

# Analysis of Stresses in Vehicle Driveline Systems using a Flexible Multibody Approach

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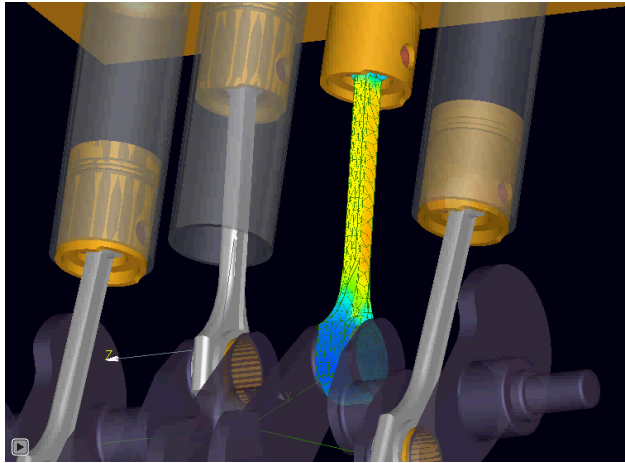
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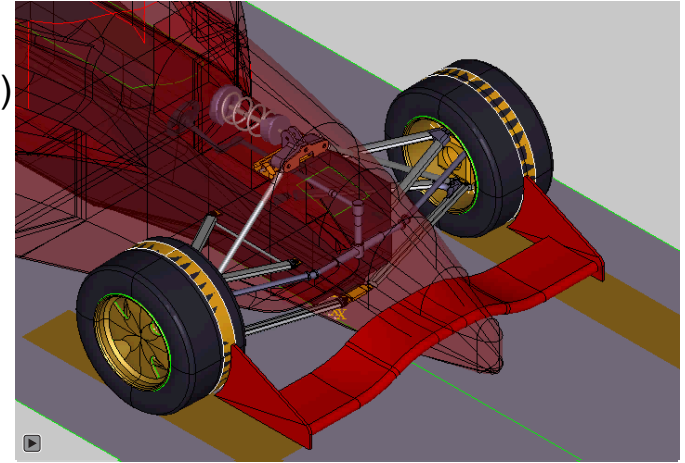
2012-07-10

# Driveline modeling

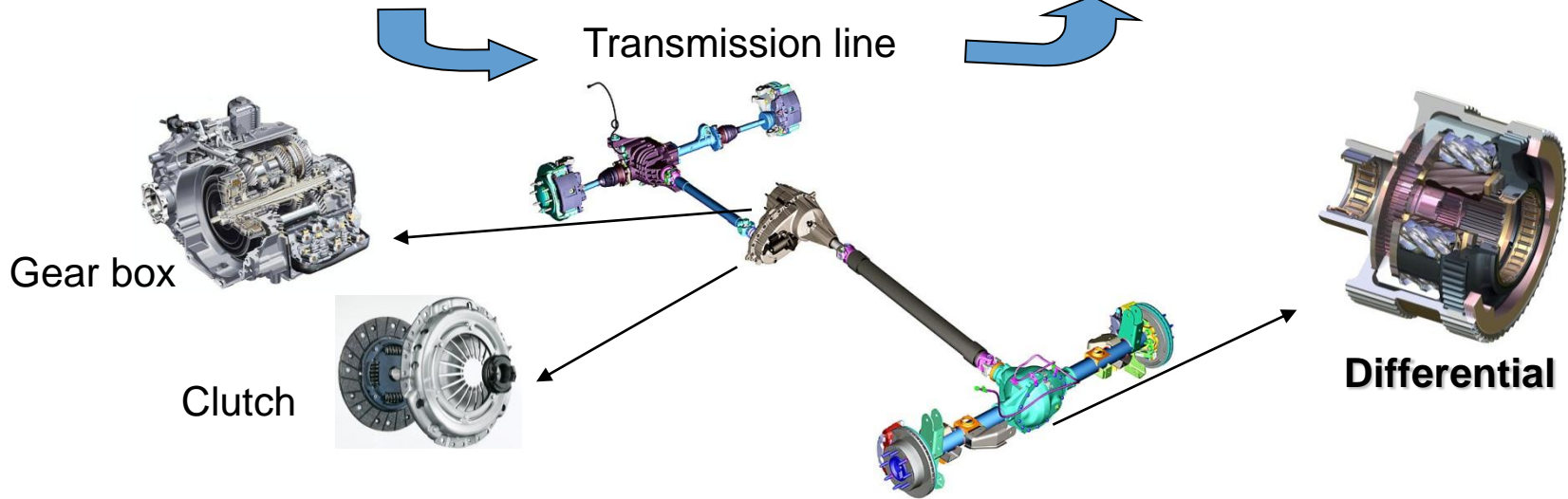


Motor

(Courtesy:  
LMS- SAMTECH)



Vehicle dynamics



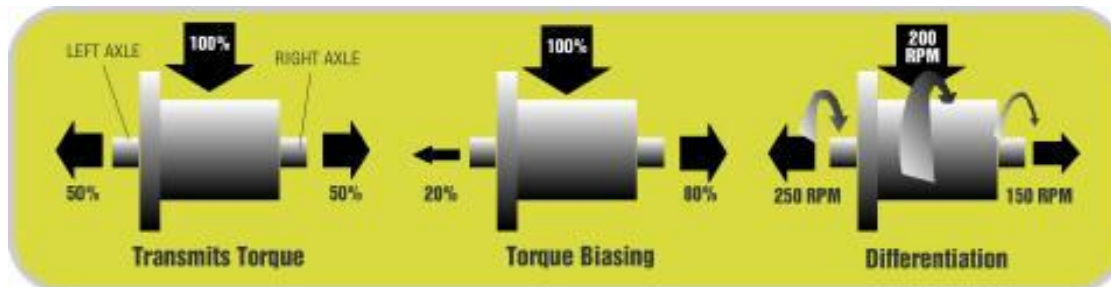
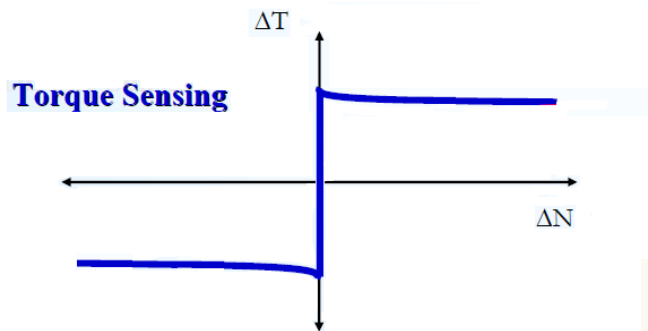
- Complex phenomena involved: backlash, stick-slip, contact, discontinuities, hysteresis, non linearities → Numerical problems

# Outline

- Description of the application: TORSEN differentials
- Gear pair model
- Contact element + lubricating squeeze film formulations
- Model description & numerical results
- Differential included in a full vehicle model
- Conclusion

# TORSEN differential

- Limited slip differential
  - Allow a variable torque distribution between the output shafts
    - avoid spinning when ground adherence is not sufficient on one driving wheel
- Torque transfer before differentiation (torque sensing)
- Full mechanical system

