained

Using Cramer's rule on the two-equations system (2.8.6)-(2.8.7), we can calculate di_M/dp as:

$$\frac{di_M}{dp} = \frac{\left(1 + \frac{i}{i_M}\right) + \left(\frac{p-\theta r - p_M}{p_M - \frac{\tau R}{2}} - 2\frac{i}{i_M}\right)}{-\left(1+\frac{i}{i_M}\right)\frac{\frac{i}{i_M^2}\left(p_M^2 - p_M\tau R + \frac{\tau^2 R^2}{3}\right)}{p_M - \frac{\tau R}{2}} - \frac{i}{i_M^2}\left(p_M - \frac{\tau R}{2}\right)\left(\frac{p-\theta r - p_M}{p_M - \frac{\tau R}{2}} - 2\frac{i}{i_M}\right)}$$

From (2.8.4), $i/i_M = (p - \theta r - p_M)/(p_M - \tau R/2)$, thus:

$$\frac{di_M}{dp} = \frac{p_M - \frac{\tau R}{2}}{-\left(1+\frac{i}{i_M}\right)\frac{i}{i_M^2}\left(p_M^2 - p_M\tau R + \frac{\tau^2R^2}{3}\right) - \frac{i}{i_M^2}\left(p_M - \frac{\tau R}{2}\right)^2\left(-\frac{i}{i_M}\right)}$$

$$\Leftrightarrow \frac{di_M}{dp} = -\frac{p_M - \frac{\tau R}{2}}{\frac{i}{i_M^2}\left(\left(p_M - \frac{\tau R}{2}\right)^2 + \left(1+\frac{i}{i_M}\right)\frac{\tau^2R^2}{12}\right)} \tag{2.8.9}$$

If p_M is larger than $\frac{\tau R}{2}$ (which is verified if the constrained are satisfied), $\frac{di_M}{dp}$ is strictly negative. □

With the results that $p_M(p_0) = p_0 - \tau r$ and $\frac{dp_M}{dp} < 1$ we have that when $p > (<)\, p_0$ then $p_M < (>)\, p_0 - \tau r$. Another implication is that when $p < p_0$ then $p_M > p_0 - \tau r > \tau R$ which implies that $i > i_M$.

2.D Proof of proposition 6

From lemmas 1 and 2, we have three possible outcomes: (i) the participation constraint is binding only for the last farmer at the equilibrium, (ii) the participation constraint is binding for all the farmers at the equilibrium, or, (iii) no constraint is binding at the equilibrium.

(i) If $g(r+R) = 0$ and $g(x) > 0\ \forall x \in [r, r+R[$. This requires $\left.\frac{\partial g(x,p_M,i_M)}{\partial x}\right|_{x=r+R} < 0$ which implies by (2.8.3) that $i_M < i$.

(ii) If $g(x) = 0\ \forall x \in [r, r+R]$, the stand-alone situation is replicated and we have that $i_M = i$.

(iii) If $g(x) > 0\ \forall x \in [r, r+R]$, then $p_M > \tau R$. From the previous section we know that then $i_M < i$. □

2.E Proof of proposition 7

Using (2.5.5) and (2.5.6) we can divide the possible outcomes into four categories depending on which constraint is binding.

First possibility: $g(r+R) > 0$**,** $i_M < i$:

Consider $(p'_M(p), i'_M(p))$ the solution to the unconstrained problem. Define $G(p) \equiv y(r+R,p) - y^0(r+R,p)$ where $y(r+R,p) = y(r+R, p'_M(p), i'_M(p))$ if $p'_M(p) \geq \tau R$ and $y(r+R,p) = 0$ if $p'_M(p) < \tau R$, with $(p'_M(p), i'_M(p))$ the solution to the system of equations (2.8.4)-(2.8.5). If $G(p) > 0$, $(p'_M(p), i'_M(p))$ respects the constraint, hence given that $i_M < i$ from the proof of proposition 6, the equilibrium is unconstrained. If $G(p) < 0$, however, $(p'_M(p), i'_M(p))$ does not respect the constraint and the equilibrium has to be constrained.

In what follows, 1) we establish that when $\tau r - \theta r > \tau R$, $G(p)$ is positive for small values of p, i.e. for $p < p_0$, and when $\tau r - \theta r \leq \tau R$, $G(p)$ is negative for all values of p; 2) we establish that $G(p)$ is negative for large values of p, i.e. for $p > p_1$; 3) We establish that $G(p)$ is strictly decreasing in p when $p_0 < p < p_1$.

Together this implies that when $\tau r - \theta r > \tau R$ for small values of p the equilibrium is unconstrained and that there exists a unique $\bar{p}$ given by $G(\bar{p}) = 0$ above which the equilibrium is constrained. It also implies that when $\tau r - \theta r \leq \tau R$ the equilibrium is constrained for all values of p.

We proceed by establishing several intermediate results: (i) $p'_M(p) < p - \tau r$ is a sufficient condition for $G(p)$ to be decreasing in p. (ii) $G(p) < 0$ for all $p > p_1$. (iii) There exists a unique p_0, with $0 < p_0 < p_1$, such that $p'_M(p_0) = p_0 - \tau r$; (iv) for $p \lesseqqgtr p_0$, $p'_M(p) \gtreqqless p - \tau r$; (v) If $\tau r - \theta r \lesseqqgtr \tau R$, then $\underline{p} \gtreqqless p_0$.

(i) We show that if $p'_M(p) < p - \tau r$, then $\frac{\partial G(p)}{\partial p} < 0$. Note that as $\underline{p} > 0$ we have $\tau R < p - \tau r$. Suppose first that $\tau R \leq p'_M(p) < p - \tau r$, using (2.8.8) and (2.8.9) we have that $\frac{\partial G(p)}{\partial p} = \frac{2(p'_M(p) - \tau R)}{i} \frac{\left(\frac{i}{i'_M(p)} + (p'_M(p) - \tau R)(p'_M(p) - \tau(R/2))\frac{6}{\tau^2 R^2}\right)}{1 + \frac{i}{i'_M(p)} + (p'_M(p) - \tau(R/2))^2 \frac{12}{\tau^2 R^2}} - \frac{2(p - \tau r - \tau R)}{i}$. It can be easily verified that the second ratio is lower that 1, thus it can be said that $\frac{\partial G(p)}{\partial p} < \frac{2(p'_M(p) - \tau R)}{i} - \frac{2(p - \tau r - \tau R)}{i}$. This is strictly negative provided that $p'_M(p) < p - \tau r$. Suppose now that $p'_M(p) < \tau R$, then $\frac{\partial G(p)}{\partial p} = -\frac{2(p - \tau r - \tau R)}{i}$ which is also negative.

(ii) Solving (2.8.4) for $i'_M(p)$ and substituting it into G(p) yields that $G(p) < 0$ if $\frac{(p'_M(p) - \tau R)^2}{p'_M(p) - \tau(R/2)} < \frac{(p - \tau r - \tau R)^2}{p - \theta r - p'_M(p)}$. A sufficient condition for that is:

$$\frac{(p'_M(p) - \tau R)^2}{p'_M(p) - \tau R} < \frac{(p - \tau r - \tau R)^2}{p - \theta r - p'_M(p)}$$

$$\Leftrightarrow (p'_M(p) - \tau R)(p - \theta r - p'_M(p)) - (p - \tau r - \tau R)^2 < 0$$

$$\Leftrightarrow w(p'_M(p)) \equiv -p'_M(p)^2 + (p - \theta r + \tau R)p'_M(p) - (p - \theta r)\tau R - (p - \tau r - \tau R)^2 < 0$$

$w(p'_M(p))$ is a polynomial of degree two in $p'_M(p)$ where the leading coefficient is strictly negative. The discriminant is given by $d(p) = (p - \theta r + \tau R)^2 - 4((p - \theta r)\tau R) - 4(p -$

$\tau r - \tau R)^2$. If this discriminant is negative then $w(p'_M(p))$ is negative for all $p'_M(p)$, hence $G(p) < 0$. $d(p)$ is a polynomial (of degree two) in p where the leading coefficient is strictly negative. It is thus negative before the first root $(p_a = \frac{1}{3}(2\tau r + \theta r) + \tau R)$ and after the second one $(p_b = 2\tau r + \tau R - \theta r)$. As $\theta < \tau$, we have $p_a < \underline{p} < p_b$. Thus $p > p_1 \equiv p_b = 2\tau r + \tau R - \theta r$ is a sufficient condition for $w(p'_M(p))$ to be negative for all $p'_M(p)$. Hence $G(p)$ is negative for all $p > p_1$.

(iii) From appendix 2.C there is a unique $p_0 \in [0, p_1]$ such that such that $p'_M(p_0) = p_0 - \tau r$.

(iv) From appendix 2.C, $\frac{\partial dp'_M}{\partial p} < 1$, hence $p'_M(p) \lesseqqgtr p - \tau r \ \forall p \gtreqqless p_0$. Substituting $p'_M(p) \geq p - \tau r$ into $G(p)$, because $i'_M(p_0) < i$, we have that $G(p) > 0 \ \forall p \leq p_0$.

(v) The value of p_0 with respect to $\underline{p}$ depends on the values of the parameters. To see this:

$$n\left(\underline{p}\right) = \frac{\tau^2 R^2}{6}\left(\tau R - (\tau r - \theta r)\right)$$

If $\tau r - \theta r > \tau R$, then $n\left(\underline{p}\right) < 0$ and $\underline{p} < p0$. If $\tau r - \theta r \leq \tau R$, then $n\left(\underline{p}\right) > 0$ and $\underline{p} > p0$. This implies that all (acceptable) values of $p > p_0$.

From (v) if $\tau r - \theta r \leq \tau R$, then all values of p are larger than p_0. From (iv) this implies that $p'_M(p) < p - \tau r$ for all values of p. This in turn implies that $p'_M(\underline{p}) < \underline{p} - \tau r = \tau R$. From the definitions we have that $y(r + R, \underline{p}) = 0$ implying that $G(\underline{p}) < 0$, this, together with from (i), $\frac{\partial G(p)}{\partial p} < 0$ for $p > \underline{p}$, is sufficient to ensure that $G(p) < 0 \ \forall p > \underline{p}$.

From (v) if $\tau r - \theta r > \tau R$, then there are values of $p \in \left[\underline{p}, p_0\right]$ such that $G(p) > 0$. For values of p larger than p_0, $\frac{\partial G(p)}{\partial p} < 0$ and with values larger than p_1 $G(p) < 0$ which implies that there is a unique $\bar{p}$ such that $G(\bar{p}) = 0$.

Second possibility: $g(r + R) = 0$, $i_M < i$:

Let $(p''_M(p), i''_M(p))$ be the solution to the maximization problem when only last farmer's participation constraint is binding, that is when $\mu = 0$ and $\lambda > 0$. Solving for λ in (2.5.4) and substituting it into (2.5.3), when $f(k) = 2\sqrt{k}$, gives:

$$i_M = i \frac{6p_M^2 - 3p_M \tau R + \tau^2 R^2}{6p_M\left(p - \tau R - \theta r\right) + 2\tau^2 R^2} \tag{2.8.10}$$

The binding participation constraint $g(r + R) = 0$ gives:

$$i_M = i \frac{(p_M - \tau R)^2}{(p - \tau r - \tau R)^2} \tag{2.8.11}$$

Prices $(p''_M(p), i''_M(p))$ are given by the intersection between the curves (2.8.10) and (2.8.11), provided $\tau R \leq p''_M(p) \leq p - \tau r$. Simplifying:

$$h(p''_M(p)) \equiv$$

$$\left(6p''^2_M - 3p''_M \tau R + \tau^2 R^2\right)(p - \tau r - \tau R)^2 - \left(p''_M - \tau R\right)^2 \left(6p''_M\left(p - \tau R - \theta r\right) + 2\tau^2 R^2\right) = 0$$

(a) $h(p''_M(p))$ is a polynomial of degree three in $p''_M(p)$ where the leading coefficient is strictly negative. This implies that $h(p''_M(p))$ has an inverse N-shape. (b) Evaluated at

$p''_M(p) = \tau R$, $h(\tau R) > 0$. (c) The first derivative of $h(p_M)$, evaluated at τR is strictly positive. This implies that τR lies in an increasing part of $h(p''_M(p))$. (d) If $\tau r - \theta r > \tau R/2$ holds, then $h(p - \tau r) < 0 \; \forall p > \underline{p}$. If $\tau r - \theta r \leq \tau R/2$ holds, then $h(p - \tau r) \lesseqqgtr 0$ holds for $p \lesseqqgtr \tilde{p}$ with $\tilde{p} = \tau r + \frac{\tau^2 R^2}{6(\tau(R/2) - \tau r + \theta r)}$.

Elements (a) to (d) are sufficient to ensure that if $\tau r - \theta r > \tau R/2$, then $h(p_M)$ has one unique root between τR and $p - \tau r$ for all $p > \underline{p}$. Hence, $\lambda > 0$ and $\mu = 0$ are possible for all the values of p we consider. If $\tau r - \theta r \leq \tau R/2$, then $h(p_M)$ has one unique root between τR and $p - \tau r$ when $p \leq \tilde{p}$ and no root between τR and $p - \tau r$ when $p > \tilde{p}$. Hence, $\lambda > 0$ and $\mu = 0$ only occur for $p \leq \tilde{p}$. Moreover, if $\tau r - \theta r < \tau R/3$, then $\tilde{p} < \underline{p}$ such that for all acceptable values of p, we have $p > \tilde{p}$.

Third possibility: $g(r + R) = 0$, $i_M = i$:

If when $g(r + R) = 0$ and $i_M = i$, with $f(k) = 2\sqrt{k}$ we have that $p_M = p - \tau r$. Replacing i_M by i and p_M by $p - \tau r$ in (2.5.3) and (2.5.4), solving for λ in (2.5.4) and substituting it into (2.5.3), we have:

$$\mu = -\frac{R}{i^2}\left((p - \tau r)\left(\tau r - \theta r - \tau\frac{R}{2}\right) + \frac{\tau^2 R^2}{6}\right) \tag{2.8.12}$$

If $\tau r - \theta r > \tau R/2$, then μ is always negative. If $\tau r - \theta r \leq \tau R/2$, then $\mu \lesseqqgtr 0$ if $p \lesseqqgtr \tilde{p}$ where $\tilde{p} = \tau r + \frac{\tau^2 R^2}{6(\tau(R/2) - \tau r + \theta r)}$ since $\mu = 0$ when $p = \tilde{p}$ and $\frac{\partial \mu}{\partial p} > 0$.

Fourth possibility: $g(r + R) > 0$, $i_M = i$:

If $g(r + R) > 0$, from appendix 2.D (iii), we have that $i_M < i$. This implies that $g(r + R) > 0$ and $i_M = i$ never occurs.

Summarizing, this means that, if $\tau r - \theta r > \tau R$, then $g(r + R) > 0$ and $i_M < i$ for $p \in [\underline{p}, \bar{p}]$ while $g(r + R) = 0$ and $i_M < i$ for $p > \bar{p}$. If $\tau R/2 < \tau r - \theta r < \tau R$ then $g(r + R) = 0$ and $i_M < i$ for any $p > \underline{p}$. If $\tau r - \theta r < \tau R/2$ then $g(r + R) = 0$ and $i_M < i$ for $p \in [\underline{p}, \tilde{p}]$ while $g(r + R) = 0$ and $i_M = i$ for $p > \tilde{p}$. $\square$

Chapter 3

Can contract farming improve smallholders' participation to the market?*

3.1 Introduction

The importance of smallholders' market participation for poverty alleviation in developing countries has been increasingly recognized (Von Braun and Kennedy, 1994, Heltberg and Tarp, 2002, Barrett, 2008 etc.). However, due to inadequate transport infrastructure and important distances between areas of production and areas of consumption, rural households are often exposed to access difficulties and fail to take part to the market. These effects are even stronger for highly perishable products such as milk. Metzger et al. (1995) report that less than 50% of milk production is commercialized in Africa, less than 25% in Senegal. In Ethiopia, Holloway et al. (2000) showed that transport cost is an important determinant for the participation to the milk market. They found that transport time has to be reduced by almost two hours to induce a representative non-participant to entry.[1]

Since the nineties, West Africa has seen the emergence of small-scale processing units called "mini-dairies" that play an intermediary role between the farmers and the market (Dieye et al., 2005, Corniaux et al., 2005). These intermediaries have some kind of advantage over the farmers to sell the products on the market. They use more efficient transport devices, such as trucks, they own bulk cooling tanks, such that they can stock the milk and do not have to transport it every day, etc. This cost advantage often requires an important fixed cost, which cannot be borne by each farmer alone. According to Dieye (2003), the presence of such intermediaries seems to improve market participation.

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[1] The same impact of transport costs on market participation is found in other sectors: see for instance Ouma et al. (2010) on the banana market in Central Africa or Heltberg and Tarp (2002) on food crops in Mozambique.

The survey he conducted in the peri-urban region of Kolda (Senegal), where several "mini-dairies" are established, show that the level of milk commercialization reaches 75% in the wet season.

In this chapter we examine under which conditions the presence of an intermediary could actually improve farmers' participation to the market. We develop a simple contract model between geographically dispersed farmers and an intermediary who has a transport cost advantage over them. Due to their spatial dispersion, an heterogeneity exists among farmers, such that the intermediary has an interest to propose a different contract to each of them. Nevertheless, spatial price discrimination might not be feasible. Notably, it implies that the intermediary collects the product himself from the farmers. There are, however, other modes of collection and hence other pricing policies that the intermediaries may choose. For instance, in Senegal, some processing units, such as "La Laiterie du Berger" (LdB) in Richard-Toll, organize milk collection and pay all the farmers the same price, independent of the distance. This corresponds to uniform pricing. In others, such as "Le Fermier" in Kolda, farmers are responsible for transport, such that the ones who are located far from the processing unit receive a considerably lower net price than the closer ones. This corresponds to mill pricing.

We observe that contracts between "mini-dairies" and farmers often involve interlinked transactions. Additionally to playing an intermediary role on the milk market, the dairy also play this role on the input market, providing the farmer the cattle feeds that are necessary for his production. In such a contract, both output and input prices are determined jointly. In the region of Kolda, DIEYE et al. (2005) report that processing units provide credit and cattle feeds to the farmers in order to increase commercial links. The two most important milk processing units in this region ("Bilaame Puul Debbo" and "Le Fermier") use three different mechanisms for linking milk purchase and feeds selling: credit for feeds purchase, direct feeds purchase for the farmer, or guarantee to the feeds seller in case of non-payment by the farmer (DIEYE, 2006). In Northern Senegal, "La Laiterie du Berger" buys large quantities of cattle feeds and resells it to the farmers at 50 percent of the market price (BATHILLY, 2007).[2]

These interlinked contracts have been shown to be efficient, at least when the intermediary is able to discriminate between farmers (GANGOPADHYAY and SENGUPTA, 1987). One may wonder if they also permit to improve farmers' participation to the market. Comparing an interlinked contract model with a simple one, we analyze if interlinked transactions do or do not permit to give access to the market to more farmers than a simple contract does.

Knowing under which conditions the presence of an intermediary could improve farmers' participation to the market, one may ask when this actually helps to reduce poverty. Using a Foster-Greer-Thorbecke indicator, we compare the outcomes of the different

[2] Numerous evidence of the use of interlinked contracts is found in other countries and sectors: see for instance WARNING and KEY (2002) for an analysis of the groundnut sector in Senegal, JAYNE, YAMANO and NYORO (2004) for examples in cash crops production in Kenya, SIMMONS, WINTERS and PATRICKS (2005) for an examination of various Indonesian sectors, or KEY and RUNSTEN (1999) for a look at Mexican frozen vegetable industry.

types of contracts/pricing policies we consider, in terms of their poverty reduction impact, in order to arrive at some policy recommendations as to the contract that should be used. In particular, we look at what a benevolent policy maker who wants to decrease poverty but is unable to impose a complex tax and subsidy scheme should impose as a contract/pricing policy to be used by intermediaries. Alternatively, our results can be seen as the contract/pricing policy that an external donor which helps set up agricultural intermediaries with a view of reducing rural poverty should impose as a condition to those intermediaries.

The chapter is structured as follows. Next section details the framework and the assumptions linked to the model. Sections 3 develops the simple contract model for a for-profit trader who only buys the product to the farmers, in the cases of spatial price discrimination, uniform pricing, and mill pricing. Section 4 presents the interlinked contract model, where the trader is also provider of inputs, for the three pricing policies. Section 5 discusses the implications of contract and pricing policy choices, related to farmers' participation to the market and to poverty reduction. Finally, section 6 concludes.

3.2 Framework

We analyze the impact of the contract choice made by the intermediary on the number of farmers who can benefit from it, in the following theoretical framework. Geographical locations are represented along a linear segment. A final good market is located at the origin 0. We consider one agricultural good whose price p is set on this market. We assume that the different agents in our model do not have any impact on this price.[3] This good is consumed at location 0 which can be assumed to be an urban center.

At a distance r from this urban center, there is a rural area over which farmers are uniformly distributed. Each farmer produces the agricultural good according to the same production function $f(k)$, where k is the quantity of input he uses. This input is sold at price i on the market at location 0. The production function has the usual properties: $f(.)$ is continuously differentiable, $f(0) = 0$, $f_k = \frac{df}{dk} > 0$, $\lim_{k\to 0} f_k = \infty$, $\lim_{k\to\infty} f_k = 0$ and $\frac{d^2 f}{dk^2} < 0$. To obtain explicit solution for the contracts, we assume $f(k) = 2\sqrt{k}$. Farmers are assumed to be profit maximizers.

To transport the agricultural good to the market, farmers face high transport costs. These costs are assumed to be linear in distance for the output. To simplify the analysis we assume that transport costs are negligible for the input and set them equal to zero. A farmer located at a distance x from the market faces a transport cost τx. At least the first farmer is assumed to be able to sell profitably on the market, which implies the following restriction: $p > \tau r$.

[3] This can be the case for example because we are in a small open economy and the price of this good is determined on world markets.

An intermediary is located at r.[4] This trader offers simple or interlinked contracts to the geographically dispersed producers. In the simple contract case, the trader sells the agricultural output from the farmers on the market located in 0, at market price p. In the interlinked contract case, he also buys input for them on this market, at price i. The intermediary is assumed to have a cost advantage. Here, we assume that the trader has an advantage to transport the good between r and 0. Transport costs for the trader are given by $t(x) = \theta r + \tau(x - r)$ per unit of output transported with $\theta < \tau$.

We consider two types of contract. When proposing a simple contract, the trader only acts as a buyer of output and proposes a price for this output to the farmer. When proposing an interlinked contract, the trader also acts as a seller of input and proposes both prices, for the input and the output, at the same time, to the farmer. In both cases, each farmer can individually accept or refuse the contract.

