A Schur Complement Method for DAE Systems in Power System Simulation

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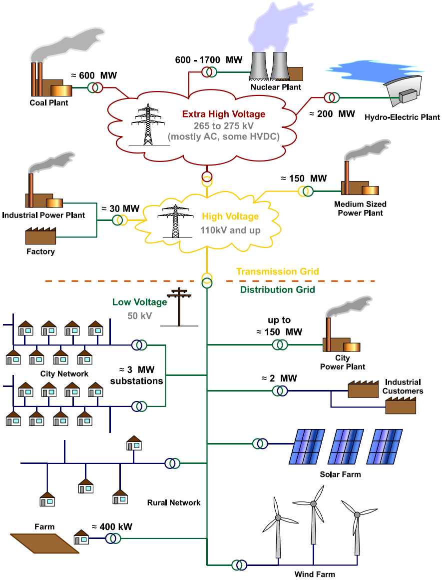
Outline

Large-Scale Electric Networks



Dynamic Simulation

Used by power system operation companies for:

- dynamic security assessment of the network by analyzing large sets of scenarios in real-time
- operator training
- scheduling day to day operation with optimal power flow studies using dynamic system response

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- interactive evaluation of changes
  - adding new transmission lines
  - increasing percentage renewable energy sources
  - implementing new control schemes
  - decommissioning old power plants
- hardware-in-the-loop simulations

Dynamic Simulation

Used by power system operation companies for:

- dynamic security assessment of the network by analyzing large sets of scenarios in real-time(Speed: Critical)
- operator training (Speed: Critical)
- scheduling day to day operation with optimal power flow studies using dynamic system response (in research stage)

Used by people involved in designing and planning for:

- interactive evaluation of changes (Speed: Desired)
    - adding new transmission lines
    - increasing percentage renewable energy sources
    - implementing new control schemes
    - decommissioning old power plants
- hardware-in-the-loop simulations (Speed: Critical, Real-time)

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Goal

Get accurate results, *faster*

Mathematical Formulation

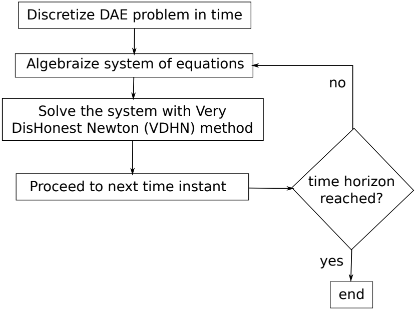
Initial value, hybrid, stiff, non-linear DAE problem:

$$\mathbf{\Gamma}\dot{\mathbf{x}} = \Xi(\mathbf{x}, \mathbf{V})$$
$$x(t_0) = x_0, V(t_0) = V_0$$

where:

- $\mathbf{V}$ is the vector of voltages through the network
- $\mathbf{x}$ is the expanded state vector containing the differential and algebraic variables (except the voltages) of the system
- $\mathbf{\Gamma}$ is a diagonal matrix with $(\mathbf{\Gamma})_{\ell\ell} = \begin{cases} 0 & \text{iff } \ell^{th} \text{ equation is algebraic} \\ 1 & \text{iff } \ell^{th} \text{ equation is differential} \end{cases}$

Standard Integrated Algorithm



Procedure Acceleration

- Sparse linear solvers
- Model Simplification/Reduction
- Better hardware (larger memory, faster CPUs, etc)