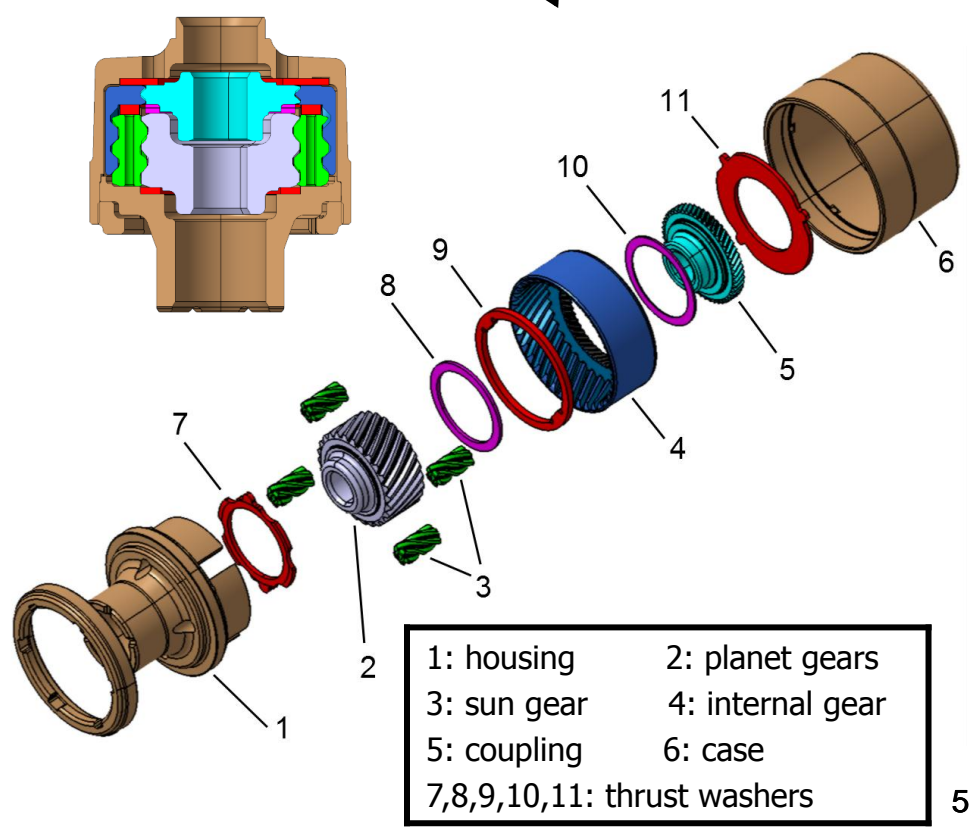
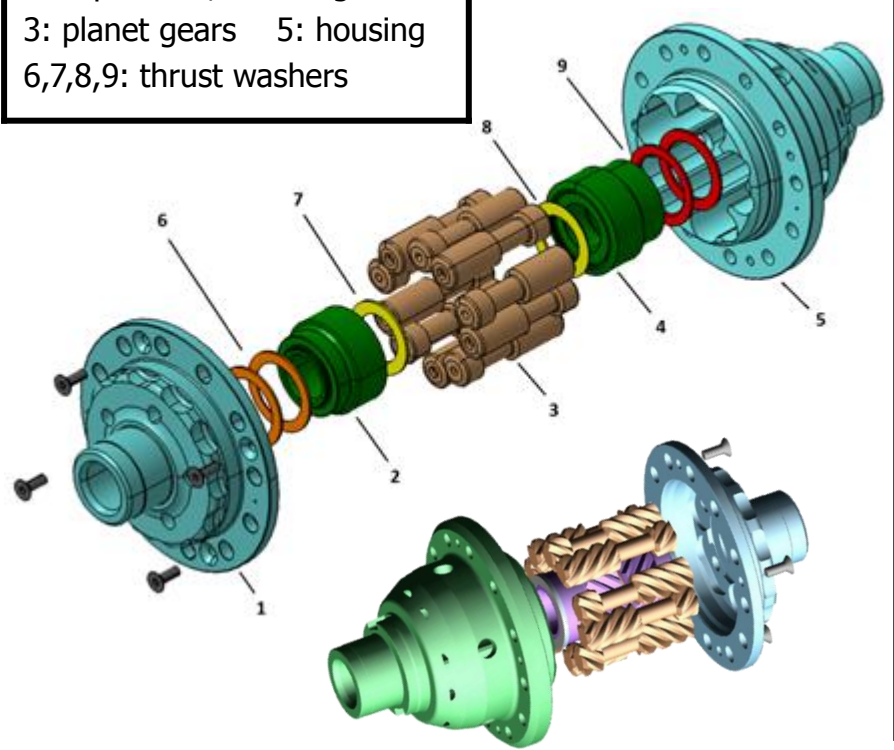




# Type B & C TORSEN differentials

- Housing, helical gear pairs and thrust washers
- Locking due to relative friction gears ↔ washers & gears ↔ housing
- 4 working modes
- Front/rear differential (type B) & central differential (type C)

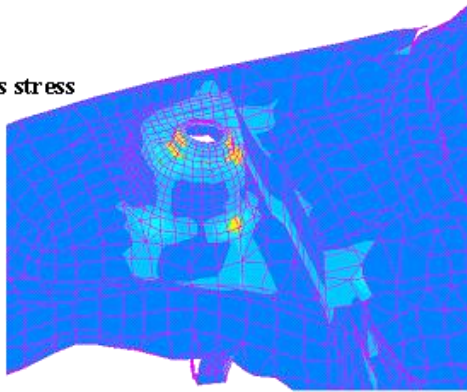
1: cap 2, 4: side gears  
3: planet gears 5: housing  
6,7,8,9: thrust washers



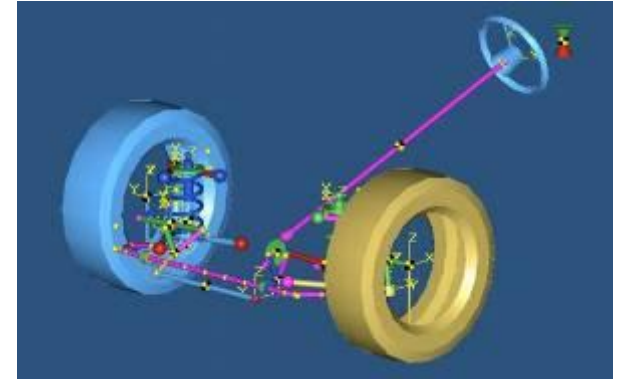
# Evolution of virtual prototyping

- Finite Element: structural analysis of components

von Mises stress



- Multibody system: mechanism of rigid bodies



- Flexible Multibody systems:  
System approach (MBS)  
& structural dynamics (FEM)



# FE coordinates

- Many interactions between transmission components are due to flexibility  
 → nonlinear finite element method based on the absolute nodal coordinates

- Software: Samcef Field/MECANO



- Rigid **and** flexible bodies
- Parametrization of rotations with the cartesian rotation vector + updated Lagrangian approach

- Equations of motion

$$M(q)\ddot{q} + g^{gyr}(q, \dot{q}) + g^{int}(q) + \Phi_q^T (p\Phi + k\lambda) = g^{ext}(t)$$