The sequence is the following. In a first step, the trader proposes a simple contract (p_F) or an interlinked contract (p_F, i_F) to the farmer. The farmer can either accept or reject the contract. In a second step, the farmer chooses the quantity of input $k^*(x)$ it is optimal for him to use depending on the prices he faces, which determines his level of production. If he has accepted the simple contract, he faces prices (p_F, i). In the interlinked contract case, he faces prices (p_F, i_F). If he has rejected the contract, his production will no longer be sold to the trader, but directly to the final market. The same applies to purchase of inputs. In this case, he chooses the optimal amount of inputs k^0 as a function of market prices (p, i) as well as of the transport cost t^0 he has to support. In a last step, the production takes place and is sold on the market, directly by the farmer (if he has rejected the contract) or via the trader (if the farmer has accepted the contract).

As a buyer of output, the profit maximizer trader pays a discriminatory price p_D (resp. uniform price p_U, mill price p_M) to the farmer. In the interlinked contract case, as a provider of input, the trader also charges a price i_D (resp. i_U, i_M) to the farmer per unit of input provided. The interlinked contract is thus defined by a couple (p_F, i_F) where p_F is the agricultural output price paid by the trader to the farmer (i.e. $p_D(x)$, p_U, or $p_M - \tau(x - r)$) and i_F is the input price paid by the farmer to the trader (i.e. $i_D(x)$, i_U, or i_M), while in the simple contract we have $i_F = i$.

We solve this problem backward. The farmer located in x maximizes his income $y(x)$ such that he has the following objective:

$$\max_{k(x)} \; y(x) = p_F(x) f(k(x)) - i_F(x) k(x) \tag{3.2.1}$$

The existence of an interior solution to this problem is guaranteed by the above assumptions on the production function. The choice of the input quantity satisfies the

[4]In developing countries, poor infrastructures in rural area reduce the incentives for firms to locate within this area. By locating just outside of the rural area, the trader has a better access to roads, electricity, water, etc. Because of the limited number of farmers involved and the potentially large investment costs, the intermediary is assumed to have monopoly/monopsony power when he trades with the farmers. On the final market, however, the intermediary is price-taker.

necessary condition $\frac{df}{dk} = \frac{i_F(x)}{p_F(x)}$, which, given the particular form of the production function ($f(k) = 2\sqrt{k}$), provides the following farmer's demand for the input

$$k^*(x) = \left(\frac{p_F(x)}{i_F(x)}\right)^2 \tag{3.2.2}$$

If the farmer refuses the contract proposed by the trader, he obtains an alternative income by directly selling the agricultural output at net price $p - \tau x$ and buying the input at price i on the market. In this case his input use $k^0(x)$ is characterized by

$$k^0(x) = \left(\frac{p - \tau x}{i}\right)^2 \tag{3.2.3}$$

All the farmers are not able to do so. Indeed, farmers located further than $r + R^0$ (where $R^0 = (p - \tau r)/\tau$) are not able to take part to the market by themselves. Indeed, the net price they would receive for the output is negative. Those farmers are assumed to have a zero alternative income, such that $y^0(x)$ is defined in two parts as:

$$y^0(x) = \begin{cases} \frac{(p-\tau x)^2}{i} & if\ x < r + R^0 \\ 0 & if\ x \geq r + R^0 \end{cases} \tag{3.2.4}$$

The trader's objective function is:

$$\max_{p_F(x), i_F(x), R} \Pi = \int_r^{r+R} (p - \theta r - \tau(x - r) - p_F(x)) f(k^*(x)) + (i_F(x) - i) k^*(x) dx \tag{3.2.5}$$

where $k^*(x)$ is the farmer's demand for the input given by (3.2.2).

This objective is subject to the following participation constraint, for each farmer: $g(x) \equiv y(x, p_F(x), i_F(x)) - y^0(x) \geq 0\ \forall x$. For the simple contract the constraint $i_F(x) = i\ \forall x$ applies. When the uniform pricing policy is used, $p_F(x) = p_U\ \forall x$ while under mill pricing policy we have $p_F(x) = p_M - \tau(x - r)\ \forall x$.

The number of farmers included may also be constrained by a physical limit of the production area, for instance, the existence of a national border or simply the absence of farmers beyond a certain distance. To take this into account we include the constraint that $R \leq \bar{R}$ where $\bar{R}$ is the maximum number of farmers who could be included ($r + \bar{R}$ represents the physical limit). We are interested in the cases where $\bar{R} > R^0$. Indeed, with $\bar{R} < R^0$, all the farmers would take part to the market by themselves and looking at the intermediary's impact on the participation would be of little interest. If $\bar{R}$ is large enough, the constraint $R \leq \bar{R}$ will not be binding at the optimum, as the intermediary has never interest in collecting the product until the farmer located in $r+\bar{R}$. In particular, when $\bar{R}$ is such that $p < \theta r + \tau \bar{R}$, the transport cost to reach the last farmer is larger than the price that can be earned on his production, such that it is never optimal for the intermediary to collect the product so far. For simplicity, hereafter we assumed that this condition is satisfied.[5]

[5]One may check that the conditions under which the presence of the intermediary increases the

3.3 Simple contract

Using a simple contract, the trader proposes a price $p_F(x)$ for the output to the farmer located in x. Regarding the input, the farmer simply faces the market price i, as the trader is not involved in the input trading. The trader's objective function is:

$$\max_{p_F(x),R} \Pi = \int_r^{r+R} (p - \theta r - \tau(x-r) - p_F(x)) f(k^*(x))dx \tag{3.3.1}$$
$$\text{s.t. } y(x, p_F(x))) \geq y^0(x) \ \forall x$$

3.3.1 Spatial price discrimination

The trader proposes a contract $(p_D(x))$ to the farmer located in x. This contract can be different, depending on the location of the farmer and the difference between two farmers' contracts does not necessarily represent the difference in transport costs between them. Each farmer can individually accept or refuse the contract proposed.

The trader's problem may be written as:

$$\max_{p_D(x),R_D} \Pi = \int_r^{r+R_D} (p - \theta r - \tau(x-r) - p_D(x)) f(k^*(x))dx$$
$$\text{s.t. } g(x) \equiv p_D(x) f(k^*(x)) - ik^*(x) - y^0(x) \geq 0 \ \forall x$$

Using $f(k) = 2\sqrt{k}$, the Lagrangian is given by:

$$\mathcal{L} = \int_r^{r+R_D} 2(p - \theta r - \tau(x-r) - p_D(x)) \frac{p_D(x)}{i} + \lambda(x) \left(\frac{p_D(x)^2}{i} - y^0(x) \right) dx$$

Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial p_D(x)} = (p - \theta r - \tau(x-r) - 2p_D(x)) \frac{2}{i} + 2\lambda(x) \frac{p_D(x)}{i} = 0 \ \forall x \tag{3.3.2}$$

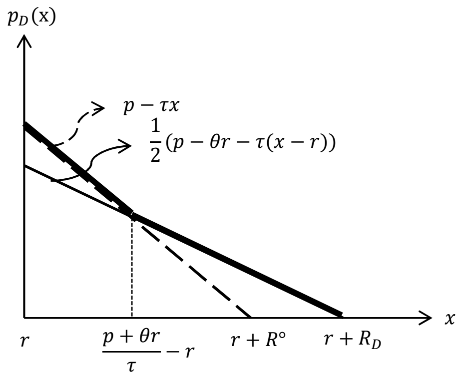
$$\frac{\partial \mathcal{L}}{\partial R_D} = 2(p - \theta r - \tau R_D - p_D(r+R)) \frac{p_D(r+R_D)}{i} + \lambda(r+R_D) \left(\frac{p_D(r+R_D)^2}{i} - y^0(r+R_D) \right) = 0 \tag{3.3.3}$$

$$\lambda(x) \geq 0, \ \frac{p_D(x)^2}{i} - y^0(x) \geq 0, \ \lambda(x) \left(\frac{p_D(x)^2}{i} - y^0(x) \right) = 0 \tag{3.3.4}$$

From (3.3.2) and (3.3.4) it can be shown that the unconstrained outcome $p_D(x) = \frac{1}{2}(p - \theta r - \tau(x-r))$ satisfies the participation constraint for $x \geq \frac{p+\theta r}{\tau} - r$. For smaller values of x, the participation constraint is binding and $p_D(x) = \sqrt{iy^0(x)}$. This result is illustrated by figure 3.1 while formal proof is given in Appendix 3.A.

participation to the market (i.e. the number of participants under the contract is higher than in the stand-alone situation) are not affected if this assumption it is not satisfied (that is, if $R^0 < \bar{R} < \frac{p-\theta r}{\tau}$). This can be formally verified by including a constraint $R \leq \bar{R}$ in the optimization problem (3.2.5). Nevertheless, the intensity of the intermediary's impact on the participation could be constrained by the physical limit. This has to be kept in mind when comparing types of contracts and pricing policies.

Figure 3.1: Spatial price discrimination (simple contract)



Thus the optimal contract is characterized[6] by:

$$\begin{cases} (i) \text{ for } x \geq \frac{p+\theta r}{\tau} - r, & p_D(x) = \frac{1}{2}(p - \theta r - \tau(x - r)) \quad \text{and} \quad g(x) > 0 \\ (ii) \text{ for } x < \frac{p+\theta r}{\tau} - r, & p_D(x) = \sqrt{iy^0(x)} \quad \text{and} \quad g(x) = 0 \end{cases} \tag{3.3.5}$$

From that, we can establish the optimal size R_D. Assume first that $\lambda(r + R_D) > 0$. This implies $p_D(r+R_D) = \sqrt{iy^0(r + R_D)}$. Substituting in (3.3.3) gives $2(p-\theta r-\tau R_D - p_D(r + R))\frac{p_D(r+R_D)}{i} = 0$, that is: $p_D(r + R) = p - \theta r - \tau R_D$. Given that the constraint is binding in this case, $p - \theta r - \tau R_D = \sqrt{iy^0(r + R_D)} = p - \tau(r + R_D) \Leftrightarrow \theta = \tau$. Given that by assumption $\tau > \theta$, this is a contradiction and hence we have that $\lambda(r+R_D) = 0$. From (3.3.5), we have $p_D(r + R_D) = \frac{1}{2}(p - \theta r - \tau R_D)$. Substituting it into (3.3.3), we have:

$$2\left(p - \theta r - \tau R_D - \frac{1}{2}(p - \theta r - \tau R_D)\right) \frac{1}{2} \frac{(p - \theta r - \tau R_D)}{i} = 0$$

$$\Leftrightarrow R_D = \frac{p - \theta r}{\tau} \tag{3.3.6}$$

This means that for all possible parameter values we have $R_D > R^0$: under spatial price discrimination, the trader always chooses to include more farmers than the number able to take part to the market in the stand alone situation. Note that, all the farmers located in $]Max[r, \frac{p+\theta r}{\tau} - r], \frac{p+\theta r}{\tau} + r[$ obtain a positive surplus. Hence, when $p < 2\tau r - \theta r$, all the farmers, except the last one, obtain a positive surplus.

[6]It can be shown that arbitrage between farmers is impossible: for any location x, the farmer has no interest in transporting the good by himself to another location z in order to benefit from the price $p_D(z)$. The potential gain from such a strategy is always more than compensated by the incurred transport cost.

3.3.2 Uniform pricing

Under uniform pricing policy, the trader is constrained to pay the same uniform price to all the farmers, independent on their location. His problem may be written as:

$$\max_{p_U,R_U} \Pi = R_U\left(p - \theta r - \frac{\tau R_U}{2} - p_U\right) f(k^*)$$

$$\text{s.t. } g(r) \equiv p_U f(k^*) - ik^* - y^0(r) \geq 0$$

Using $f(k) = 2\sqrt{k}$, the Lagrangian is given by:

$$\mathcal{L} = 2R_U\left(p - \theta r - \frac{\tau R_U}{2} - p_U\right)\frac{p_U}{i} + \lambda(p_U - p + \tau r)$$

Kuhn-Tucker conditions are:

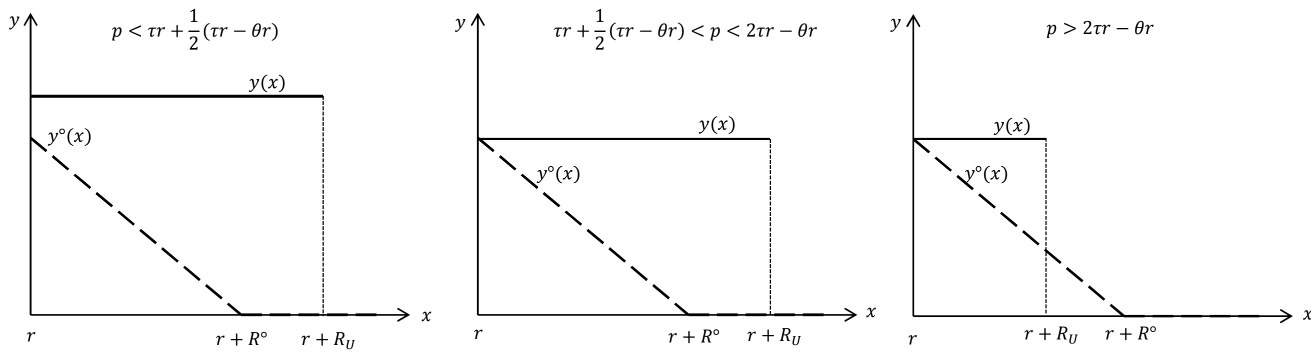
$$\frac{\partial \mathcal{L}}{\partial p_U} = \frac{2R_U}{i}\left(p - \theta r - \frac{\tau R_U}{2} - 2p_U\right) + \lambda = 0 \qquad (3.3.7)$$

$$\frac{\partial \mathcal{L}}{\partial R_U} = 2\left(p - \theta r - \frac{\tau R_U}{2} - p_U\right)\frac{p_U}{i} - \tau R_U\frac{p_U}{i} = 0$$

$$\Leftrightarrow \tau R_U = p - \theta r - p_U \qquad (3.3.8)$$

$$\lambda \geq 0,\ p_U \geq p - \tau r,\ \lambda(p_U - p + \tau r) = 0 \qquad (3.3.9)$$

Figure 3.2: Uniform pricing (simple contract)



It can be shown (see Appendix 3.A) that the unconstrained outcome ($p_U = \frac{1}{3}(p-\theta r)$, $R_U = \frac{2}{3}\frac{p-\theta r}{\tau}$) satisfies the participation constraint for low values of p. For larges values of p, the participation constraint is binding and $R_U = \frac{\tau r-\theta r}{\tau}$. The optimal contract is thus characterized by:

$$\begin{cases} (i)\ p_U = \frac{1}{3}(p-\theta r), \quad g(r) > 0, \quad R_U = \frac{2}{3}\frac{p-\theta r}{\tau} > R^0 \quad \text{if } p < \tau r + \frac{1}{2}(\tau r - \theta r) \\ (ii)\ p_U = p - \tau r, \quad g(r) = 0, \quad R_U = \frac{\tau r - \theta r}{\tau}\begin{cases} > R^0 & \text{if } \tau r + \frac{1}{2}(\tau r - \theta r) \leq p < 2\tau r - \theta r \\ \leq R^0 & \text{if } p \geq 2\tau r - \theta r \end{cases} \end{cases} \qquad (3.3.10)$$

Depending on the parameter values we have $R_U > R^0$ or $R_U < R^0$: the trader chooses to include more farmers than the number able to take part to the market in the stand alone situation, only if the market price is low compared to his advantage in transport cost. Since for all $x > r$ we have $g(x) > g(r)$, all the farmers included (except the closest one when p is large) obtain a positive surplus from dealing with the trader. Figure 3.2 illustrates these results.

3.3.3 Mill pricing

Under mill pricing policy, the trader pays the same mill price to all farmers, that is the same fixed price minus the transport cost from the farmer's location. He has to propose the same contract (p_M) to all farmers (where p_M is independent of x) but farmers have to support the transport costs. Thus, the net price received by the farmer for the output is $p_F(x) = p_M - \tau(x - r)$. The trader's problem may be written as:

$$\max_{p_M, R_M} \Pi = \int_r^{r+R_M} (p - \theta r - p_M) f(k^*(x))dx$$

$$\text{s.t. } g(x) \equiv p_M f(k^*(x)) - ik^*(x) - y^0(x) \geq 0$$

Using $f(k) = 2\sqrt{k}$, and noting that the satisfaction of the participation constraint for the first farmer is sufficient to ensure it is satisfied for all farmers[7], the Lagrangian is given by:

$$\mathcal{L} = \frac{2R_M}{i} (p - \theta r - p_M) \left(p_M - \frac{\tau R_M}{2} \right) + \lambda(p_M - p + \tau r)$$

Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial p_M} = \frac{2R_M}{i} \left(p - \theta r + \frac{\tau R_M}{2} - 2p_M \right) + \lambda = 0 \tag{3.3.11}$$

$$\frac{\partial \mathcal{L}}{\partial R_M} = 2(p - \theta r - p_M) \frac{(p_M - \tau(R_M/2))}{i} - \frac{\tau R_M}{i} (p - \theta r - p_M) = 0 \tag{3.3.12}$$

$$\lambda \geq 0, \; p_M \geq p - \tau r, \; \lambda(p_M - p + \tau r) = 0 \tag{3.3.13}$$

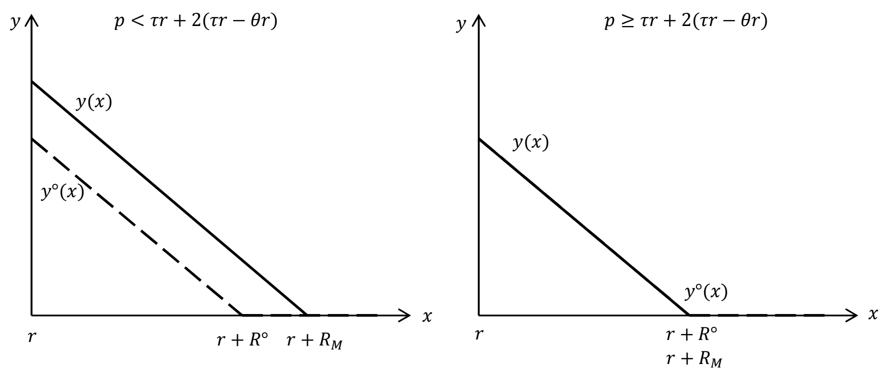
It can be shown (see Appendix 3.A) that the unconstrained outcome ($p_M = \frac{2}{3}(p - \theta r)$, $R_M = \frac{2}{3}\frac{p-\theta r}{\tau}$) satisfies the participation constraint for low values of p. For larges values of p, the participation constraint is binding and $R_M = \frac{p - \tau r}{\tau}$. The optimal contract is thus characterized by:

$$\begin{cases} (i) \; p_M = \frac{2}{3}(p - \theta r), \quad g(x) > 0 \; \forall x, \quad R_M = \frac{2}{3}\frac{p-\theta r}{\tau} > R^0 & \text{if } p < \tau r + 2(\tau r - \theta r) \\ (ii) \; p_M = p - \tau r, \quad g(x) = 0 \; \forall x, \quad R_M = \frac{p-\tau r}{\tau} = R^0 & \text{if } p \geq \tau r + 2(\tau r - \theta r) \end{cases} \tag{3.3.14}$$

[7] Indeed, using $f(k) = 2\sqrt{k}$, $y(x) = \frac{(p_M - \tau(x-r))^2}{i}$. A sufficient condition for $y(x) \geq y^0(x)$ is $p_M - \tau(x - r) \geq p - \tau x \Leftrightarrow p_M \geq p - \tau r$.

This means that for all possible parameter values we have $R_M \geq R^0$: using the mill price policy, the trader always chooses to include at least all the farmers who are able to take part to the market in the stand alone situation. If the market price is low compared to his advantage in transport cost, he chooses to include even more farmers. However, the farmers obtain a positive surplus from dealing with the trader only when the market price is low enough. Otherwise, they receive from the trader exactly their reservation price. Figure 3.3 illustrates these results.

Figure 3.3: Mill pricing (simple contract)



3.4 Interlinked contract

The trader proposes an interlinked contract $(p_F(x), i_F(x))$ to the farmer located in x. His objective function is:

$$\max_{p_F(x), i_F(x), R} \Pi = \int_r^{r+R} (p - \theta r - \tau(x-r)) f(k^*(x)) - p_F(x) f(k^*(x)) + (i_F(x) - i) k^*(x) dx$$
$$\text{s.t. } y(x, p_F(x), i_F(x)) \geq y^0(x) \ \forall x \tag{3.4.1}$$

3.4.1 Spatial price discrimination

The trader's problem may be written as:

$$\max_{p_D(x), i_D(x), R_D} \Pi = \int_r^{r+R_D} (p - \theta r - \tau(x-r)) f(k^*(x)) - i k^*(x) - (p_D(x) f(k^*(x)) - i_D(x) k^*(x))\, dx$$

$$\text{s.t. } g(x) \equiv p_D(x) f(k^*(x)) - i_D(x) k^*(x) - y^0(x) \geq 0 \ \forall x$$

Using $f(k) = 2\sqrt{k}$, the Lagrangian is given by:

$$\mathcal{L} = \int_r^{r+R_D} 2(p - \theta r - \tau(x-r)) \frac{p_D(x)}{i_D(x)} - i \left(\frac{p_D(x)}{i_D(x)} \right)^2 + (\lambda(x) - 1) \frac{p_D(x)^2}{i_D(x)} - \lambda(x) y^0(x) dx \tag{3.4.2}$$

Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial p_D(x)} = (p - \theta r - \tau(x-r))\frac{2}{i_D(x)} - 2i\frac{p_D(x)}{i_D(x)^2} + 2(\lambda(x)-1)\frac{p_D(x)}{i_D(x)} = 0 \ \forall x \quad (3.4.3)$$

$$\frac{\partial \mathcal{L}}{\partial i_D(x)} = -(p - \theta r - \tau(x-r))2\frac{p_D(x)}{i_D(x)^2} + 2i\frac{p_D(x)^2}{i_D(x)^3} - (\lambda(x)-1)\frac{p_D(x)^2}{i_D(x)^2} = 0 \ \forall x \quad (3.4.4)$$

$$\frac{\partial \mathcal{L}}{\partial R_D} = 2(p - \theta r - \tau R_D)\frac{p_D(r+R_D)}{i_D(r+R_D)} - i\left(\frac{p_D(r+R_D)}{i_D(r+R_D)}\right)^2$$

$$+ (\lambda(r+R_D)-1)\frac{p_D(r+R_D)^2}{i_D(r+R_D)} - \lambda(r+R_D)y^0(r+R_D) = 0 \quad (3.4.5)$$

$$\lambda(x) \geq 0, \ \frac{p_D(x)^2}{i_D(x)} - y^0(x) \geq 0, \ \lambda(x)\left(\frac{p_D(x)^2}{i} - y^0(x)\right) = 0 \quad (3.4.6)$$

Solving (3.4.4) for $\lambda(x)$ and substituting in (3.4.3) gives

$$\frac{i_D(x)}{p_D(x)} = \frac{i}{p - \theta r - \tau(x-r)} \quad (3.4.7)$$

Substituting (3.4.7) in (3.4.4) gives $\lambda(x) = 1$. Substituting $\lambda(r+R_D) = 1$ and (3.4.7) in (3.4.5) gives:

$$\frac{(p - \theta r - \tau R_D)^2}{i} = y^0(r+R_D) \quad (3.4.8)$$

Equation (3.4.8) has no solution for $R_D < R^0$. Indeed, $y^0(r+R_D)$ would be given by $(p - \tau r - \tau R_D)^2/i$, which cannot be equal to $(p - \theta r - \tau R_D)^2/i$ as $\theta < \tau$ by assumption. If $R_D \geq R^0$, then we have $y^0(r+R_D) = 0$. Substituting in (3.4.8) gives the following value for R_D:

$$R_D = \frac{p - \theta r}{\tau} \quad (3.4.9)$$

which is similar to (3.3.6). Under spatial price discrimination, the optimal number of farmers R_D is the same whatever the contract is simple or interlinked, and is larger than R^0. However, in the interlinked contract, as $\lambda(x) = 1 \ \forall x$, we have $g(x) = 0 \ \forall x$ and no farmer is able to obtain a positive surplus from dealing with the trader.[8] Although more farmers are able to take part to the market in the presence of the trader, this does not increase their income. Indeed, using an interlinked contract, the trader who is able to discriminate is able to capt all the efficiency gain, pushing all the farmers' incomes down to their reservation levels.

