Full discontinuous Galerkin formulation of shells in large deformations with parallel and fracture mechanics applications

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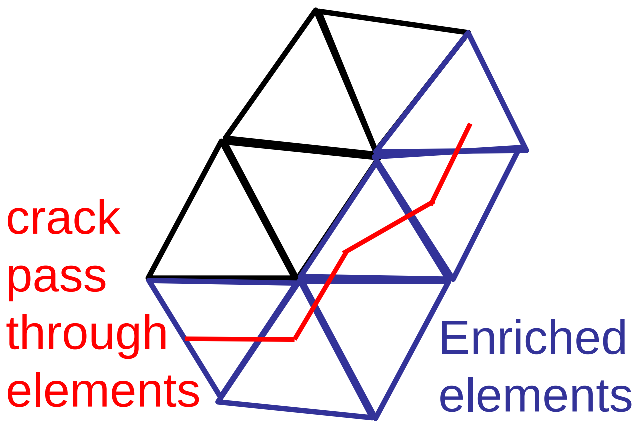
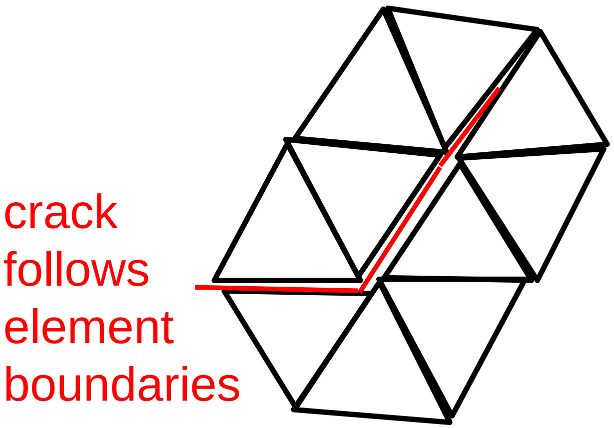
- The fracture process is modeled by cohesive elements to study
  - Dynamic crack propagation
  - Fragmentation

  - XFEM
  - Interface elements



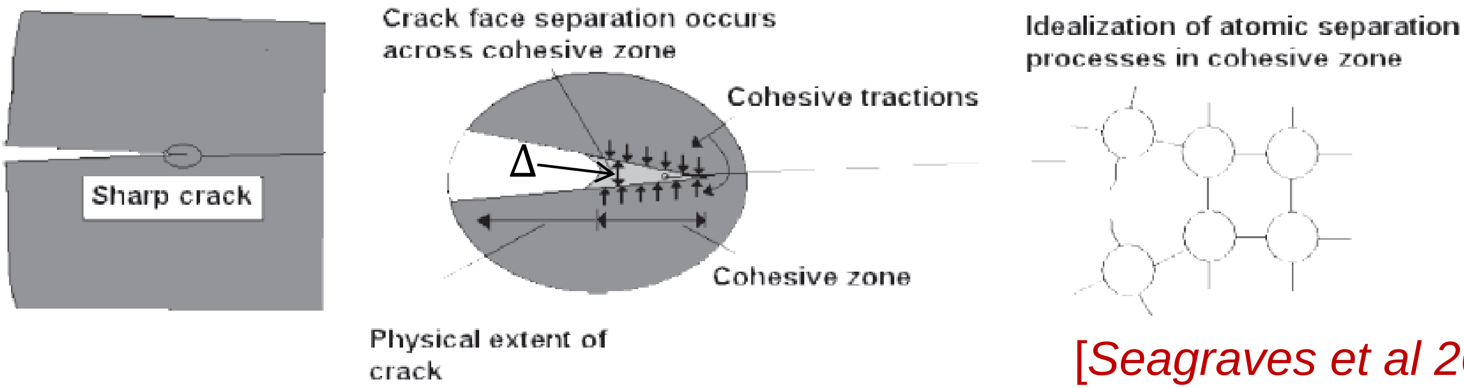
Commonly used for crack propagation



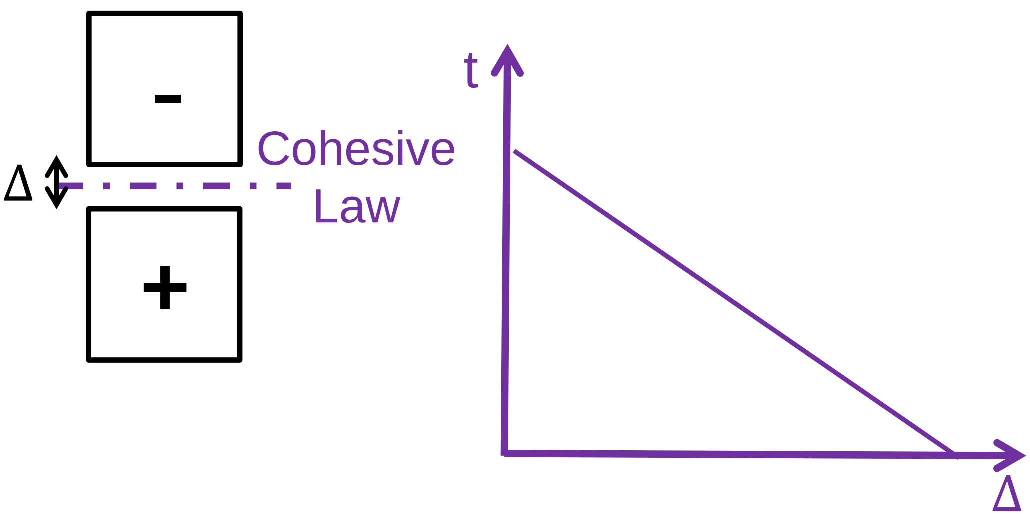
Dynamic phenomena (crack propagation due to impact, fragmentation)

- Cohesive zone model is very appealing to model crack initiations in a numerical model

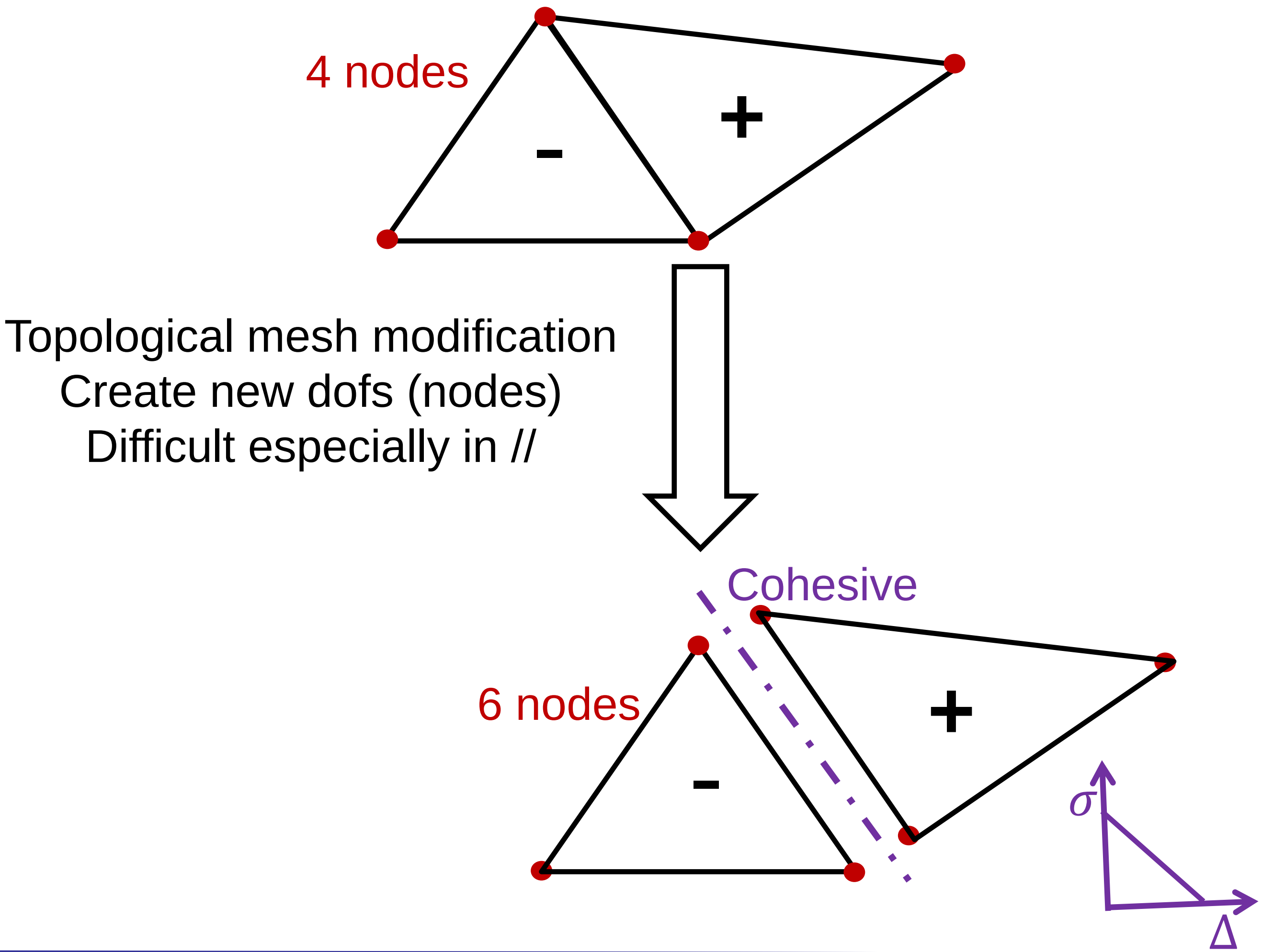
  - Model the separation of crack lips in brittle materials



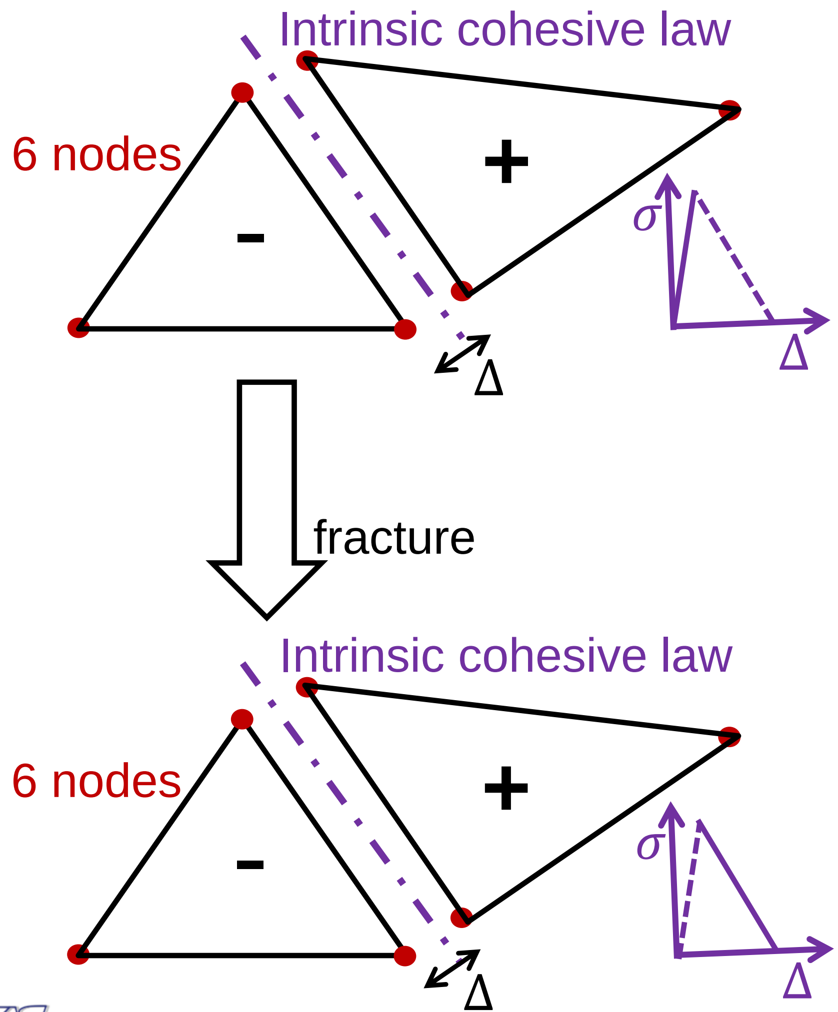
[*Seagraves et al 2010*]