$$k \Phi(q, \dot{q}, t) = 0$$

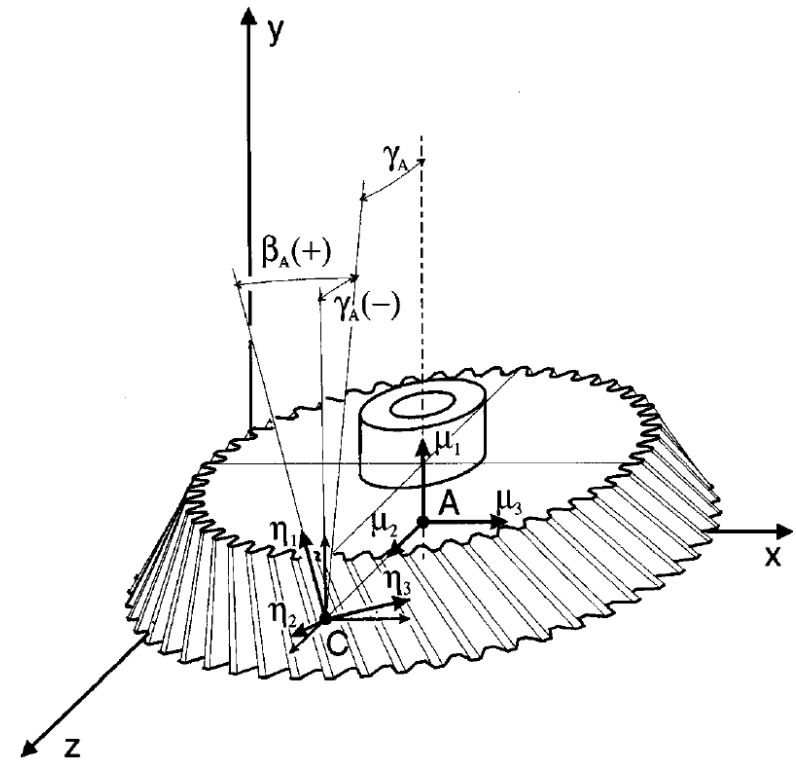
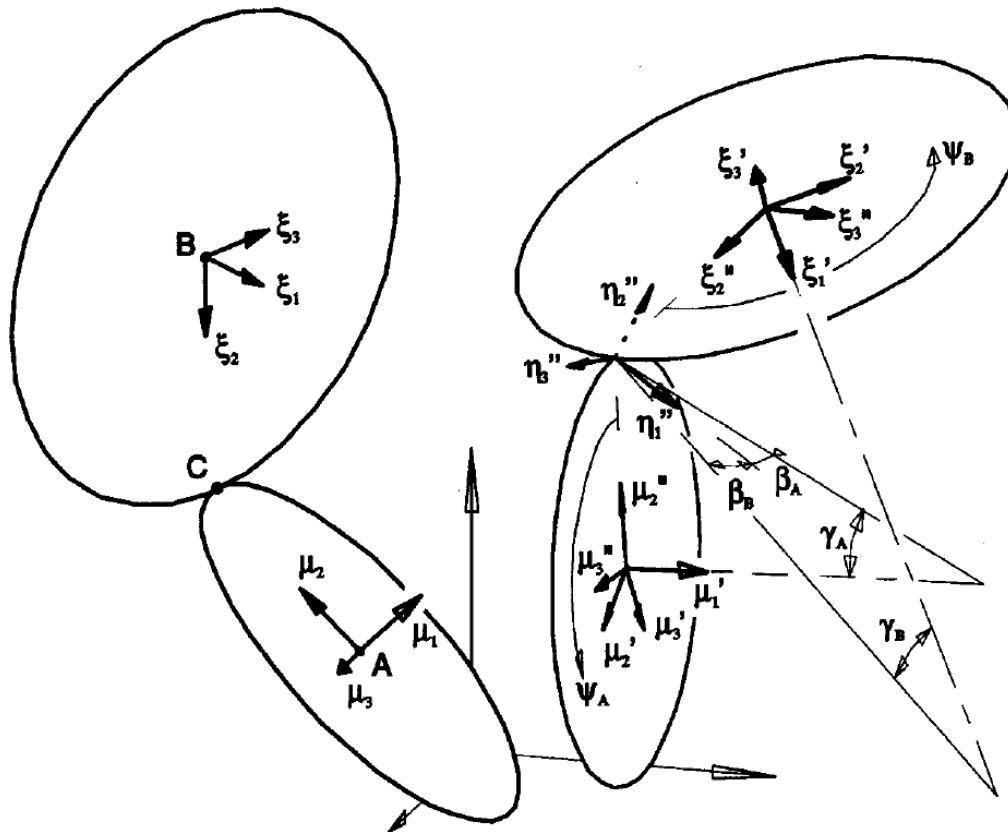
Penalty factor

Scaling factor

Constraints: joints, rigidity

# Gear pair element

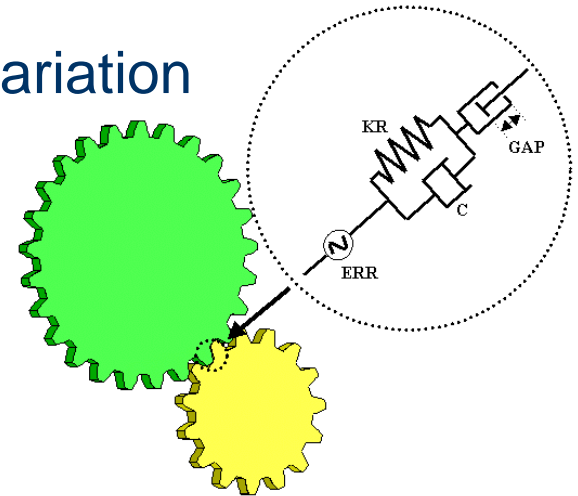
- Flexible joint between two physical nodes: one at the center of each wheel (rigid body).
- Any kind of gear pairs : spur gear, bevel gear, helical gear, worm gears...



(A. Cardona, 1995)

# Gear pair element

- Flexibility : spring (KR) and damper (C)
- Time fluctuation of mesh stiffness due to variation of number of teeth in contact (ISO 6336)
- Backlash (GAP)
- Load transmission error (ERR)
- Misalignment
- 15 variables



$$q = \{ \underline{x_A^T} \quad \underline{\Theta_A^T} \quad x_B^T \quad \underline{\Theta_B^T} \quad \underline{\psi_A} \quad \underline{\psi_B} \quad u_m \}$$

12 dof

3 redundant coordinates

3 constraints

Deformation of the gear mesh in the hoop direction

Normal contact force  $\mathcal{F} = k\lambda_1$

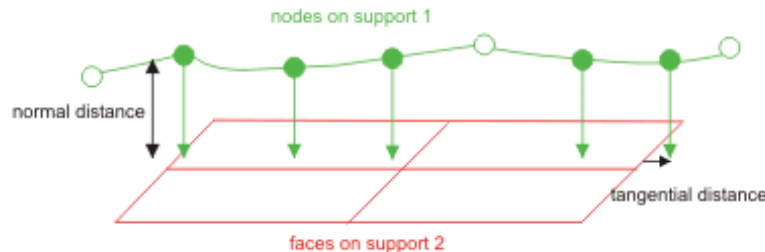
$$\phi_1 = (-\psi_A z_A + \psi_B z_B) \frac{m_n \cos \alpha_n}{2} + u_m \cos \alpha_n = 0$$

$$\phi_2 = (x_C^A - x_C^B) \cdot \eta_3''^A = 0$$

$$\phi_3 = \eta_2''^A \cdot \eta_3''^B = 0$$

# Contact condition

- Augmented lagrangian method
- Flexible/rigid or flexible/flexible contact
- 2 steps : - projection of slave nodes on master surface(s)

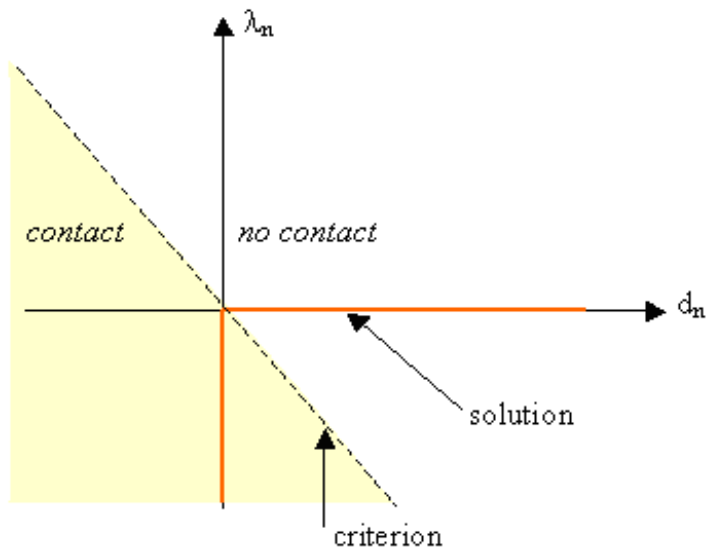


$$\delta d_n = \underline{n}^T B \delta \underline{q}$$

$$\delta \Delta u_1 = \underline{t}_1^T B \delta \underline{q}$$

$$\delta \Delta u_2 = \underline{t}_2^T B \delta \underline{q}$$

- definition of the contact condition



## Contact criteria

$$\sigma_n = k\lambda_n + p d_n$$

$$\sigma_{t_1} = k\lambda_1 + p \Delta u_1$$

$$\sigma_{t_2} = k\lambda_2 + p \Delta u_2$$

(k = scaling factor , p=regularisation parameter)