[8]Note that it can be shown that arbitrage between farmers is impossible: for any location x, the farmer has no interest in transporting the good by himself to another location z in order to benefit from the prices $(p_D(z), i_D(z))$. The potential gain from such a strategy is always more than compensated by the incurred transport cost.

3.4.2 Uniform pricing

The trader's problem may be written as:

$$\max_{p_U, i_U, R_U} \Pi = \int_r^{r+R_U} (p - \theta r - \tau(x-r)) f(k^*) - ik^* - (p_U f(k^*) - i_U k^*) dx$$

$$\text{s.t. } g(x) \equiv p_U f(k^*) - i_U k^* - y^0(x) \geq 0 \ \forall x$$

The continuum of constraints $g(x) \geq 0$ may be replaced by the unique constraint $g(r) \geq 0$ (see chapter 2). Using $f(k) = 2\sqrt{k}$, the Lagrangian is given by:

$$\mathcal{L} = R_U \left(\left(p - \theta r - \tau \frac{R_U}{2} \right) 2\frac{p_U}{i_U} - i \left(\frac{p_U}{i_U} \right)^2 \right) + (\lambda - R_U) \frac{p_U^2}{i_U} - \lambda y^0(r) \tag{3.4.10}$$

Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial p_U} = R_U \left(\left(p - \theta r - \frac{\tau R_U}{2} \right) \frac{2}{i_U} - 2i \frac{p_U}{i_U^2} \right) + 2(\lambda - R_U) \frac{p_U}{i_U} = 0 \tag{3.4.11}$$

$$\frac{\partial \mathcal{L}}{\partial i_U} = R_U \left(- \left(p - \theta r - \frac{\tau R_U}{2} \right) 2\frac{p_U}{i_U^2} + 2i \frac{p_U^2}{i_U^3} \right) - (\lambda - R_U) \frac{p_U^2}{i_U^2} = 0 \tag{3.4.12}$$

$$\frac{\partial \mathcal{L}}{\partial R_U} = \left(\left(p - \theta r - \tau \frac{R_U}{2} \right) 2\frac{p_U}{i_U} - i \left(\frac{p_U}{i_U} \right)^2 \right) - \frac{\tau R_U}{2} 2\frac{p_U}{i_U} - \frac{p_U^2}{i_U} = 0 \tag{3.4.13}$$

$$\lambda \geq 0, \ \frac{p_U^2}{i_U} - y^0(r) \geq 0, \ \lambda \left(\frac{p_U^2}{i_U} - y^0(r) \right) = 0 \tag{3.4.14}$$

Solving (3.4.12) for λ and substituting in (3.4.11) gives

$$\frac{i_U}{p_U} = \frac{i}{p - \theta r - \frac{\tau R_U}{2}} \tag{3.4.15}$$

Substituting (3.4.15) in (3.4.12) give $\lambda = R_U$. Substituting $\lambda = R_U$ and (3.4.15) in (3.4.13) gives:

$$\frac{(p - \theta r - \tau(R_U/2))^2}{i} - \tau R_U \frac{(p - \theta r - \tau(R_U/2))}{i} - y^0(r) = 0$$

As by assumption $p > \tau r$, the optimal number of farmers R_U is characterized by:

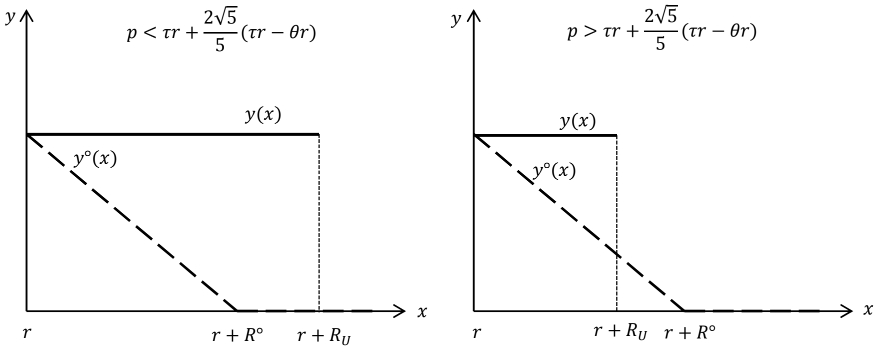
$$(p - \theta r - \tau(R_U/2))^2 - \tau R_U (p - \theta r - \tau(R_U/2)) - (p - \tau r)^2 = 0$$

Depending on the parameters of the model, this leads to a number of farmers higher or lower than in the stand-alone situation. Indeed, the optimal number of farmers is given by:

$$R_U = \frac{4}{3} \frac{p - \theta r}{\tau} - \frac{2}{3\tau} \sqrt{(p - \theta r)^2 + 3(p - \tau r)^2} \begin{cases} > R^0 \text{ if } p < \tau r + \frac{2\sqrt{5}}{5}(\tau r - \theta r) \\ \leq R^0 \text{ if } p \geq \tau r + \frac{2\sqrt{5}}{5}(\tau r - \theta r) \end{cases} \tag{3.4.16}$$

As $\lambda = R_U > 0$, we have $g(r) = 0$ by (3.4.14). Since for all $x > r$ we have $g(x) > g(r)$, all the farmers included who are located further than r obtain a positive surplus from dealing with the trader. Figure 3.4 illustrates this result.

Figure 3.4: Uniform pricing (interlinked contract)



3.4.3 Mill pricing

The trader's problem may be written as:

$$\max_{p_M, i_M, R_M} \Pi = \int_r^{r+R_M} (p - \theta r - p_M) f(k^*(x)) + (i_M - i) k^*(x) dx$$

$$\text{s.t. } g(x) \equiv (p_M - \tau(x-r)) f(k^*(x)) - i_M k^*(x) - y^0(x) \geq 0 \ \forall x$$

From chapter 2, using $f(k) = 2\sqrt{k}$,

$$\max_{p_M, i_M, R_M} \Pi = \int_r^{r+R_M} (p - \theta r - p_M) 2 \frac{p_M - \tau(x-r)}{i_M} + (i_M - i) \left(\frac{p_M - \tau(x-r)}{i_M} \right)^2 dx$$

$$\text{s.t. } \ g(r + R_M) \equiv \frac{(p_M - \tau R_M)^2}{i_M} - y^0(r + R_M) \geq 0$$

$$\text{and } \ i - i_M \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = \int_r^{r+R_M} (p - \theta r - p_M) 2 \frac{p_M - \tau(x-r)}{i_M} + (i_M - i) \left(\frac{p_M - \tau(x-r)}{i_M} \right)^2 dx$$

$$+ \lambda \left(\frac{(p_M - \tau R_M)^2}{i_M} - y^0(r + R_M) \right) + \mu(i - i_M) \quad (3.4.17)$$

Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial p_M} = \frac{2}{i_M} \left(\frac{-i}{i_M} R_M \left(p_M - \frac{\tau R_M}{2} \right) + R_M(p - \theta r - p_M) + \lambda(p_M - \tau R_M) \right) = 0 \quad (3.4.18)$$

$$\frac{\partial \mathcal{L}}{\partial i_M} = -2(p - \theta r - p_M) \frac{R_M}{i_M^2} \left(p_M - \frac{\tau R_M}{2} \right)$$

$$+ \left(p_M^2 - p_M \tau R_M + \frac{\tau^2 R_M^2}{3} \right) \frac{R_M}{i_M^2} \left(2 \frac{i}{i_M} - 1 \right) - \lambda \frac{(p_M - \tau R_M)^2}{i_M^2} - \mu = 0 \quad (3.4.19)$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R_M} = 2(p-\theta r-p_M)\frac{(p_M-\tau R_M)}{i_M} + \left(1-\frac{i}{i_M}\right)\frac{(p_M-\tau R_M)^2}{i_M} \\
+\lambda\left(-2\tau\frac{(p_M-\tau R_M)}{i_M} - \frac{\partial y^0(r+R_M)}{\partial R_M}\right) = 0
\end{aligned} \tag{3.4.20}
$$

$$
\lambda \geq 0,\ \frac{(p_M-\tau R_M)^2}{i_M} - y^0(r+R_M) \geq 0,\ \lambda\left(\frac{(p_M-\tau R_M)^2}{i_M} - y^0(r+R_M)\right) = 0 \tag{3.4.21}
$$

$$
\mu \geq 0,\ i-i_M \geq 0,\ \mu(i-i_M) = 0 \tag{3.4.22}
$$

From (3.4.21) two cases are possible: either $g(r+R_M) > 0$, either $g(r+R_M) = 0$. Assume $g(r+R_M) > 0$ hence $\lambda = 0$. Substituting in (3.4.18) and (3.4.20) gives:

$$
(p-\theta r-p_M) = \frac{i}{i_M}\left(p_M - \frac{\tau R_M}{2}\right) \tag{3.4.23}
$$

$$
2(p-\theta r-p_M)(p_M-\tau R_M) + \left(1-\frac{i}{i_M}\right)(p_M-\tau R_M)^2 = 0 \tag{3.4.24}
$$

Suppose first that $p_M = \tau R_M$. If $R_M \geq R^0$, then $g(r+R_M) = 0$ which is a contradiction. If $R_M < R^0$, then $g(r+R_M) = 0 - \frac{p-\tau r-\tau R_M}{i} < 0$, which is also a contradiction. Suppose now that $p_M \neq \tau R_M$, substituting (3.4.23) in (3.4.24), we have:

$$
\tau R_M = \left(1+\frac{i}{i_M}\right)p_M \tag{3.4.25}
$$

Equation (3.4.25) implies $\tau R_M > p_M$. This implies that $g(r+R_M) < 0$ for any value of R_M, which contradicts (3.4.21). Hence $g(r+R_M) > 0$ is excluded, thus $g(r+R_M) = 0$.

From (3.4.22) two cases are possible: either $i_M = i$, either $\mu = 0$. Suppose $i_M = i$. Suppose first that $R_M > R^0$. In that case, $g(r+R_M) = 0$ implies that $p_M = \tau R_M$. Substituting $i_M = i$ and $p_M = \tau R_M$ in (3.4.18) gives $R_M = \frac{2}{3}\frac{p-\theta r}{\tau}$. Substituting $i_M = i$, $p_M = \tau R_M$ as well as the previous value for R_M in (3.4.19) gives $\mu < 0$ which contradicts (3.4.22). Suppose now that $R_M < R^0$. In that case, $g(r+R_M) = 0$ and $i_M = i$ imply that $p_M = p - \tau r$. Substituting $i_M = i$ and $p_M = p - \tau r$ in (3.4.20) gives:

$$
(\tau r - \theta r)(p - \tau r - \tau R_M) = 0 \tag{3.4.26}
$$

As $\tau > \theta$, (3.4.26) implies that $p - \tau r = \tau R_M$, that is $R_M = R^0$, which is a contradiction. Suppose finally that $R_M = R^0$. In that case, $g(r+R_M) = 0$ implies that $p_M = \tau R_M = p - \tau r$. Substituting $i_M = i$, $p_M = p - \tau r$ and $R_M = (p-\tau r)/\tau$ in (3.4.19) gives:

$$
\mu = \frac{(p-\tau r)^2}{\tau i^2}\left(-(\tau r - \theta r) + \frac{p-\tau r}{3}\right) \tag{3.4.27}
$$

From (3.4.22), $\mu \geq 0$. This is only true for $p \geq \tau r + 3(\tau r - \theta r)$. Thus $i_M = i$ is only possible for these values of the parameters and leads to $R_M = R^0$.

Suppose now that $\mu = 0$. Suppose first that $R_M \geq R^0$. In that case, $g(r+R_M) = 0$ implies that $p_M = \tau R_M$. Substituting $\mu = 0$ and $p_M = \tau R_M$ in (3.4.19) gives:

$$
\frac{2}{3}\tau R_M\left(\frac{i}{i_M}+1\right) = p - \theta r \tag{3.4.28}
$$

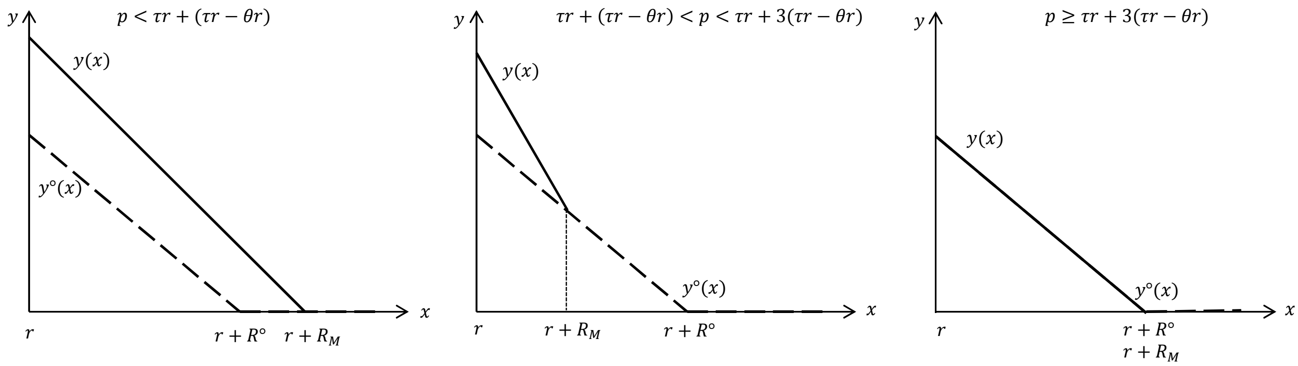
Substituting (3.4.28) and $p_M = \tau R_M$ in (3.4.18) gives $i_M = i/2$. Substituting into (3.4.28) gives $R_M = \frac{p-\theta r}{2\tau}$. From that, $R_M \geq R^0$ is true for $p \leq 2\tau r - \theta r$. Suppose now that $R_M < R^0$. Numerical simulations show that such a solution is only possible for $2\tau r - \theta r < p < \tau r + 3(\tau r - \theta r)$.

Hence the optimal size is characterized by:

$$R_M \begin{cases} = \frac{p-\theta r}{2\tau} > R^0 & \text{if } p < \tau r + (\tau r - \theta r) \\ = R^0 & \text{if } p = \tau r + (\tau r - \theta r) \\ < R^0 & \text{if } \tau r + (\tau r - \theta r) < p < \tau r + 3(\tau r - \theta r) \\ = R^0 & \text{if } p \geq \tau r + 3(\tau r - \theta r) \end{cases} \tag{3.4.29}$$

Depending on the parameter values, R_M may be lower or larger than R^0: the trader chooses to include more farmers than the number able to take part to the market in the stand alone situation, only if the market price is low compared to his advantage in transport cost. In any case, $g(r + R_M) = 0$ which means that the last farmer included do not obtain any surplus from dealing with the trader. Nevertheless, when $p < \tau r + 3(\tau r - \theta r)$, all the farmers located before $r + R_M$ obtain a positive surplus from the contract. When p is larger, then the trader simply replicates the stand alone situation and all the farmers included are indifferent between the trader's contract and their stand alone initial situation. Figure 3.5 illustrates these results.

Figure 3.5: Mill pricing (interlinked contract)



3.5 Discussion of the results

Due to high transport costs, remote farmers in developing countries are not able to take part to the market by themselves. Concluding a contract with an intermediary, who has a cost advantage over the farmers, may allow them to participate. In the previous analysis, we look at the conditions under which the presence of an intermediary permit the participation of farmers who were unable to take part to the market by themselves.

In particular, we look at what a benevolent policy maker who wants to increase participation should impose as a type of contract or pricing policy to be used by intermediaries. Similarly, we look at the conditions an external donor should impose to the intermediaries he promotes, when his aim is improving participation.

Remote farmers are generally poorer than the less isolated one (see for instance Jacoby (2000) for an empirical example). By increasing participation, the intermediary helps to reduce poverty as these farmers can get a positive income from selling their production while they were unable to do so by themselves. Even in the case where the intermediary does not improve participation, he may reduce poverty. Indeed, being able to sell their production on the market does not prevent farmers to be poor. By increasing the incomes of these farmers, the intermediary may also contribute to reduce poverty. In the following analysis, we thus look separately at both aspects: participation improvement and poverty reduction.

3.5.1 Participation

Table 3.1: Summary of results

Contract	Optimal number of farmers R_P^{C**}	Condition for $R_P^C > R^{0***}$	
Simple - Discrim.	$R_D^S = \frac{p-\theta r}{\tau}$	$\forall p$	$a \in [0, \infty[$
Simple - Uniform	$R_U^S = \begin{cases} \frac{2}{3}\frac{p-\theta r}{\tau} & \text{if } a \in [0, 1/2[\\ \frac{\tau r-\theta r}{\tau} & \text{if } a \in [1/2, \infty[\end{cases}$	$p < \tau r + (\tau r - \theta r)$	$a \in [0, 1[$
Simple - Mill	$R_M^S = \begin{cases} \frac{2}{3}\frac{p-\theta r}{\tau} & \text{if } a \in [0, 2[\\ \frac{p-\theta r}{\tau} & \text{if } a \in [2, \infty[\end{cases}$	$p < \tau r + 2(\tau r - \theta r)$	$a \in [0, 2[$
Linked - Discrim.	$R_D^L = \frac{p-\theta r}{\tau}$	$\forall p$	$a \in [0, \infty[$
Linked - Uniform	$R_U^L = \frac{4}{3}\frac{p-\theta r}{\tau} - \frac{2}{3\tau}\sqrt{(p-\theta r)^2 + 3(p-\tau r)^2}$	$p < \tau r + \frac{2\sqrt{5}}{5}(\tau r - \theta r)$	$a \in \left[0, 2\sqrt{5}/5\right[$
Linked - Mill	$R_M^L \begin{cases} \frac{p-\theta r}{2\tau} & \text{if } a \in [0, 1] \\ < R^0 & \text{if } a \in]1, 3[\\ = R^0 & \text{if } a \in [3, \infty[\end{cases}$	$p < \tau r + (\tau r - \theta r)$	$a \in [0, 1[$

* $a = (p - \tau r)/(\tau r - \theta r)$ is an indicator of the relative capacity for the farmers, as a whole, to take part profitably to the market by themselves. It is inversely related to trader's cost advantage.

** The superscript C stands for type of contract (i.e. linked (L) or simple (S)) and the subscript P stands for the pricing policy (i.e. discrimination (D), uniform pricing (U) or mill pricing (M)).

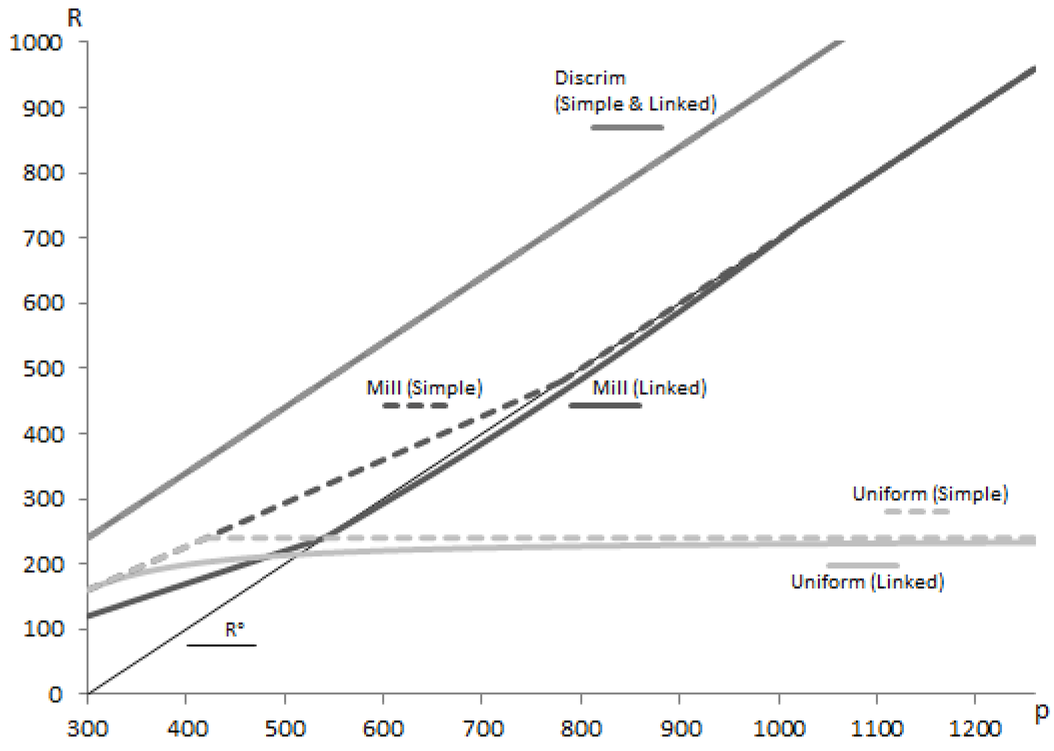
*** Number of farmers that have access to the market in the absence of the trader is given by $R^0 = \frac{p-\tau r}{\tau}$.

High transport costs in developing countries may impede remote farmers to take part to the market by themselves. The presence of an intermediary, who has a cost advantage over the farmers, may allow remote farmers to take part to the market, through a contract with the intermediary. However, except if the trader is able to perfectly discriminate, this is only the case when the difference in transport costs between the trader and the farmer (i.e. $\tau r - \theta r$) is large compared to the output price. It is only when the intermediary has a *sufficient* cost advantage over the farmers that he chooses to include more farmers than those able to reach the market by themselves.

Nevertheless, the importance of the difference needed for the trader to improve market participation depends on the type of contract as well as of the pricing policy used. In such a situation, using appropriate regulations, a policy maker may be willing to encourage the types of contracts that lead the for-profit trader to contract with a higher number of farmers.