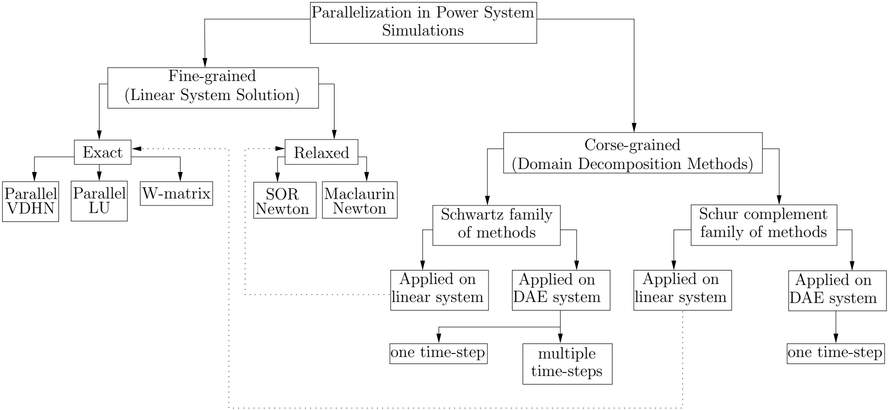
Increased Demands

- Bigger models
- Higher accuracy
- More complex, hybrid, components

Result

Dynamic simulations of large-scale systems *remain* time consuming

DDMs and Parallel Processing techniques

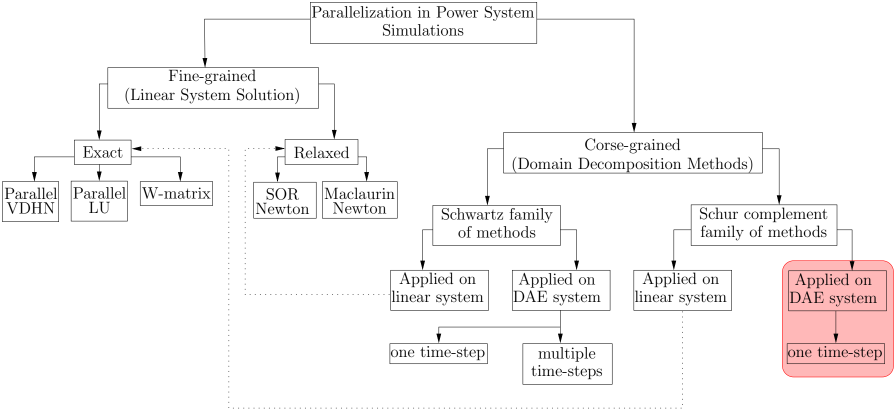


[Kron (1963)]

[Ilic'-Spong, Crow and Pai (1987)]

[La Scala, Sbrizzai and Torelli (1991)]

DDMs and Parallel Processing techniques



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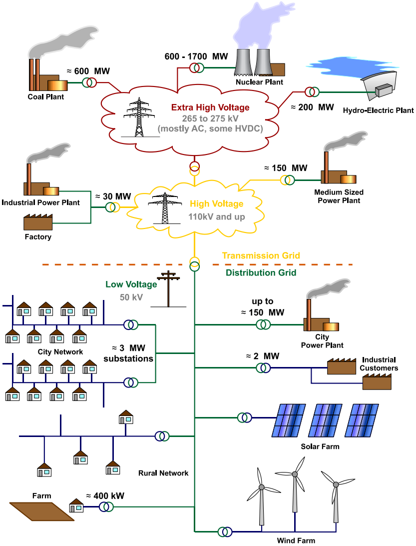
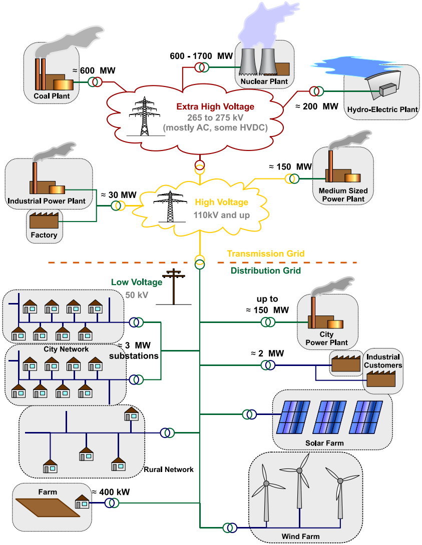
Decomposition Scheme