- The insertion of cohesive elements during the simulation is difficult to implement as it requires topological mesh modifications
  - Extrinsic cohesive approach

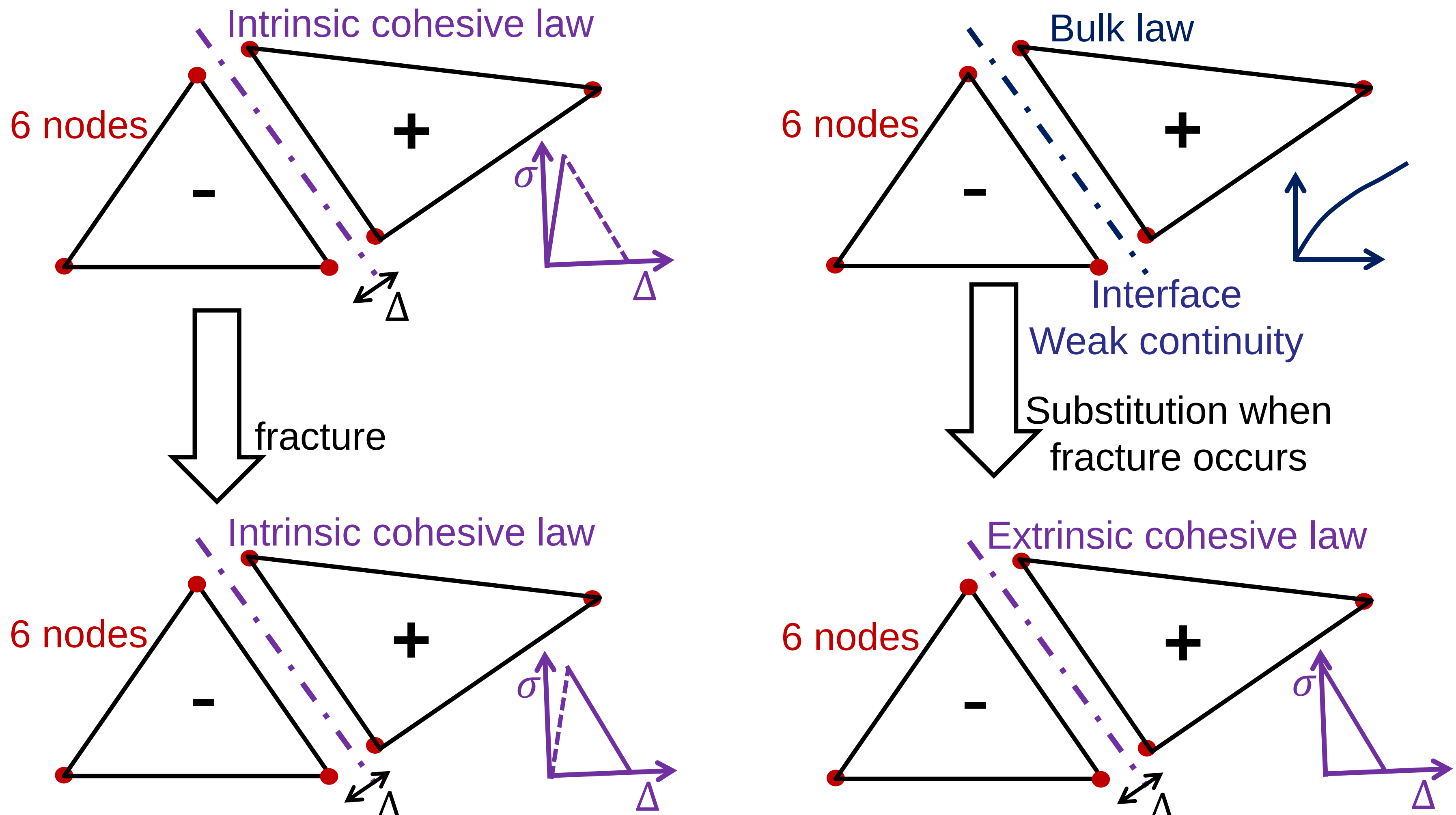


- A recourse to an intrinsic cohesive law is generally done with FEM
  - Intrinsic cohesive approach



- Intrinsic cohesive law leads to numerical problems [*Seagraves et al 2010*]
  - Spurious stress wave propagation
  - Mesh dependency
  - Crack propagation rate too high

- Use of extrinsic cohesive law is easier when coupled with DG but is only developed for 3D elements
  - FEM (continuous Galerkin)
  - Discontinuous Galerkin



Plan

- Develop a discontinuous Galerkin method for shells
  - One-field formulation
- Discontinuous Galerkin / Extrinsic Cohesive law framework
  - Develop a suitable cohesive law for thin bodies
- Applications
  - Fragmentations, crack propagations under blast loadings

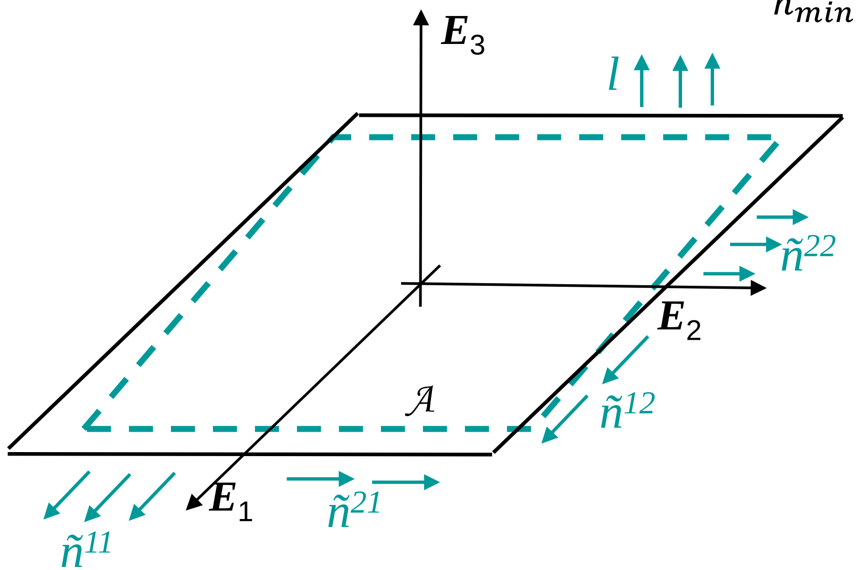
- The stress tensor $\boldsymbol{\sigma}$ is integrated on the thickness in the convected basis
  - Reduced stresses

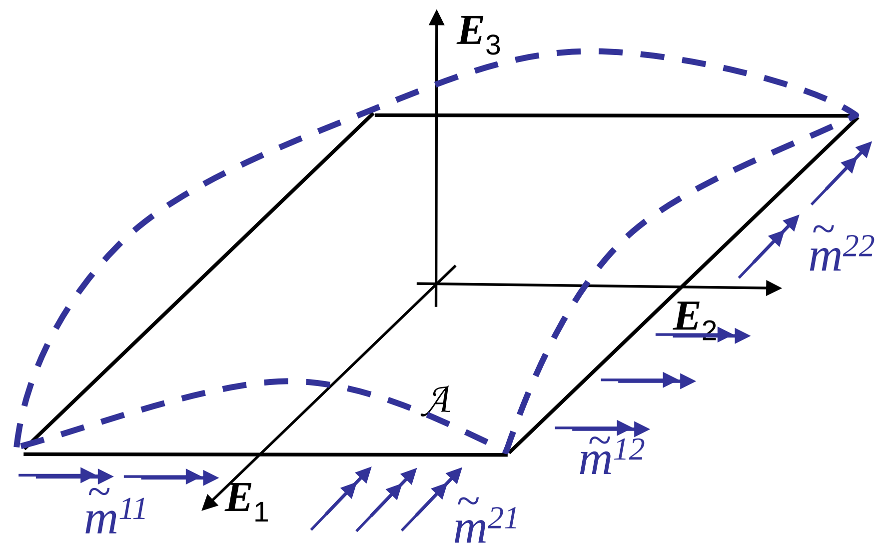
coupling

$$\boldsymbol{n}^{\alpha} = \frac{1}{\bar{j}} \int_{h_{min}}^{h_{max}} j\boldsymbol{\sigma} \cdot \boldsymbol{g}^{\alpha} d\xi^{3} = \left(\tilde{n}^{\alpha\beta} + \lambda_{\mu}^{\beta} \tilde{m}^{\alpha\mu}\right) \boldsymbol{\varphi}_{,\beta}$$

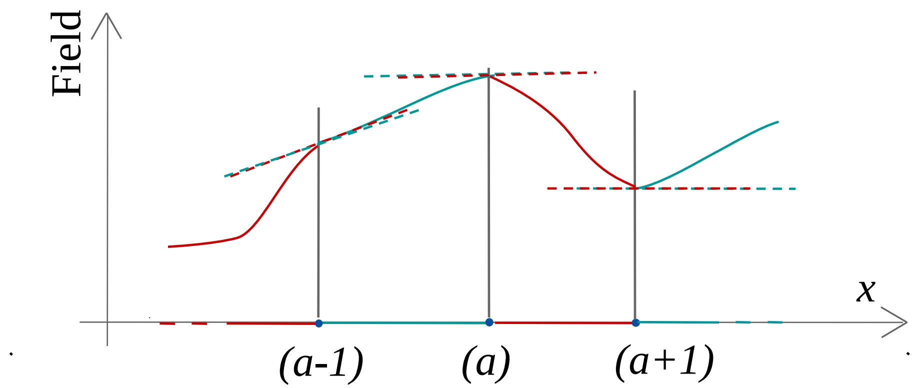
$$\tilde{\boldsymbol{m}}^{\alpha} = \frac{1}{\bar{j}} \int_{h_{min}}^{h_{max}} j\xi^{3} \boldsymbol{\sigma} \cdot \boldsymbol{g}^{\alpha} d\xi^{3}$$

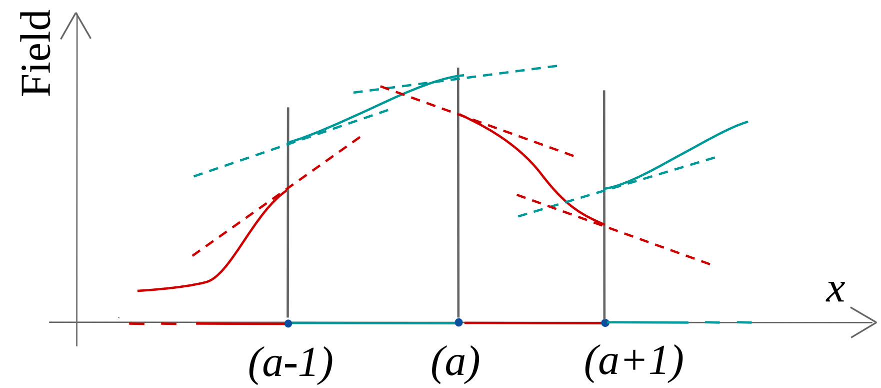
$$\boldsymbol{l} = \frac{1}{\bar{j}} \int_{h_{min}}^{h_{max}} j\boldsymbol{\sigma} \cdot \boldsymbol{g}^{3} d\xi^{3} \approx 0$$





- FEM (Continuous Galerkin)
- Discontinuous Galerkin





Integration by parts on the structure

$$\sum_e \int_{A_e} \left[\bar{j}\boldsymbol{n}^\alpha \cdot \delta\boldsymbol{\varphi}_{,\alpha} + \bar{j}\widetilde{\boldsymbol{m}}^\alpha \cdot \lambda_h \delta\boldsymbol{t}_{,\alpha} - \bar{j}\boldsymbol{l} \cdot \lambda_h \delta\boldsymbol{t}\right] dA = 0$$

Integration by parts on each element (unusual on $\boldsymbol{l}$)

$$\sum_e \left\{ \int_{A_e} \left[\left(\bar{j}\boldsymbol{n}^\alpha\right)_{,\alpha} \cdot \delta\boldsymbol{\varphi} + \left(\bar{j}\widetilde{\boldsymbol{m}}^\alpha\right)_{,\alpha} \cdot \lambda_h \delta\boldsymbol{t} - (\bar{j}\boldsymbol{l})_{,\alpha} \cdot \int_\alpha \lambda_h \delta\boldsymbol{t} d\alpha' \right] dA \right.$$