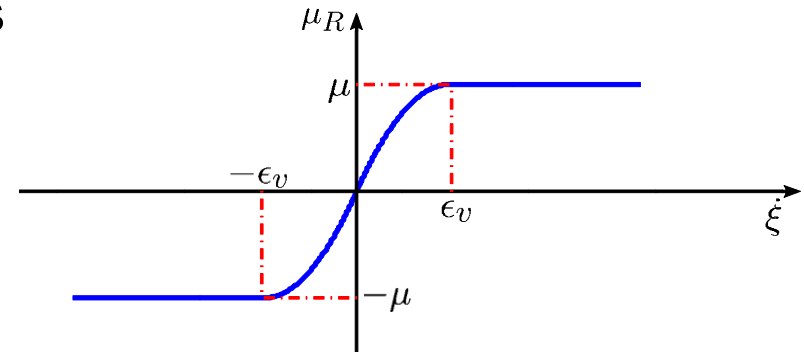
# Contact condition

- Friction

$$F_{fr} = \mu_R |F_{norm}|$$

Regularization to avoid discontinuities

$$\mu_R(\dot{\xi}) = \begin{cases} \mu(2 - \frac{|\dot{\xi}|}{\epsilon_v}) \frac{\dot{\xi}}{\epsilon_v} & |\dot{\xi}| < \epsilon_v \\ \mu \frac{\dot{\xi}}{|\dot{\xi}|} & |\dot{\xi}| \geq \epsilon_v \end{cases}$$



- Contact formulation not adapted in case of high relative axial velocity at contact establishment → impacts problems

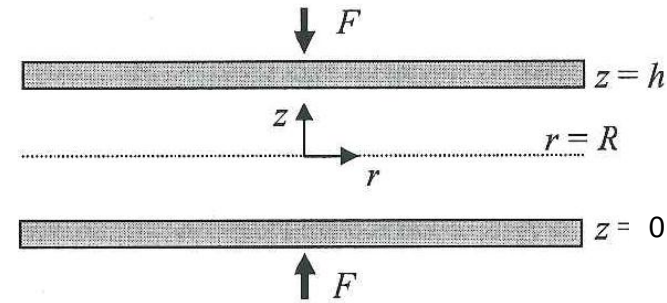
➤ Solutions:

- ✓ Penalty method to allow a small penetration between the two contacting bodies and relax slightly the discontinuity
- ✓ Squeeze film modelling of the lubricating oil

# Squeeze film modelling

- Assumptions:

- 2 plates ( $2h \ll R$ )
- Newtonian fluid
- Isothermic fluid in steady state
- No sliding on walls
- Axisymmetric (cylindrical coordinates)



- Continuity equation 
$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{\partial v_z}{\partial z} = 0$$

- Momentum equation (radial component): inertia terms, elongation gradient and  $\frac{v_r}{r^2}$  neglected)

$$\frac{\partial p}{\partial r} = \mu \frac{\partial^2 v_r}{\partial z^2}$$



# Squeeze film modelling

- Boundary and initial conditions

$$\begin{array}{ll} v_r = 0, & z = 0, h \\ p = p_{atm}, & r = R \end{array} \qquad \begin{array}{ll} \frac{\partial v_r}{\partial z} = 0, & z = 0 \\ h = h_0, & t = 0 \end{array}$$

- Momentum equation  $\rightarrow$  velocity profile

$$\frac{\partial p}{\partial r} = \mu \frac{\partial^2 v_r}{\partial z^2} \quad \xrightarrow{\int \int dz^2} \quad v_r = -\frac{1}{2\mu} \frac{\partial p}{\partial r} [h z - z^2]$$

- Continuity equation  $\rightarrow$  pressure profile

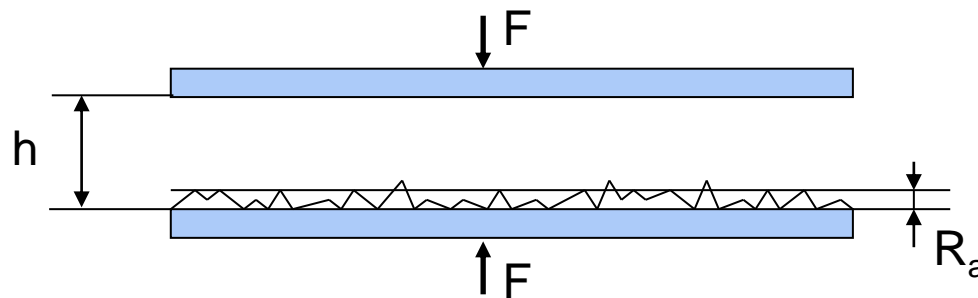
$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{\partial v_z}{\partial z} = 0 \quad \xrightarrow{\int \int dr dz} \quad \frac{\partial p}{\partial r} = 6\mu \frac{\dot{h}}{h^3} r$$

- Force applied on plates

$$F(h, \dot{h}) = -\frac{3\pi\mu R^4 \dot{h}}{2h^3}$$

# Squeeze film modelling

- Thrust washers of TORSEN differentials are rough  
→ contact between the two metallic bodies when  $h < R_a$

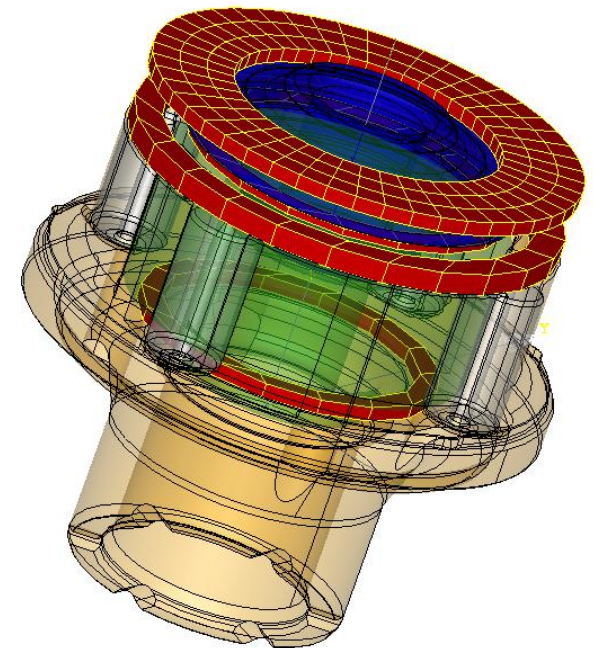
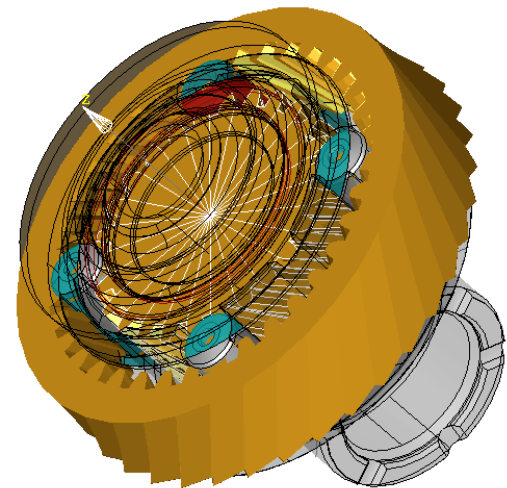


$$F(h, \dot{h}) = \begin{cases} \frac{3\pi\mu R^4 \dot{h}}{2h^3} & \text{if } h > R_a \\ \frac{3\pi\mu R^4 \dot{h}}{2h^3} + F_{\text{contact}(metal-metal)} & \text{if } h < R_a \end{cases}$$

- *Remark:* avoid the bad numerical conditioning of the squeeze film model when  $h, \dot{h} \cong 0$