Decomposition Scheme



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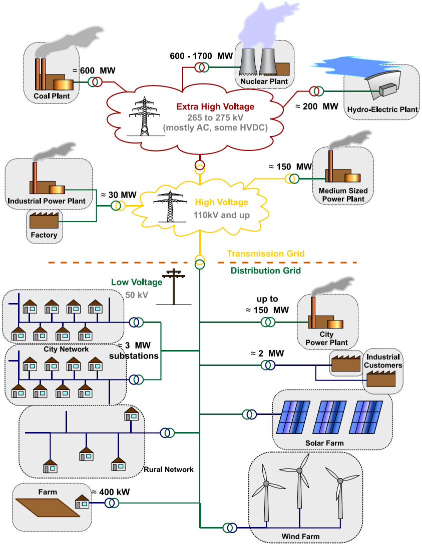

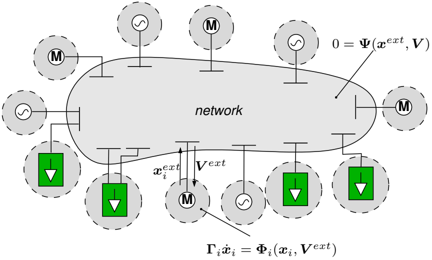

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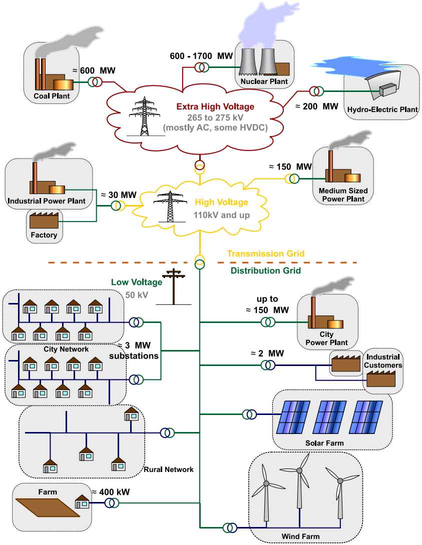
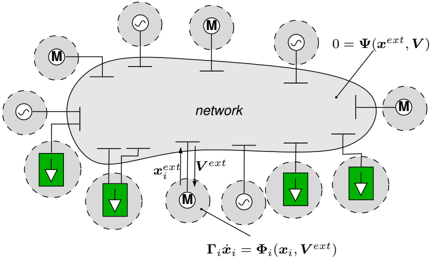




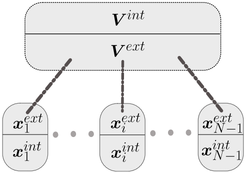
Network sub-domain:

$$0 = \mathbf{\Psi}(\mathbf{x}^{ext}, \mathbf{V})$$

Component i (*injector*) sub-domain:

$$\mathbf{\Gamma}_i \dot{\mathbf{x}}_i \quad = \mathbf{\Phi}_i(\mathbf{x}_i, \mathbf{V}^{ext})$$

Decomposition Scheme



where:

- $\mathbf{x}_i = \begin{pmatrix} \mathbf{x}_i^{int} \\ \mathbf{x}_i^{ext} \end{pmatrix}$, injector i sub-domain variables
  - $\mathbf{x}_i^{int}$: interior variables, coupled only with local equations
  - $\mathbf{x}_i^{ext}$: local Interface variables, coupled with both local and non-local (external to the sub-domain) equations
- $\mathbf{V} = \begin{pmatrix} \mathbf{V}^{int} \\ \mathbf{V}^{ext} \end{pmatrix}$, network sub-domain variables
  - $\mathbf{V}^{int}$: interior variables, coupled only with local equations
  - $\mathbf{V}^{ext}$: local Interface variables, coupled with both local and non-local (external to the sub-domain) equations

Solution Steps

Based on the Schur complement DDM:

1. Build sub-domain local systems (apply integration formula, compute Newton method matrices, etc)
2. Eliminate the interior variables from the sub-domains and build a global reduced system involving only the interface variables
3. Solve the reduced system to obtain the interface variables
4. Backward substitute the interface variables into the sub-domain local systems and solve to obtain interior variables
5. Repeat until all sub-domain local systems have converged