Additional interface terms

$$\left. - \int_{\partial A_e} \left[\bar{j}\boldsymbol{n}^\alpha \cdot \delta\boldsymbol{\varphi}\nu_\alpha^- + \bar{j}\widetilde{\boldsymbol{m}}^\alpha \cdot \lambda_h \delta\boldsymbol{t}\nu_\alpha^- - \bar{j}l \cdot \int_\alpha \lambda_h \delta t d\alpha' \ \nu_\alpha^- \right] dA \right\} = 0$$

- The equation of the full-DG formulation [*Becker et al cmame2011, Becker et al ijnme2012*]

$$\sum_e \int_{A_e} \left[\left(\bar{j}\boldsymbol{n}^\alpha\right)_{,\alpha} \cdot \delta\boldsymbol{\varphi} + \left(\bar{j}\tilde{\boldsymbol{m}}^\alpha\right)_{,\alpha} \cdot \lambda_h \delta\boldsymbol{t} \right] dA \; +$$

FEM (CG) equation

$$\sum_s \int_s \left[\langle \bar{j}\boldsymbol{n}^\alpha \rangle \cdot [\![\delta\boldsymbol{\varphi}]\!] + [\![\boldsymbol{\varphi}]\!] \cdot \langle \delta(\bar{j}\boldsymbol{n}^\alpha) \rangle + [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_{\bar{\delta}} \left\langle \frac{\beta_2 \mathcal{H}_n^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{\varphi}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \right] \nu_{\bar{\alpha}} \, d\partial A_e \; +$$

$$\sum_s \int_s \left[\langle \bar{j}\tilde{\boldsymbol{m}}^\alpha \rangle \cdot [\![\lambda_h \delta\boldsymbol{t}]\!] + [\![\boldsymbol{t}]\!] \cdot \langle (\bar{j}\lambda_h \tilde{\boldsymbol{m}}^\alpha) \rangle + [\![\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_{\bar{\delta}} \left\langle \frac{\beta_1 \mathcal{H}_m^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \right] \nu_{\bar{\alpha}} \, d\partial A_e \; +$$

$$\sum_s \int_s [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t} \nu_{\bar{\beta}} \left\langle \frac{\beta_3 \mathcal{H}_s^{\alpha\beta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t} \; \nu_{\bar{\alpha}} \, d\partial A_e \; = 0$$

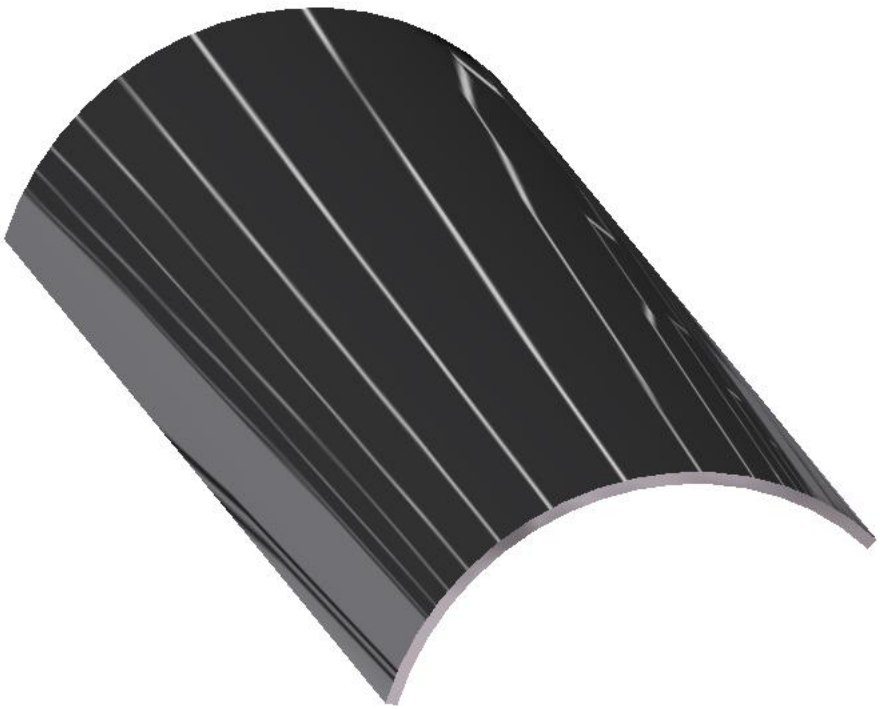
Consistency terms

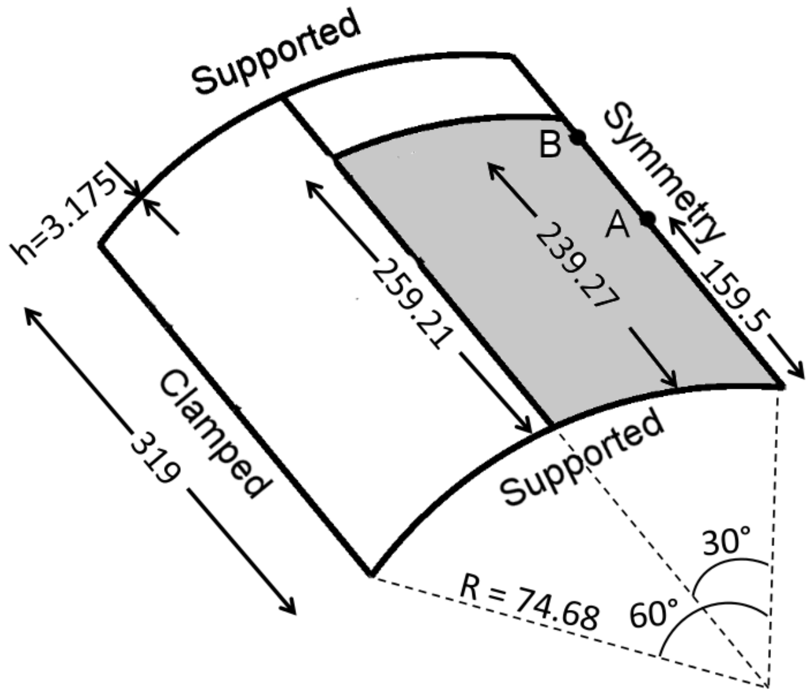
Symmetrization terms

Stabilization terms

  – Application of the DG method gives 2 Bulk, 2 consistency, 2 symmetrization and 3 stabilization terms

- A benchmark to prove the ability of the full-DG formulation to model continuous mechanics
  - J_2-linear hardening (elasto-plastic large deformations) panel loaded dynamically (explicit Hulbert-Chung scheme)





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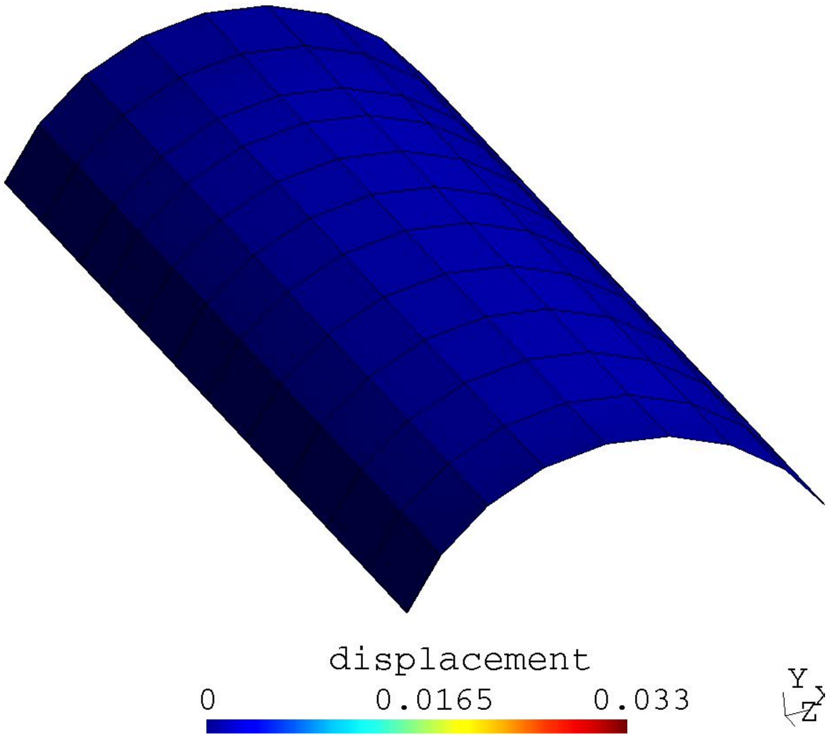

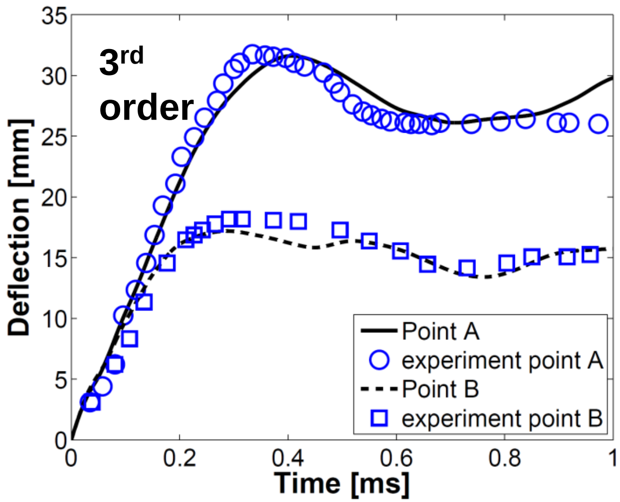

  - The results match experimental data

Plan

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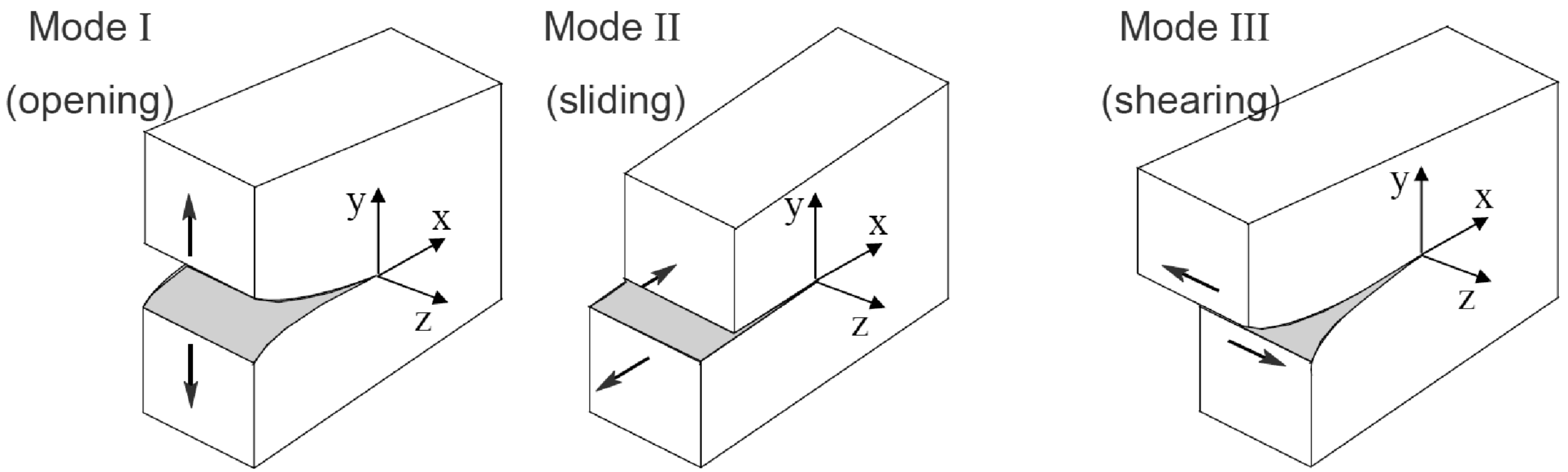
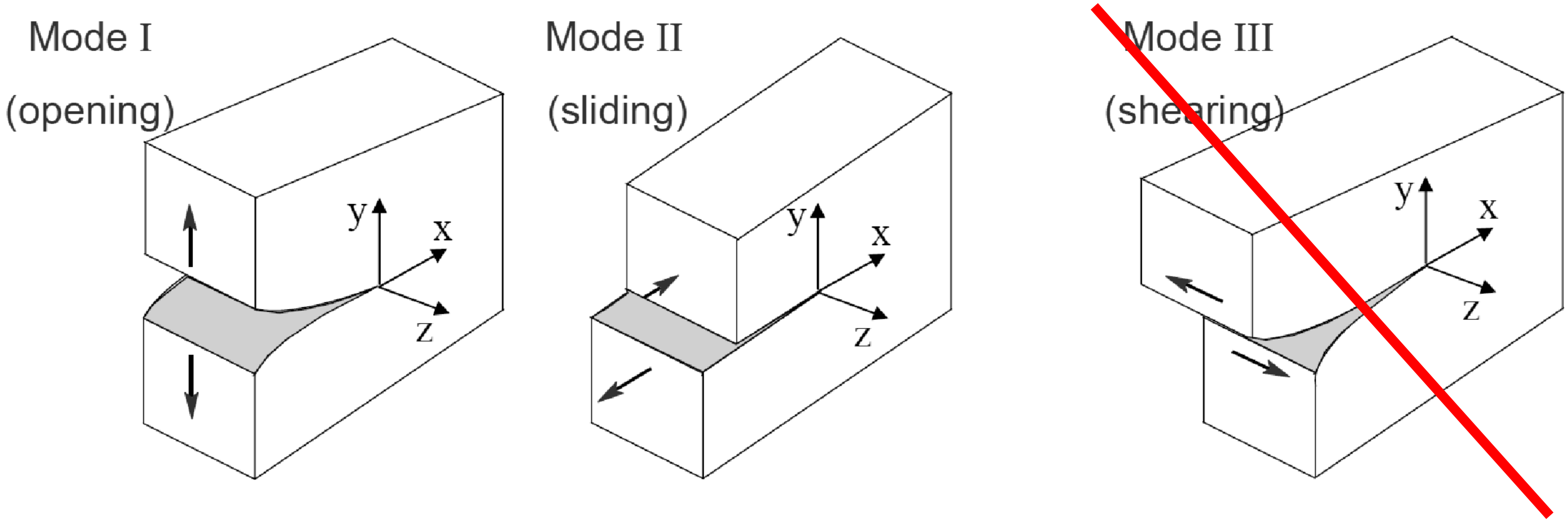
- There are 3 fracture modes in fracture mechanics



