

# Persistency of lactation yields: a review

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## Abstract

Persistency of yields during the lactation is an important lactation curve parameter. It can be defined as the ability to maintain the level of production during the lactation. Real persistency, for a constant level of production, can be distinguished from apparent persistency. Persistency was defined in many different ways. Three major mathematical definitions are based: a) on mathematical lactation curve models; b) on ratios of partial, total or other yields during the lactation and c) on the variation of those yields. Definition of persistency can be extended to fat and protein yields. Heritability values for persistency are found between 0.01 and  $> 0.30$ , with most values around 0.10. Most values reported for measures based on variation and on longer periods were higher than those for ratio measures. Heritability for fat and especially protein persistencies are lower than those reported for milk persistency. Importance of persistency is double. It affects accuracy of yield evaluations based on incomplete lactation records (e.g., test-day models) and it has an economic importance on its own; as it reduces feed and other management costs. Genetic evaluations for persistency can be conceived as separate evaluation, combined (multiple-trait) with or deduced from yield evaluation.

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## 1. Introduction

The main income for most dairy farmers is based on milk, fat and protein yields of their cows. Especially since the introduction of milk quotas in the European Union, maximum improvement of yields is not necessarily economically optimum. As a matter of fact, if profits are a function of returns minus costs, reduction of costs must be considered to improve profits when increases in returns are limited. A way to reduce costs is to distribute the same total yield more equally over the whole lactation. The distribution of lactation yield is known under the name of persistency of lactation yields, often simply called persistency. The precise definition is often not well worked out, but in general we say that the lactation of a cow is more persistent if, for the same total yield, the animal peaks lower and the lactation curve is flatter. Better persistency is considered advantageous for a certain number of reasons. The two most important are the better use of cheap roughage (e.g., Sölkner and

Fuchs, 1987) and the reduction of stress due to high peak production (e.g., Zimmermann and Sommer, 1975).

The concept of persistency is also often related to the mathematical description of the lactation curve. Several different measures are classically used to describe persistency. We will describe the most important persistency measures and we will deal with the genetic aspects of persistency and its introduction in current and future selection schemes for yield traits.

## 2. Definition of persistency

### 2.1. Persistency of milk yields

Different approaches exist to define persistency and there is a certain ambiguity. In general persistency can be defined as the ability to maintain a more or less constant yield during the lactation. If we consider that the shape of the curve is different from one animal to another (Danell, 1982), such

difference exists even at a constant level of production (Grossman et al., 1986). Therefore we can define persistency as a function of the flatness of the lactation curve. For an animal this means that it is more persistent as another if the lactation curve has a flatter shape. In the literature this concept is called persistency of the lactation, or persistency of the lactation curve or persistency of milk yields or persistency. It is clear that the shape of the lactation curve depends also on the total yield, represented by the surface below. Therefore Gengler (1995) distinguished between apparent or observed persistency and real persistency. The first is defined without considering total yields constant

### 2.2. Persistency of fat and protein yields

It is obvious that the same concepts as for milk yields can be generalized for fat and protein, even if such studies have been rather seldom in the literature (e.g., Kandzi and Glodek, 1990; Bouloc and Boichard, 1991; Swalve, 1994).

## 2. Lactation curves

The milk production of a cow can be subdivided into three parts. An ascending phase between calving and peak yield, a near constant production around peak and a descending part after peak (e.g., Gengler, 1990). Lactations failing to show the first phase, or showing steadily increasing production are called atypical lactations. Shanks et al. (1981) summarized proportions reported in different papers. The literature is not very consistent concerning the relative importance of atypical curves as proportions from 15% for Ferris et al. (1985) to 45% for Schneeberger (1978) are given. These differences are due to different definitions of atypical, differences in population studies, management of animals and recording procedures (e.g., Belgium). But the existence of atypical lactations is a fundamental problem mathematical models used must deal with. Lactation curves are also important in the field

of incomplete lactation extension (e.g., Keown and Van Vleck, 1973; Schaeffer and Burnside, 1976) and of test-day models (e.g., Ptak and Schaeffer, 1993).