These results are summarized in table 3.1. One may note that the conditions under which the intermediary includes more farmers than those able to reach the market by themselves depend on his cost advantage $(\tau r - \theta r)$ *relative* to the first farmer's reservation price $(p - \tau r)$. If the net price that farmers can get on the market is low and/or if the intermediary's cost advantage is important, then he is able to give access to the market to farmers who were not able to participate by themselves. For presentation purpose, we define $a = (p - \tau r)/(\tau r - \theta r)$ as a measure of the relative importance of alternatives opportunities for the farmers on the market. [9]

Figure 3.6: Optimal number of farmers



* Choice of the parameters: $r = 300$, $\tau = 1$ $\theta = 0.2$ and $i = 100$

Figure 3.6 illustrates the main results. It depicts, for both types of contracts and for the three pricing policies, the number of farmers it is optimal for the trader to contract with, as a function of the output price p. As it has been shown, under spatial price

[9]Note that $1/a$ may also be interpreted as the intrinsic importance of the space on the market, as defined by ZHANG and SEXTON (2001).

discrimination, this number of farmers is the same, whenever the contract is simple or interlinked. This number is also higher than R^0, the number of farmers able to take part to the market in the absence of the trader, and this is valid for any p.

Spatial price discrimination is not always feasible. And if it is, the policy maker may want to avoid it. Indeed, while leading to the highest participation, this policy implies that all the farmers' incomes are pushed down to their reservation levels. It allows to include farmers that were excluded from the market in the stand-alone situation, but except from potential spillover effects, they are not better off than in the stand alone situation.

Among the other possibilities, the simple mill pricing contract always leads to a participation greater or equal than that with other contracts. It can be seen on figure 3.6 that the number of farmers included in this contract constitutes an upper envelope of the set of possibilities. Moreover, the simple mill pricing contract contributes to increase market participation with respect to the stand alone situation, as long as $p < \tau r + 2(\tau r - \theta r)$. After this threshold, the number of farmers included in the contract is equal to R^0, but never below. However, the stand alone situation is simply replicated such that no farmer is able to increase his income by dealing with the trader.

It is interesting to note that, in terms of participation, for any given pricing policy, the interlinked contract is always dominated by the simple one. The interlinked contract, while having various advantages, does not allow to increase the number of participants, with respect to a simple contract. A policy maker interested in increasing the number of farmers included may be willing to promote the simple contract against the interlinked one. Obviously, the interlinked contract leads to a higher profit for the trader[10], hence an intervention is needed to restrict the use of the interlinkage.

When interlinkage is allowed, uniform pricing dominates mill pricing in terms of participation when p is low. Thus, when p is small, or alternatively, when the difference in transport costs between the farmer and the trader is large (precisely, when $a < \frac{3}{4\sqrt{3}-3}$), uniform interlinked contract is preferred to the mill pricing one, in terms of market participation. After this threshold, the reverse is true. When $\frac{2\sqrt{5}}{5} < a < 1$, uniform pricing even leads to a number of farmers lower than R^0 while mill pricing still allows to increase market access. When $a > 1$ both policies implies $R < R^0$ but mill pricing still includes more farmers than uniform does.[11]

Nevertheless, when $a > 3$, the trader using the mill pricing policy simply replicates the stand alone situation such that no farmer can obtain any surplus from dealing with him. This does not occur under uniform pricing: although less farmers are included, they are all (except the first one) able to obtain a positive surplus from the contract.

[10] Otherwise, the optimal interlinked contract would look like a simple one.

[11] This last point cannot be proved formally, but is assessed by numerical simulation.

3.5.2 Poverty

Following Foster et al. (1984), we adopt the following poverty indicator:

$$Pov = \frac{1}{\bar{R}} \int_{r+q}^{r+\bar{R}} \left(\frac{z - y(x)}{z} \right)^2 dx \qquad (3.5.1)$$

where $z > 0$ is poverty line, $(z - y(x)$ is the income shortfall of the farmer located in x), $\bar{R} - q$ is the number of poor farmers (having income no greater than z) and $\bar{R}$ is the maximum number of farmers to be considered (limited by any physical limit). Larger is Pov, higher is the poverty.

We will compare the outcomes of the different pricing policies in terms of this indicator, in order to derive which policy used by the intermediary performs better in reducing poverty with respect the stand-alone situation. In particular, we defined the "gain" in terms of poverty reduction as:

$$G_P^C = Pov^0 - Pov_P^C \qquad (3.5.2)$$

where the superscript C stands for type of contract (i.e. linked (L) or simple (S)) and the subscript P stands for the pricing policy (i.e. discrimination (D), uniform pricing (U) or mill pricing (M)) while Pov^0 represents the level of poverty in the stand-alone situation and is defined as: $Pov^0 = \frac{1}{\bar{R}} \int_{r+q^0}^{r+\bar{R}} \left(\frac{z-y^0(x)}{z} \right)^2 dx$ where $\bar{R} - q^0$ is the number of poor in the absence of the intermediary.

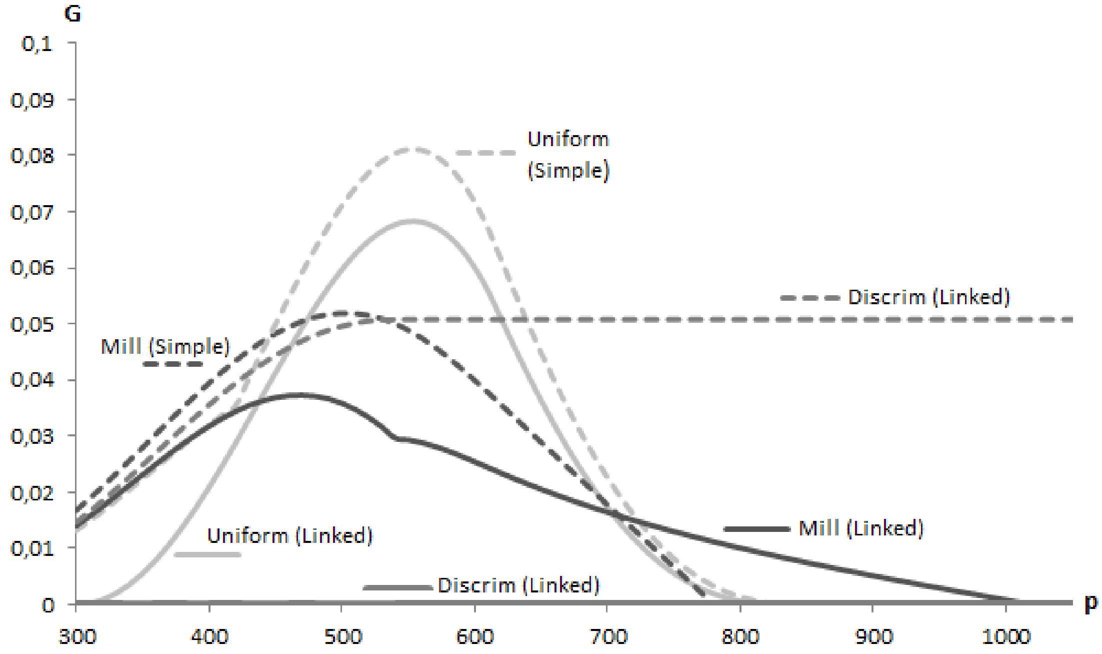
As explained before, we only consider $\bar{R}$ large enough such that $p < \theta r + \tau \bar{R}$. Under this condition, it can be shown that the physical limit has no impact on the ranking of the different contracts considered in terms of poverty reduction. In particular, if the limit is increased to $\bar{R} + W$, the difference in poverty reduction between two types of contracts $G_1 - G_2$ is decreased to $\frac{\bar{R}}{\bar{R}+W}(G_1 - G_2)$, but, as $\frac{\bar{R}}{\bar{R}+W} > 0$, the ranking between G_1 and G_2 is not affected. As we are only interested in this ranking, we can arbitrarily choose $\bar{R}$ in the following numerical analysis.

Figure 3.7 illustrates the poverty reduction effect of the different types of contracts we consider. As expected, an interlinked contract with spatial price discrimination does not contributes to poverty reduction, as it does not permit to increase the income of any farmers. If the contract is simple, however, spatial price discrimination contributes to poverty reduction. Indeed, under this policy, all the farmers located in $[Max[r, \frac{p+\theta r}{\tau} - r], \frac{p+\theta r}{\tau}]$ get a positive surplus from such a contract. As soon as some of them are poor, the presence of the intermediary contributes to reduce poverty.

Numerical simulations show that the type of contract that has to be encouraged in order to decrease poverty crucially depends on the output market price. In particular, comparing uniform and mill pricing interlinked contracts, uniform pricing tends to perform better in reducing poverty for moderate values of p, while mill pricing tends to perform better for low and large values of p.

The indicator G given by (3.5.2) (forgetting the subscripts and superscripts) can be decomposed into two parts. On the one hand, the presence of the intermediary may help to reduce poverty for farmers who were already able to take part to the market by

Figure 3.7: Poverty reduction



* Choice of the parameters: $r = 300$, $\tau = 1$, $\theta = 0.2$, $i = 100$ and $z = 1000$.

themselves. On the other hand, poverty may also be reduced by the participation of farmers who were unable to participate by themselves. Defining the first gain in poverty reduction as $G(in)$ and the latter as $G(out)$, we have $G = G(in) + G(out)$ with:

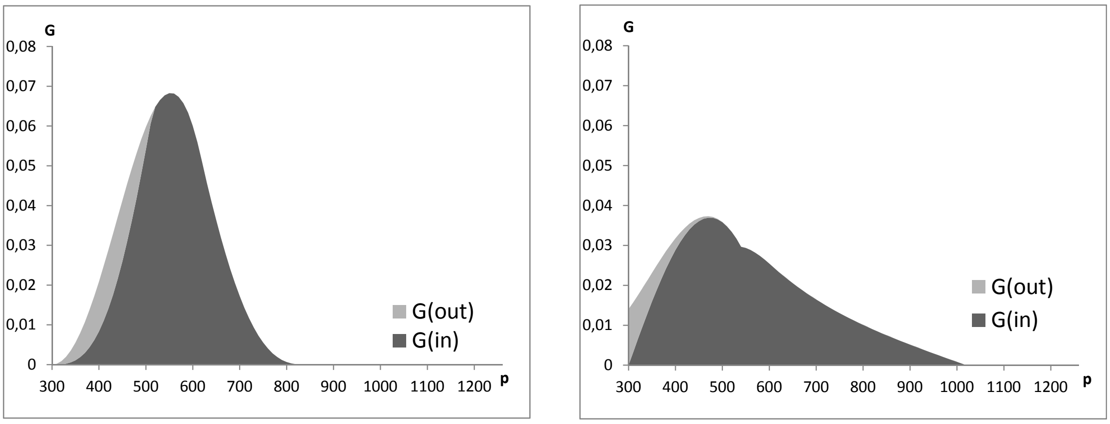
$$G(in) = \frac{1}{\bar{R}} \left(\int_{r+q^0}^{r+R^0} \left(\frac{z - y^0(x)}{z} \right)^2 dx - \int_{r+q}^{r+R^0} \left(\frac{z - y(x)}{z} \right)^2 dx \right) \tag{3.5.3}$$

$$G(out) = 1 - \frac{1}{\bar{R}} \int_{r+R^0}^{r+R} \left(\frac{z - y(x)}{z} \right)^2 dx \tag{3.5.4}$$

Figure 3.8 illustrates this decomposition for uniform and mill pricing policies in the interlinked contract case (the same pattern appears in the simple contract case). When the output market price is low, the major part of the poverty reduction comes from the increased participation. This part is progressively reduced as p increases such that for a large p, the poverty reduction is only due to the increase of the incomes of the farmers who were already able to take part to the market by themselves, as the presence of the intermediary does not help to increase participation anymore.

Figure 3.8: Decomposition of the poverty reduction effect

(a) Uniform pricing (Linked) (b) Mill pricing (Linked)



* Choice of the parameters: $r = 300$, $\tau = 1$, $\theta = 0.2$, $i = 100$ and $z = 1000$.

3.6 Conclusions

In a developing country context, where transport costs are high, remote farmers are not able to take part to the market by themselves. The presence of a trader, who has a cost advantage, may help them to participate to the market. In this chapter we show that, except if the intermediary is able to perfectly discriminate, his presence helps to increase market participation only if he has a *sufficient* cost advantage over the farmers.

Moreover, his impact on participation depends on the type of contract and on the pricing policy used. With this respect, the mill pricing simple contract dominates all other kinds of arrangements, except spatial price discrimination.

As it has been discussed in chapter 2, interlinked contracts have various advantages. One may ask if they also permit to improve farmers' participation to the market. Comparing an interlinked contract model with a simple one, the previous analysis show it is not the case: interlinked transactions do not permit to give access to the market to more farmers than a simple contract does.

Based on a Foster-Greer-Thorbecke indicator, we compare the different types of contracts/pricing policies in terms of their poverty reduction impact. The analysis show that the arrangement that performs better in reducing poverty greatly depends on the output market price, or alternatively, on the trader's cost advantage. Hence, no general policy recommendation can be made regarding the arrangement that should be encouraged by a policy maker willing to reduce poverty.

The link between participation improvement and poverty reduction is analyzed by decomposing the poverty reduction effect between an effect for farmers who were already able to take part to the market in the absence of the intermediary and farmers who are able to participate thanks to the intermediary. It appears that, for low values of the output price, most of the poverty reduction induced by the intermediary's presence comes from the inclusion of newly participants. When p is larger, this effect is reduced, until a point where the poverty reduction only comes from the increase of the incomes of farmers who were already participating.

The present study gives elements for future researches. First, this model should be extended to a more general production function, such as an iso-elastic one. We expect that the results deduced here are not restrictive and also apply to more general specifications. Second, more general, non linear, transport cost functions should be considered, notably giving the possibility for economies when transporting both input and output. Finally, rather than considering monopsony/monopoly intermediary, the entry of competitors could be analyzed.

Appendices

3.A Simple contract

Proof of (3.3.5)

From (3.3.4), for each x, two cases are possibles. (i) Either $\lambda(x) = 0$ which implies from (3.3.2) that $p_D(x) = \frac{1}{2}(p - \theta r - \tau(x - r))$. This only respects the participation constraint for $x \geq \frac{p+\theta r}{\tau} - r$. (ii) Or $\lambda(x) > 0$ which implies that the participation constraint is binding: $\frac{p_D(x)^2}{i} = y^0(x)$, that is $p_D(x) = \sqrt{iy^0(x)}$. Substituting in (3.3.2), we have: $\lambda(x) = -\frac{p-\theta r-\tau(x-r)-2\sqrt{iy^0(x)}}{\sqrt{iy^0(x)}}$. This is only positive for $x \leq \frac{p+\theta r}{\tau} - r$. □

Proof of (3.3.10)

From (3.3.9) two cases are possibles. (i) Assume $\lambda = 0$. From (3.3.7), we have $p_U = \frac{1}{2}(p - \theta r - \tau(R_U/2))$. Substituting in (3.3.8), we have $R_U = \frac{2}{3}\frac{p-\theta r}{\tau}$. Replacing in the previous expression for p_U, this gives $p_U = \frac{1}{3}(p - \theta r)$. This only respects the participation constraint if $p \leq \frac{1}{2}(3\tau r - \theta r)$. (ii) Assume the participation constraint is binding: $p_U = p - \tau r$. From (3.3.8), $\tau R_U = \tau r - \theta r$. Substituting in (3.3.7), we have $\lambda = \frac{\tau r - \theta r}{\tau i}(2p + \theta r - 3\tau r)$. This is only positive if $p \geq \frac{1}{2}(3\tau r - \theta r)$. □

Proof of (3.3.14)

From (3.3.12), either $p_M = \tau R_M$ or $p_M = p - \theta r$. In this last case however, trader's profit is zero, which may be excluded. Thus:

$$p_M = \tau R_M \tag{3.6.1}$$

From (3.3.13) two cases are possibles. (i) Assume $\lambda = 0$. From (3.3.11), $p_M = \frac{1}{2}(p - \theta r + \tau(R_M/2))$. Substituting in (3.6.1), we have $R_M = \frac{2}{3}\frac{p-\theta r}{\tau}$. Hence, $p_M = \frac{2}{3}(p - \theta r)$. This only respects the participation constraint if $p \leq 3\tau r - 2\theta r$. (ii) Assume the participation constraint is binding: $p_M = p - \tau r$. From (3.6.1), $\tau R_M = p - \tau r$. Substituting in (3.3.11), we have $\lambda = \frac{p-\tau r}{\tau i}(p + 2\theta r - 3\tau r)$. This is only positive if $p \geq 3\tau r - 2\theta r$. □

Chapter 4

Entry and expansion strategy of an intermediary under uncertainty*†

4.1 Introduction

In Senegal, as in other West African countries, rural households involved in the livestock sector only obtain low income from the sale of the milk they produce. Indeed, this production takes place in a pastoral or an agropastoral system in which each farmer has only some cows and each one provides low quantity of milk (Duteurtre, 2006). Moreover, due to inadequate transport infrastructure and important distances between areas of production and areas of consumption, they face high transport costs. As the milk is highly perishable, it has to be sold on a daily basis, which implies that transport and transaction costs are important compared to the low quantity of product sold. In some cases, these costs are so important that farmers fail to participate to the market by themselves.

Since the nineties, we have seen the emergence of small-scale processing units, called mini-dairies, that play an intermediary role between the farmers and the market (Dieye et al., 2005, Corniaux et al., 2005). These intermediaries seem to rapidly expand (see figure 1, page 8). They have some kind of advantage over the farmers to sell the products on the market. They use more efficient transport devices, such as trucks, they own bulk cooling tanks, such that they can stock the milk and do not have to transport it every day, etc. This cost advantage often requires an important fixed cost, which cannot be borne by each farmer alone.

The presence of such intermediaries has the potential to increase farmers' income (see chapter 2) and participation to the market (see chapter 3). Hence we analyze under what conditions these intermediaries will find it profitable to enter the market and whether

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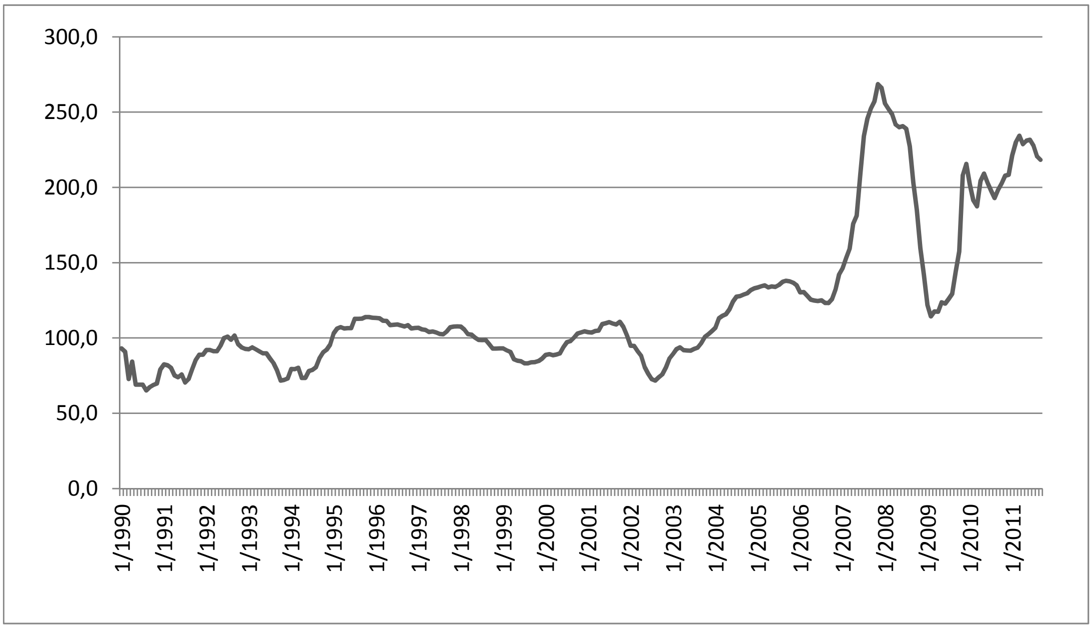
policy recommendations can be made to a policy maker willing to encourage their entry. On the one hand, they need to make costly irreversible investments in order to deal with farmers who are geographically isolated and dispersed. On the other hand, prices of food products are characterized by important volatility which leads to uncertainty and creates an environment which tends to discourage investment by profit-seeking agents. This is particularly true in the milk sector, as it may be seen on figure 4.1 that represents the evolution of the FAO dairy price index since the nineties.

Figure 4.1: FAO Dairy Price Index



* Source: FAO (2011)

Once an intermediary decides to enter the market, another question is the number of farmers who can benefit from his entry, that is the size of the milk collection area the mini-dairy decides to work with. This size can evolve with the time, hence we analyze the factors that determines this size as well as its evolution in future periods.

In what follows we establish what is the optimal investment strategy for an intermediary who buys an input from geographically dispersed farmers and who sells this transformed input on a market characterized by price volatility. Uncertainty faced by such intermediaries has been considered in the literature but more interest has been given to the stochastic characteristic of farmer's supply (for instance, see Lofgren, 1992). When market price uncertainty is considered, the contract components, such as the optimal input price, are investigated. For instance, Chaudhuri and Gupta (1995) look at the effect of the output market price uncertainty on the optimal credit-product interlinked contract. However, the intermediary is assumed to be already installed and few is done about the effect of the uncertainty on his entry decision. In this chapter, we determine at what price it is optimal for the intermediary to invest and, given that the number of farmers will have an impact on the cost of investment, we also establish what is the optimal initial size of collection area. The higher is this optimal price the less likely is the entry by the intermediary on the market; the bigger the collection area, the more farmers will benefit from the presence of this intermediary. We will also establish whether it is likely that the intermediary increases his collection area in the future.

Our analysis will determine what are the factors which influence the decision to invest and whether it is likely or not to see these intermediaries appear on the market. This means that we study the profitability of an intermediary's investment project in a context of market price uncertainty. Given the irreversible nature of the investment and the uncertainty linked to the agricultural prices volatility, we use the real option theory to define the optimal plan of investment for such a firm. Indeed, the net present value rule leads to overoptimistic results in this context (Dixit and Pindyck, 1994). Using data on the milk sector in Senegal, we check the profitability of two existing mini-dairies and explore whether the project of transforming local milk into powder could be profitable.

In a perspective of improving farmers' income and participation to the market, public authorities or external donors may be interested in helping the firm to invest. In this chapter, we compare the impact of various economic policies that can be implemented. Policies that induce a larger number of farmers included in the project benefit directly to the marginal farmers. Policies that leads to a shorter investment delay benefit more rapidly to the farmers included. Depending on the donor's objective, we discuss the relevance of policies such as help for initial investment or aid for incremental investment. Using data on the milk sector in Senegal, we simulate these effects in a real context.

The chapter is structured as follows. The next section develops the theoretical model regarding investment timing and size choice, taking into account the possibility of including at a later date other farmers after the initial investment has been made. Section 3 discusses comparative statics results. Section 4 is devoted to study cases in the milk sector in Senegal. Finally, section 5 concludes.