Full-DG/ECL framework

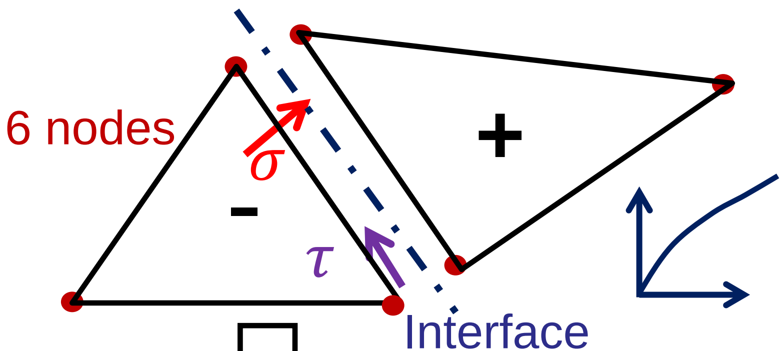
- Only modes I and II can be modeled by Kirchhoff-Love theory
  - Kirchhoff-Love → out-of-plane shearing is neglected



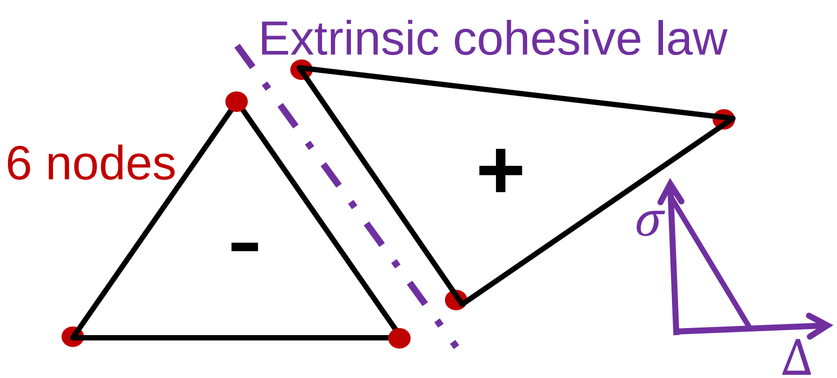
  - Model restricted to problems with negligible 3D effects at the crack tip

- Fracture criterion based on an effective stress

  – Camacho & Ortiz Fracture criterion [*Camacho et al ijss1996*]

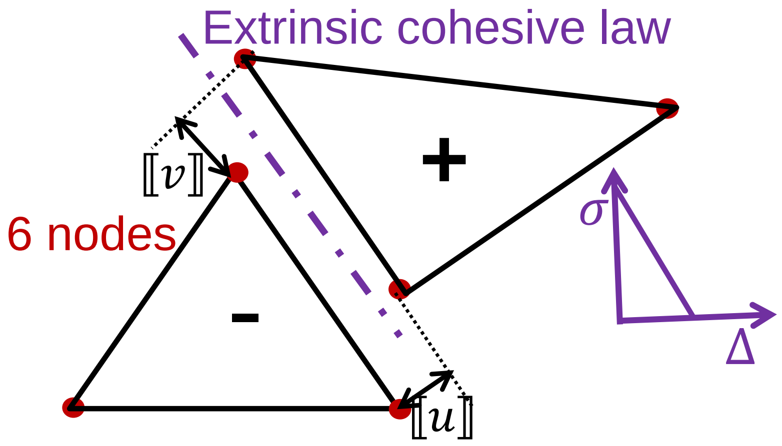


$$\sigma_{eff} > \sigma_c \quad \text{with} \quad \sigma_{eff} = \begin{cases} \sqrt{\sigma^2 + \beta^{-2}\tau^2} \text{ if } \sigma \geq 0 & \text{Traction} \\ \frac{1}{\beta} \ll |\tau| - \mu_c|\sigma| \text{ if } \sigma < 0 & \text{Compression} \end{cases}$$



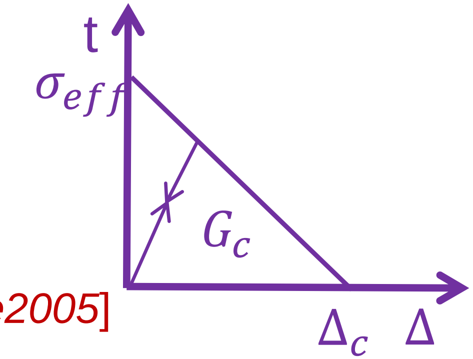
σ_c, β and μ_c are material parameters

- The cohesive law is formulated in terms of an effective opening
  - Camacho & Ortiz Fracture criterion [*Camacho et al ijss1996*]

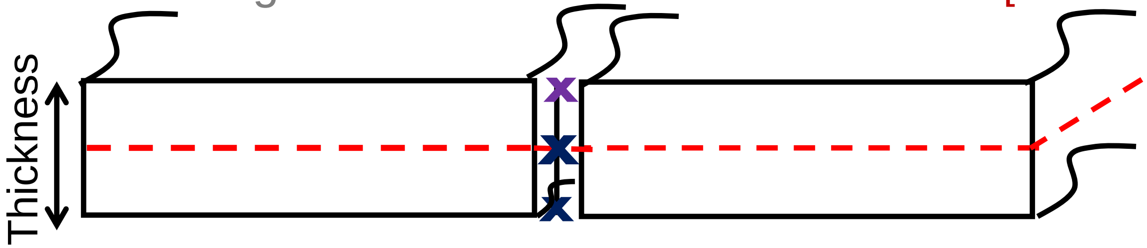


$$\Delta = \sqrt{[\![u]\!] + \beta^2 [\![v]\!]}$$

- The through-the-thickness crack propagation is not straightforward with shell elements

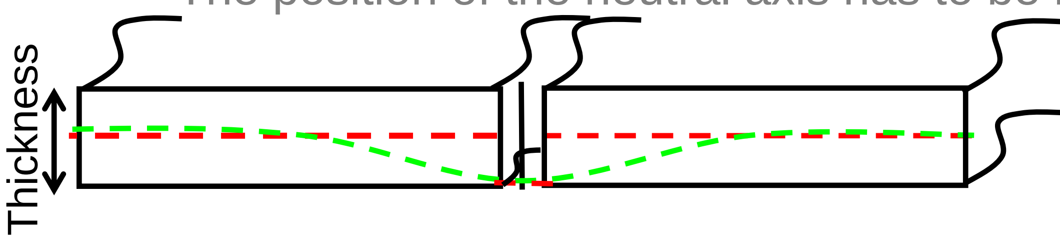


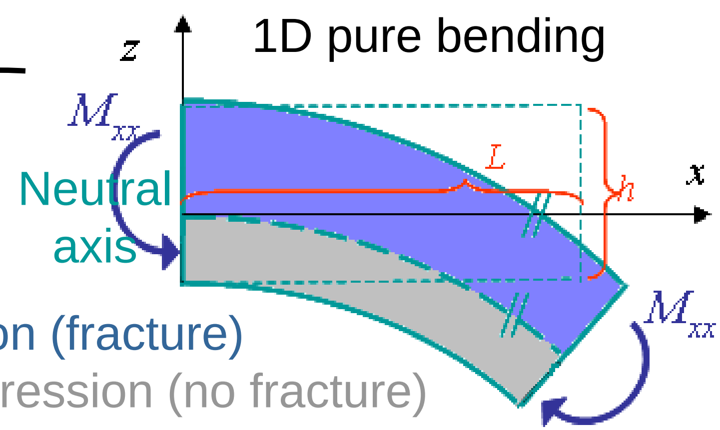
  - No elements on thickness
  - Integrate the 3D TSL on the thickness *[Cirak et al cmame2005]*



Fracture criterion is met
→ cohesive law
Unreached fracture
→ bulk law

  - The position of the neutral axis has to be recomputed to propagate the crack





Discontinuity
Continuity (Computation ?)

Tension (fracture)
Compression (no fracture)

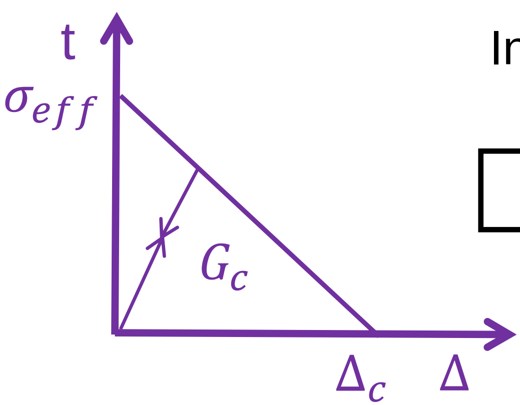
- The cohesive law can be formulated in terms of reduced stresses
  - Same as shell equations

Bulk law
Stress tensor $\boldsymbol{\sigma}$

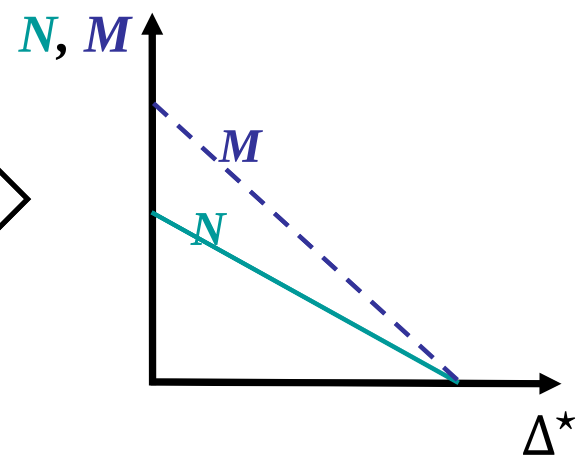
Integration on thickness

$$\boldsymbol{n}^{\alpha} = \frac{1}{\bar{j}} \int_{h_{min}}^{h_{max}} j\boldsymbol{\sigma} \cdot \boldsymbol{g}^{\alpha} d\xi^{3}$$

$$\tilde{\boldsymbol{m}}^{\alpha} = \frac{1}{\bar{j}} \int_{h_{min}}^{h_{max}} j\xi^{3}\boldsymbol{\sigma} \cdot \boldsymbol{g}^{\alpha} d\xi^{3}$$