# Model description

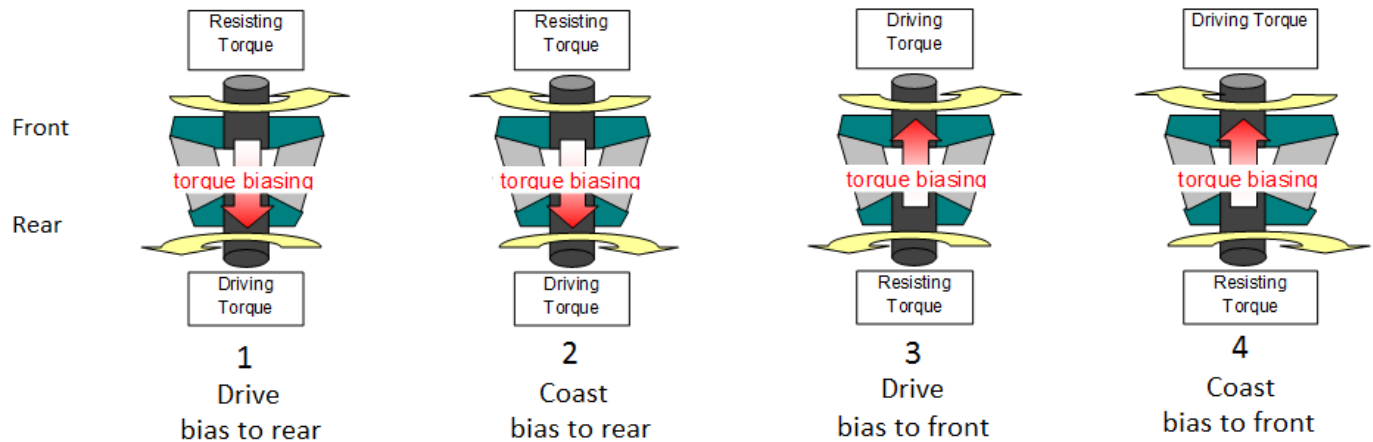
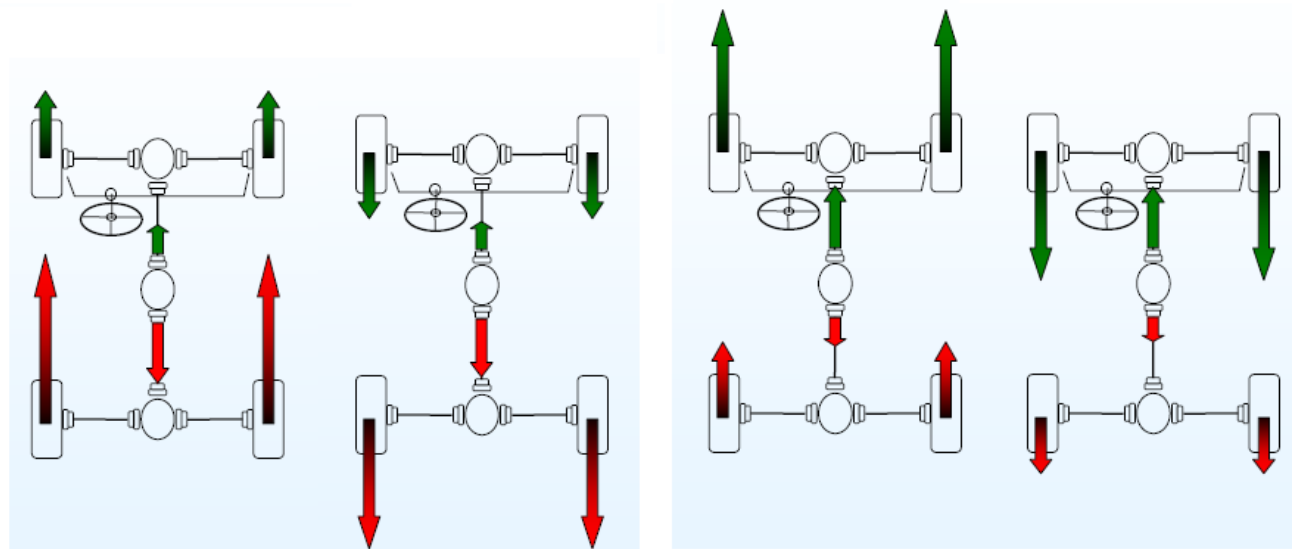
- Assumptions: - joints between Planet gears and housing modeled as cylindrical joints  
- contact SG/washer 3 and CPL/washer 4 neglected
- 18 bodies:
  - 9 rigid: gear wheels, housing
  - 9 flexible: thrust washers, case, driveshafts
- $\approx$  43000 generalized coordinates
- Constraints :
  - 8 gear pair elements
  - 5 contact relations
  - 4 hinges
  - 1 screw joint



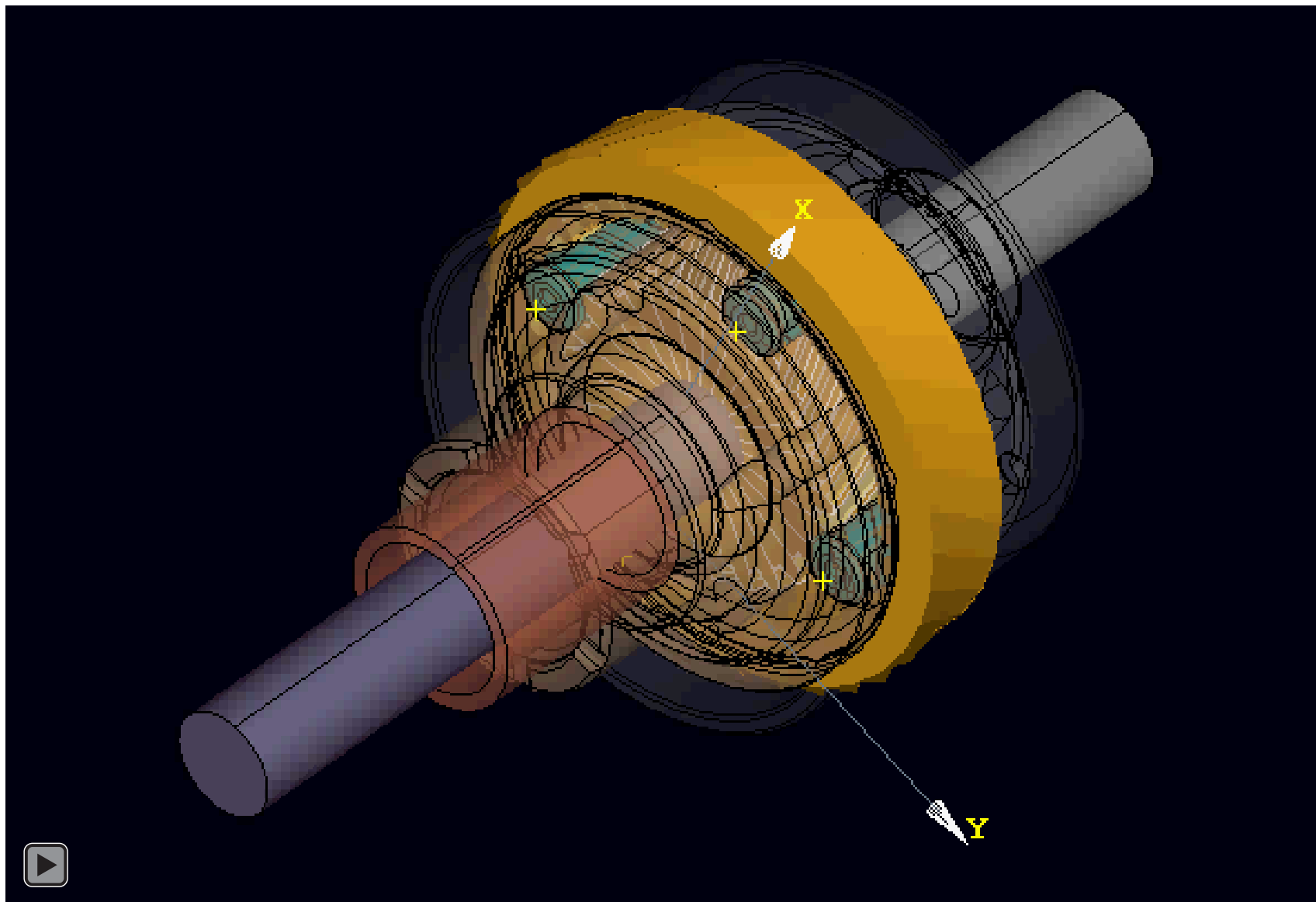
# TDR computation for the 4 locking modes