4.2 The model

We consider a situation in which farmers are geographically dispersed and produce an agricultural product. Each farmer's supply is assumed to be price inelastic due to the intrinsic characteristics of agricultural products, (namely, they are perishable while the production period is long) and produces each period a quantity s of the product that can be sold. Each farmer has the possibility to sell his product on the final market which can be, for instance, an urban center. For this, he has to incur a transport cost which is increasing with distance. The farmer located at a distance $x \geq 0$ from the final market faces a transport cost $\upsilon(x)$ per unit of product with $\partial \upsilon(x)/\partial x > 0$ and $\upsilon(0) = 0$. On the market, the farmer has to incur transaction costs due to imperfect information. We assume these costs are proportional to the price and lower the price effectively received by the farmer on the market (Key et al., 2000) from p_t to ψp_t with $\psi \in [0, 1]$. The farmer's expected net price per unit at each period is thus given by $\psi p_t - \upsilon(x)$. Formally, we assume that farmers are uniformly distributed along a linear segment with origin at 0 and that the final good market is located at the origin. The further away farmers are from urban centers, the lower is their net expected income. This results for the farmers into lower incomes and higher risk of poverty.

In this chapter, we establish whether it is beneficial for intermediaries to enter the market and whether farmers will benefit from this intermediary. The intermediary collects the product from the farmers, pays each farmer the same price irrespective of his location and sells the product on the final goods market. The intermediary is located at the origin 0 and hence does not face any transport cost for his output.[1] Depending on the transport cost for the input, the intermediary chooses the optimal size of the collection area R, that is, the number of farmers from whom he buys the agricultural input. The collection area is represented by the segment $[0, R]$. The price on the urban market for the good is assumed to be stochastic and influenced by the price on world markets. We assume that the market price evolves following a geometric Brownian motion:

$$dp = \alpha \; p_t \; dt + \; \sigma \; p_t \; dz \tag{4.2.1}$$

where α is the drift and σ is the volatility of p. $dz = \epsilon_t \sqrt{dt}$, (where $\epsilon_t \sim N(0,1)$ is a white noise) is the Wiener increment, which satisfies $E[dz] = 0$. The deterministic part $\alpha p_t dt$ represents the trend of world prices and the stochastic part $\sigma p_t dz$ represents the volatility of the market price.

The assumption of the geometric Brownian motion seems reasonable for agricultural products including milk. Turvey and Power (2006) have performed test for ordinary Brownian motion on historical data from 17 commodity futures contracts and have found that the null hypothesis of ordinary Brownian motion cannot be rejected for 14 of the 17 series. Fluid Milk is shown to be consistent with a geometric Brownian motion at all confidence levels.

We are interested in the optimal investment strategy for an intermediary who faces a stochastic market price. This means that we establish "when", that is, at what price, it is optimal for the trader to invest in a plant as well as what is the optimal size of the area on which the agricultural input will be collected. To set-up a collection area, investments have to be made and these costs are assumed to be sunk and hence irreversible. After entering the market with an initial size for the collection area, the trader can expand this area in future periods by incurring additional costs.

Once installed, the intermediary gives the same price to all the farmers. As a farmer's reservation price is decreasing with distance to the urban center, the trader has to give all farmers a price at least equal to the reservation price of the farmer located the closest to the urban center, that is ψp_t. All the farmers further away obtain a positive surplus from selling their product to the intermediary rather than themselves on the final market. The intermediary has a constant operating marginal cost c and faces a collection cost $T(R_t)$ per unit of agricultural input transported, with $T(0) = 0$ and $T'(R) > 0$. The first cost c is independent of the distance and represents outlays such as, for instance, electricity, output packaging, etc. Collection cost $T(R_t)$ increases with the size of the collection area and includes costs such as fuel, driver's wage, etc.

[1] The same reasoning holds for an intermediary who is located at a distance $r > 0$ from the market. In that case, the operating cost c hereafter also includes the output transport cost from the location r to the final good market 0.

The intermediary's operating profit is given by $\Pi(p_t, R_t) = sR_t\,(p_t - \psi p_t - c - T(R_t))$, that is:

$$\Pi(p_t, R_t) = sR_t\,((1-\psi)p_t - c - T(R_t))$$

As farmers are distributed on a segment, each additional farmer included increases the length of the segment, which increases the transport cost $T(R)$. For simplicity, we assume that this cost is linear in R such that $T(R_t) = \tau R_t$ with τ the unit transport cost per unit of distance. In that case:

$$\Pi(p_t, R_t) = sR_t\,((1-\psi)p_t - c - \tau R_t) \tag{4.2.2}$$

Because the price is variable over time, we can have periods during which $p_t \leq (c + \tau R_t)/(1-\psi)$, which implies that the operating profit is negative.

The investment cost has two parts. First, building the plant costs a fixed amount I. Second, extending the collection area costs κ per unit of distance. Indeed, the decision of including more farmers in the area is costly. In the milk sector for instance, one way to attract farmers to the network is to encourage artificial inseminations in order to increase their production. In that case, sheds have also to be constructed to protect these animals that cannot resist to high temperatures. Formations about hygiene, animal welfare, and animal health have to be organized to guarantee the quality of the product. All these costs, as well as the search and information costs for concluding the contract are represented by κ. Dudez and Broutin (2003) report that it is difficult to enforce the contract concluded with the farmers such that they supply milk to the processing unit on a regular basis. One way to do this is by providing them with technical or medical assistance. The initial cost is thus $I + \kappa R^*(p^*)$ and each expansion of the area costs κ per additional unit.

As the profit of the intermediary is uncertain and investment costs are irreversible, we analyze this problem of investment strategy using real options theory (Dixit and Pindyck, 1994). The investment strategy of the trader describes the optimal price at which the trader should enter the market, the optimal initial capacity and the optimal future capacity expansion profile. Several models of capacity choice have been developed. We use insights of incremental investment models, such as Pindyck (1988) and Dixit (1995), as well as models on fixed capacity choice such as Dangl (1999) or Bøckman et al. (2008).

The real options theory takes into account the fact that an investor with an opportunity to invest has the possibility to wait before investing in that project. When investment costs are irreversible and future environment is uncertain, this option has a certain value and any investment involves giving up that option. Once this option is given up, the firm cannot disinvest if the market conditions change dramatically. This lost option has to be included as a part of the total investment cost. When postponing the investment (that is keeping the option to invest) the firm not only obtains the "capital appreciation" of the non-invested money, but also value the fact of avoiding future losses. The real options theory says that, if the value of the option to invest is higher than the net value of the project, the firm's best strategy is to wait. The project should be undertaken when the stochastic price reaches the threshold at which the option to invest has exactly the same value as the net value of the plant once constructed. To determine this threshold, we need an expression for the option to invest F and an expression for the value of the project once installed V.

The firm not only faces this trade-off for the initial investment, but also for each additional unit of size. Thus, in our model we take into account the value of the option to increase this size in future periods, that is the value of postponing the incremental investment. Given this, the value of the project once installed V depends on the value on the size, as well as on the option to increase this size in the future.

The optimal investment strategy is described by three elements. First, a threshold price p^*: it is optimal for the firm to invest in the project when the stochastic market price is above this threshold. Second, the initial size of the collection area: when the stochastic market price crosses the threshold p^*, it is optimal for the firm to invest in a project with a size R^*. Third, the optimal size $R^*(p)$ that will be installed depending on the evolution of the market price. For instance, when the stochastic market price increases from $\underline{p}$ to $\bar{p}$ (with $\bar{p} > \underline{p} > p^*$) it is optimal for the firm to increase the size from $R^*(\underline{p})$ to $R^*(\bar{p})$.

The optimal investment strategy is determined as follows. First, we determine the value V of the installed project and the value F of the option to invest. Then we calculate the price at which F and the net value of the project $V - I - \kappa R$ are equal. This threshold price $p^*(R)$ depends on the size R of the installed project. Second, we determine the optimal size $R^*(p)$ as a function of the realization of the stochastic market price. Finally, we use $R^*(p)$ and $p^*(R)$ to calculate the threshold p^* at which it is optimal to make the initial investment, as well as the initial size R^*.

4.2.1 Value of the installed project V

The value of the installed project $V_t(p_t, R_t)$ at time t is the sum of the current operating profit over the interval $(t, t+dt)$ and the continuation value (the expected discounted value of future operating profits) after $t + dt$. Note that, at period t, p_t is known but p_{t+dt} is unknown and depends on p_t by the stochastic process (4.2.1).

$$V_t(p_t, R_t) = \Pi_t(p_t, R_t)dt + E_t[V_{t+dt}(p_{t+dt}, R_{t+dt})e^{-\rho dt}]$$

where $e^{-\rho dt}$ is the discount factor (the present value in t of one monetary unit to be received in $t + dt$) with ρ the fixed continuously compounded discount rate. The term $E_t[V_{t+dt}(p_{t+dt}, R_{t+dt})]$ takes into account all the possible $V_{t+dt}(p_{t+dt}, R_{t+dt})$ and their respective probabilities. From that, we have the following Bellman equation:

$$V_t(p_t, R_t) = \Pi_t(p_t, R_t)dt + e^{-\rho dt}E_t[V_{t+dt}(p_{t+dt}, R_{t+dt})] \tag{4.2.3}$$

Approximating $e^{-\rho dt}$ by $1 - \rho dt$, adding and substracting V_t between the square brackets, applying Ito's Lemma to $E_t[dV]$ defined as $E_t[V_{t+dt}(p_{t+dt}, R_{t+dt}) - V_t(p_t, R_t)]$, ignoring terms which are small relative to dt, reorganizing the terms, and finally dividing by dt yields the following non-homogeneous differential equation:

$$\frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 V_t(p_t, R_t)}{\partial p^2} + \alpha p_t \frac{\partial V_t(p_t, R_t)}{\partial p} - \rho V_t(p_t, R_t) + \Pi_t(p_t, R_t) = 0 \tag{4.2.4}$$

which is non-stochastic. Indeed, it depends only on p_t and R_t which are known at time t. From (4.2.2), this can be written as:

$$\frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 V_t(p_t,R_t)}{\partial p^2} + \alpha p_t \frac{\partial V_t(p_t,R_t)}{\partial p} - \rho V_t(p_t,R_t) + sR_t\left((1-\psi)p_t - c - \tau R_t\right) = 0 \quad (4.2.5)$$

The Appendix 4.A shows that the particular solution to this non-homogeneous differential equation is given by:

$$V_t(p_t,R_t) = \underbrace{B_1(R_t)p_t^{\beta_1} + B_2(R_t)p_t^{\beta_2}}_{\text{option to expand R: } G(p_t,R_t)} + \underbrace{sR_t\left(\frac{(1-\psi)p_t}{\rho-\alpha} - \frac{c+\tau R_t}{\rho}\right)}_{\text{fundamental}} \quad (4.2.6)$$

where $B_1(R_t)$ and $B_2(R_t)$ are to be determined, $\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}$ and $\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}$.

$G(p_t,R_t)$ is the value of the firm's "growth options", i.e. given the current size R_t and the current value of p_t, $G(p_t,R_t)$ is the present value of any additional profits that might result of future size expansion (PINDYCK, 1988: 971).[2] This value is given by $G(p_t,R_t) = B_1(R_t)p_t^{\beta_1} + B_2(R_t)p_t^{\beta_2}$. However, the likelihood of expanding the size becomes very small when the market price becomes small, so the value of the options to expand should be zero as p goes to zero. Therefore, the coefficient B_2, corresponding to the negative root β_2, should be equal to zero, such that (4.2.6) becomes:

$$V_t(p_t,R_t) = B_1(R_t)p_t^{\beta_1} + sR_t\left(\frac{(1-\psi)p_t}{\rho-\alpha} - \frac{c+\tau R_t}{\rho}\right) \quad (4.2.7)$$

The interpretation of (4.2.7) is the following. The last terms represent the expected present value of the profit the intermediary would obtain if he kept the size of the collection area constant at the level R_t forever. $B_1(R_t)p_t^{\beta_1}$ is the value of the intermediary's options to expand this area in the future. It may be socially difficult to reduce the size of the collection area (i.e. excluding farmers). This is why we take the investment in the collection area as being irreversible. Once the area has been extended, it cannot be reduced. Thus the value of the installed project (4.2.7) does not take into account any option of decreasing the size.

4.2.2 Value of the option to invest F

Knowing the value $V_t(p_t,R_t)$ of the installed project as a function of the current price p_t, we could use the diffusion process of p_t and Ito's Lemma to obtain the diffusion

[2] Assume for a moment that the units of size are discrete, in this case, $G(p_t,R_t) = \Delta G(p_t,R_t) + \Delta G(p_t,R_t+1) + \Delta G(p_t,R_t+2) + \ldots$ where, given the current size of R_t, $\Delta G(p_t,R_t)$ is the value of the option of expanding the size by one unit, that is reaching the size R_t+1. $\Delta G(p_t,R_t+1)$ is the value of the option of expanding the size from R_t+1 to R_t+2 and so on. These options must be exercised sequentially, so the total value of the firm's options to expand is $G(p_t,R_t) = \sum_{j=R_t}^{\infty} \Delta G(p_t,j)$ (PINDYCK, 1988: 971). With these incremental units becoming infinitesimally small, we have: $G(p_t,R_t) = \int_{R_t}^{\infty} \Delta G(p_t,z)dz$.

process of $V_t(p_t, R_t)$. From that, we could find the value F of the option to invest in the project as a function of V. However, it would be difficult to solve the differential equation linking F and V, as the drift and diffusion parameters of V are complicated expressions. Following Dixit and Pindyck (1994: 182), we use an alternative approach. We determine the value of the option to invest as the function of the price, $F_t(p_t)$. Then we use the solution for $V_t(p_t, R_t)$ given by (4.2.7) as the boundary condition that holds at the optimal price threshold.

The option to invest, which can be seen as the value of the inactive firm, takes into account the possibility to postpone the investment. While the value of the installed project depends on the size R_t, the value of the option to invest does not depend on this size, because, by definition, the option to invest only has a value when the project is not installed, i.e. it has no size. As long as the investment is not undertaken, holding the option to invest yields no cash flow, thus the only return it yields is its capital appreciation. The value is thus given by:

$$F_t(p_t) = e^{-\rho dt} E[F_t(p_{t+dt})] \tag{4.2.8}$$

Using Ito's lemma, $F_t(p_t)$ satisfies the following differential equation

$$\frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 F_t(p_t)}{\partial p^2} + \alpha p_t \frac{\partial F_t(p_t)}{\partial p} - \rho F_t(p_t) = 0 \tag{4.2.9}$$

The particular solution is $F_t(p_t) = A_1 p_t^{\beta_1} + A_2 p_t^{\beta_2}$. This solution is valid over the range of prices for which it is optimal to hold the option. This range is defined by boundary conditions. One natural boundary condition is 0. Since $p = 0$ is an absorbing barrier, the option to invest has no value for very small values of p_t. This indicates that the constant A_2, corresponding to the negative root β_2, must be equal to zero:

$$F_t(p_t) = A_1 p_t^{\beta_1} \tag{4.2.10}$$

where A_1 is a constant to be determined.

The other boundary of that region is $p^*(R)$, the price at which it is optimal to exercise the option (i.e. to invest). This boundary can be described as a "free boundary" (Dixit and Pindyck, 1994: 109 and 141). Indeed, $p^*(R)$ is endogenous and must be determined simultaneously with $F(p)$. It means that A_1 will be determined as a part of the solution, simultaneously with threshold $p^*(R)$ above which it is optimal to invest. This is the object of the next section.

4.2.3 Threshold for the initial investment $p^*(R)$

We look for the threshold price $p^*(R)$ above which ($p_t > p^*(R)$) it is optimal to invest in the project. The investment strategy is determined by the following trade-off. On one hand, investing later saves the interest on the investment cost $I + \kappa R$. On the other hand, investing now yields an immediate cash flow plus the opportunity to expand the size (so to increase the cash flow) but eliminates the opportunity to avoid losses if the market price decreases. This investment strategy satisfies a value matching condition and a smooth pasting condition.

The value matching condition indicates that at the threshold price $p^*(R)$ the firm is indifferent between investing in a project of size R, and not investing, that is: $F(p^*(R)) = V(p^*(R), R) - \kappa R - I$. The smooth pasting condition is given by $\frac{\partial F(p^*(R))}{\partial p} = \frac{\partial V(p^*(R),R)}{\partial p}$. From (4.2.10) and (4.2.7), we have[3]:

$$A_1 p^*(R)^{\beta_1} = B_1(R) p^*(R)^{\beta_1} + sR\left(\frac{(1-\psi)p^*(R)}{\rho-\alpha} - \frac{c+\tau R}{\rho}\right) - \kappa R - I \tag{4.2.11}$$

$$\beta_1 A_1 p^*(R)^{\beta_1 - 1} = \beta_1 B_1(R) p^*(R)^{\beta_1 - 1} + sR\frac{(1-\psi)}{\rho-\alpha} \tag{4.2.12}$$

From (4.2.11) and (4.2.12), we have:

$$p^*(R) = \frac{\beta_1(\rho-\alpha)}{(\beta_1-1)(1-\psi)}\left(\frac{\kappa}{s} + \frac{c+\tau R}{\rho} + \frac{I}{sR}\right) \tag{4.2.13}$$

The interpretation of $p^*(R)$ is the following: the intermediary is willing to spend the initial investment cost $I + \kappa R$ in order to invest in a project of size R when the market price reaches the threshold $p^*(R)$.

4.2.4 Threshold curve $p^{\#}(R)$ and optimal size $R^*(p)$

We look for the price threshold $p^{\#}(R)$ that must be reached in order to extend the size up to R.[4] The threshold satisfies two boundary conditions: a value matching condition and a smooth pasting condition (Dixit and Pindyck, 1994: 364).

The value matching condition is given by $\partial V_t(p_t, R_t)/\partial R = \kappa$. From (4.2.7), this is equivalent to

$$\frac{\partial B_1(R_t)}{\partial R} p_t^{\beta_1} + s\left(\frac{(1-\psi)p_t}{\rho-\alpha} - \frac{c+2\tau R_t}{\rho}\right) = \kappa \tag{4.2.14}$$

When the firm decides to expand the size up to $R_t + 1$, it gives up the option $\Delta G(p_t, R_t)$, because once exercised, the option is dead (Pindyck, 1988: 971) and so on for the following units. Equation (4.2.14) says that the firm should expand the size until the value of marginal unit of size is equal to the cost of this marginal unit: the purchase cost κ and the opportunity cost $\Delta G(p_t, R_t) = -(\partial B_1(R_t)/\partial R) p_t^{\beta_1}$ (Pindyck, 1988: 972).

The smooth pasting condition is given by $\partial^2 V_t(p_t, R_t)/\partial R \partial p = 0$. From (4.2.7), this is equivalent to:

$$\beta_1 \frac{\partial B_1(R_t)}{\partial R} p_t^{\beta_1 - 1} + \frac{s(1-\psi)}{\rho-\alpha} = 0 \tag{4.2.15}$$

[3] Note that the value of A_1 is found by substituting (4.2.13) in (4.2.12) and that $B_1(R)$ will be determined in the following section.

[4] The fact that $\partial^2\Pi/\partial R^2 < 0$ ensures that the marginal unit of size R can be treated independently independently of any other unit (Dixit and Pindyck, 1994: 366).

These boundary conditions give us the curve of thresholds $p^{\#}(R)$ that must be reached to expand the size up to R:

$$p^{\#}(R) = \frac{\beta_1(\rho - \alpha)}{(\beta_1 - 1)(1 - \psi)} \left(\frac{\kappa}{s} + \frac{c + 2\tau R}{\rho} \right) \tag{4.2.16}$$

This threshold curve for the incremental investments $p^{\#}(R)$ has to be distinguished from the threshold for the initial investment $p^{*}(R)$ given by (4.2.13) . When the price reaches $p^{\#}(R)$, the intermediary (who is already installed) is willing to spend the investment cost κ in order to increase the size until R.

This threshold curve allow us to determine the optimal size $R^{*}(p_t)$ such that, at the market price p_t, the firm is just indifferent between extending the capacity by one unit or not. This optimal size is given by the inverse of the thresholds curve (4.2.16). Precisely,

$$R^{*}(p_t) = \frac{1}{2\tau} \left(\frac{\beta_1 - 1}{\beta_1} \frac{\rho}{\rho - \alpha} (1 - \psi) p_t - \frac{\kappa \rho}{s} - c \right) \tag{4.2.17}$$

Equations (4.2.14) and (4.2.15) also give $b_1(R_t) = \partial B_1(R_t)/\partial R$. Value matching says that, at the optimal size level, the marginal discounted profit has to be equal to the marginal cost of increasing the size. This cost includes monetary cost κ as well as opportunity cost $-b_1(R_t)p_t^{\beta_1}$. When the firm exercises its option to install the Rth unit of size, it gives up the marginal option value $-b_1(R_t)p_t^{\beta_1}$. With this definition, $b_1(R_t)$ is negative. Replacing p by (4.2.16) in the smooth pasting condition (4.2.15) we obtain:

$$b_1(R_t) = -\frac{s(\beta_1 - 1)^{\beta_1 - 1}(1 - \psi)^{\beta_1}}{\beta_1^{\beta_1}(\rho - \alpha)^{\beta_1}} \left(\frac{\kappa}{s} + \frac{c + 2\tau R_t}{\rho} \right)^{1 - \beta_1} \tag{4.2.18}$$

This allows us to determine $B_1(R_t)$. Indeed, as $B_1(R_t)p_1^{\beta}$ represents the value of the firm's growth options, it is given by the integration of the marginal value $-b_1(R)p_t^{\beta_1}$ (Pindyck, 1988: 972, Dixit and Pindyck, 1994: 365): $B_1(R_t) = \int_{R_t}^{\infty} -\frac{\partial B_1(z)}{\partial z} dz$.

Under the assumption[5] that $\rho > 2\alpha + \sigma^2$, we have:

$$B_1(R_t) = \frac{1}{2(\beta_1 - 2)} \frac{s}{\tau} \frac{(\beta_1 - 1)^{\beta_1 - 1}\rho(1 - \psi)^{\beta_1}}{\beta_1^{\beta_1}(\rho - \alpha)^{\beta_1}} \left(\frac{\kappa}{s} + \frac{c + 2\tau R_t}{\rho} \right)^{2 - \beta_1} \tag{4.2.19}$$

[5] $B_1(R_t) = \int_{R_t}^{\infty} \frac{s(\beta_1 - 1)^{\beta_1 - 1}(1 - \psi)^{\beta_1}}{\beta_1^{\beta_1}(\rho - \alpha)^{\beta_1}} \left(\frac{\kappa}{s} + \frac{c + 2\tau z}{\rho} \right)^{1 - \beta_1} dz$. This integral converges if the power of the integrand is lower than -1. Indeed, $\lim_{z \to \infty} b_1(z) = 0$ only if the the power of z is lower than -1 (in order to get something divided by ∞). This is the case when $\beta_1 > 2$. Assuming that α and σ are given, this condition may be rewritten has $\rho > 2\alpha + \sigma^2$. In other words, the firm has to be impatient enough for the option of expanding the size to be finite. Otherwise, the value of the option to expand the size is infinite and the firm never exercises it.

