On The Evaluation Of The Through Thickness Residual Stresses Distribution Of Cold Formed Profiles

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Abstract. The aim of this research is to evaluate the through thickness residual stresses distribution in the walls and in the corners of a cold-formed open section made of a material presenting a non linear hardening behaviour. To get results as close as possible to the reality, the complete process is modeled, including coiling and uncoiling of the sheet before the cold bending of the corner itself. The elastic springback after flattening as well as after final shaping are also taken into account. In order to validate the model in predicting the residual stresses distribution, the presented results are confronted to experimental measurements and FE results collected from the literature.

Keywords: Residual stresses, Process, Nonlinear hardening behaviour, Cold-formed

INTRODUCTION

In the design of structural cold-formed steel members, residual stresses count for a lot and may be decisive in the evaluation of the ultimate load in steel structures. The current paper presents theoretical equations aiming at evaluating the through thickness residual stresses distribution in cold-formed members due to coiling, uncoiling and cold braking of the corners. Through these equations, the effect of the radius of coiling and folding on the stresses and on the final radius after springback can be examined for carbon steel as well as for material presenting a non linear hardening behaviour as stainless steel. This is of great interest for the analysis of ultimate load in structural cold-formed members whilst compressive residual stresses cause a direct redistribution of the stresses during the loading and alters the behaviour in carrying.

COMPUTATIONS OF RESIDUAL STRESSES

Coiling, Flattening And Springback

In this section, the equations proposed by W.M. Quach et al. (4) are modified in order to add the non linear hardening behaviour. Due to the complexity of the theoretical solution, MATLAB is used to solve the equations numerically.

It is assumed that the flat sheet is free of stresses before its storage thanks to the annealing step. In this modified approach, the steel can exhibit an isotropic non-linear hardening behaviour. The longitudinal direction, denoted x, is the coiling direction, y corresponds to the thickness of the sheet and z is the transverse direction. As the width of the plate is assumed to be large enough to disregard the presence of transverse strains, the problem can be summarized as the *plane strain pure bending of a sheet in the x-y plane*.

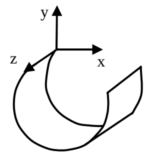

FIGURE 1. Plane strain pure bending of a sheet in the x-y plane

Theoretical Model

The procedure followed in these investigations is quickly described here. Hooke's law is chosen to characterise transversely isotropic material behaviour and one needs a longitudinal and transverse Young's modulus E and E_t as well as Poisson's ratio to describe it. As soon as ε_z and γ_{yx} are equal to zero, the incremental part of the strain field $\underline{d\varepsilon}$ and the corresponding stress field $\underline{d\sigma}$ are given by:

$$\underline{d\varepsilon} = \begin{bmatrix} d\varepsilon_x \\ d\varepsilon_z \end{bmatrix} = \begin{bmatrix} d\varepsilon \\ 0 \end{bmatrix} = \begin{bmatrix} d\kappa . y \\ 0 \end{bmatrix} \quad (1)$$

$$\underline{d\sigma} = \underline{\underline{C_e}}^{-1} \underline{d\varepsilon} - \underline{\underline{C_e}}^{-1} \underline{d\varepsilon^p} \quad (2)$$

One assumes that $\underline{d\varepsilon} = \underline{d\varepsilon^e} + \underline{d\varepsilon^p}$ .and $\underline{\underline{C_e}} = \begin{bmatrix} 1/E & -\nu_t/E_t \\ -\nu_t/E_t & 1/E_t \end{bmatrix}$ and $\underline{d\varepsilon^p} = \begin{bmatrix} d\varepsilon_x^{\ p} \\ d\varepsilon_z^{\ p} \end{bmatrix}$ is the unknown part of the problem.

Since plastic strains occur, Hill's quadratic flow surface $F_p(\underline{\sigma}, \underline{X}, \alpha_k, \underline{\underline{L}})$ and the corresponding consistency equation have to be fulfilled.

$$F_p = \sigma_{equ} - \sigma_F = 0 \quad (3)$$

$$\frac{d\varepsilon^p{}_x}{d\varepsilon^p{}_z} = \frac{\dot{\lambda}\left(\dfrac{\partial F_p}{\partial \underline{\sigma}}\right)_x}{\dot{\lambda}\left(\dfrac{\partial F_p}{\partial \underline{\sigma}}\right)_z} = \frac{\left(\underline{\underline{L}}\underline{\sigma}\right)_x}{\left(\underline{\underline{L}}\underline{\sigma}\right)_z} \quad (4)$$

with $\sigma_{equ} = \sqrt{\frac{1}{2}(\underline{\sigma})^T \underline{\underline{L}} (\underline{\sigma})}$; $\underline{\underline{L}}$ is the tensor of the anisotropic parameters : $\underline{\underline{L}} = \begin{bmatrix} G+H & -H \\ -H & H+F \end{bmatrix}$.