[Saad (2003)]

Injector sub-domain local system

$$\underbrace{\begin{pmatrix} \mathbf{A}_{1i} & \mathbf{A}_{2i} \\ \mathbf{A}_{3i} & \mathbf{A}_{4i} \end{pmatrix}}_{\mathbf{A}_i} \underbrace{\begin{pmatrix} \triangle \mathbf{x}_i^{int} \\ \triangle \mathbf{x}_i^{ext} \end{pmatrix}}_{\triangle \mathbf{x}_i} + \begin{pmatrix} 0 \\ \mathbf{B}_i \triangle \mathbf{V}^{ext} \end{pmatrix} = \underbrace{\begin{pmatrix} \mathbf{f}_i^{int}(\mathbf{x}_i^{int}, \mathbf{x}_i^{ext}) \\ \mathbf{f}_i^{ext}(\mathbf{x}_i^{int}, \mathbf{x}_i^{ext}, \mathbf{V}^{ext}) \end{pmatrix}}_{\mathbf{f}_i}$$

- $\mathbf{A}_{1i}$: coupling between interior variables, $\mathbf{A}_{4i}$: coupling between local interface variables,
- $\mathbf{A}_{2i}$ and $\mathbf{A}_{3i}$: coupling between the local interface and the interior variables,
- $\mathbf{B}_i$: coupling between the local interface variables and external interface variables of network sub-domain

Matrix $\mathbf{A}_i$

- General
- Dense
- Size: $2 \div 40$

Injector sub-domain local system

$$\underbrace{\begin{pmatrix} \mathbf{A}_{1i} & \mathbf{A}_{2i} \\ \mathbf{A}_{3i} & \mathbf{A}_{4i} \end{pmatrix}}_{\mathbf{A}_i} \underbrace{\begin{pmatrix} \triangle \mathbf{x}_i^{int} \\ \triangle \mathbf{x}_i^{ext} \end{pmatrix}}_{\triangle \mathbf{x}_i} + \begin{pmatrix} 0 \\ \mathbf{B}_i \triangle \mathbf{V}^{ext} \end{pmatrix} = \underbrace{\begin{pmatrix} \mathbf{f}_i^{int}(\mathbf{x}_i^{int}, \mathbf{x}_i^{ext}) \\ \mathbf{f}_i^{ext}(\mathbf{x}_i^{int}, \mathbf{x}_i^{ext}, \mathbf{V}^{ext}) \end{pmatrix}}_{\mathbf{f}_i}$$

Elimination of *injector* sub-domain interior variables

$$\mathbf{S}_i \triangle \mathbf{x}_i^{ext} + \mathbf{B}_i \triangle \mathbf{V}^{ext} = \tilde{\mathbf{f}}_i$$

- $\mathbf{S}_i = \mathbf{A}_{4i} - \mathbf{A}_{3i}\mathbf{A}_{1i}^{-1}\mathbf{A}_{2i}$: the "local" Schur complement matrix
- $\tilde{\mathbf{f}}_i = \mathbf{f}_i^{ext} - \mathbf{A}_{3i}\mathbf{A}_{1i}^{-1}\mathbf{f}_i^{int}$

Network sub-domain local system

$$\underbrace{\begin{pmatrix} \mathbf{D}_1 & \mathbf{D}_2 \\ \mathbf{D}_3 & \mathbf{D}_4 \end{pmatrix}}_{\mathbf{D}} \underbrace{\begin{pmatrix} \triangle\mathbf{V}^{int} \\ \triangle\mathbf{V}^{ext} \end{pmatrix}}_{\triangle\mathbf{V}} + \begin{pmatrix} 0 \\ \sum_{j=1}^{N-1} \mathbf{C}_j \triangle\mathbf{x}_j^{ext} \end{pmatrix} = \underbrace{\begin{pmatrix} \mathbf{g}^{int}(\mathbf{V}^{int}, \mathbf{V}^{ext}) \\ \mathbf{g}^{ext}(\mathbf{V}^{int}, \mathbf{V}^{ext}, \mathbf{x}^{ext}) \end{pmatrix}}_{\mathbf{g}}$$