4.2.5 Investment strategy

The threshold for investment, for any given size, is given by $p^*(R)$ defined by (4.2.13), while the optimal size, for any given market price is given by $R^*(p)$ defined by (4.2.17). The solution to this system of two equations in the two unknowns p and R gives us p^* and R^*, that are the threshold price for the initial investment and the initial optimal size. Above which this threshold ($p_t > p^*$) it is optimal to invest in a project of size R^*.

Substituting (4.2.17) in (4.2.13), we have:

$$p^* = \frac{\beta_1(\rho-\alpha)}{(\beta_1-1)(1-\psi)}\left(\frac{\kappa}{s}+\frac{c}{\rho}+2\sqrt{\frac{\tau I}{s\rho}}\right) \tag{4.2.20}$$

At that level, the firm establishes the plant (which costs I) with a capacity $R^*(p^*)$ (that costs $\kappa R^*(p^*)$). Substituting (4.2.20) in (4.2.17), this initial size is given by:

$$R^* = \sqrt{\frac{\rho I}{\tau s}} \tag{4.2.21}$$

At the following periods, the intermediary expands R when p increases, following the curve $R^*(p)$ given by (4.2.17).

4.3 Discussion

Consider p^* and R^* defined by (4.2.20) and (4.2.21). One may note that these results are the solution to the system of two equations (4.2.13) and (4.2.16) whose curves always intersect at the minimum of $p^*(R)$. It means that R^* is the size that minimizes the threshold for investment.

This size R^* is nothing else that the size which minimizes the average initial cost per unit produced. Consider the total discounted cost $sR\frac{c+\tau R}{\rho}+\kappa R+I$. Intuitively, the threshold for the initial investment $p^*(R)$ depends directly on the average cost per unit produced $\frac{c+\tau R}{\rho}+\frac{\kappa}{s}+\frac{I}{sR}$. In the same way, the threshold for the incremental investment $p^{\#}(R)$ depends directly on the marginal cost per unit produced $\frac{c+2\tau R}{\rho}+\frac{\kappa}{s}$. Before R^*, the marginal cost being lower than the average one, increasing the size allows to decrease the average cost. After R^*, the opposite is true and any increase in the size of the collection area also increases the average cost. Hence, the intermediary chooses the initial size of the collection area as if he was simply cost-minimizer. Doing so, he chooses the initial size such as he is able to make the initial investment as soon as possible.

This result is due to the presence of the option to increase the size. The intermediary's choice for the initial investment is only driven by timing considerations: he may choose a small size in order to invest sooner, knowing that he can expand the size in the future. In the absence of such a possibility, the intermediary would have chosen a larger size as well as a longer delay for the initial investment.

Indeed, assume that the option to increase the size does not exist.[6] Then the optimal

[6] This corresponds to setting $B_1(R)$ equal to zero in the previous sections.

size is fixed: once it has been chosen, it cannot be changed. It is the case for instance when considering building a hotel. The number of rooms is chosen at the time of investment and additional rooms cannot be easily added after the hotel is build. DANGL (1999) and BØCKMAN et al. (2008) consider such models. In this case, the optimal size, for any observed price, is such that the intertemporal profit net from investment cost is maximized, that is:

$$\max_R \; sR\left(\frac{(1-\psi)p}{\rho-\alpha} - \frac{c+\tau R}{\rho}\right) - (I+\kappa R) \tag{4.3.1}$$

The first-order condition is given by:

$$s\left(\frac{(1-\psi)p}{\rho-\alpha} - \frac{c+2\tau R}{\rho}\right) = \kappa \tag{4.3.2}$$

The optimal size, for any given price, is thus:

$$\tilde{R}^*(p) = \frac{1}{2\tau}\left(\frac{\rho}{\rho-\alpha}(1-\psi)p - \frac{\kappa\rho}{s} - c\right) \tag{4.3.3}$$

Comparing (4.2.17) and (4.3.3) we have $\tilde{R}^*(p) > R^*(p)$ as $(\beta_1 - 1)/\beta_1$ (the inverse of the "option value multiple") is lower than 1. The absence of the option to expand the size however does not affect $p^*(R)$, the threshold for initial investment for any given size. The solution to the system of equations (4.2.13) and (4.3.3) is:

$$\tilde{R}^* = \frac{\rho}{2(\beta_1-2)\tau s}\left(\left(\frac{\kappa}{s}+\frac{c}{\rho}\right)s + \sqrt{\left(\frac{\kappa}{s}+\frac{c}{\rho}\right)^2 s^2 + \frac{4\beta_1(\beta_1-2)\tau s I}{\rho}}\right) \tag{4.3.4}$$

Comparing (4.2.21) and (4.3.4) we have that $\tilde{R}^* > R^*$.[7]

When the intermediary has no option to expand the size in the future, he faces a trade-off between investing rapidly in a smaller collection area and waiting in order to invest in a larger project. When he has the option to increase the size, this trade-off disappears, as he can invest rapidly in a small collection area and expand it in the future. In that case, minimizing the threshold for the initial investment is the optimal strategy.

In the absence of the option to expand the size, each change in the external environment of the intermediary may affect the result in two ways. On the one hand, there is an entry threshold effect: the change induces the firm to advance or postpone the initial investment, for any given initial size. The entry threshold effect of a change of the parameter x is given by the first (partial) derivative of (4.2.13) with respect to x: $\partial p^*(R)/\partial x$. It is the effect of x on the threshold for the initial investment in a project of

[7] A sufficient condition for $\tilde{R}^* > R^*$ is $\left(\frac{\kappa}{s}+\frac{c}{\rho}\right)s + \sqrt{\left(\frac{\kappa}{s}+\frac{c}{\rho}\right)^2 s^2 + \frac{4\beta_1(\beta_1-2)\tau s I}{\rho}} > \sqrt{\frac{4(\beta_1-2)^2\tau s I}{\rho}}$ which is always satisfied as $\left(\frac{\kappa}{s}+\frac{c}{\rho}\right)s > 0$ and $\beta_1 > \beta_1 - 2$.

size R. On the other hand, we have an effect on the optimal size: given the current price, the optimal size of the collection area is affected by the external change. This effect is given by the first (partial) derivative of (4.3.3) with respect to x: $\partial\tilde{R}^*(p)/\partial x$ that is the effect of x on the optimal size, for any given p. As both effects go in opposite directions, the total effect is a priori ambiguous. For instance, a higher uncertainty tends to delay the investment for any initial size, but also tends to reduce the optimal size, for any given price. As the threshold for investment is lower when the initial size is smaller, one cannot a priori determine whether the threshold for investment will be higher or lower, and whether the initial size will by larger or smaller.

However, in the presence of the option to expand the size, only the entry threshold effect matters when determining p^* as this price is always the minimum of $p^*(R)$. The effect on R^* depends on the way the parameter affects the curve $p^*(R)$. If the effect is the same for any size, then R^* is not affected. However if the parameter affects the threshold for investment $p^*(R)$ differently depending whether the size is small or large, then the initial size is affected by a change of the parameter. Therefore, there are two questions of interest in terms of economic policy. First, how does the entry threshold react to an external change? Second, is the initial size of the collection area affected? If this is the case, are the effects on p^* and R^* going in the same or opposite directions? Table 4.1 reports these effects for various parameters of interest.

Table 4.1: Effects on the entry threshold and on the optimal size

	Entry threshold effect $\frac{\partial p^*(R)}{\partial x}$	Effect on the optimal size $\frac{\partial R^*(p)}{\partial x}$	Effect on p^* $\frac{dp^*}{dx}$	Effect on R^* $\frac{dR^*}{dx}$
Uncertainty σ	$+$	$-$	$+$	0
Transport cost τ	$+$	$-$	$+$	$-$
Supply s	$-$	$+$	$-$	$-$
Invest. cost I	$+$	0	$+$	$+$
Extension cost κ	$+$	$-$	$+$	0

Derivatives for the total effects are given in Appendix 4.B.

Interestingly, the price volatility has no effect on the initial size of the collection area R^*. In an uncertain context, the intermediary tends to invest later but in the same collection area as in a certain environment. When the uncertainty is high (i.e. σ is large), the expected cash flow from investing is large, as the expected profit is unlimited for p large and limited for p small. However, the value of the option to invest, that is the value of waiting before investing, is also large, thus the opportunity cost of investing is large. The latter effect dominates the first, such that when uncertainty is high the firm postpones investments and waits for p to be higher before investing. For any given size of the collection area, $p^*(R)$ is larger, thus p^* is larger: when the uncertainty increases, the intermediary chooses to postpone the initial investment ($dp^*/d\sigma > 0$). Uncertainty affects the investment threshold $p^*(R)$ independently of the size of the project, hence, whatever is σ, the initial size is always the same ($dR^*/d\sigma = 0$). Note that, while the uncertainty has no effect on the initial size of the project, the optimal size for any given market price is smaller when the uncertainty is higher. Thus, under high uncertainty, the intermediary waits until the market price reaches a threshold such that the initial size is the same as under low uncertainty.

As explained before, milk production in Senegal (as lots of agricultural productions in developing countries) takes place in system that involves low quantity supplied by each farmer and high transport costs. These elements depress farmers' incomes in the absence of intermediaries, but also tend to delay the emergence of such intermediaries. Indeed, both low s and high τ increase p^*. This would not be such a worry if the initial size of the collection area was larger. In this case, the intermediary would simply wait longer in order to make an investment that benefits to more farmers. The results above show it is not the case for transport cost: with large τ, the intermediary tends to have smaller collection area ($\partial R^*(p)/\partial \tau < 0$) as collecting the product on this area is costly. He also waits longer ($\partial p^*(R)/\partial \tau > 0$) as a higher output price is necessary to compensate the higher cost. This effect is even larger for large sizes, such that the initial size is reduced ($dR^*/d\tau < 0$). Hence, the transport costs reduce farmers' access on both sides as the intermediary invests later (p^* increases) and in a smaller collection area (R^* decreases).

The low supply, however, increases the initial size of the collection area R^*. Facing farmers with low individual supply (low s), the intermediary has to wait for a higher market price before that contracting with an additional farmer becomes profitable ($p^{\#}(R)$ is larger). Hence, the optimal size of the collection area for any given market price $R^*(p)$ is lower. In the same way, the intermediary has to wait longer before making the initial investment, for any given initial size $p^*(R)$ is higher. This effect is even more important when considering a small initial collection area. Indeed, it is more difficult to cover the investment cost incurred when the total level of production (sR) is low. Hence, facing low supply, the intermediary invests later (p^* is higher) but in a larger initial size R^* in order to compensate for the low level of production coming from the low individual supply.

An external donor can intervene in favor of the intermediaries with various instruments. We analyze two of them here. First, he can provide aid in the form of a support to the intermediary for the investment. This consists in financing part of the investment cost I. Decreasing I has no effect on the optimal size of the collection area for a given market price ($R^*(p)$). However, it decreases the threshold for initial investment p^*. When the initial investment takes places, the size of the collection area $R^*(p^*)$ is smaller. Nevertheless, when the output market price increases, this size also increases. When the market price eventually crosses the threshold that was relevant before the donor's intervention, the size also reaches a level equivalent to the initial size before intervention. From the farmers' point of view, a donor financing a part of the investment cost helps the intermediary to propose a contract to some of them sooner. For the most distant farmers, however, this intervention has no effect, as the price has to cross the without intervention threshold for them to be included.

Second, the aid for the investment may be allocated to increase the number of farmers included rather than to the initial investment. In this case, the external donor finances a part of κ. This increases the optimal size $R^*(p)$ for any given market price. Moreover, decreasing κ also decreases the threshold for initial investment p^*, while keeping the initial size R^* unchanged. This means that the initial investment takes place sooner, but that no less farmers are included, contrarily to what happens when the donor finances directly the initial investment.

Depending on the donor's objective, one or the other instrument may be more efficient. For instance, when the donor wants to improve remote farmers' participation to the market, helping the intermediary to reduce κ deserves the donor's aim. If he wants to improve market participation for only few farmers, but more rapidly, then the aid for investment is relevant. It can be shown that a benevolent planner who want to maximize the social welfare (defined by the sum of the intermediary's profit and the farmers' income) is willing to advance the initial investment compared to what is chosen by the for-profit intermediary ($p^{SO*} < p^*$).[8] However, the initial size chosen by the for-profit intermediary is socially optimal ($R^{SO*} = R^*$). Financing the initial investment I, an external donor permits to decrease the threshold for the initial investment p^* but the initial size R^* is also reduced. When financing the incremental investments cost κ, R^* is not affected, but the decrease in p^* is expected to be less important. In the following section, we simulate the impacts of the donor's aid, and compare the effects of the two instruments in a cost-benefit analysis perspective.

4.4 Case studies: Senegalese milk sector

Although milk consumption in Africa is still low compared to the rest of the world, dairy products make now part of the consumption habits of most of the African households. In Senegal, the two most consumed dairy products are sour milk (that can be made either with fresh local milk or with imported powder) and milk powder (up to now, only imported). Currently, the Senegalese demand for dairy products is mainly satisfied by imports, mostly from Europe. Indeed, local production only covers 32% of the demand (MINEFI, 2006). This may be explained by three factors. First, most of the Senegalese milk sector is characterized by a pastoral or agro-pastoral system of production. Farmers are distributed in large rural areas, while consumers are concentrated in the main urban centers. The high transport costs prevent the farmers to access the market by themselves. Second, it is often claimed that the cheap imports, sometimes subsidized by the exporter countries, hamper the development of the local sector. Third, as any agricultural product, milk faces high price volatility. The uncertainty it creates tends to reduce the investments in the sector.

Since the nineties, we have seen the emergence of small-scale processing units that play an intermediary role between the farmers and the market. These intermediaries also faces high transport costs, low market price and uncertainty, but are able to support some fixed investment costs that cannot be borne by each farmer alone. Moreover, external donors who want to improve farmers' access to the market provide aid to set-up such firms.

[8] Precisely, $p^{SO*} = \frac{\beta_1(\rho-\alpha)}{(\beta_1-1)}\left(\frac{\kappa}{s} + \frac{c}{\rho} + 2\sqrt{\frac{\tau I}{s\rho}}\right) < p^*$.

We apply our theoretical model to two of these processing firms, as well as to the case of a new project that is still under consideration for investment. For the existing processing units, we want first to assess if the investment already made was appropriate in the sense of our model and second to determine the future growth of the collection area, that is the improvement of remote farmers' access to the market. Regarding the new project, we would like to assess whether it is profitable or not and what would be the collection area if it is appropriate to build such a business. For all the cases considered, we also look at the effects of the potential intervention of external donors interested in improving farmers' market access.

4.4.1 Le Fermier

The small-scale milk processing unit "Le Fermier" is located at Kolda in Southern Senegal, since 1997. This unit produces sour milk and pasteurized milk, using fresh milk from the farmers located in the countryside around Kolda. The products are mainly sold to the consumers in Kolda. Le Fermier is the most important processing unit in the region of Kolda. It treats more than 40% of milk collected in this region (Dieye, 2006: 97). Since 2001, Le Fermier is involved in a loyalty system with the farmers that supply fresh milk. It progressively increases the number of regular suppliers (from 9 villages in 2001 to 12 in 2002 and 15 in 2003).

In what follows we determine the price p^* at which the project should have been established and check that the current market price is actually above this threshold, i.e. that the investment was indeed an appropriate strategy. We also look at the optimal size of the collection area, depending on the market price, and verify that the observed size corresponds to what is predicted by our model.

Bakhoum (2006: 29) estimated that the total investment has cost between 8 and 10 millions F CFA. From that, we assume 8.2 millions were devoted to the creation of the unit ($I = 8200000$) and 1.8 millions to the capacity expansion until 15 villages, that is an investment of 120000 F CFA per village ($\kappa = 120000$).

Dieye (2003: 39) reports that the input transport cost ranges from 10.8 to 29 F CFA per liter. We know that the villages where milk is collected are from 7.39 to 18 km from Kolda (Dieye, 2006: 101 and Dia, 2002: 90-91). We can reasonably assume that the highest transport cost (29 CFA) corresponds to the largest distance (18 km) and the smallest cost (10.8 CFA) to the smallest distance (7.39 km). From that, we assume that input transport cost is approximately 1.5 F CFA per liter per km (we can, with more certainty assert that the average input transport cost lies between 0.6 and 3.9 F CFA per liter per km). The average distance between the villages currently involved in Le Fermier's collection area is 1.5 km. For that reason, we use 1.5 km as unit of distance. Hence the input transport costs is $\tau = 2.25$ F CFA per liter per unit of distance.

In 2001, the price paid to the farmers was 200 F CFA per liter during the wet season and 245 F CFA during the dry season (Dieye, 2003: 40) while the market price ranged between 350 and 450 CFA (Dieye, 2003: 44 and Diao et al., 2002: 226). From that we estimates that the price paid to the farmer is around 55% of the market price, such that we use $\psi = 0.55$.

From the income statement of Le Fermier (Dieye, 2003: 44), we calculate that the operating cost (including the costs of sugar, sachets, gas, electricity as well as the output transport cost) is 119 F CFA per liter. Input transport from the first village (located at 7.39 km) to the plant is independent of the size of the collection area, such that we include this cost (1.5*7.39 = 11) in the operating cost. Thus, we use $c = 130$.

Dia (2002: 34) observed that the individual supply is around 10 liters per day, that we multiply by the average number of producers per village (3.7 from Dia, 2002: 34) to get the village supply. Hence $s = 37$.

We estimate the drift of the milk price distribution process as the mean of the monthly milk price index (IHPC lait) real growth rate[9] in Senegal (from November 2005 to May 2010) divided by 30: $\alpha = 0.0000773$ which correspond to an annual real growth of around 2.8%. As an estimation of σ, we use the square root of the variance of this index, that we divide by 30: $\sigma = 0.001016$ (this corresponds to 0.389 annualy). Finally, we choose $\rho = 0.00018$. This daily interest rate corresponds to a 6.6% annual interest rate.

Table 4.2: Le Fermier

β_1	=	2.30644
$p^{\#}(R)$	=	$292.299 + 10.0728R$
$R^*(p)$	=	$-29.0186 + 0.0992772p$
p^*	=	334.712
R^*	=	4.21067
$R^*(400)$	=	10.6923
$R^*(450)$	=	15.6561

$I = 8200000$, $\kappa = 120000$, $\psi = 0.55$, $c = 130$, $s = 37$, $\tau = 2.25$, $\alpha = 0.0000773$, $\sigma = 0.001016$ and $\rho = 0.00018$.

Main results are given in table 4.2. The option to invest should be exercised when the output market price is above $p^* = 334.712$. At this price, the firm must built an initial collection area of 4 villages ($R^*(p^*) = 4.21067$).

We have noted that the actual market price in 2001 ranges between 350 and 450 CFA. Our calibration shows that, if the price is 400 F CFA, the firm should include 11 villages ($R^*(400) = 10.6923$). This fits with the number of villages actually included by Le Fermier in 2001 (9 villages).

Our results are robust to changes in most of the parameters values. For the parameters α, σ, ρ, τ, κ, I and s, a 10% increase in the value of the parameter affects p^* by less than 0.7%. The effects of c and ψ, however, are more important, a 10% increase in c (resp. ψ) leading p^* to increase by 8.7% (resp. 13.9%).

When choosing to enter at the price $p^* = 334.712$ with an initial size $R^* = 4.21067$, the firm takes into account the possibility of further expand the size, what it actually did in reality. While models with fixed capacity are relevant in some contexts, they

[9] To obtain real growth rate, we deduct the nominal growth rate of the general IHPC index from the nominal growth rate of the specific IHPC lait index. As rates are lower than 5%, this represents a good approximation.

are certainly not in the agricultural production context. Accounting for the possibility to increase the size thus gives us a more accurate view of the firm's optimal strategy. But, more importantly, not taking this possibility into account would lead to unrealistic predictions. Indeed, in a fixed capacity model, we calculate that the intermediary has interest to enter when the market price reaches the threshold $\tilde{p}^* = 1250.15$. In that case the size of the collection area would be $\tilde{R}^* = 190.093$. This is due to the fact that this model consider the trade-off between size and threshold, which does not exist in reality.

Table 4.3: Le Fermier: impact of aid

	I	κ	p^*	R^*	$R^*(450)$
Financing initial investment	6304000	120000	329.487	3.69192	15.6561
			(-1.59%)	(-14.05%)	(0%)
Financing incremental invest.	8200000	0	333.405	4.2167	15.7859
			(-0.39%)	(0%)	(+0.82%)

Cost for the donor in the two cases: 1896000 FCFA (assuming market price is 450 F CFA).

Other parameters: $\psi = 0.55$, $c = 130$, $s = 37$, $\tau = 2.25$, $\alpha = 0.0000773$, $\sigma = 0.001066$ and $\rho = 0.00018$.

Variation with respect to the results without intervention (table 4.2) are reported in parentheses.

An example of the impact of an external donor's intervention is given in table 4.3. Two polices that have the same cost are compared: financing the initial investment (i.e. decreasing I by 1896000 FCFA) and financing the incremental investments (i.e. decreasing κ by 120000 FCFA). The choice of the instrument depends on the donors' objective. If he wants to increase the actual number of villages involved, he has to finance the incremental investment and help the firm to decrease κ. For instance he may set-up an artificial insemination program, exclusively devoted to Le Fermier's suppliers. Or he may finance medical assistance given by the firm to the farmers. If however he wants the firm to invest sooner, it is better for him to finance the initial investment.

4.4.2 La Laiterie du Berger

"La Laiterie du Berger" (LdB) produces dairy products in Richard-Toll (Northern Senegal) since 2006. It buys fresh milk to farmers dispersed on an area of 50 km around the plant. We want to calculate the threshold price p^* at which the project should have been installed, as well as the optimal size of the collection area that should have been established. As the processing unit has been constructed with the aid of external donors, we analyze the impact of this intervention on the investment strategy.

According to the managers of LdB (personal interview, 2009), the costs of transporting, cleaning and testing the raw material are 100 F CFA per liter. From that, we assume that half of the costs comes from transport such that input transport cost is estimated at 50 F CFA per liter. As milk is transported on average on 25 km, we use $\tau = 2$. The transformation at the plant (pasteurization, packaging, etc.) costs 150 F CFA per liter. The output transport cost is estimated from the data of the firm Nestlé that was previously operating in Senegal, as LdB uses the same kind of transport devices, that is a refrigerated truck. Dieye (2006: 36) estimated that Nestlé's transport costs were 135

F CFA per liter from the collection area of Dahra to the consumption center of Dakar at 265 km, that is 0.5 F CFA per liter and per kilometer. As the LdB mainly sells its products at Dakar at 365 km from Richard-Toll, we estimate output transport costs 182.5 F CFA per liter. The operating cost is calculated as the sum of transformation cost, cleaning and testing cost and output transport cost, such that $c = 382, 5$.