The numerical solution is thus summarised in a system of two equations with two unknowns ($d\varepsilon_x{}^p$ and $d\varepsilon_z{}^p$) and is solved in MATLAB. One divides the thickness into several layers; at each step of the numerical computation, the curvature increases and the system of equations is solved; σ_F is described by Swift law $\sigma_F = K(\varepsilon_0 + \varepsilon^p_{equ})^n$ (chosen for representing the non linear hardening behaviour where $\varepsilon^p{}_{equ}$ is the equivalent plastic strain), and is re-evaluated at each step (according to the increased ε^p_{equ}) and the flow surface F_p grows up isotropically. Under reverse yielding, one supposes isotropic hardening behaviour in each case developed in this work. For further information, see (3) and (4).

Finite Element Model

A finite element model of the bending of a steel sheet has also been conducted. The boundary conditions of the finite element model are depicted below.

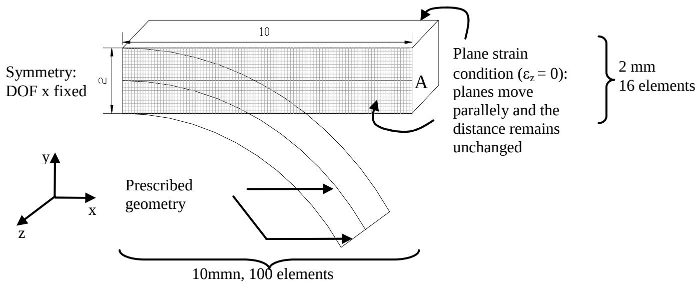

FIGURE 2. Boundary conditions for the FE model

The displacements are directed by two lines; *the neutral axis* of the sheet (x direction) is located at the half-width and fits an arc of circle (the radius of which being variable during the coiling); *the right edge* (y direction) remains all the time perpendicular to the neutral axis at the point A. The plans in the z direction are constrained to follow the displacement parallely with respect to the hypothesis of the problem. For symmetry reasons, only one half of the sheet is modelled. The present model has been implemented in the LAGAMINE finite element code, which has been developed at the University of Liège for more than 20 years. For more information see (2).

Residual Stresses Distribution

In the current paper, the so-called DP1000 (dual phase) steel has been chosen. This steel presents a non-linear hardening behaviour and authors (7) give material parameters of Swift law in order to characterise the hardening behaviour.

Table 1. Material parameters for DP 1000

F	G	H	N	K	n	ε_0
1.051	1.036	0.925	3.182	1626	0.17	0.00487

K, ε_0, n are material parameters determined by tensile test in the rolling direction z.

Moreover, such steel presents an anisotropic yield locus which can be described by means of Hill's 1948 quadratic equation (equ. (3)) using the material parameters given in Table 1.

MATLAB results and finite element results have been reported on the graph (**FIGURE 3**: left graph), where the left part and the right part represent respectively: *the stresses (obtained by numerical resolution) due to the coiling of the sheet; the stresses (obtained by numerical and finite element resolution) due to the coiling (s_c) and the uncoiling (s_r) of the shee. I*n the core of the sheet (y<0.8mm), where the stresses remained elastic after coiling, the uncoiling process leaves no stresses. After the coiling and uncoiling process, the resulting moment, corresponding to the flexural stresses remaining, can be easily evaluated. Unloading via a reverse bending moment produces elastic strains and stresses corresponding to the springback of the sheet (**FIGURE 3**: right graph).