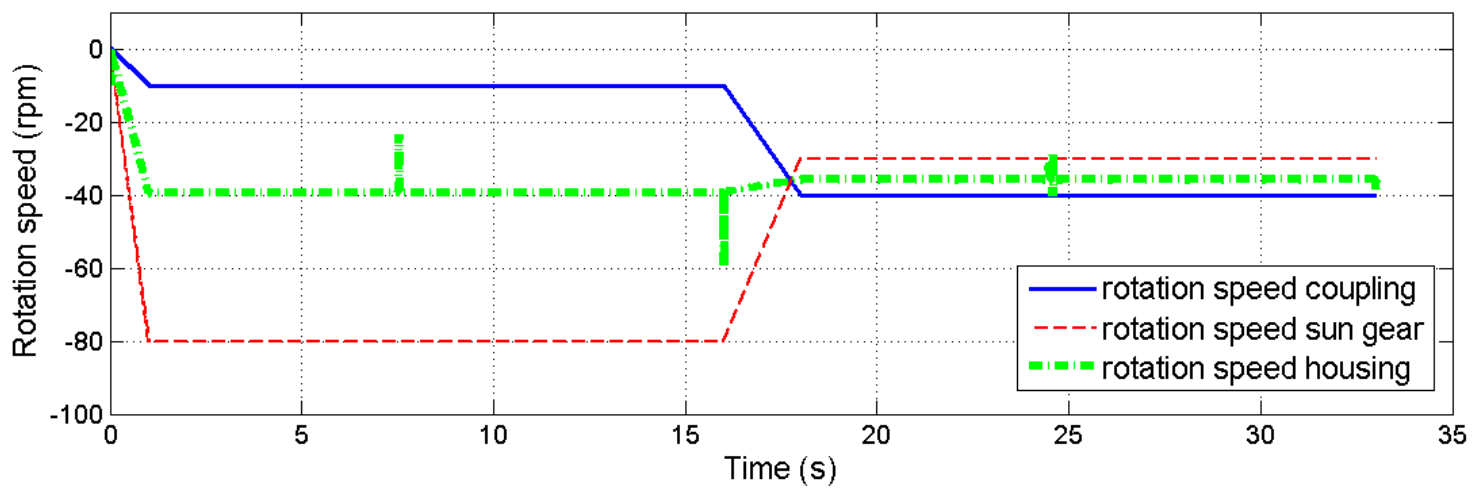
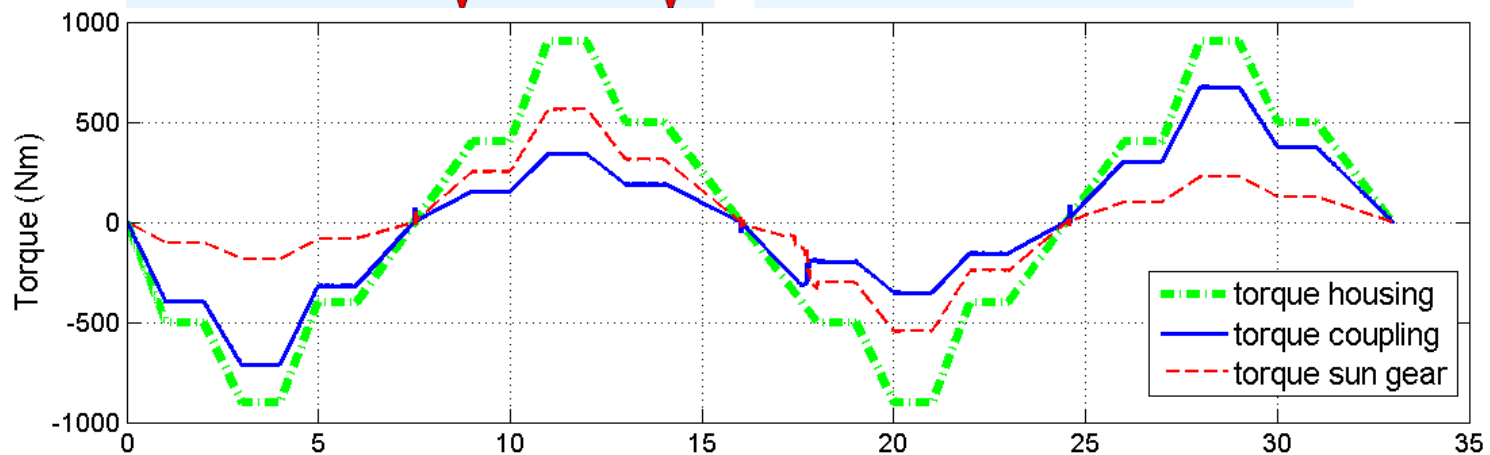
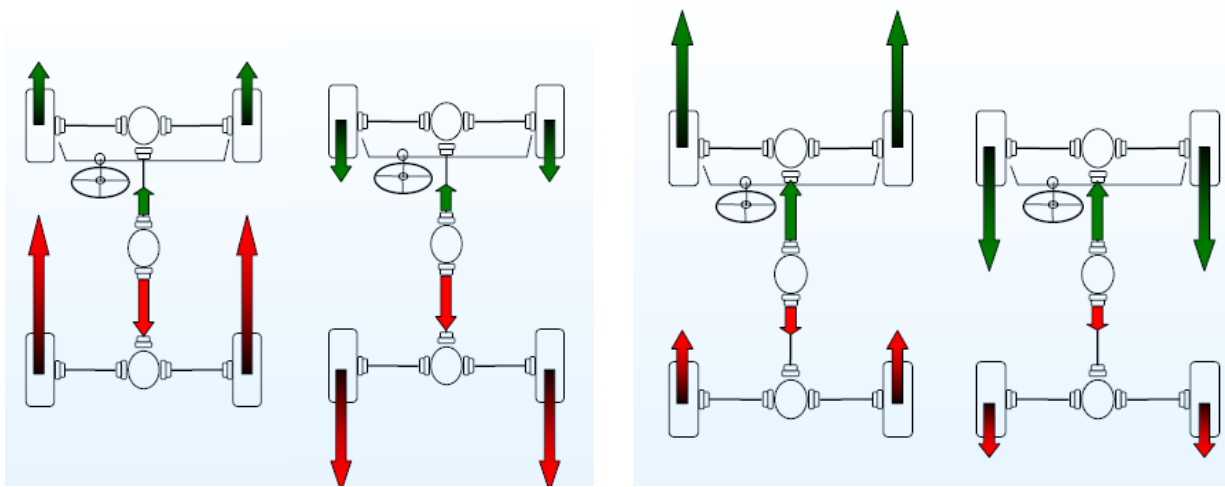
- TDR : Torque Distribution Ratio