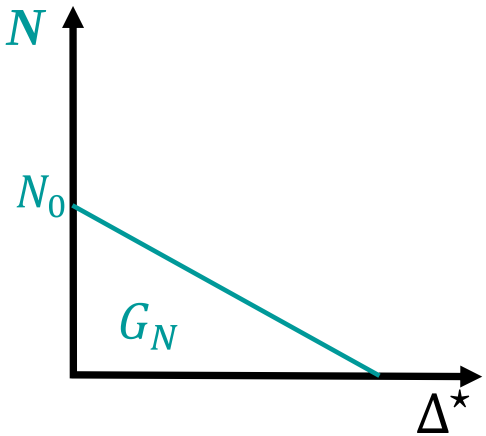
Integration on thickness

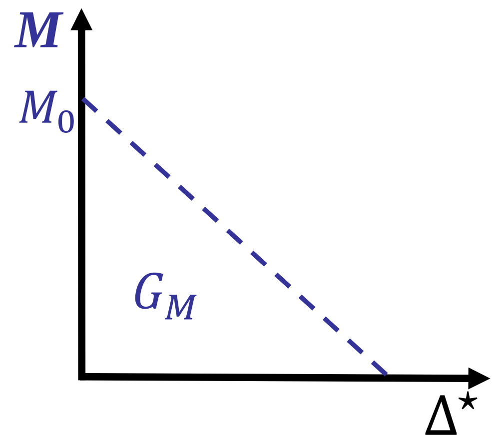


  - Similar concept suggested by Zavattieri [*Zavattieri jam2006*]

Full-DG/ECL framework

- Define $\Delta^{\star}$ and $N(\Delta^{\star}), M(\Delta^{\star})$ to dissipate an energy equal to hG_c during the fracture process [*Becker et al ijnme2012, Becker et al ijf2012*]
  - Integration on thickness

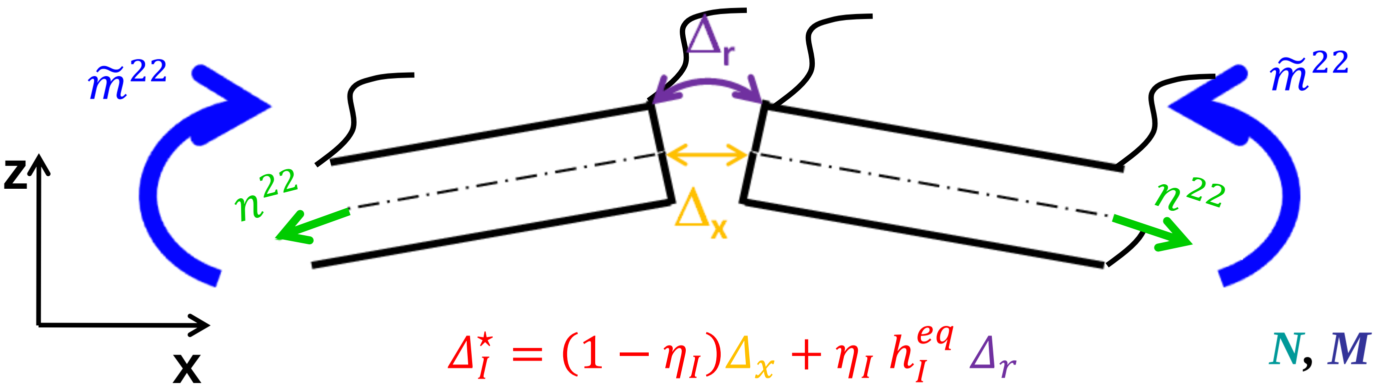




$$G_N + G_M = hG_c$$

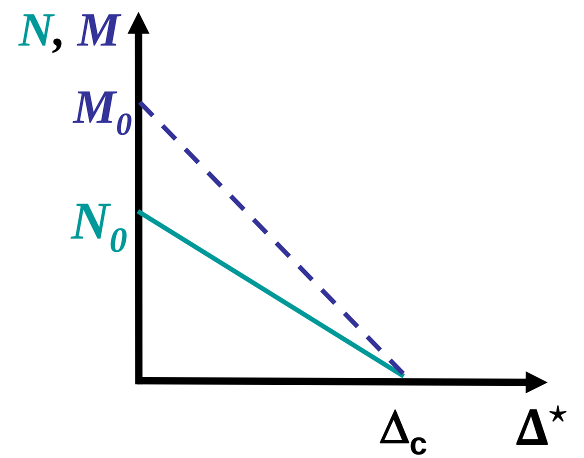
- Using the superposition principle the energy released for any couple N,M is equal to hG_c [*Becker et al ijnme2011*]

  – Pure mode I



$$\Delta_I^\star = (1-\eta_I)\Delta_x + \eta_I\, h_I^{eq}\, \Delta_r$$

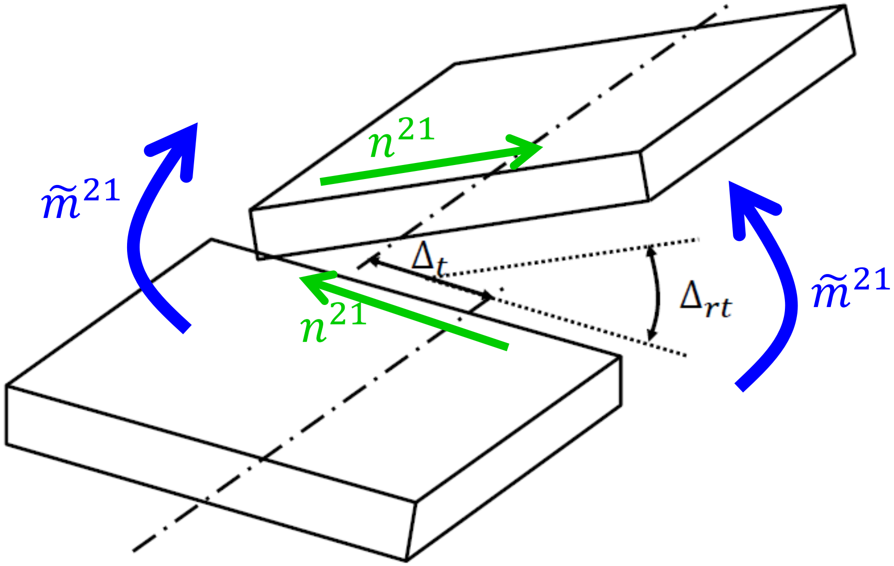
$$h_I^{eq} = \frac{M_0}{h\sigma_c - N_0}$$

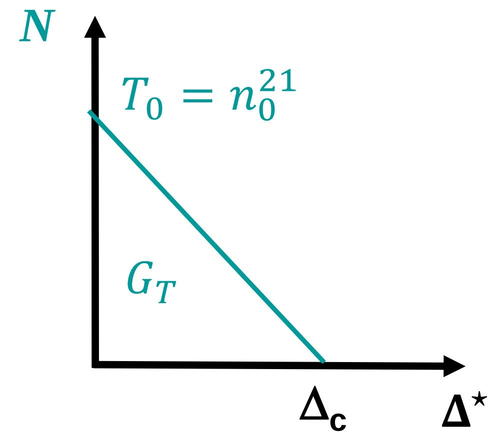


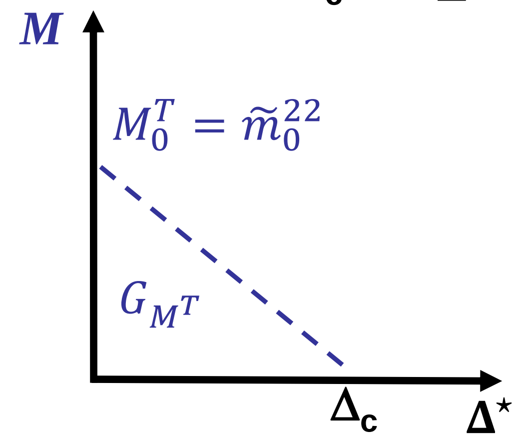
  – Coupling parameter

$$\eta_I = \frac{\left|1/h_I^{eq} M_0\right|}{N_0 + \left|1/h_I^{eq} M_0\right|} = \frac{\mathrm{h}\sigma_c - N_0}{h\sigma_c}$$

- The cohesive model for mode I can be extended to mode II







$$G_T + G_{M^T} = h\beta G_c$$

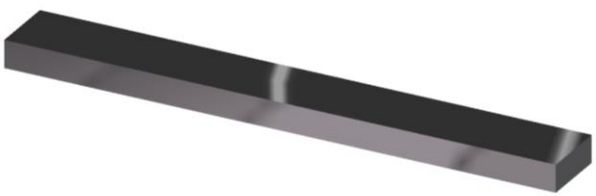
$$\Delta_{II}^{\star} = (1 - \eta_{II})\Delta_t + \eta_{II} h_{II}^{eq} \Delta_{rt}$$

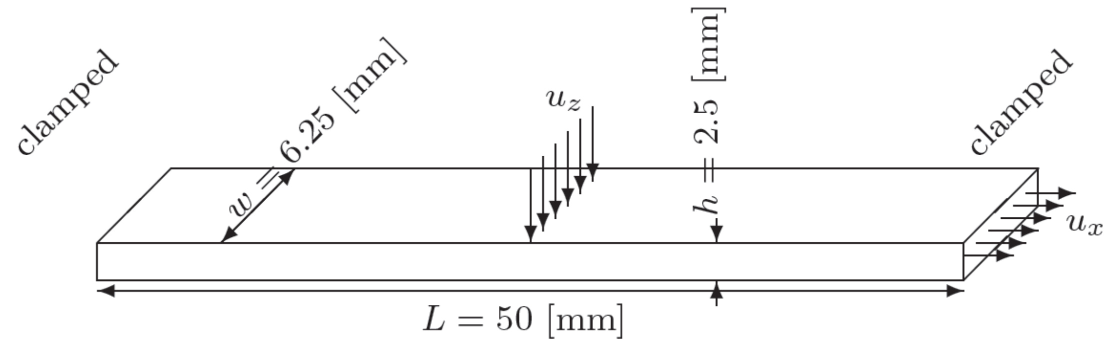
$$h_{II}^{eq} = \frac{M_0^T}{h\beta\sigma_c - T_0}$$

– Coupling parameter

$$\eta_{II} = \frac{\left|1/h_{II}^{eq} M_0^T\right|}{T_0 + \left|1/h_{II}^{eq} M_0^T\right|} = \frac{h\beta\sigma_c - T_0}{h\beta\sigma_c}$$

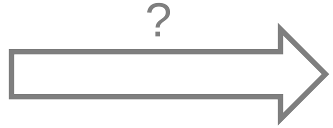
- The transition between uncracked to fully cracked body depends on ΔE_{int}
  - Double clamped elastic beam loaded in a quasi-static way