- $\mathbf{D}_1$: coupling between interior variables, $\mathbf{D}_4$: coupling between local interface variables,
- $\mathbf{D}_2$ and $\mathbf{D}_3$: coupling between the local interface and the interior variables,
- $\mathbf{C}_j$: coupling between the local interface variables and external interface variables of sub-domain j

Matrix $\mathbf{D}$

- Structurally symmetric
- Sparse
- Size: up to $20\,000 \div 30\,000$

Network sub-domain local system

$$\underbrace{\begin{pmatrix} \mathbf{D}_1 & \mathbf{D}_2 \\ \mathbf{D}_3 & \mathbf{D}_4 \end{pmatrix}}_{\mathbf{D}} \underbrace{\begin{pmatrix} \triangle\mathbf{V}^{int} \\ \triangle\mathbf{V}^{ext} \end{pmatrix}}_{\triangle\mathbf{V}} + \begin{pmatrix} 0 \\ \sum_{j=1}^{N-1} \mathbf{C}_j \triangle\mathbf{x}_j^{ext} \end{pmatrix} = \underbrace{\begin{pmatrix} \mathbf{g}^{int}(\mathbf{V}^{int}, \mathbf{V}^{ext}) \\ \mathbf{g}^{ext}(\mathbf{V}^{int}, \mathbf{V}^{ext}, \mathbf{x}^{ext}) \end{pmatrix}}_{\mathbf{g}}$$

Elimination of *network* sub-domain interior variables

1. Eliminating the interior variables of the network sub-domain to reduce the size of the global reduced system, requires building the local Schur complement matrix $\mathbf{S}_N = \mathbf{D}_4 - \mathbf{D}_3\mathbf{D}_1^{-1}\mathbf{D}_2$ which is in general a **dense** matrix. Thus, destroying the matrix $\mathbf{D}$ sparsity.
2. Not eliminating the interior variables of the network sub-domain but including them in the global reduced system, retains the matrix $\mathbf{D}$ sparsity but increases the size of the reduced system.

Network sub-domain local system

$$\underbrace{\begin{pmatrix} \mathbf{D}_1 & \mathbf{D}_2 \\ \mathbf{D}_3 & \mathbf{D}_4 \end{pmatrix}}_{\mathbf{D}} \underbrace{\begin{pmatrix} \triangle\mathbf{V}^{int} \\ \triangle\mathbf{V}^{ext} \end{pmatrix}}_{\triangle\mathbf{V}} + \begin{pmatrix} 0 \\ \sum_{j=1}^{N-1}\mathbf{C}_j\triangle\mathbf{x}_j^{ext} \end{pmatrix} = \underbrace{\begin{pmatrix} \mathbf{g}^{int}(\mathbf{V}^{int},\mathbf{V}^{ext}) \\ \mathbf{g}^{ext}(\mathbf{V}^{int},\mathbf{V}^{ext},\mathbf{x}^{ext}) \end{pmatrix}}_{\mathbf{g}}$$

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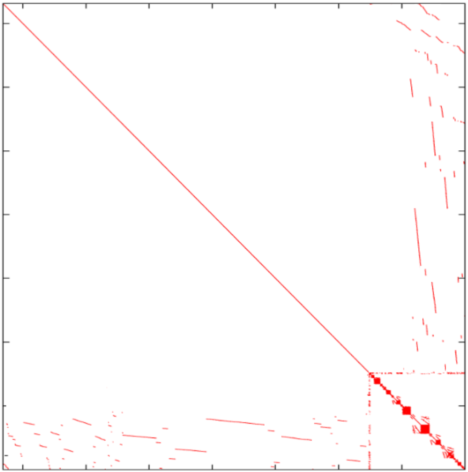
Global Reduced System