$$TDR = \frac{T_1}{T_2}$$

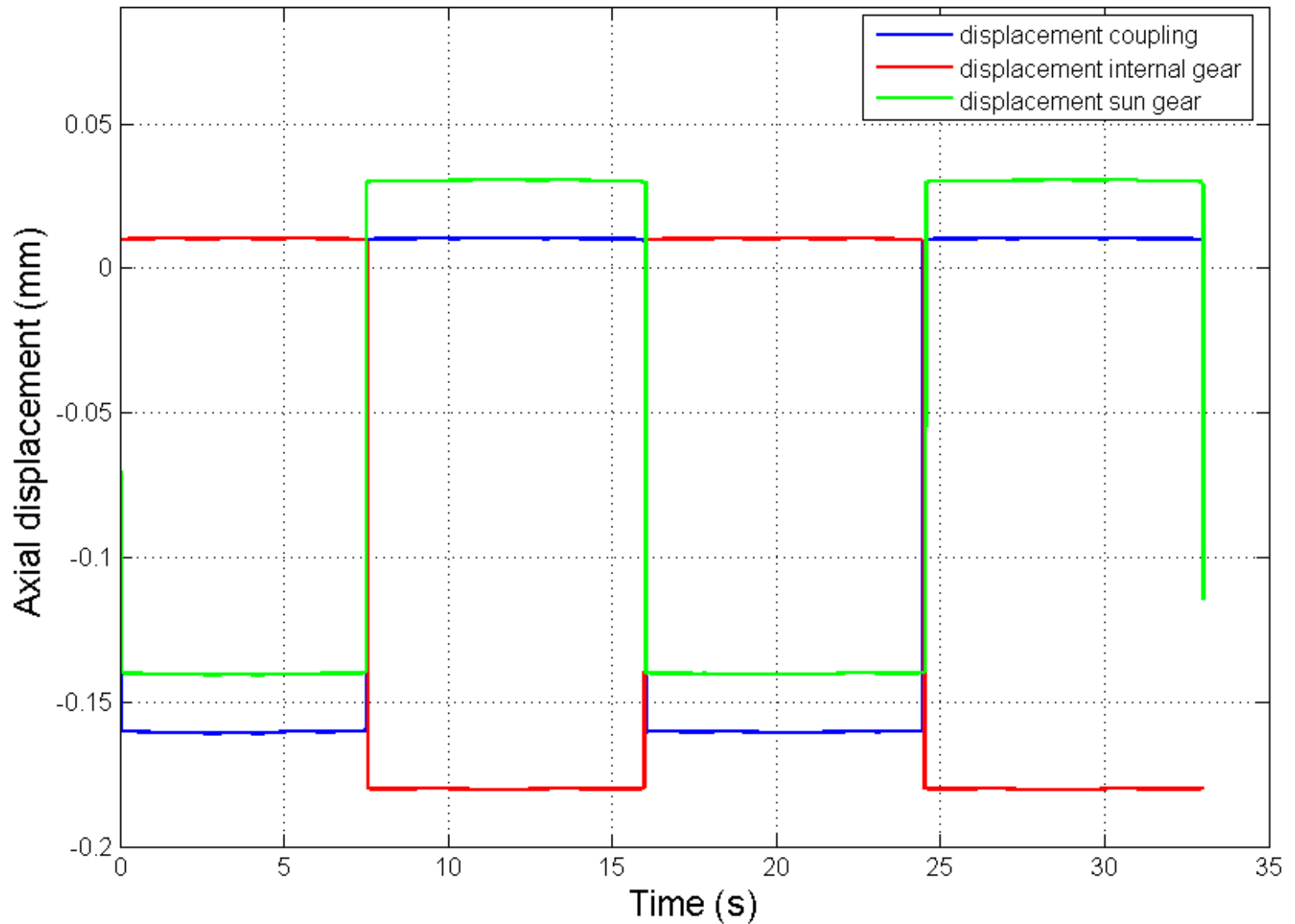


# Configuration on vehicle

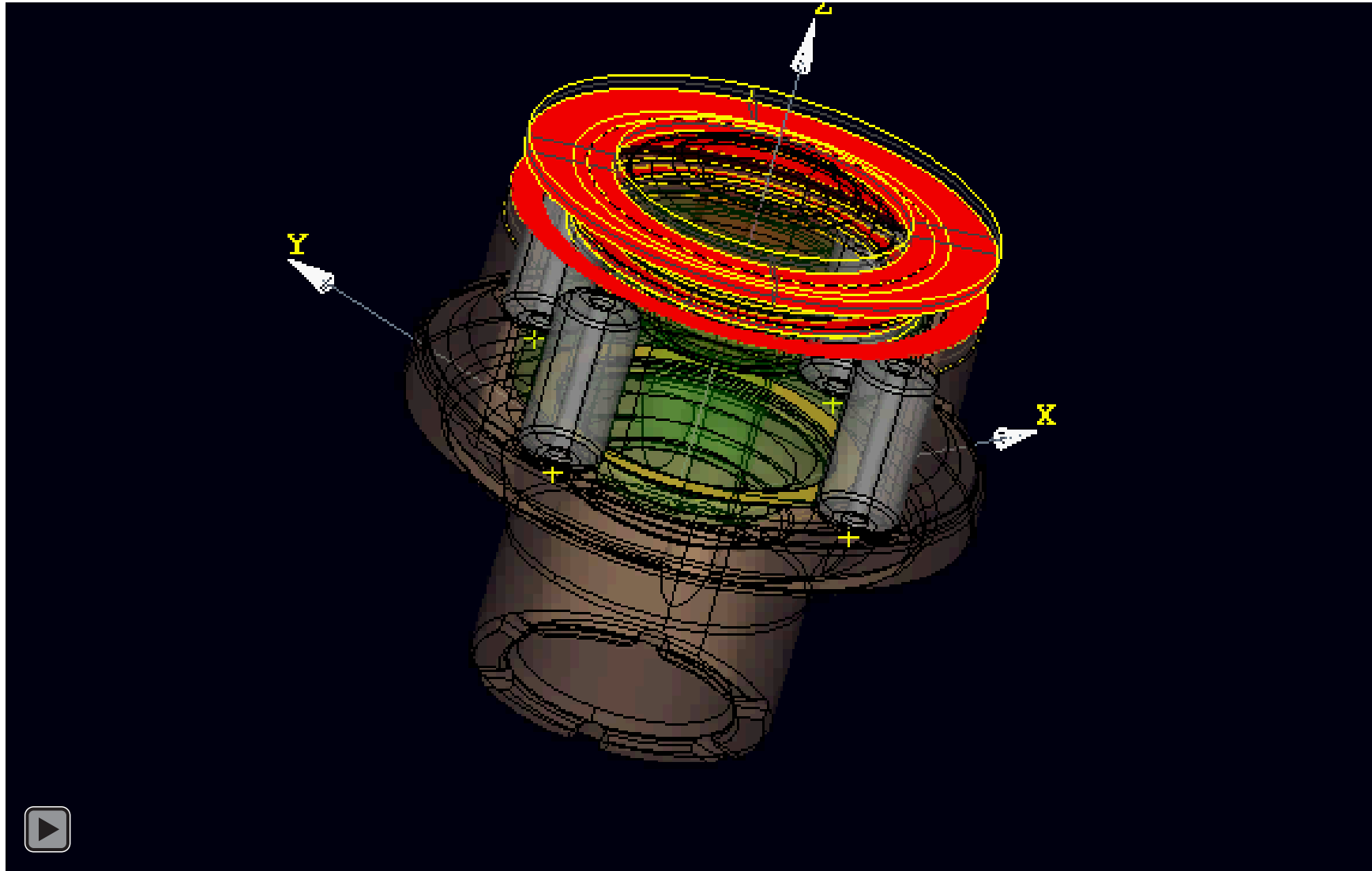




# Axial displacements of gear wheels



# Contact pressure





# Model validation

- TDR comparison for each mode with experimental data

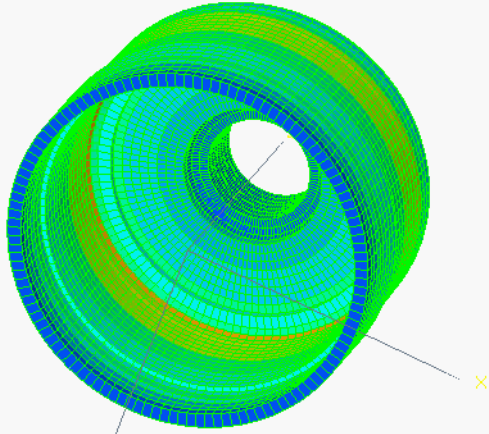
Type C  
(center diff)

<b>TDR</b>	<b>Mode 1</b>	<b>Mode 2</b>	<b>Mode 3</b>	<b>Mode 4</b>
	Drive bias to rear	Coast bias to rear	Drive bias to front	Coast bias to front
experimental	4,02	2,82	1,57	1,62
simulation	3,9	2,94	1,56	1,65
error (%)	2,98	4,25	0,64	1,85

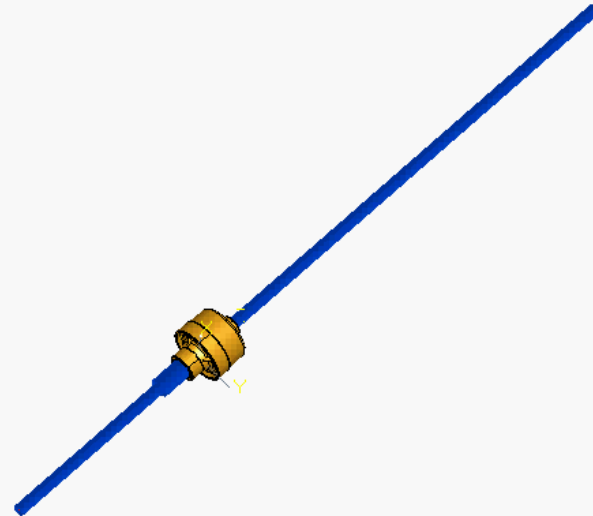
Type B  
(front diff)