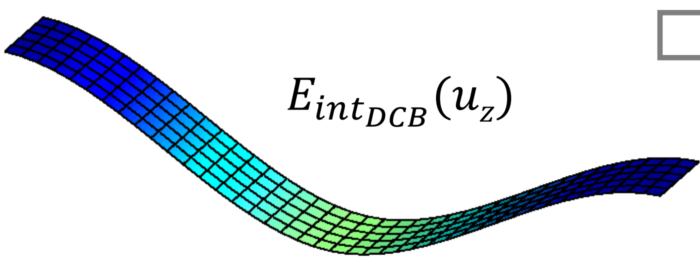




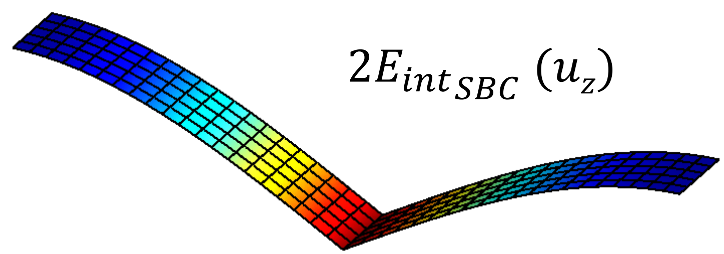
No fracture

Fully broken





$E_{int_{DCB}}(u_z)$



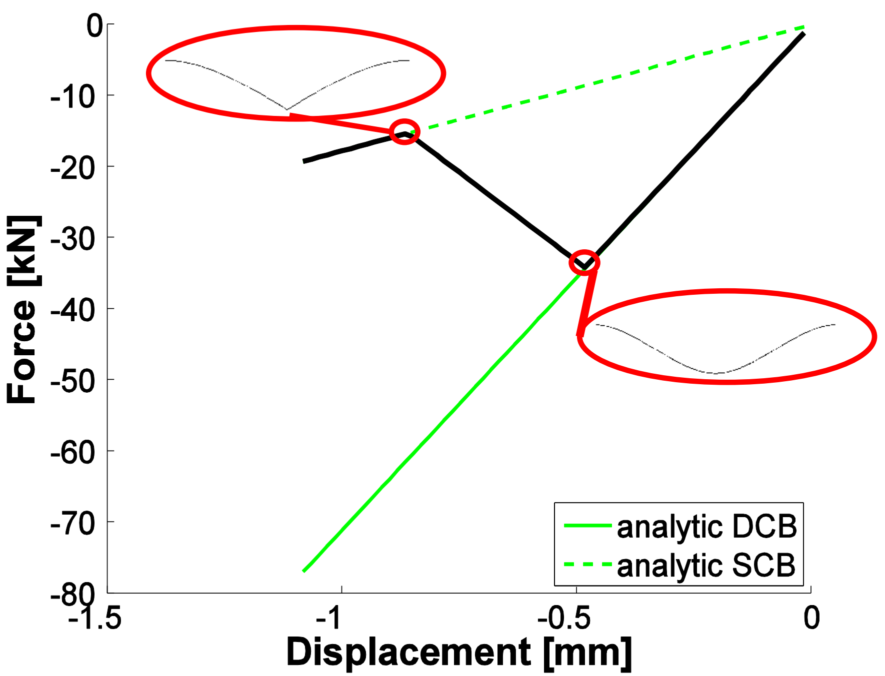
$2E_{int_{SBC}}(u_z)$

$$\Delta E_{int}(u_z) = E_{int_{DCB}}(u_z) - 2E_{int_{SBC}}(u_z)$$

- The framework can model stable/unstable crack propagation
  - Geometry effect (no pre-strain)

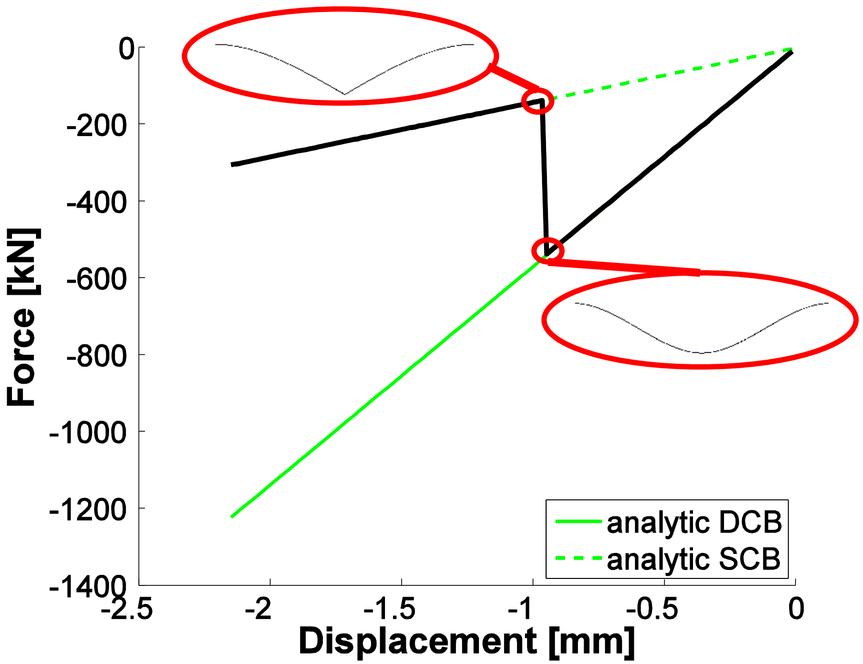
Stable transition

$$\Delta E_{int}\,(u_z) \;<\; hGc$$



Unstable transition

$$\Delta E_{int}\,(u_z) \;>\; hGc$$



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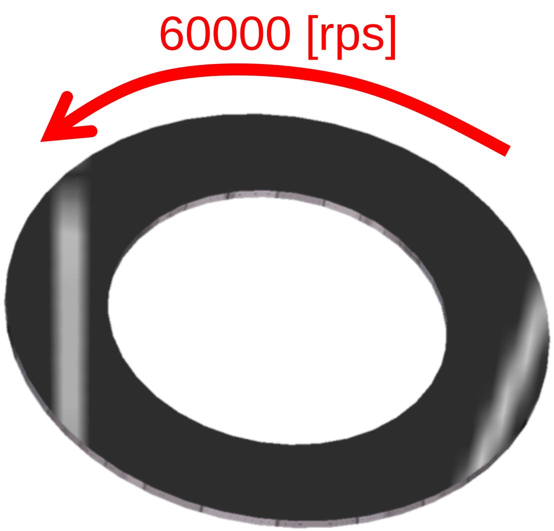
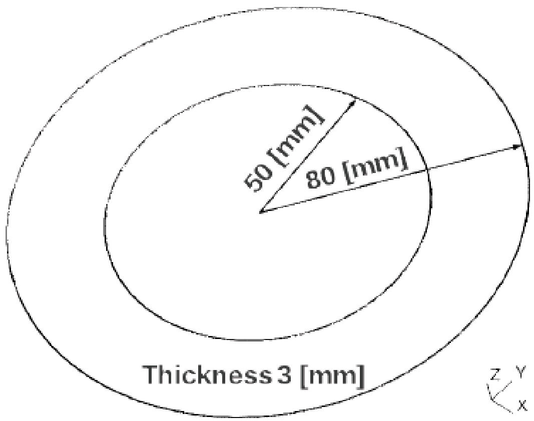
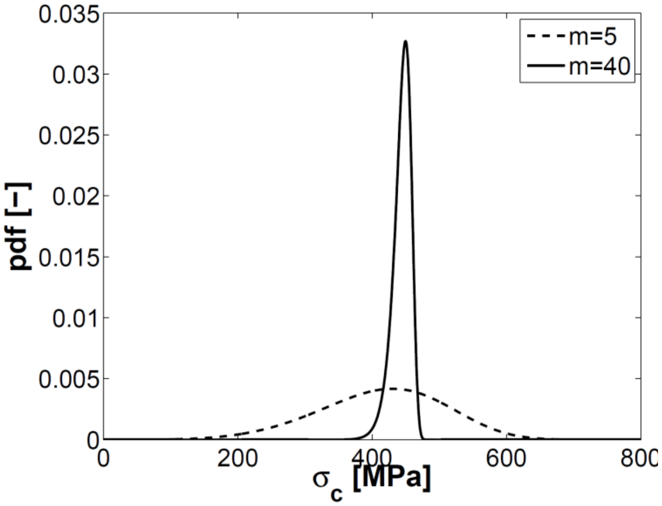
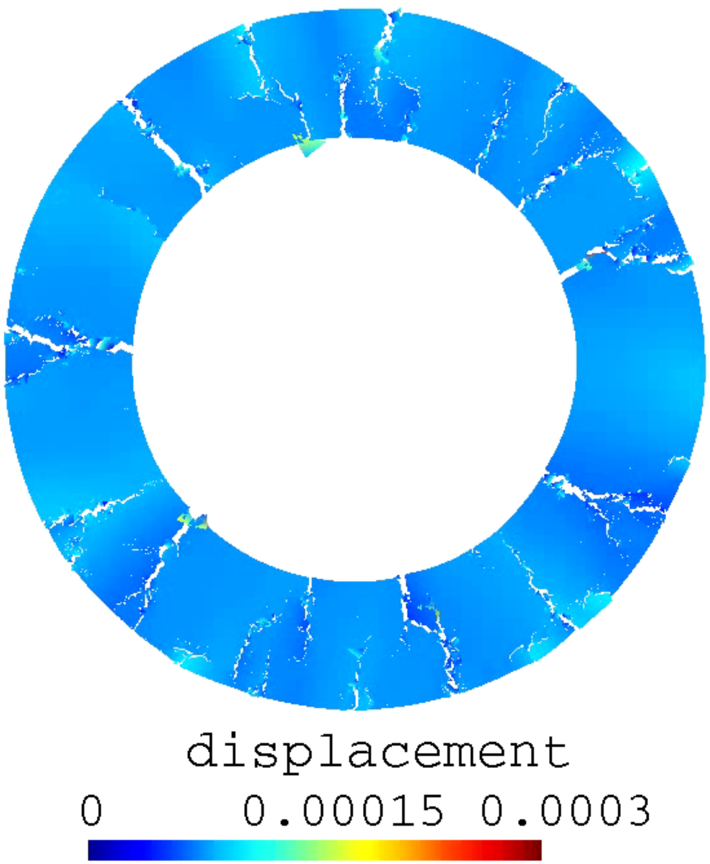
- A benchmark to investigate the fragmentation
  - Elastic plate ring loaded by a centrifugal force

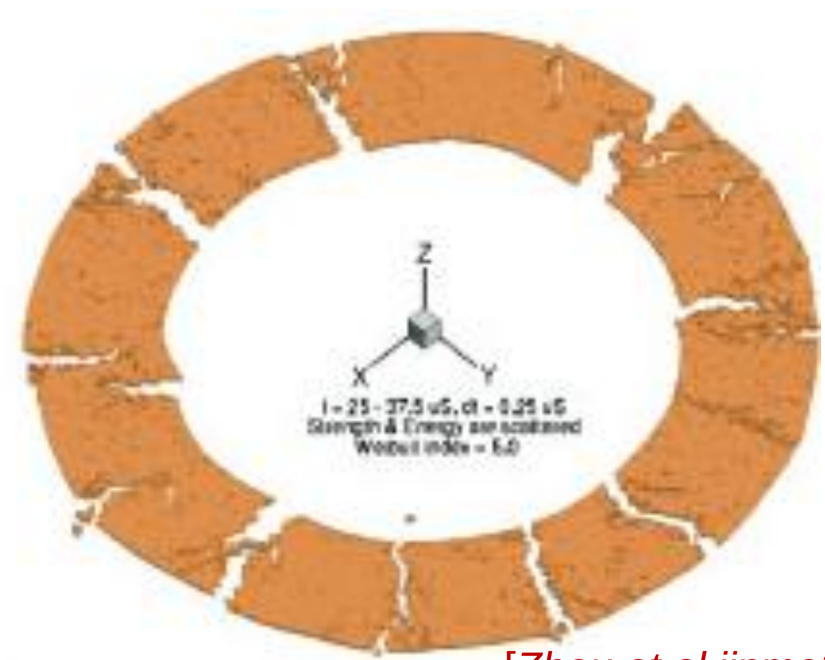






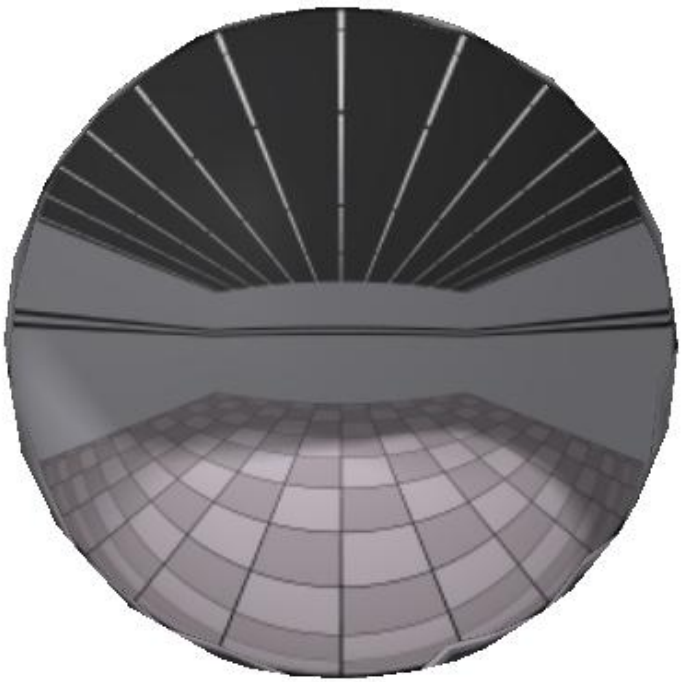
- Fragmentation phenomena can also be studied by the full-DG/ECL framework
  - Results are compared with the literature [*Zhou et al ijnme2004*]