$$\begin{pmatrix} \mathbf{S}_1 & 0 & 0 & \cdots & 0 & \mathbf{B}_1 \\ 0 & \mathbf{S}_2 & 0 & \cdots & 0 & \mathbf{B}_2 \\ 0 & 0 & \mathbf{S}_3 & \cdots & 0 & \mathbf{B}_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \mathbf{D}_1 & \mathbf{D}_2 \\ \mathbf{C}_1 & \mathbf{C}_2 & \mathbf{C}_3 & \cdots & \mathbf{D}_3 & \mathbf{D}_4 \end{pmatrix} \begin{pmatrix} \triangle \mathbf{x}_1^{ext} \\ \triangle \mathbf{x}_2^{ext} \\ \triangle \mathbf{x}_3^{ext} \\ \vdots \\ \triangle \mathbf{V}^{int} \\ \triangle \mathbf{V}^{ext} \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{f}}_1 \\ \tilde{\mathbf{f}}_2 \\ \tilde{\mathbf{f}}_3 \\ \vdots \\ \mathbf{g}^{int} \\ \mathbf{g}^{ext} \end{pmatrix}$$

Global Reduced System

$$\begin{pmatrix} \mathbf{S}_1 & 0 & 0 & \cdots & 0 & \mathbf{B}_1 \\ 0 & \mathbf{S}_2 & 0 & \cdots & 0 & \mathbf{B}_2 \\ 0 & 0 & \mathbf{S}_3 & \cdots & 0 & \mathbf{B}_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \mathbf{D}_1 & \mathbf{D}_2 \\ \mathbf{C}_1 & \mathbf{C}_2 & \mathbf{C}_3 & \cdots & \mathbf{D}_3 & \mathbf{D}_4 \end{pmatrix}$$



Global Reduced System

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$$\underbrace{\begin{pmatrix} \mathbf{D}_1 & \mathbf{D}_2 \\ \mathbf{D}_3 & \mathbf{D}_4 - \sum_{i=1}^{N-1} \mathbf{C}_i \mathbf{S}_i^{-1} \mathbf{B}_i \end{pmatrix}}_{\widetilde{\mathbf{D}}} \underbrace{\begin{pmatrix} \Delta\mathbf{V}^{int} \\ \Delta\mathbf{V}^{ext} \end{pmatrix}}_{\Delta\mathbf{V}} = \underbrace{\begin{pmatrix} \mathbf{g}^{int} \\ \mathbf{g}^{ext} - \sum_{i=1}^{N-1} \mathbf{C}_i \mathbf{S}_i^{-1} \tilde{\mathbf{f}}_i \end{pmatrix}}_{\widetilde{\mathbf{g}}}$$

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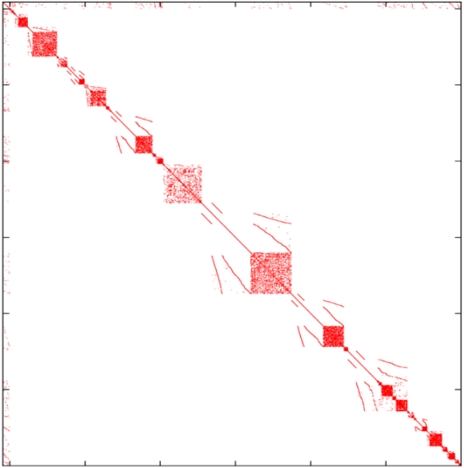
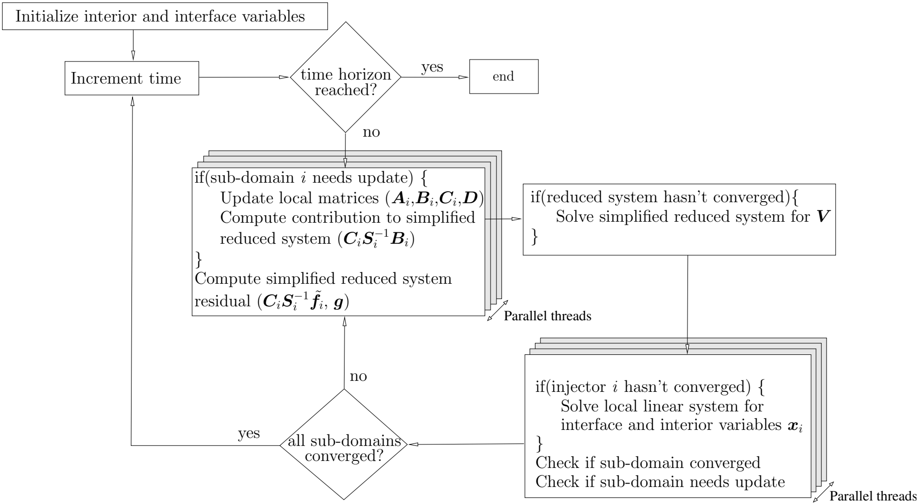