## 3. Mathematical lactation curve models

### 3.1. Introduction

Different approaches have been used to find a function to fit observed daily yields. Different objectives of these functions also exist. The three most important are: 1) the adjustment for individual cows to describe a given curve and eventually the persistency, 2) the adjustment of a curve for groups of cows, which is often done for management reasons and 3) the estimation of 305 day yields using the adjusted curve, the total yield being only the surface below the curve.

Several different models exist (e.g., Wood 1967; Grossman and Kooops, 1988; Cobby and Le Du, 1978) and we will try to group them to make the comprehension of the models easier.

### 3.2. Exponential functions

The use of exponential functions goes back to Gaines (1927). But Wood (e.g., 1967) popularized this approach. It was then used by several other authors (e.g., Kellogg et al., 1977). His basic function is often called Wood's incomplete gamma function. If we define the daily yield at day  $t$  by  $y_t$ , the model is:

$$y_t = at^b e^{-ct} \quad (1)$$

where  $a$ ,  $b$  and  $c$  are parameters,  $a$  being linked to peak yield,  $b$  to the increasing phase slope and  $c$  to the decreasing phase. Parameters can be estimated using the natural logarithmic transformation (e.g., Shanks et al., 1981; Grossman et al., 1986; Batra et al., 1987; Congleton and Everett, 1980a and 1980b):

$$\ln(y_t) = \ln(a) + b \ln(t) - ct \quad (2)$$

This transformation yields a multiple regression of  $\ln(y_t)$  on  $\ln(t)$  and  $t$  with parameters  $\ln(a)$ ,  $b$  and  $c$  that can be estimated by ordinary least squares. This function has the weakness that seasonal effects have a strong influence on  $y_t$  and therefore the following modification was proposed (e.g., Grossman et al., 1986; Batra et al., 1987):

$$y_t = at^b e^{-ct} (1 + u \sin(x) + v \cos(x)) \quad (3)$$

where  $u$  and  $v$  are additional parameters associated with seasonal variation other than season of calving and  $x$  is the day of the year expressed as radians. With certain assumptions we can also use the logarithmic transformation:

$$\ln(y_t) = \ln(a) + b \ln(t) - ct + u \sin(x) + v \cos(x) \quad (4)$$

The problem with this transformation is that the number of parameters is augmented so therefore the fit should be better, but we lose residual degrees of freedom, or from the more practical point of view, we need to know at least 6 test-days.

Schneeberger (1981) proposed another modification of Wood's gamma function to take the time of initial production ( $t_0$ ) into account:

$$y_t = a(t - t_0)^b e^{-c(t - t_0)} \quad (5)$$

Schaeffer et al. (1977) used a more complex exponential function, called a one-compartment open model.

### 3.3. Multiphasic models

As the lactation can be easily subdivided in different phases, the idea was developed to use multiphasic models. An example is given by Grossman and Koops (1988):

$$y_t = \sum_{i=1}^n \left\{ a_i b_i \left[ 1 - \tanh^2(b_i(t - c_i)) \right] \right\} \quad (6)$$

where  $y_t$  is the production at day  $t$ ;  $a_i$ ,  $b_i$  and  $c_i$  are parameters associated with phase  $i$ ;  $\tanh$

is the hyperbolic tangent and  $n$  the number of phases. This method works well if 2 or 3 phases are used (Grossman and Koops, 1988), but 6 or 9 parameters must be estimated. This fact limits the use of this approach to situations where enough observations are known.

This method has a certain number of advantage but has been seldom used. The main reason is that despite its theoretical advantages, it can not be linearized, so that parameter estimation is difficult.