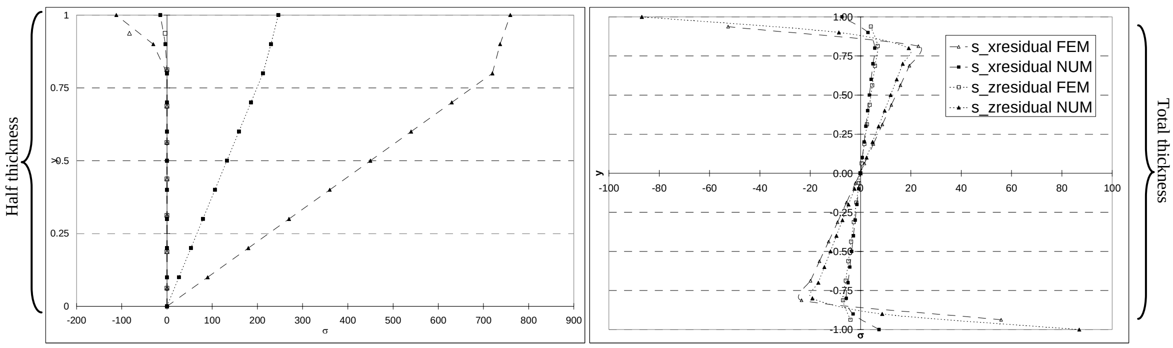

FIGURE 3. Flexural stresses due to coiling and uncoiling and residual stresses after springback (Rc=250mm)

Residual Stresses Due To Press Braking

Theoretical Model

In case of press braking, the sheet is set into a die and is shaped by means of a punch (press braking on a V press). First, one supposes that press braking can again be modelled as pure bending, thus the final stress distribution in the corner can easily be found out by means of the considerations developed previously. Such assumption is not far from reality if the length of the bent profile allows ignoring deformation through the longitudinal direction. Moreover, in reality, loads are applied via a punch and they could be more accurately modelled by means of a uniformly distributed internal pressure. In addition, the radius of the fold decreases a lot and the previous calculations can not be conducted anymore because of the presence of the radial (through thickness) stresses. Nevertheless, in the frame of the present research, one has chosen to keep the previous hypothesis disregarding the presence of the radial stresses.

The calculations are conducted with considering an initial state of stress (*sz_residualNUM* and *sx_residualNUM)* and plastic deformations corresponding to process preceding the press-braking. In that part of the problem, *x* indicates the longitudinal direction of the fold (initial state of stresses: *sz_residualNUM*) and *z* indicates the transverse direction (ISS: *sx_residualNUM*) as the fold is bent in a plane perpendicular to the plane of coiling-uncoiling.

Residual Stresses Distribution

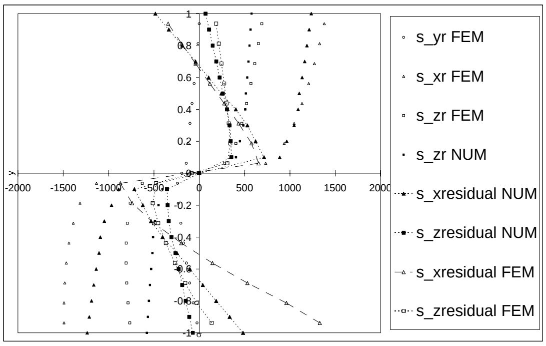

FIGURE 4. Residual stresses due to coiling - uncoiling -springback - bending - springback: R_c=250mm, R_f=6mm. results for a DP1000 steel.

As explained, the presence of radial stresses influences the distribution of stresses through the thickness during bending and after springback (**FIGURE 4**). Nevertheless, after springback, the distribution of stresses approached by the theoretical (MATLAB resolution) method is quite good in regards with the amplitude of stresses provided by the FE calculations.

In Hill's quadratic yield locus equation (equ. (3)) developed as equ. (5), one can notice that σ_x and σ_z are influenced by σ_y. Moreover σ_y increases the equivalent plastic strain, which also modifies the value of σ_F. In the tensed part of the sheet, theoretical investigation recommends a stronger hardening while in the compressed part, the conclusion is the opposite.

$$\frac{d\varepsilon^{p}{}_{x}}{d\varepsilon^{p}{}_{z}} = \frac{\dot{\lambda}\left(\frac{\partial F_p}{\partial \underline{\sigma}}\right)_x}{\dot{\lambda}\left(\frac{\partial F_p}{\partial \underline{\sigma}}\right)_z} = \frac{\left(\underline{\underline{L}}\,\underline{\sigma}\right)_x}{\left(\underline{\underline{L}}\,\underline{\sigma}\right)_z} \qquad (5)$$

Discrepancies between the calculations are also due to the distribution of the elastic stresses after springback (**FIGURE 5**). In the case of the bending of a curved sheet, this distribution is not linear as described in 8.