Solution Algorithm



Implementation on RAMSES[1] platform

- General
  - Standard Fortran 95/2003
  - OpenMP shared-memory parallel programing API
  - HSL41 Sparse Linear Solver
  - BLAS-LAPACK (Intel MKL)
- Tested platforms
  - Intel/AMD, UMA/NUMA, 32/64 bit
  - Windows/Linux
  - Intel/GNU Fortran

[1] "Relaxable Accuracy Multithreaded Simulator of Electric power Systems"

Test case

Power System Features	
# of buses	15226
# of branches	21765
# of injectors	10694
DAE Systems features	
# of Differential states	72293
# of Algebraic states	73946
Total	146239

Disturbance

- Short circuit near a bus lasting for 5 cycles (100ms)
- Cleared by opening a double-circuit line
- System is simulated over a period of 240s

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Results: Overview

Machine No	1.	2.	3.	4.
No of Cores	2	4	8	24
Speedup	2.9	3.3	4.1	4.5
Scalability	1.4	1.7	1.8	2.3

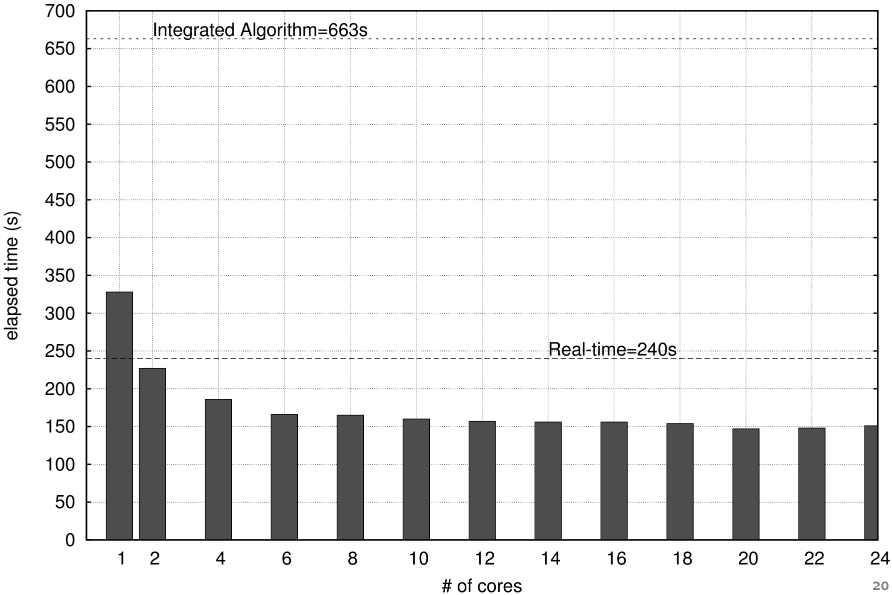
Platforms

1. Intel Core2 Duo CPU T9400 @ 2.53GHz, 3.9GB, Microsoft Windows 7
2. Intel Core i7 CPU 2630QM @ 2.90GHz, 7.7GB, Microsoft Windows 7
3. Intel Xeon CPU L5420 @ 2.50GHz, 16GB, Scientific Linux 5
4. AMD Opteron Interlagos CPU 6238 @ 2.60GHz, 64GB, Debian Linux 6