### 3.4. Polynomial and inverse polynomial models

Ali and Schaeffer (1987) used an inverse quadratic polynomial model first described by Nelder (1966) and used by Yadav et al. (1977) for dairy cattle. This model is written as:

$$y_t^{-1} = \beta_1 + \beta_0 t^{-1} + \beta_2 t \quad (7)$$

where the parameter  $\beta$  represents peak yield,  $\beta_0$  is associated with increasing and  $\beta_2$  with decreasing phase and  $y_t^{-1}$  is the inverse of yield at day  $t$ . The model is written as a multiple regression of  $y_t^{-1}$  on  $t$  and  $t^{-1}$ . It has the advantage that the number of parameters is limited to 3.

### 3.5. Multiple regression

Another interesting way to describe the lactation curve is the multiple regression approach first described by Ali and Schaeffer (1987). The regression model has the form:

$$y_t = p_0 + p_1 \gamma_t + p_2 \gamma_t^2 + p_3 \omega_t + p_4 \omega_t^2 \quad (8)$$

where:  $\gamma_t = t/305$  and  $\omega_t = \ln(305/t)$ . The parameters are again linked to the lactation curve:  $p_0$  with the peak yield,  $p_1$  and  $p_2$  with decreasing and  $p_3$  and  $p_4$  with increasing phase. This model permits a good description of the curve according to results presented by Ali and Schaeffer (1987), but it has 5 parameters and therefore needs at least 6

observations to estimate these parameters. This model was used in the modeling of test-day (TD) genetic evaluation as correction for days in milk at test-day by Ptak and Schaeffer (1993). In this context corrections are done for all TD records in lactations of cows grouped in given age-season classes and therefore the number of parameters is not important.

A new development is the random regression model (e.g., Jamrozik and Schaeffer, 1995) where the lactation curve is described phenotypically and genetically using Ali and Schaeffer (1987) multiple regression. Fixed regression coefficients are estimated for given region-age-season classes and random (genetic) regression coefficients for an animal using the genetic covariances among coefficients and the numerator relationship matrix among animals.

### 3.6. Other models

Several other models can be used and have been described in the literature. An example is the vibration model reported by Hayashi et al. (1986). Cobby and Le Du (1978) described a certain number of alternative models, essentially modified exponential function based on Wood's function (e.g., 1967).

## 4. Persistency measures

### 4.1. Measures based on mathematical lactation curve models

The most common mathematical model is based on Wood (1967). This model has the particularity that it is rather easy to develop a simple formula of parameters associated with persistency. This simple formula is:

$$P = c^{-(b+1)} \quad (9)$$

Another way persistency can be deduced out of such an mathematical model is given by Grossman and Koops (1988). They associated with their multiphasic model, persistency to the duration of their second phase:

$$P = 2b\bar{2} \quad (10)$$

For multiple regression models persistency can be linked to slope of the decreasing phase of lactation.

Random regression models (e.g., Jamrozik and Schaeffer, 1995) can provide breeding values for individual day yields and for partial, total or other lactation yields. Persistency measures can than be developed out of these values.

Other formulas exist to describe persistency out of lactation curve models (Rowlands et al., 1982), but their importance for the description of persistency is rather limited. Another problem is that one should be able to describe all types of lactations, including atypical ones.

### 4.2. Measures based on ratios between total, partial, maximum, or other yields

The idea to use ratios as measures of persistency is old, it goes at least back to

Sanders (1930). He defined persistency as the ratio between mean and peak yield. This idea was used by Sölkner and Fuchs (1987). They defined two persistency measures,  $P_{TOMAX2}$  and  $P_{TOMAX3}$ , as:

$$P_{TOMAX2} = \frac{\text{Max. yield}_{\text{days 1-200}}}{\text{Mean yield}_{\text{days 1-200}}} * 100 \quad (11)$$

$$P_{TOMAX3} = \frac{\text{Max. yield}_{\text{days 1-305}}}{\text{Mean yield}_{\text{days 1-305}}} * 100 \quad (12)$$

Higher values for  $P_{TOMAX2}$  and  $P_{TOMAX3}$ , are associated with lower persistency. This not very logical fact must be remembered when comparing them to other methods.