The French Agency for Development[10] reports that the initial investment for the LdB was 1100000 euros, that is 7216000000 F CFA. From that, we assume that 1000000 euros are devoted to build the plant ($I = 656000000$) and 100000 to the initial capacity. We estimate the initial size of the collection segment was 25 km[11] such that $\kappa = 2624000$.

Individual supply is assumed to be higher than for Le Fermier, as LdB's farmers are assumed to have more productive cows (due to better feed, inseminations, medical care, etc.). We use $s = 15$ which corresponds to what is observed in reality (personal interview, 2009).

The market price for the products of the LdB is between 750 and 1200 CFA, depending on the volume of the package (personal observations, 2009), while each farmer received 200 F CFA per liter for the milk he provides to the LdB, that is between 17% and 27% of the market price. From that, we use $\psi = 0.2$. This is lower than for the farmers contracting with Le fermier because LdB's providers are more isolated than Le Fermiers' one. Indeed, Kolda is an urban center where fresh milk can be sold directly to the consumers, while Richard-Toll is much more smaller. The closest urban center from Richard-Toll is Saint-Louis located at 120 km from the LdB.

Parameters of the milk price distribution process are calculated similarly to what has been done in the case of Le Fermier: $\alpha = 0.0000773$ and $\sigma = 0.001066$. Also, $\rho = 0.00018$.

Table 4.4: La Laiterie du Berger

β_1	$=$	2.30644
$p^{\#}(R)$	$=$	$521.252 + 5.0364R$
$R^*(p)$	$=$	$-103.497 + 0.198554p$
p^*	$=$	837.224
R^*	$=$	62.7375
$R^*(975)$	$=$	90.0936

$I = 656000000$, $\kappa = 2624000$, $\psi = 0.2$, $c = 382.5$, $\tau = 2$, $s = 15$, $\alpha = 0.0000773$, $\sigma = 0.001066$ and $\rho = 0.00018$.

Table 4.4 summarizes the main results. The firm should invest when the output market price crosses the threshold $p^* = 837.224$. At this price, the firm must build an initial collection area of 60 kilometers ($R^* = 62.7375$). The current market price for the products of the LdB is between 750 and 1200 CFA, depending on the volume of the

[10] http://www.afd.fr/home/presse-afd/projets_emblematiques/pid/1138

[11] Indeed, the number of farmers involved has doubled between 2006 and 2009: from 200 (http://www.rsesenegal.com/portail/main.php?page=projet&id=6) to 400 (personal interview, 2009). So we assume the size of the collection area has also doubled.

package (personal observations, 2009) which we cannot assert is above the investment threshold[12].

Table 4.5: La Laiterie du Berger: impact of actual aid

	I	κ	p^*	R^*	$R^*(975)$
Financing initial investment (cost=356000000)	300000000	2624000	734.929 (-13.91%)	42.4264 (-47.87%)	90.0936 (0%)

$\psi = 0.2$, $c = 382.5$, $\tau = 2$, $s = 15$, $\alpha = 0.0000773$, $\sigma = 0.001066$ and $\rho = 0.00018$.

Variation with respect to the results without intervention (table 4.4) are reported in parentheses.

Nevertheless, the investment took place, mainly thanks to the aid of the French Agency for Development (AFD). We assume external donors provided financial aid for 356000000 F CFA.[13] Table 4.5 shows the impact of this intervention.

Note again that not accounting for the option to increase the size would lead to unrealistic predictions. Indeed, in that case the threshold for the initial investment (with the full cost $I = 656000000$) would have been $\tilde{p}^* = 2277.85$ and the size $\tilde{R}^* = 694.97$.

4.4.3 Powder project at La Laiterie du Berger

Senegalese farmers' organizations and external donors have recently considered the possibility of producing milk powder in Senegal. Indeed, this product is less perishable than fresh milk, and is largely consumed in Dakar. As farmers complain that the local fresh milk suffers from the "unfair" competition from imported milk powder, this has been considered as a way to increase farmers' access to the market. In what follows, we analyze the opportunity of investing in such a project for a firm such as La Laiterie du Berger.

The cost of the spray drying equipment is estimated to 385000 euros, that is $I = 250250000$ F CFA. Transporting powder is less costly than transporting fresh milk, as it does not need refrigerated devices, and needs less space. However, transforming fresh milk into powder is more costly than transforming it in pasteurized or sour milk, what the LdB currently does. This is due to the important electricity consumption of the equipment as well as to the need for a sterile environment. To take this into account, we assume $c = 300$.

Contrarily to sour milk, powder market price is driven by the international price, as the powder currently consumed in Senegal is imported. We estimate the parameters of

[12] Indeed, as we do not know the distribution of market prices between the minimum 750 and the maximum 1200, it is difficult to say if the average price is above 837.224. Nevertheless, we can have some doubts about this.

[13] Indeed, the director of the LdB reports he has to borrow 300000000 F CFA to build the plant (`http://www.rsesenegal.com/portail/main.php?page=projet&id=6`). The remaining amount has been provided by external donors such as the AFD.

the distribution process from the FAO dairy price index[14] (from November 2005 to May 2010): $\alpha = 0.00001186$ and $\sigma = 0.002292$. This correspond to 0.043 and 0.836 annually. For the other parameters, we use similar assumptions as in the previous section.

Table 4.6: Powder project

β_1	$=$	1.50105
$p^{\#}(R)$	$=$	$423.438 + 5.10954R$
$R^*(p)$	$=$	$-82.872 + 0.195712p$
p^*	$=$	1055.48
R^*	$=$	123.699
$R^*(375)$	$=$	0

$I = 250250000$, $\kappa = 2624000$, $\psi = 0.2$, $c = 300$, $s = 15$, $\tau = 2$, $\alpha = 0.00001186$, $\sigma = 0.002292$ and $\rho = 0.00018$.

Main results are given in table 4.6. The option to invest should be exercised when the output market price is above $p^* = 1055.48$. At this price, the firm should invest in the powder technology and devote a collection area of 124 km to the production of powder. This threshold price is a bit higher than the one obtained in the previous section. However, the market price of powder, driven down by imports, is much lower than the one of liquid milk. In 2009 (personal observations), the price for 1kg of powder lied between 2500 and 3500 F CFA. As 8 liters of fresh milk are necessary to make one kilogram of powder, this means that the powder price for the equivalent of 1 liter of milk was between 312.5 and 437.5 F CFA, which is largely under the threshold for initial investment. Even the intervention of an external donor financing all the investment cost is not able to drive p^* under the market price (with $I = 0$, $p^* = 423.438$). From that, it is clear that investing in the powder technology is not an interesting strategy to expand the milk sector in Senegal, given the low market prices.

4.5 Conclusions

In this chapter, we study the investment decisions of an intermediary who buys an input from geographically dispersed farmers and who sells this transformed input on a market characterized by price volatility. Due to the irreversible nature of the investment and to the uncertainty linked to price volatility, we use the real options theory to determine at what price it is optimal for the intermediary to invest as well as the number of farmers who are included in the collection area, both initially and in future periods.

If the possibility to increase the size of the collection area in future periods was not taken into account, the firm would face a trade-off between investing rapidly in a project

[14] To obtain real growth rate, we deduct the nominal growth rate of the general (Senegalese) IHPC index from the nominal growth rate of the FAO dairy price index. As rates are lower than 5%, this represents a good approximation.

that includes a small number of farmers or waiting in order to invest in a larger project. However, when the option to expand the area is taken into account, our results show that this trade-off disappears. Indeed, it is possible to invest rapidly in a small collection area and expand it in the future. In that case, minimizing the delay for the initial investment is the optimal strategy for the investor and the initial size is determined by this strategy.

Higher volatility is shown to postpone the initial investment, while the initial size is not affected by this factor. Hence, under higher uncertainty, the firm postpones the investment until the market price reaches a threshold sufficient for the firm to invest in the same collection area. From the farmers' point of view, higher uncertainty thus means that while the same number of them can initially benefit from the intermediary's entry, this entry takes place later. Note that, for any given market price, higher is the uncertainty, lower is the number of farmers included.

Furthermore, our model shows that the intermediary's entry is delayed by high transport costs. In the context of developing countries, large transport costs crucially explain farmers' low participation to the market, but also tend to decrease intermediaries' entry. While external donors may be unable to reduce transport costs either for the farmer or the intermediary, they may provide some aid for the investment. Our analysis shows that an intervention that reduces the initial investment cost helps the intermediary to enter more rapidly (that is, when the market price is lower) with a smaller collection area. However, at any given market price, the number of farmers included is not affected. An intervention that reduces the expansion costs is less efficient in reducing the delay for investment: it helps the intermediary to invest sooner but the initial collection area is not affected, such that the threshold for initial investment is higher than if the intervention focuses on the initial investment cost. Nevertheless, the reduction of the expansion costs allow to include more farmers at any given market price.

We apply our theoretical model to three case studies in the milk sector in Senegal. The results show that the milk processing unit "Le Fermier" implanted in Kolda (Southern Senegal) since 1997 results indeed from a profitable investment decision. The actual evolution of the number of suppliers involved corresponds to the predictions of our model. Regarding "La Laiterie du Berger" established in Richard-Toll (Northern Senegal) since 2006, the profitability is less obvious. It is likely that the initial investment would not have been profitable without the aid of external donors. Finally, the project of producing milk powder has been recently considered by Senegalese farmers' organizations and external donors. Our analysis shows that such a project is not profitable due to the low price of milk powder, more than to the large investment cost incurred for such a project.

The inclusion of the option to expand the size of the collection area dramatically changes the results, compared to a situation where the intermediary is assumed to choose the optimal size at the moment of the initial investment. None of the considered projects in the Senegalese milk sector would turn to be profitable if the option to expand the size was not considered. A model of fixed size can be relevant for some industries, for instance when considering building an hotel, the number of rooms is chosen at the time of investment and additional rooms cannot be easily add after the hotel is build. In the agricultural sector however, the option to increase the size has to be considered.

The model developed here gives potential avenues for future research.[15] First, in the previous analysis, we consider that the intermediary is unable to decrease the number of farmers included. It means that he takes his investment decisions knowing that his operating profit can become negative if the output price becomes too small. A possible extension of our model consists in the inclusion of an option to decrease the size, or at least to suspend the operation at the remote locations should the output price fall. An alternative is to consider the option to suspend the whole operation, if the operating profit becomes negative. Including such options should decrease the threshold for initial investment by offering a (not costless) protection against negative profit. Second, for simplicity, we only have considered linear transport costs, while it is likely that actual transport costs are convex. More general profit function should be considered in future work. Third, the accuracy of the model could be improved by considering other diffusion processes for the market price than the simple geometric Brownian motion. In particular it does not account for the surges of food imports[16] that many developing countries have recently experienced. Examples[17] are numerous and these episodes seem to be more frequent since 1994 with the implementation of trade liberalization measures under the Uruguay Round Agreements (FAO, 2006). Combining a jump (Poisson) process with the Brownian motion would more closely represent these import surges that decreases dramatically the price with a non zero probability.

[15] In appendices 4.C, 4.D and 4.E, we draw some outlines for possible extensions.

[16] While there is no unique definition of import surge we retain the one given by the WTO Agreement on Safeguards (Article 2): "When a product is imported into a country in such increased quantities, absolute or relative to domestic production, and under such conditions as to cause or threaten to cause serious injury to the domestic industry that produces like or directly competitive products".

[17] For instance, the rise of chicken legs imports in Senegal between 1997 and 2003 have depressed retail prices by 15% (Duteurtre et al., 2005). In 2004 in Philippines, domestically produced onions prices were about one-third of the 1999 level, after sharp increases of imports in 1999, 2001 and 2003 (FAO, 2007a). In Honduras, farm gate prices for rice collapsed by 30 % in 1992, following the 1991 import surge (FAO, 2007b).

Appendices

4.A Appendix A

The differential equation (4.2.5) can be written as:

$$\frac{1}{2}\sigma^2 p^2 V_{pp} + \alpha p V_p - \rho V + \gamma p + \omega = 0$$

where $V_{pp} = \partial^2 V_t(p_t, R_t)/\partial p^2$, $V_p = \partial V_t(p_t, R_t)/\partial p$, $\gamma = sR(1-\psi)$ and $\omega = sR(-c-\tau R)$.

We guess that the particular solution has the following form:

$$V_{part}(p) = C\gamma p + D\omega$$

Plugging into the differential equation:

$$\frac{1}{2}\sigma^2 p^2 0 + \alpha p C\gamma - \rho(C\gamma p + D\omega) = -\gamma p - \omega$$

$$\Leftrightarrow (\alpha - \rho)pC\gamma + \gamma p = \rho D\omega - \omega$$

A possible solution (such as both sides equal zero) is:

$$C = \frac{1}{\rho - \alpha} \quad , \quad D = \frac{1}{\rho}$$

Thus,

$$V_{part}(p) = \frac{p\gamma}{\rho - \alpha} + \frac{\omega}{\rho}$$

Following Dixit and Pindyck (1994: 180), the solution of the homogeneous part of the equation can be expressed as a linear combination of any two independent solutions:

$$V_{homo}(p) = B_1 p^{\beta_1} + B_2 p^{\beta_2}$$

where $\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}$ and $\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}$ are respectively the positive and the negative root of the quadratic equation: $\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - \rho$.

The solution to the equation (4.2.5) is given by the solution to the homogeneous part of the equation to which we add the the particular solution of the full equation:

$$V(p) = B_1 p^{\beta_1} + B_2 p^{\beta_2} + \frac{p\gamma}{\rho - \alpha} + \frac{\omega}{\rho}$$

Replacing γ and ω, we get:

$$V(p) = B_1 p^{\beta_1} + B_2 p^{\beta_2} + \frac{psR(1-\psi)}{\rho - \alpha} + \frac{sR(-c-\tau R)}{\rho}$$

$$V(p) = B_1 p^{\beta_1} + B_2 p^{\beta_2} + sR\left(\frac{p(1-\psi)}{\rho - \alpha} - \frac{(c+\tau R)}{\rho}\right)$$

4.B Appendix B

$$\frac{dp^*}{d\sigma} = \left(\frac{\kappa}{s} + \frac{c}{\rho} + 2\sqrt{\frac{\tau I}{s\rho}}\right) \frac{\rho - \alpha}{1 - \psi} \frac{-1}{(\beta_1 - 1)^2} \frac{\partial \beta_1}{\partial \sigma} > 0$$

$$\frac{dp^*}{dI} = \frac{\beta_1(\rho - \alpha)}{(\beta_1 - 1)(1 - \psi)} \sqrt{\frac{\tau}{s\rho I}} > 0$$

$$\frac{dp^*}{d\kappa} = \frac{\beta_1(\rho - \alpha)}{(\beta_1 - 1)(1 - \psi)} \frac{1}{s} > 0$$

$$\frac{dp^*}{d\tau} = \frac{\beta_1(\rho - \alpha)}{(\beta_1 - 1)(1 - \psi)} \sqrt{\frac{I}{\tau s\rho}} > 0$$

$$\frac{dp^*}{ds} = \frac{\beta_1(\rho - \alpha)}{(\beta_1 - 1)(1 - \psi)^2} \left(\frac{-\kappa}{s^2} + \frac{-1}{s^2}\sqrt{\frac{\tau I s}{\rho}}\right) < 0$$

$$\frac{dR^*}{d\sigma} = 0$$

$$\frac{dR^*}{dI} = \frac{1}{2I}\sqrt{\frac{\rho I}{\tau s}} > 0$$

$$\frac{dR^*}{d\kappa} = 0$$

$$\frac{dR^*}{d\tau} = \frac{-1}{2\tau}\sqrt{\frac{\rho I}{\tau s}} < 0$$

$$\frac{dR^*}{ds} = \frac{-1}{2s}\sqrt{\frac{\rho I}{\tau s}} < 0$$

4.C Appendix C: Inclusion of a suspension option

We have noted that, because the price is variable over time, we can have periods during which the operating profit is negative. In the framework of the previous analysis, once he has invested, the intermediary has no possibility to avoid these periods of operating losses. Nevertheless, it would be optimal for him to suspend the operation during these periods. In particular, it would be optimal to suspend the collection of the agricultural product from the most distant farmers. Hence, decreasing the collection cost, he could increase his profit until it becomes positive.

One may assume that the intermediary is able to temporarily suspend the collection at some locations, when they lead to a negative profit. In what follow, we assume the intermediary has the option to suspend, without any cost, the collection from some farmers in case of negative profit. Following Dixit and Pindyck (1994: 186), we consider that the intermediary has to "re-activate" these farmers (still without any cost) as soon as the market price is such that he can make positive profit by collecting their product.

We first determine the optimal strategy regarding the inclusion of an additional farmer, depending on the market price. That is, we determine $p^{\#}(R)$, the threshold price at which it is optimal to incur the investment cost κ in order to include an additional farmer. Then, we determine the optimal strategy regarding the initial investment. That is, we look at $p^*(R)$, the threshold at which it is optimal to spend the cost $I + \kappa R$ in order to launch a project of size R. As in the previous analysis, the initial size R^* and the threshold for the initial investment p^* are determined by the equality between $p^{\#}(R)$ and $p^*(R)$.

Threshold curve $p^{\#}(R)$

Assume the intermediary has the possibility to "suspend" a farmer would the profit become negative. This will affect the intermediary's decision regarding the inclusion of an additional farmer.[18] This additional farmer does not need to be utilized (Pindyck, 1988: 974), in the sense that the intermediary can make the investment necessary to expand the collection area, but is not obliged to collect the agricultural product from all the farmers in that area. The profit generated by the marginal farmer is given by $\partial \Pi_t(p_t, R_t)/\partial R_t$ where $\Pi_t(p_t, R_t)$ is given by (4.2.2). If this is positive, then the intermediary buys the agricultural product from the incremental farmer, however, if it is negative, the farmer is just kept idle for some time. Thus the profit flow yield by the considered farmer is given by:

$$\pi_t(p_t, R_t) = \begin{cases} s\left((1-\psi)p_t - c - 2\tau R_t\right) \text{ if } p_t \geq (c + 2\tau R_t)/(1-\psi) \\ 0 \text{ if } p_t < (c + 2\tau R_t)/(1-\psi) \end{cases} \tag{4.5.1}$$

The value of this installed incremental farmer is given by:

$$v_t(p_t, R_t) = \pi_t(p_t, R_t)dt + E_t[v_{t+dt}(p_{t+dt}, R_{t+dt})e^{-\rho dt}]$$

From that, we have the following Bellman equation:

$$v_t(p_t, R_t) = \pi_t(p_t, R_t)dt + e^{-\rho dt}E_t[v_{t+dt}(p_{t+dt}, R_{t+dt})] \tag{4.5.2}$$

Approximating $e^{-\rho dt}$ by $1 - \rho dt$, adding and substracting v_t between the square brackets, applying Ito's Lemma to $E_t[dv]$ defined as $E_t[v_{t+dt}(p_{t+dt}, R_{t+dt}) - v_t(p_t, R_t)]$, ignoring terms which are small relative to dt, reorganizing the terms, and finally dividing by dt yields the following non-homogeneous differential equation:

$$\frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 v_t(p_t, R_t)}{\partial p^2} + \alpha p_t \frac{\partial v_t(p_t, R_t)}{\partial p} - \rho v_t(p_t, R_t) + \pi_t(p_t, R_t) = 0 \tag{4.5.3}$$

[18] Hereafter, to keep some consistency with real option theory of incremental investment, we use the terms "to install the farmer" and "the farmer installed" to say that the intermediary spend an investment cost (κ) to include the farmer in the collection area.

In the region where $p_t < (c + 2\tau R_t)/(1 - \psi)$, from (4.5.1), we have $\pi_t(p_t, R_t) = 0$ and only the homogeneous part of the differential equation remains. Thus the general solution is given by

$$v_t(p_t, R_t) = d_1(R_t)p^{\beta_1} + d_2(R_t)p^{\beta_2} \tag{4.5.4}$$

where $d_1(R_t)$ and $d_2(R_t)$ have to be determined using appropriate boundary conditions, $\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}$ and $\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}$. (4.5.4) represents the value of the option to "activate" the farmer located in R_t. However, the likelihood of activating this farmer becomes very small when the market price becomes small, so the value of the option to activate should be zero as p goes to zero. Therefore, the coefficient $d_2(R_t)$, corresponding to the negative root β_2, should be equal to zero.

In the region where $p_t \geq (c + 2\tau R_t)/(1 - \psi)$, from (4.5.1), we have that $\pi_t(p_t, R_t) = s\left((1 - \psi)p_t - c - 2\tau R_t\right)$ and the particular solution to the non-homogeneous differential equation (4.5.3) is given by:

$$v_t(p_t, R_t) = c_1(R_t)p_t^{\beta_1} + c_2(R_t)p_t^{\beta_2} + s\left(\frac{(1 - \psi)p_t}{\rho - \alpha} - \frac{c + 2\tau R_t}{\rho}\right) \tag{4.5.5}$$

where $c_1(R_t)$ and $c_2(R_t)$ have to be determined using appropriate boundary conditions. In (4.5.5), $c_1(R_t)p_t^{\beta_1} + c_2(R_t)p_t^{\beta_2}$ represents the value of the option to "suspend" the farmer located in R_t. However, the likelihood of suspending this farmer becomes very small when the market price becomes large, so the value of the option to suspend should be zero as p goes to infinity. Therefore, the coefficient $c_1(R_t)$, corresponding to the positive root β_1, should be equal to zero.

From (4.5.4) and (4.5.5), the value of the installed marginal farmer is given by:

$$v(p_t, R_t) = \begin{cases} d_1(R_t)p_t^{\beta_1} & if\ p_t < (c + 2\tau R_t)/(1 - \psi) \\ c_2(R_t)p_t^{\beta_2} + s\left(\frac{(1-\psi)p_t}{\rho-\alpha} - \frac{c+2\tau R_t}{\rho}\right) & if\ p_t > (c + 2\tau R_t)/(1 - \psi) \end{cases} \tag{4.5.6}$$

where $d_1(R_t)$ and $c_2(R_t)$ are unknown constants to be determined.