<b>TDR</b>	<b>Mode 1</b>	<b>Mode 2</b>	<b>Mode 3</b>	<b>Mode 4</b>
	Drive bias to right	Coast bias to right	Drive bias to left	Coast bias to left
experimental	1,6	1,7	1,6	1,7
simulation	1,58	1,66	1,61	1,64
error (%)	3,20	2,35	0,62	3,53

# Flexibility of driveshafts and case

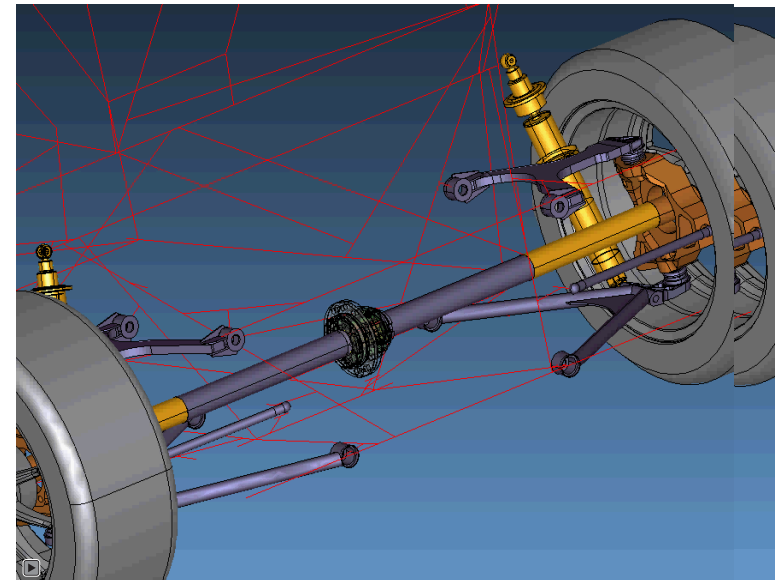
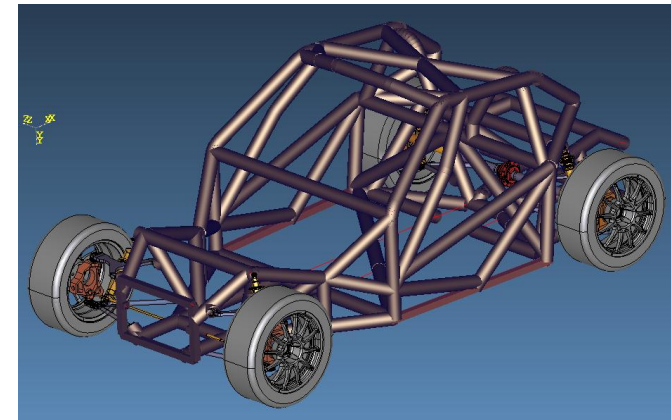


Time evolution of deformations (highly amplified) and equivalent stresses in the case and driveshafts

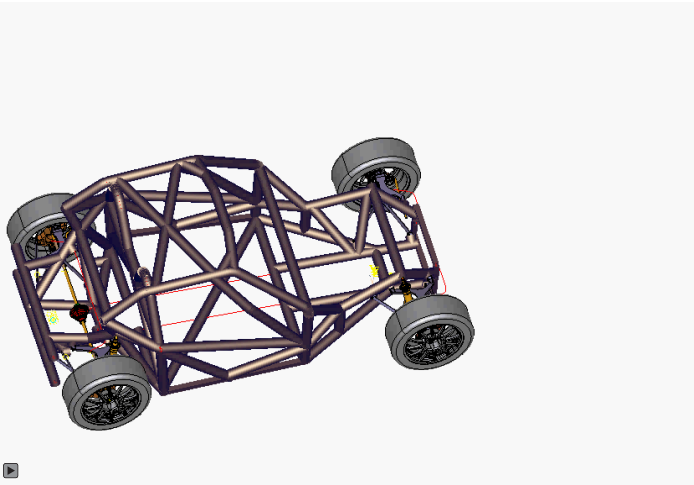


# Differential in vehicle model

- Rear differential included in full vehicle model with:
  - Rigid driveshafts
  - Flexible chassi (beams)
  - Suspensions fixed on chassi with bushings
  - Tyre models (Pacejka)
- TORSEN Type B
  - 20 rigid bodies
  - 20 gear pairs
  - 26 contacts
  - 10 cylindric joints
- 12730 generalized coordinates

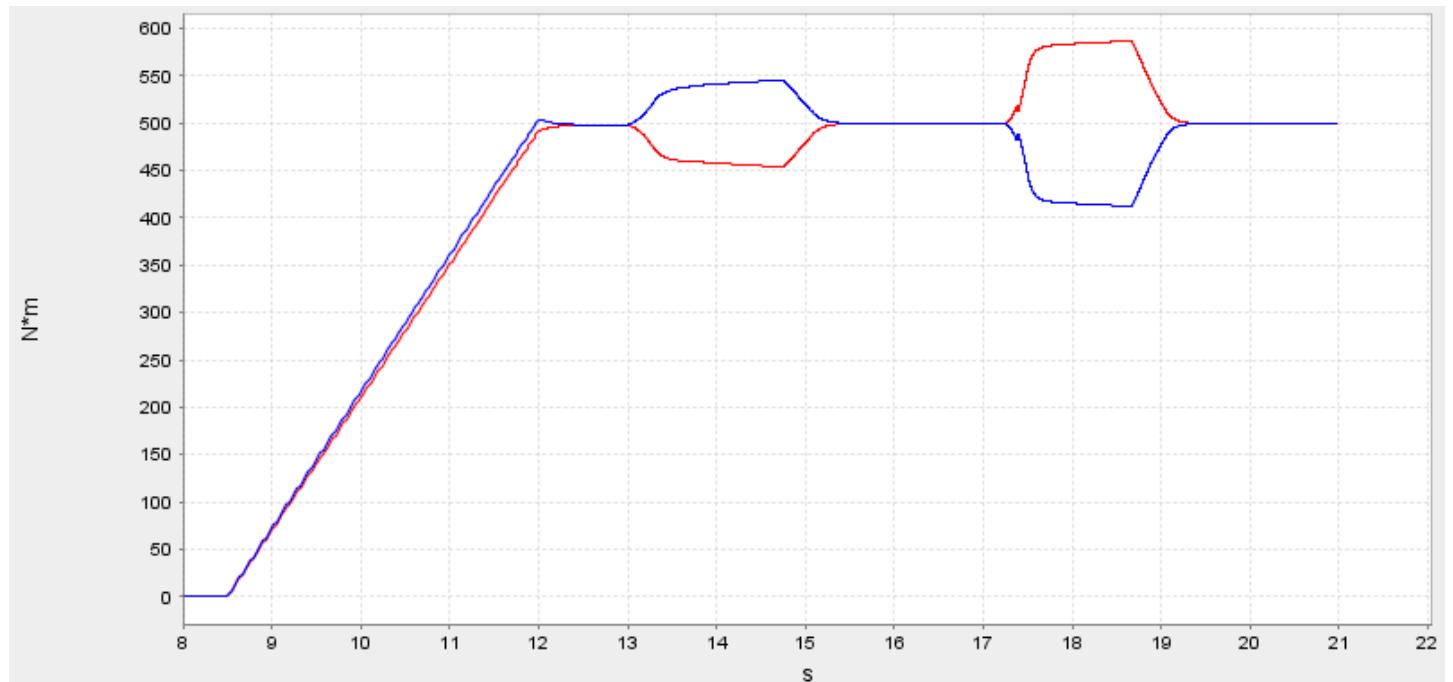


# Vehicle model



- Torque transfer of TORSEN differential when a vehicle accelerates on a slippery surface

Torque on right and left rear wheel



# Conclusion & outlook

- Dynamic TORSEN differentials modelling:
  - Gear pairs and contact condition (+ squeeze film model )
  - Global validation :comparison with experimental data (TDR)
  - Included in a full prototype vehicule model (RWD)
- Specific kinematic joint to model the link between planet gears and housing
- Optimisation of differential case to reduce the weight:

*Topology Optimization of Flexible Components in Multibody Systems:  
Application to the Housing of an Automotive Differential, Emmanuel Tromme*

Session MS-34,2: Optimisation in Nonlinear Solid Mechanics, Wednesday,  
10:49 room K6

Thank you for your attention !

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