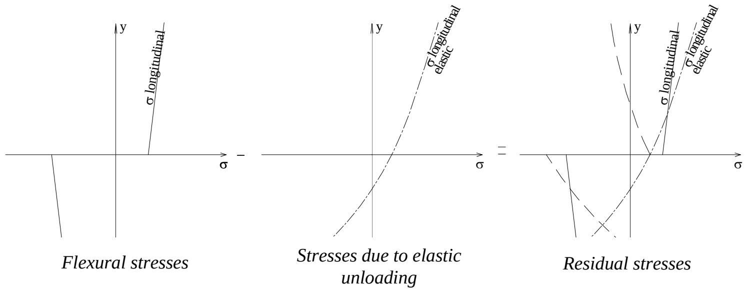

FIGURE 5. Residual stresses due to pure bending followed by springback: case of a small radius

Agreement With Experimental Measurements

Finite element predictions given by Quach et al. (5) are confronted here with Weng and White's (6) measurements and put side by side with theoretical results in **FIGURE 6**. These results concern the cold-bending of a HY-80 steel (high yield strength, low carbon, low alloy steel with nickel, molybdenum and chromium). The HY8 steel behaviour is assimilated to a non linear one, the Swift law parameters have been fitted on a stress-strain curve corresponding to a tensile test in the longitudinal direction (see Table 2). The yield strength is equal to 593.3MPa.

Table 2. Material parameters for HY80

K	n	ε_0
1035	0.1063	0.002

The finite element simulations model the complete press-braking process. They provide accurate results and overcome difficulties of measurement but such model requires a certain expertise and are time consuming. Additionally, in particular cases, they also suffer bad convergence and accurate results cannot be accomplished. Otherwise, theoretical results are in good agreement with the measurements in regard with the total amplitude of the residual stresses diagram.

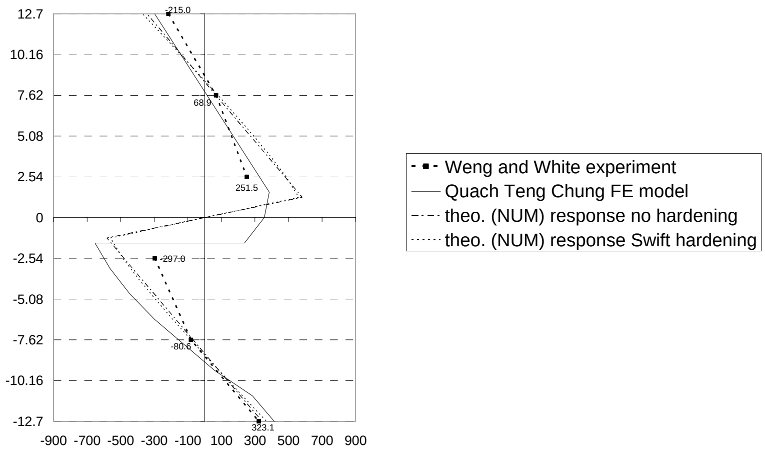

FIGURE 6. Residual stresses due to press-braking: Rf=139.7mm

CONCLUSIONS

The theoretical calculations of residual stresses in cold-formed steel members including non-linear hardening behaviour are complex and require a certain number of assumptions. In this paper, theoretical (equations solved via MATLAB) and numerical (FE) calculations have been conducted to model the whole process: the coiling-uncoiling operations followed by natural springback and the press braking process also followed by elastic springback. Plane strain condition is assumed both in the coiling-uncoiling and in the press-braking process. In the theoretical calculations, no radial stresses are taken into account in the calculations. The material is supposed to be anisotropic and to obey to Hill's quadratic yield surface and the subsequent plastic flow rule. Non linear hardening behaviour is taken into account in the calculations.

The results confirm a complex distribution of flexural stresses and show good agreement with measurements collected in literature.

In cold formed column, residual stresses cause a direct redistribution of the stresses during the loading and alter the behaviour in carrying (6). Consequently, a good knowledge of the amplitude of the residual stresses and of their distribution is of great interest for the analysis of ultimate loads in structural cold-formed members. Pragmatic convention is to model residual stresses in cold formed profile as the sum of a membrane type and flexural type. This assumption is unreasonable while the calculations show more complex variation of the distribution through the thickness.

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