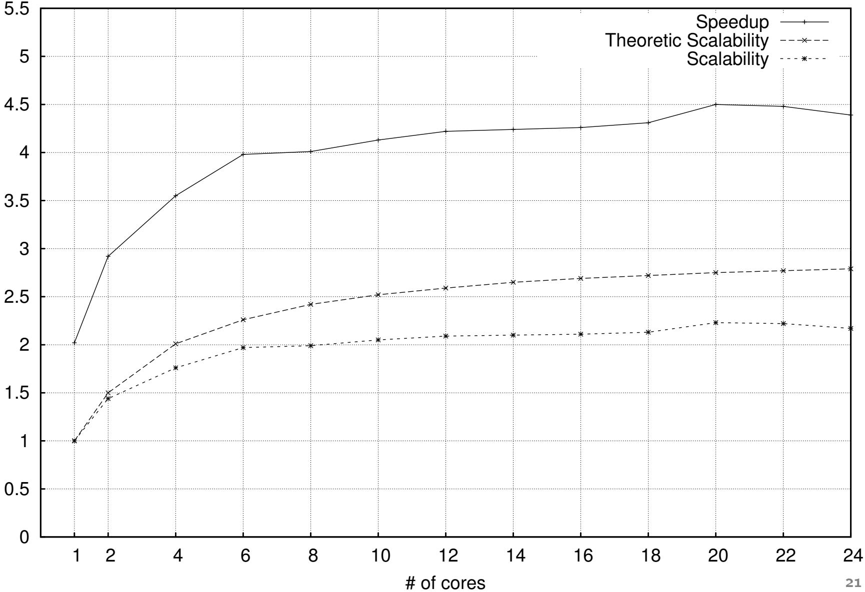
$$Speedup(M) = \frac{time\ elapsed\ integrated\ algorithm\,(1\ core)}{time\ elapsed\ parallel\ decomposed\ algorithm\,(M\ cores)}$$

$$Scalability(M) = \frac{time\ elapsed\ parallel\ decomposed\ algorithm\,(1\ core)}{time\ elapsed\ parallel\ decomposed\ algorithm\,(M\ cores)}$$

Results: Elapsed Time



Results: Speedup and Scalability



Results: Operations

Integrated

Time steps	4833
Integrated Jacobian update	1373
Integrated Newton solutions	10975

Decomposed

Time steps	4833
Global reduced system updates	32
Global reduced system solutions	8565
Local system updates *	40
Local system solutions *	5765

(*) Average per injector sub-domain (N=10694 injector sub-domains)

Conclusions

- Using a DDM for dynamic simulation of electric power systems allows the acceleration of the procedure
  - Numerically, by exploiting the locality of the sub-domain systems and avoiding the unnecessary computations (factorizations, evaluations, solutions)
  - Computationally, by exploiting the parallelization opportunities inherent to these methods
- The proposed domain decomposition algorithm
  - is accurate, DAE system is solved exactly, until global convergence, with no relaxation
  - is robust, applies to general power systems, for a great variety of disturbances without dependency on the exact decomposition
  - exhibits high convergence rate, each sub-problem is solved using a VDHN method with updated and accurate interface values during the whole procedure, convergence does not depend on number of sub-domains

Conclusions

- The implementation
  - is portable, as it can be executed on any platforms supporting the OpenMP API and a standard Fortran compiler
  - can handle general power systems, as no hand-crafted, system specific, optimizations were used
  - exhibits good sequential and parallel performance on a wide range of shared-memory, multi-core computers

The end

Thank you!