Since the Brownian motion of p_t can diffuse freely across the boundary $(c+2\tau R_t)/(1-\psi)$, the value function cannot change abruptly across it (Dixit and Pindyck, 1994: 188). Equating the values and derivatives of the two component solutions at $p_t = (c + 2\tau R_t)/(1 - \psi)$, we have a system of two linear equations in the two unknowns $d_1(R_t)$ and $c_2(R_t)$:

$$d_1(R_t)\left(\frac{c + 2\tau R_t}{1 - \psi}\right)^{\beta_1} = c_2(R_t)\left(\frac{c + 2\tau R_t}{1 - \psi}\right)^{\beta_2} + s\left(\frac{c + 2\tau R_t}{\rho - \alpha} - \frac{c + 2\tau R_t}{\rho}\right) \tag{4.5.7}$$

$$\beta_1 d_1(R_t)\left(\frac{c + 2\tau R_t}{1 - \psi}\right)^{\beta_1 - 1} = \beta_2 c_2(R_t)\left(\frac{c + 2\tau R_t}{1 - \psi}\right)^{\beta_2 - 1} + s\frac{1 - \psi}{\rho - \alpha} \tag{4.5.8}$$

Solving this system gives us the value of $d_1(R_t)$ and $c_2(R_t)$. As we will need this result later on also, the result for $c_2(R_t)$ is:

$$c_2(R_t) = \frac{\beta_1}{\beta_1 - \beta_2}\left(\frac{1 - \psi}{c + 2\tau R_t}\right)^{\beta_2} s(c + 2\tau R_t)\left(\frac{1}{\rho} - \frac{\beta_1 - 1}{\beta_1(\rho - \alpha)}\right) \tag{4.5.9}$$

Following DIXIT and PINDYCK (1994: 189), one may check that the last term in (4.5.9) is positive, hence $c_2(R)$ is positive and decreasing in R, for any given value of R.

Knowing $c_2(R_t)$ and $d_1(R_t)$, we know the value v of the marginal installed farmer as a function of the current price p_t (given by (4.5.6)). In order to find the price threshold at which it is optimal to invest in the additional farmer, we define the value of the option to invest as the function of the price, $f_t(p_t, R_t)$ and use the solution for $v_t(p_t, R_t)$ as the boundary condition that holds at the optimal price threshold.

The option to invest, by definition, has a value when the investment has not yet been undertaken. As long as the investment is not undertaken, holding the option to invest yields no cash flow, thus the only return it yields is its capital appreciation. The value is thus given by:

$$f_t(p_t, R_t) = e^{-\rho dt} E_t[f_{t+dt}(p_{t+dt}, R_{t+dt})] \tag{4.5.10}$$

Using Ito's lemma, $f_t(p_t, R_t)$ satisfies the following differential equation

$$\frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 f_t(p_t, R_t)}{\partial p^2} + \alpha p_t \frac{\partial f_t(p_t, R_t)}{\partial p} - \rho f_t(p_t, R_t) = 0 \tag{4.5.11}$$

The particular solution is $f_t(p_t, R_t) = b_1(R_t)p_t^{\beta_1} + b_2(R_t)p_t^{\beta_2}$. This solution is valid over the range of prices for which it is optimal to hold the option. This range is defined by some boundary conditions. One natural boundary condition is 0. Since $p = 0$ is an absorbing barrier, the option to invest has no value for very small values of p_t. This indicates that the constant $b_2(R_t)$, corresponding to the negative root β_2, must be equal to zero:

$$f_t(p_t, R_t) = b_1(R_t)p_t^{\beta_1} \tag{4.5.12}$$

The other boundary of that region is $p^{\#}(R)$, the price at which it is optimal to exercise the option (i.e. to invest in the incremental farmer located in R). This boundary can be described as a "free boundary" (DIXIT and PINDYCK, 1994: 109 and 141). Indeed, $p^{\#}(R)$ is endogenous and must be determined simultaneously with f. It means that $b_1(R_t)$ will be determined as a part of the solution, simultaneously with threshold $p^{\#}(R)$ above which it is optimal to invest to install the marginal farmer.

At this threshold, two boundary conditions, a value matching condition and a smooth pasting condition (DIXIT and PINDYCK, 1994: 364), have to be satisfied. The value matching condition indicates that at the threshold price $p^{\#}(R)$ the firm is indifferent between investing to install the farmer located in R, and not investing for that, that is: $f(p_t, R_t) = v(p_t, R_t) - \kappa$. The smooth pasting condition is given by $\partial f(p_t, R_t)/\partial p = \partial v(p_t, R_t)/\partial p$. From (4.5.6) and (4.5.12), this is equivalent to

$$b_1(R)p^{\beta_1} = c_2(R)p^{\beta_2} + s\left(\frac{(1-\psi)p}{\rho - \alpha} - \frac{c + 2\tau R}{\rho}\right) - \kappa \tag{4.5.13}$$

$$\beta_1 b_1(R)p^{\beta_1 - 1} = \beta_2 c_2(R)p^{\beta_2 - 1} + \frac{s(1-\psi)}{\rho - \alpha} = 0 \tag{4.5.14}$$

where $c_2(R)$ is given by (4.5.9). There is no reason to incur the investment cost only to keep the farmer idle for some time. This is why, in equation (4.5.6) we use the solution for v in the region where the marginal farmer is active.

Together, (4.5.13) and (4.5.14) define the price threshold curve $p^{\#}(R)$ which gives us for each R the critical price that has to be reached in order for the collection area to be optimally expanded till R. It is implicitly defined by:

$$Q(p(R)) \equiv \frac{\beta_1 - \beta_2}{\beta_1} c_2(R) p^{\beta_2} + \frac{\beta_1 - 1}{\beta_1} s \frac{1-\psi}{\rho - \alpha} p - s \frac{c + 2\tau R}{\rho} - \kappa = 0 \tag{4.5.15}$$

where $c_2(R)$ is given by (4.5.9).

As $\beta_1 - \beta_2 > 0$, $\beta_1 > 0$ and $c_2(R) > 0$, comparing (4.5.15) with (4.2.16), it can be seen that this threshold $p^{\#}(R)$ is lower than the one previously obtained, for any given R, as (4.2.16) is obtained by setting $c_2(R) = 0$ in (4.5.15). The intermediary is willing to invest sooner in the establishment of any farmer, knowing that he has the possibility to suspend this farmer in case of negative profit.

Threshold for initial investment $p^*(R)$

The value of the installed project is given by:

$$V_t(p_t, R_t) = \Pi_t(p_t, R_t)dt + E_t[V_{t+dt}(p_{t+dt}, R_{t+dt})e^{-\rho dt}]$$

From that, we have the following Bellman equation:

$$V_t(p_t, R_t) = \Pi_t(p_t, R_t)dt + e^{-\rho dt} E_t[V_{t+dt}(p_{t+dt}, R_{t+dt})] \tag{4.5.16}$$

Approximating $e^{-\rho dt}$ by $1 - \rho dt$, adding and substracting V_t between the square brackets, applying Ito's Lemma to $E_t[dV]$ defined as $E_t[V_{t+dt}(p_{t+dt}, R_{t+dt}) - V_t(p_t, R_t)]$, ignoring terms which are small relative to dt, reorganizing the terms, and finally dividing by dt yields the following non-homogeneous differential equation:

$$\frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 V_t(p_t, R_t)}{\partial p^2} + \alpha p_t \frac{\partial V_t(p_t, R_t)}{\partial p} - \rho V_t(p_t, R_t) + \Pi_t(p_t, R_t) = 0 \tag{4.5.17}$$

The particular solution to this non-homogeneous differential equation is:

$$V_t(p_t, R_t) = B_1(R_t) p_t^{\beta_1} + C_2(R_t) p_t^{\beta_2} + sR_t \left(\frac{(1-\psi)p_t}{\rho - \alpha} - \frac{c + \tau R_t}{\rho} \right) \tag{4.5.18}$$

At the time of the initial investment, the interpretation of (4.5.18) is the following. The last term is the expected present value of the profit the intermediary would get if he kept the number of farmers constant at the level R_t forever. $C_2(R_t)p_t^{\beta_2}$ is the value of his options to suspend some farmers in the future. Finally, $B_1(R_t)p_t^{\beta_1}$ is the value of the intermediary's options to install more farmers in the future, that is to expand the collection area further than R_t. There is no reason to incur an investment cost only to keep some farmers idle for some time. Thus, we can assume that the farmers initially included are also active. This is why the option to activate some farmers is not taken into account in (4.5.18).

We look for the threshold price $p^*(R)$ above which ($p_t > p^*(R)$) it is optimal to invest in the project. The value matching condition indicates that at the threshold price $p^*(R)$ the firm is indifferent between investing in a project of size R, and not investing, that is keeping the option to invest, defined, as before, by (4.2.10): $F(p^*(R)) = V(p^*(R), R) - \kappa R - I$. The smooth pasting condition is given by $\frac{\partial F(p^*(R))}{\partial p} = \frac{\partial V(p^*(R),R)}{\partial p}$. From (4.2.10) and (4.5.18), we have:

$$A_1 p^{\beta_1} = B_1(R)p^{\beta_1} + C_2(R)p^{\beta_2} + sR\left(\frac{(1-\psi)p}{\rho-\alpha} - \frac{c+\tau R}{\rho}\right) - \kappa R - I \quad (4.5.19)$$

$$\beta_1 A_1 p^{\beta_1 - 1} = \beta_1 B_1(R)p^{\beta_1-1} + \beta_2 C_2(R)p^{\beta_2-1} + sR\frac{(1-\psi)}{\rho-\alpha} \quad (4.5.20)$$

Using (4.5.19) and (4.5.20), $p^*(R)$ is implicitly defined by:

$$Z(p(R)) \equiv \frac{\beta_1-\beta_2}{\beta_1}C_2(R)p^{\beta_2} + sR\left(\frac{\beta_1-1}{\beta_1}\frac{(1-\psi)}{\rho-\alpha}p - \frac{c+\tau R}{\rho}\right) - \kappa R - I = 0 \quad (4.5.21)$$

where $C_2(R) = \int_0^R c_2(z)dz$ where $c_2(.)$ is defined by (4.5.9).

As $\beta_1 - \beta_2 > 0$, $\beta_1 > 0$ and $C_2(R) > 0$, comparing (4.5.21) with (4.2.13), it can be seen that this threshold $p^*(R)$ is lower than the one previously obtained, for any given R, as (4.2.13) is obtained by setting $C_2(R) = 0$ in (4.5.21). The intermediary is willing to invest sooner, knowing that he has the possibility to suspend the furthest farmers if they become unprofitable.

Investment strategy

The threshold for investment, for any given size, is given by $Z(p(R)) = 0$ defined by (4.5.21), while the optimal size, for any given market price is given by $Q(p(R)) = 0$ defined by (4.5.15). The solution to this system of two equations in the two unknowns p and R gives us p^* and R^*, that are the threshold price for the initial investment and the initial optimal size. The first time the price p reaches this threshold p^*, it is optimal to invest in a collection area of size R^*.

Using the implicit function theorem on (4.5.21), we have: $\frac{dp^*(R)}{dR} = -\frac{\partial Z(p(R))/\partial R}{\partial Z(p(R))/\partial p}$. Using (4.5.15), we have that $\frac{\partial Z(p(R))}{\partial R} = Q(p(R))$. As, at the optimum R^*, $Q(p(R^*))$ has to be equal to zero, we have that the numerator of the above derivative is zero. From this result, one may expect that the result obtained before still holds, that is: R^* is such that $p^*(R)$ is minimized. If this is true, then p^* is lower when the option to suspend the size is taken into account (compared to (4.2.20)). Indeed, as $p^*(R)$ is lower for any given R (as shown in (4.5.21)), and R^* is at the minimum of $p^*(R)$ then $p^* = p^*(R^*)$ is also lower.

4.D Appendix D: Elastic supply function

Assume that the farmer's supply function is increasing in the price received. For simplicity, we assume it follows the linear function $s(w_t) = -a + bw_t$ where w_t is the price paid by the intermediary at period t while $a \geq 0$ and $b > 0$ are given.

The intermediary's operating profit is given by:

$$\Pi(p_t, R_t) = (-a + bw_t)R_t\left(p_t - c - w_t - \tau R_t\right) \tag{4.5.22}$$

The price w_t is endogenously determined at each period by the intermediary as a function of the current output market price, in order to maximize profit:

$$\frac{\partial\Pi(p_t, R_t)}{\partial w_t} = 0 \Leftrightarrow w_t = \frac{1}{2}\left(p_t - c - \tau R_t + \frac{a}{b}\right) \tag{4.5.23}$$

Substituting (4.5.23) in (4.5.22), the intermediary's operating profit is given by:

$$\Pi(p_t, R_t) = \frac{bR_t}{4}\left(p_t - c - \tau R_t - \frac{a}{b}\right)^2 \tag{4.5.24}$$

Threshold for initial investment $p^*(R)$

From (4.5.24), the differential equation (4.2.4) can be written as:

$$\frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 V_t(p_t, R_t)}{\partial p^2} + \alpha p_t \frac{\partial V_t(p_t, R_t)}{\partial p} - \rho V_t(p_t, R_t) + \frac{bR_t}{4}\left(p_t - c - \tau R_t - \frac{a}{b}\right)^2 = 0 \tag{4.5.25}$$

The particular solution to this is given by:

$$V_t(p_t, R_t) = B_1(R_t)p_t^{\beta_1} + \frac{bR_t}{4(\rho - 2\alpha - \sigma^2)}p_t^2 - \frac{bR_t(c + \tau R_t + (a/b))}{2(\rho - \alpha)}p_t + \frac{bR_t(c + \tau R_t + (a/b))^2}{4\rho} \tag{4.5.26}$$

Note that the coefficient corresponding to the negative root is zero as the likelihood to increase the size becomes very small when p goes to zero.

We look for the threshold price $p^*(R)$ above which it is optimal to invest in the project. The value matching condition indicates that at the threshold price $p^*(R)$ the firm is indifferent between investing in a project of size R, and not investing, that is keeping the option to invest, defined, as before, by (4.2.10): $F(p^*(R)) = V(p^*(R), R) - \kappa R - I$. The smooth pasting condition is given by $\frac{\partial F(p^*(R))}{\partial p} = \frac{\partial V(p^*(R), R)}{\partial p}$. From (4.2.10) and (4.5.26), we have:

$$A_1 p^{\beta_1} = B_1(R)p^{\beta_1} + \frac{bR}{4(\rho - 2\alpha - \sigma^2)}p^2 - \frac{bR(c + \tau R + (a/b))}{2(\rho - \alpha)}p + \frac{bR(c + \tau R + (a/b))^2}{4\rho} - \kappa R - I \tag{4.5.27}$$

$$\beta_1 A_1 p^{\beta_1 - 1} = \beta_1 B_1(R)p^{\beta_1 - 1} + \frac{bR}{2(\rho - 2\alpha - \sigma^2)}p - \frac{bR(c + \tau R + (a/b))}{2(\rho - \alpha)} \tag{4.5.28}$$

From (4.5.27) and (4.5.28), we have that $p^*(R)$ is implicitly defined by:

$$Z(p(R)) \equiv$$

$$\frac{\beta_1 - 2}{\beta_1}\frac{bR}{4(\rho - 2\alpha - \sigma^2)}p^2 - \frac{\beta_1 - 1}{\beta_1}\frac{bR(c + \tau R + (a/b))}{2(\rho - \alpha)}p + \frac{bR(c + \tau R + (a/b))^2}{4\rho} - \kappa R - I = 0 \tag{4.5.29}$$

Threshold curve $p^{\#}(R)$

We look for the successive thresholds $p^{\#}(R)$ that must be reached to extend the size until the successive R. These thresholds satisfy two boundary conditions, a value matching condition given by $\partial V_t(p_t, R_t)/\partial R = \kappa$ and a smooth pasting condition is given by $\partial^2 V_t(p_t, R_t)/\partial R \partial p = 0$. From (4.5.26), this is equivalent to

$$\frac{\partial B_1(R)}{\partial R} p^{\beta_1} + \frac{b}{4(\rho - 2\alpha - \sigma^2)} p^2 - \frac{b(c + 2\tau R + (a/b))}{2(\rho - \alpha)} p + \frac{b(c + \tau R + (a/b))^2}{4\rho} + \frac{b\tau R(c + \tau R + (a/b))}{2\rho} = \kappa \tag{4.5.30}$$

$$\beta_1 \frac{\partial B_1(R)}{\partial R} p^{\beta_1 - 1} + \frac{b}{2(\rho - 2\alpha - \sigma^2)} p - \frac{b(c + 2\tau R_t + (a/b))}{2(\rho - \alpha)} = 0 \tag{4.5.31}$$

From (4.5.30) and (4.5.31), we have that $p^{\#}(R)$ is implicitly defined by:

$$Q(p(R)) \equiv \frac{\beta_1 - 2}{\beta_1} \frac{b}{4(\rho - 2\alpha - \sigma^2)} p^2 - \frac{\beta_1 - 1}{\beta_1} \frac{b(c + 2\tau R + (a/b))}{2(\rho - \alpha)} p + \frac{b(c + \tau R + (a/b))^2}{4\rho} + \frac{b\tau R(c + \tau R + (a/b))}{2\rho} - \kappa = 0 \tag{4.5.32}$$

Investment strategy

The threshold for investment, for any given size, is given by $p^*(R)$ defined by (4.5.29), while the optimal size, for any given market price is given by $p^{\#}(R)$ defined by (4.5.32). The solution to this system of two equations in the two unknowns p and R gives us p^* and R^*, that are the threshold price for the initial investment and the initial optimal size. Above which this threshold it is optimal to invest in a project of size R^*.

Using the implicit function theorem on (4.5.29), we have: $\frac{dp^*(R)}{dR} = -\frac{\partial Z(p(R))/\partial R}{\partial Z(p(R))/\partial p}$. Using (4.5.32), we have that $\frac{\partial Z(p(R))}{\partial R} = Q(p(R))$. As, at the optimum R^*, $Q(p(R^*))$ has to be equal to zero, we have that the numerator of the above derivative is zero. Numerical simulations show that, as before, R^* is such that $p^*(R)$ is minimized.

4.E Appendix E: Alternative collection cost function

Hereafter we assume a more general function for the collection cost, precisely $T(R_t) = \tau R_t^{\gamma}$ with $\gamma > 0$. This function covers the cases of convex ($\gamma > 1$) and concave ($\gamma < 1$) transport costs. When $\gamma = 1$, the transport cost is linear, such that the previous analysis is a specific case of the following analysis.

Threshold for initial investment $p^*(R)$

With $T(R_t) = \tau R_t^{\gamma}$, the differential equation (4.2.4) can be written as:

$$\frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 V_t(p_t, R_t)}{\partial p^2} + \alpha p_t \frac{\partial V_t(p_t, R_t)}{\partial p} - \rho V_t(p_t, R_t) + sR_t\left((1 - \psi)p_t - c - \tau R_t^{\gamma}\right) = 0 = 0 \tag{4.5.33}$$

The particular solution to this is given by:

$$V_t(p_t, R_t) = B_1(R_t)p_t^{\beta_1} + sR_t\left(\frac{(1-\psi)p_t}{\rho-\alpha} - \frac{c+\tau R_t^{\gamma}}{\rho}\right) \tag{4.5.34}$$

Note that the coefficient corresponding to the negative root is zero as the likelihood to increase the size becomes very small when p goes to zero.

We look for the threshold price $p^*(R)$ above which it is optimal to invest in the project. The value matching condition indicates that at the threshold price $p^*(R)$ the firm is indifferent between investing in a project of size R, and not investing, that is keeping the option to invest, defined, as before, by (4.2.10): $F(p^*(R)) = V(p^*(R), R) - \kappa R - I$. The smooth pasting condition is given by $\frac{\partial F(p^*(R))}{\partial p} = \frac{\partial V(p^*(R),R)}{\partial p}$. From (4.2.10) and (4.5.34), we have:

$$A_1 p^{\beta_1} = B_1(R)p^{\beta_1} + sR\left(\frac{(1-\psi)p}{\rho-\alpha} - \frac{c+\tau R^{\gamma}}{\rho}\right) - \kappa R - I \tag{4.5.35}$$

$$\beta_1 A_1 p^{\beta_1-1} = \beta_1 B_1(R)p^{\beta_1-1} + sR\frac{1-\psi}{\rho-\alpha} \tag{4.5.36}$$

From (4.5.35) and (4.5.36), we have that $p^*(R)$ is defined by:

$$p^*(R) = \frac{\beta_1(\rho-\alpha)}{(\beta_1-1)(1-\psi)}\left(\frac{\kappa}{s} + \frac{c}{\rho} + \frac{\tau R^{\gamma}}{\rho} + \frac{I}{sR}\right) \tag{4.5.37}$$

One may note that $p^*(R)$ is increasing in γ. When the collection costs are convex, the intermediary waits longer before investing in the project for any given size, compared to the linear case. Indeed, he incurs an additional cost compared to the case of linear collection costs. When collection costs are concave, the intermediary is able to invest sooner.

Threshold curve $p^{\#}(R)$

We look for the successive thresholds $p^{\#}(R)$ that must be reached to extend the size until the successive R. These thresholds satisfy two boundary conditions, a value matching condition given by $\partial V_t(p_t, R_t)/\partial R = \kappa$ and a smooth pasting condition is given by $\partial^2 V_t(p_t, R_t)/\partial R\partial p = 0$. From (4.5.34), this is equivalent to

$$\frac{\partial B_1(R)}{\partial R}p^{\beta_1} + s\left(\frac{(1-\psi)p}{\rho-\alpha} - \frac{c}{\rho} - \frac{(1+\gamma)\tau R^{\gamma}}{\rho}\right) = \kappa \tag{4.5.38}$$

$$\beta_1\frac{\partial B_1(R)}{\partial R}p^{\beta_1-1} + \frac{s(1-\psi)}{\rho-\alpha} = 0 \tag{4.5.39}$$

From (4.5.38) and (4.5.39), we have that $p^{\#}(R)$ is defined by:

$$p^{\#}(R) = \frac{\beta_1(\rho-\alpha)}{(\beta_1-1)(1-\psi)}\left(\frac{\kappa}{s} + \frac{c}{\rho} + \frac{(1+\gamma)\tau R^{\gamma}}{\rho}\right) \tag{4.5.40}$$

One may note that $p^{\#}(R)$ is increasing in γ. When the collection costs are convex, the intermediary waits longer before including an additional farmer, compared to the linear case, as the cost incurred for collecting the product at this location is larger. When collection costs are concave, the intermediary is able to include this additional farmer sooner.

Investment strategy

The threshold for investment, for any given size, is given by $p^*(R)$ defined by (4.5.37), while the optimal size, for any given market price is given by $p^{\#}(R)$ defined by (4.5.40). The solution to this system of two equations in the two unknowns p and R gives us p^* and R^*, that are the threshold price for the initial investment and the initial optimal size. Above this threshold it is optimal to invest in a project of size R^*. Precisely,

$$R^* = \left(\frac{\rho I}{\gamma \tau s}\right)^{\frac{1}{\gamma+1}} \tag{4.5.41}$$

$$p^* = \frac{\beta_1(\rho - \alpha)}{(\beta_1 - 1)(1 - \psi)} \left(\frac{\kappa}{s} + \frac{c}{\rho} + \frac{(1+\gamma)\tau}{\rho} \left(\frac{\rho I}{\gamma \tau s}\right)^{\frac{\gamma}{\gamma+1}}\right) \tag{4.5.42}$$

As expected, R^* is smaller and p^* is higher when γ is larger. When the collection costs are convex (resp. concave), the intermediary invests later (resp. sooner), in a smaller (resp. larger) collection area, compared to the linear case. One may check that, for any value of $\gamma > 0$, R^* is such that $p^*(R)$ (defined by (4.5.37)) is minimized.

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