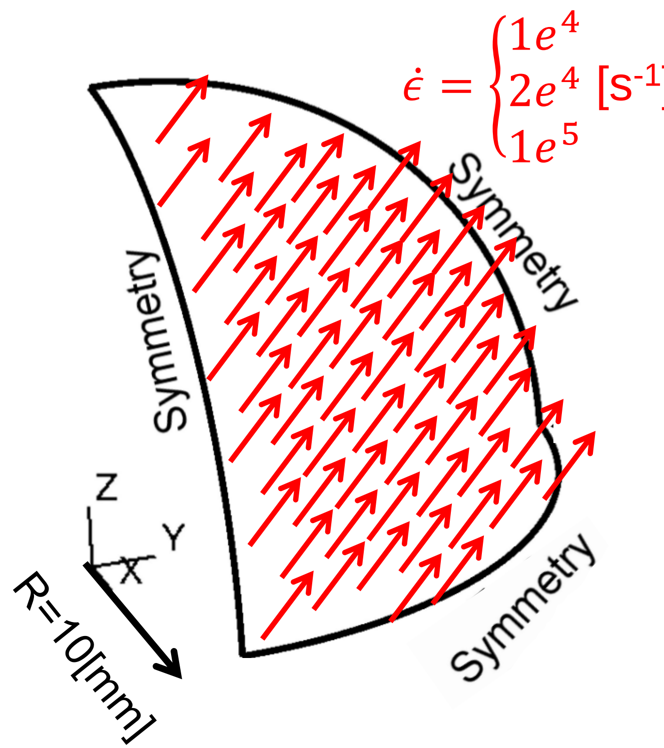




[*Zhou et al ijnme2004*]

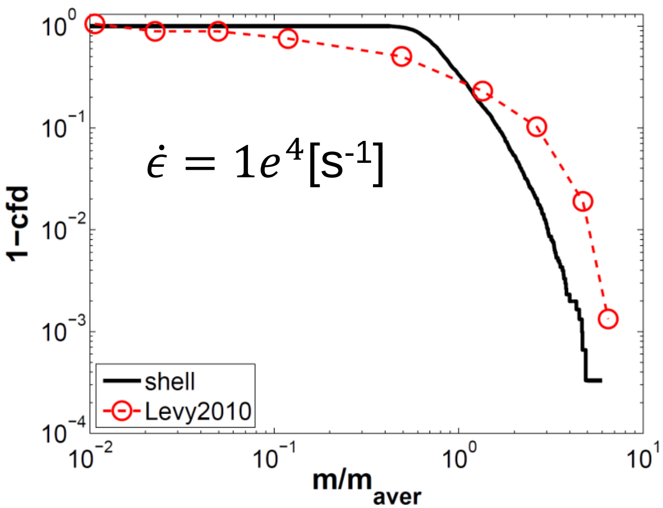
- Application to the dynamic fragmentation of a sphere
  - Elastic sphere under radial uniform expansion

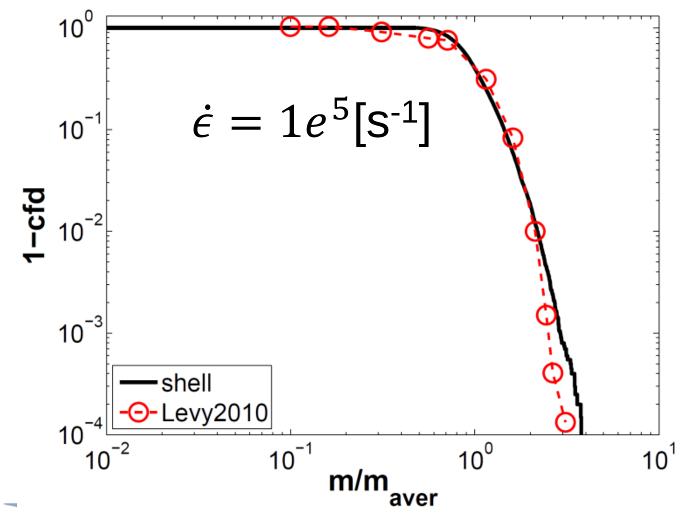


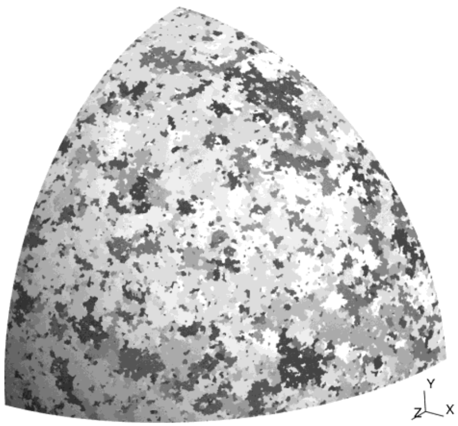


Applications of the DG/ECL framework

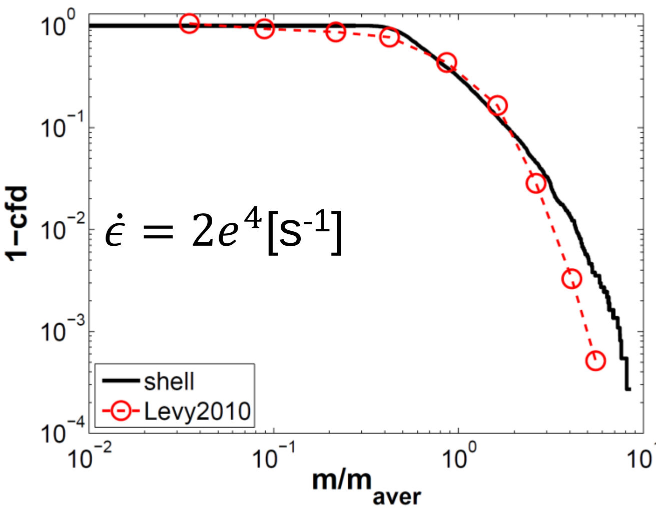
- The distribution of fragments and the number of fragments are in agreement with the literature [*Levy EPFL2010*]
  - Levy uses 3D elements

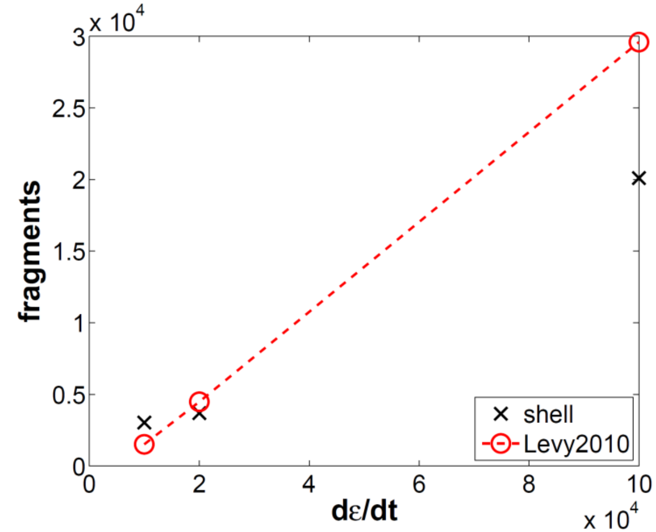




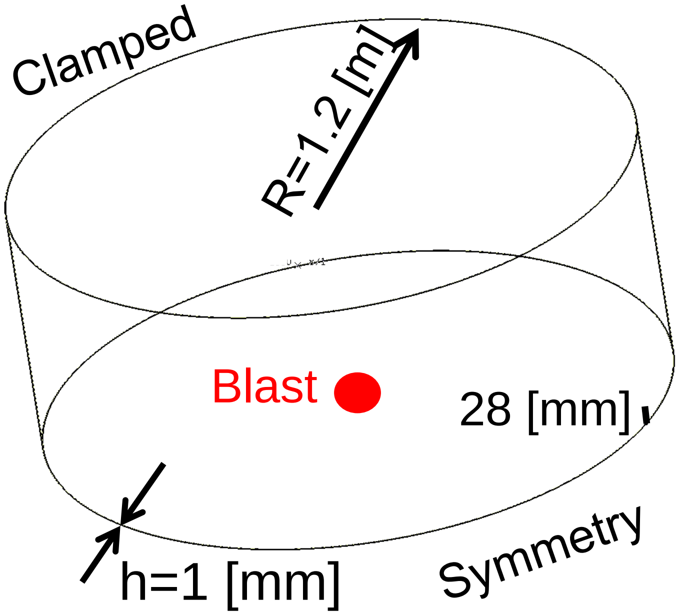


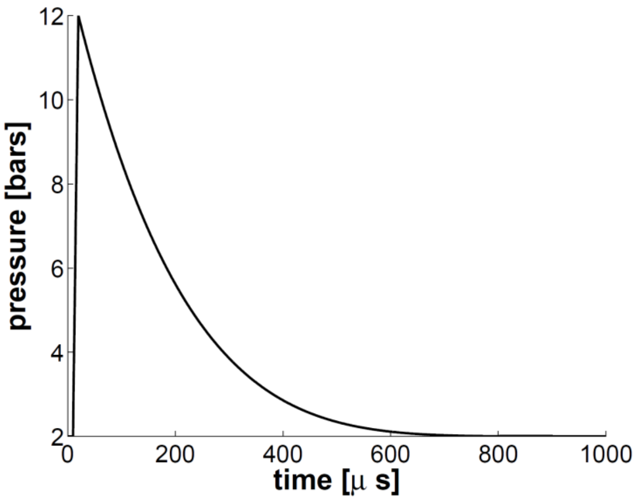
$\dot{\epsilon} = 1e^4[s^{-1}]$
2 588 265 Dofs
±48h on 32 cpus
(±17h for $\dot{\epsilon} = 1e^5[s^{-1}]$)



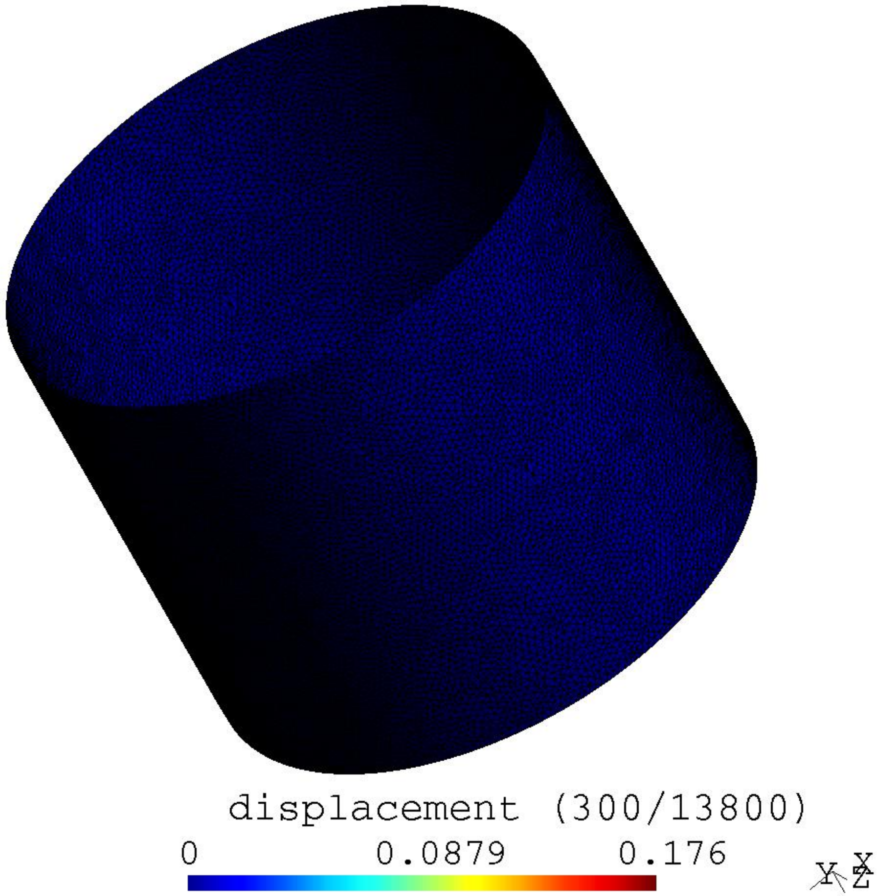


- Blast of an axially notched elasto-plastic cylinder (large deformations)

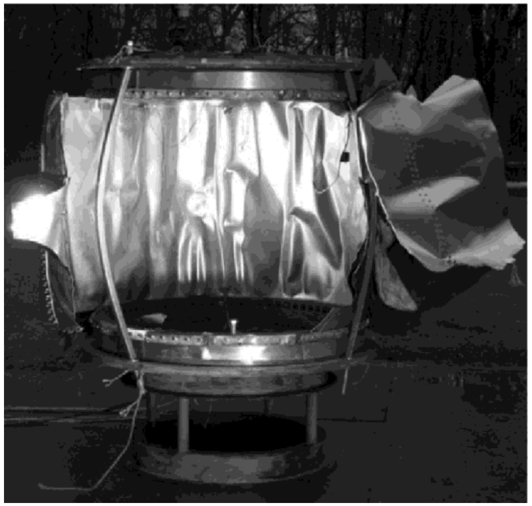


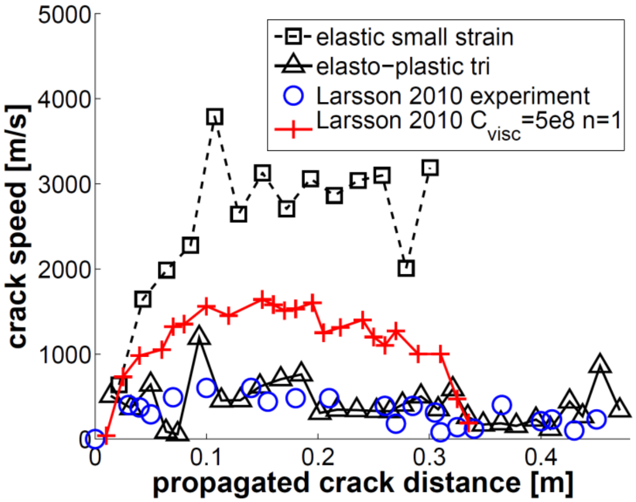


- Accounting for plasticity allows capturing the crack speed
  - Compare with the literature [*Larson et al ijnme2011*]

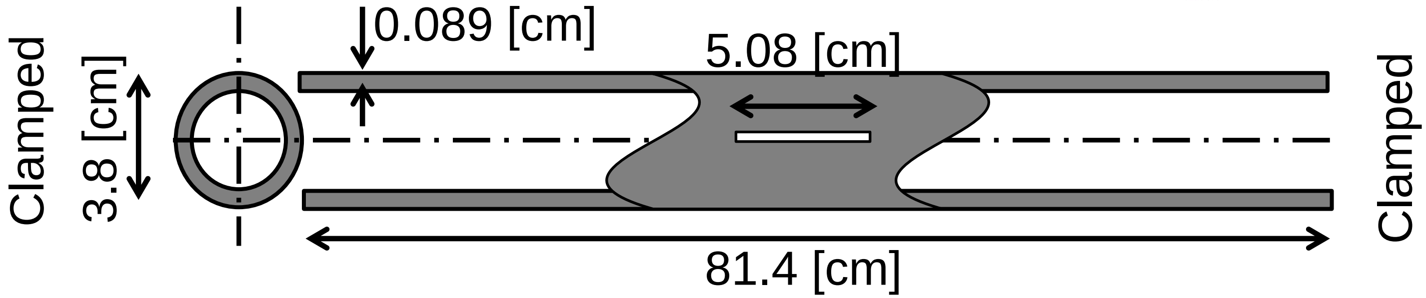


556 080 Dofs
±72h on 16 cpus



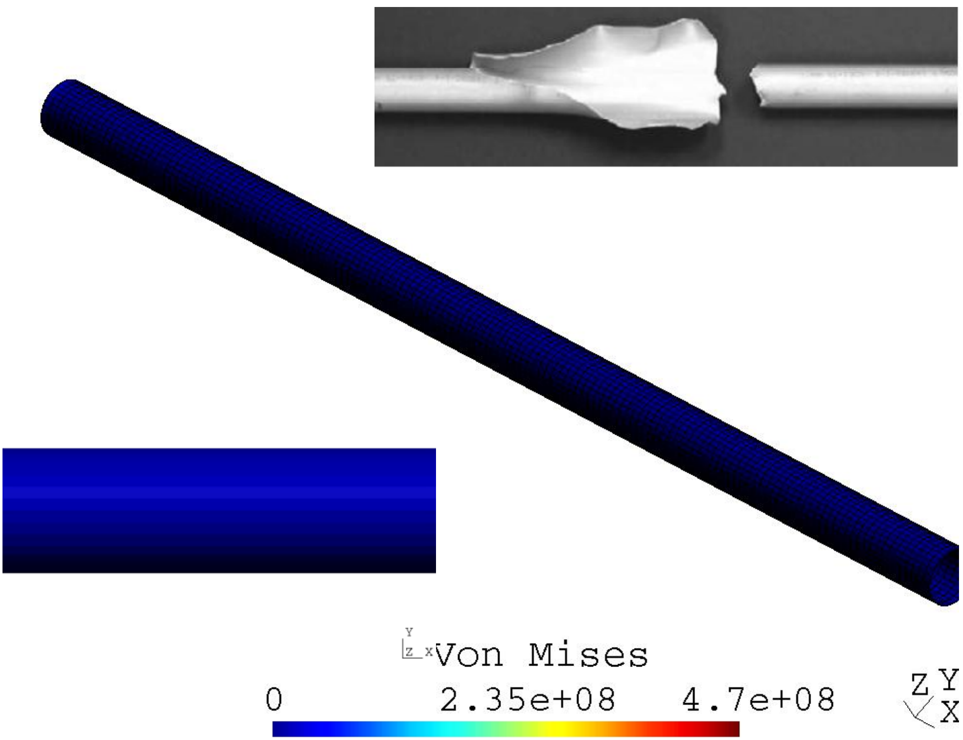


- Pressure wave passes through an axially notched elasto-plastic pipe (large deformations)

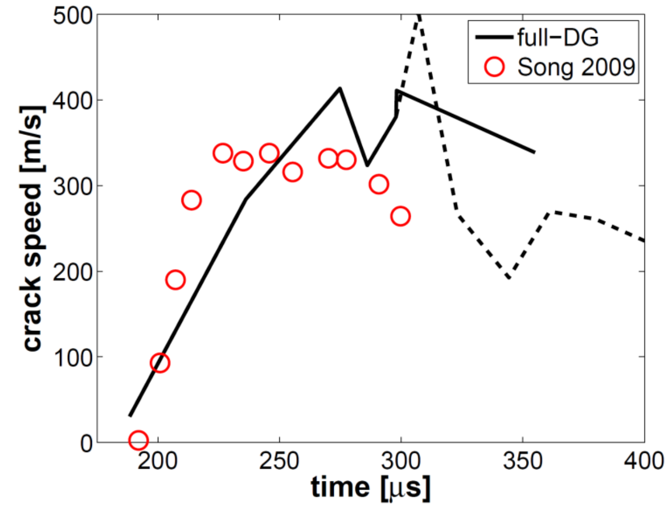


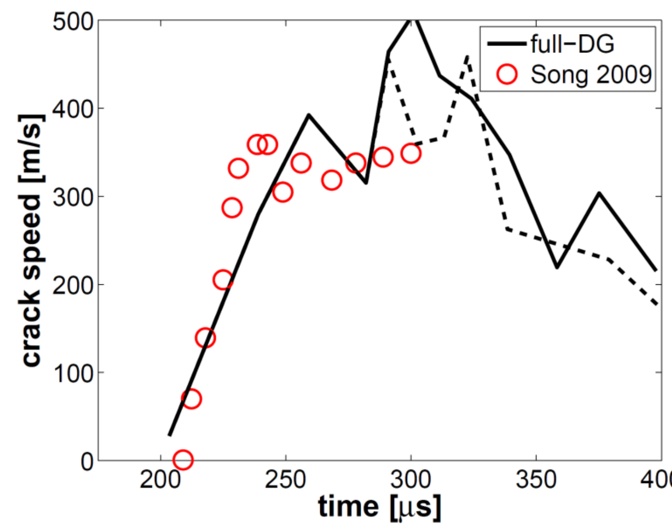
Applications of the DG/ECL framework

- Crack path and speed are well captured by the framework
  - Compare with the literature
    [*Song et al jam2009*]



224 256 Dofs
±21 h on 12 cpus





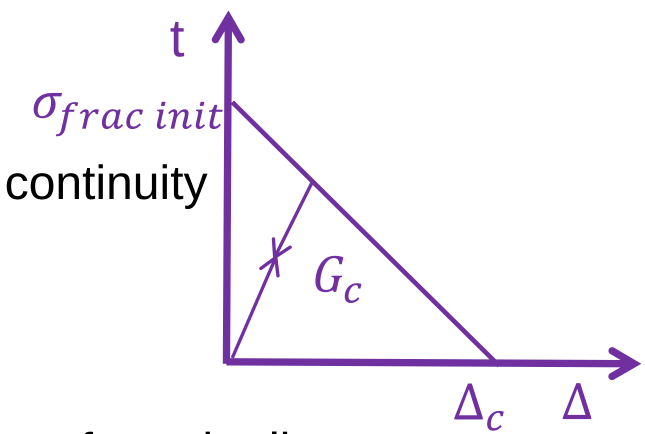
Conclusions

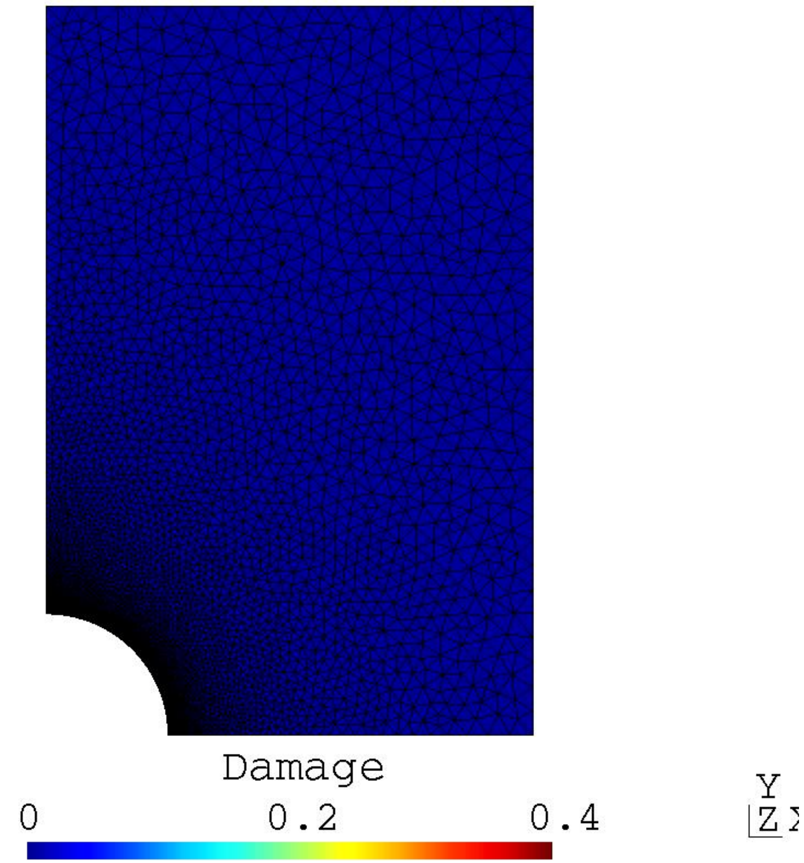
- Full-DG / ECL framework allows accounting for fracture in dynamic simulations of thin bodies
  - Crack propagation as well as fragmentation
  - Recourse to an elasto-plastic model is mandatory to capture crack speed
  - Affordable computational time for large models (using // implementation)

Future work

- Model the damage to crack transition by coupling a damage law with the full-DG/ECL framework
  - Replace the criterion based on an effective stress by a criterion based on the damage
  - Define the shape of the cohesive law

- The benchmark shows encouraging perspectives
  - Linear damage theory
  - Fracture criterion $D > D_c$
  - Cohesive shape





G_c from the literature [Mazars et al ijss1996]

Future work

- The benchmark shows encouraging perspectives but many improvements are required
  - Non local damage model
  - Account for stress triaxiality (and out-of-plane shearing)
  - Shape of the cohesive law
  - …

Thank you for your attention