Keown et al. (1986) used an modification of this method:

$$P = \frac{\text{Max. yield}_{\text{days 1-300}}}{\text{Yield}_{\text{at day 300}}} * 100 \quad (13)$$

This measure is also linked negatively to persistency and shares therefore the same intuitive weakness as  $P_{TOMAX2}$  and  $P_{TOMAX3}$ .

Others such as Johansson and Hansson (1940) introduced ratios between partial yields. These methods were popular and many variants of their two measures,  $P_{2:1}$  and  $P_{3:1}$ , exist:

$$P_{2:1} = \frac{\text{Partial yield}_{\text{days 101-200}}}{\text{Partial yield}_{\text{days 1-100}}} \quad (14)$$

$$P_{3:1} = \frac{\text{Partial yield}_{\text{days 201-300}}}{\text{Partial yield}_{\text{days 1-100}}} \quad (15)$$

Some scientist (e.g., Ericson et al., 1988) used the same ratios but expressed them as percentages.

Others modified them, as the persistency measure defined by Mahadevan (1951):

$$P = \frac{\text{Partial yield}_{\text{days 1-180}} - \text{Partial yield}_{\text{days 1-70}}}{\text{Partial yield}_{\text{days 1-70}}} \quad (16)$$

Danell (1982) also modified  $P_{2:1}$ , but with the intention of creating a measure associated with ascending and descending phases of lactation called  $P_1$  and  $P_2$ . She defined:

$$P_1 = \frac{\text{Partial yield}_{\text{days 91-180}}}{\text{Partial yield}_{\text{days 1-90}}} * 100 \quad (17)$$

$$P_2 = \frac{\text{Partial yield}_{\text{days 121-210}}}{\text{Partial yield}_{\text{days 31-120}}} * 100 \quad (18)$$

All the ratio methods based on  $P_{2:1}$  and  $P_{3:1}$  associate higher values to better persistency.

Other ratio methods exist, but all share the same basic definitions of ratios between certain partial, peak, daily, total or other yields.

#### 4.3. Measures based on variation of yields

The definition of persistency as flatness of the curve can be easily described by measures of dispersion. This rather simple idea seems to have been seldom used in the past. Only since its use by Sölkner and Fuchs in 1987, it has been reconsidered by other scientists (Gengler, 1990; Swalve, 1994).

Sölkner and Fuchs (1987) defined two persistency measures based on the standard deviations of test-day yields:

$$P_{SD2} = \sqrt{\frac{1}{p} \sum_{i=1}^p (TD_i - \mu_{TD})^2} \quad (19)$$

where  $TD_i$  is the  $i$ th test-day yield,  $\mu_{TD}$  is the mean test-day yield and  $p$  is the number of the last test-day before 200 days in milk.

$$P_{SD3} = \sqrt{\frac{1}{p} \sum_{i=1}^p (TD_i - \mu_{TD})^2} \quad (20)$$

where  $TD_i$  is the  $i$ th test-day yield,  $\mu_{TD}$  is the mean test-day yield and  $p$  is the number of the last test-day before 305 days in milk. This measure has a weak point, the lactation needs to be finished, or at least considered finished. Both methods are affected by the fact that a

given test-day does not necessary take place at the same stage of lactation.

An important advantage of these measures is that their distribution can be empirically considered nearer to a normal one than for ratio measures (Sölkner and Fuchs, 1987). But at the same time all persistency measures based on variation methods share the disadvantage that their values become nearer to zero with higher persistency.

But according to Gengler (1990) and Swalve (1994), the measures based on the variation of test-days are influenced by erratic variations that are observed from test-day to test-day. Gengler et al., 1995 used therefore the persistency measure called  $P_{YV}$  measured as yield variation and defined as:

$$P_{YV} = \left\{ \frac{1}{305} \left[ \frac{(M_1)^2}{100} + \frac{(M_2)^2}{100} + \frac{(M_3)^2}{105} - \frac{(M_T)^2}{305} \right] \right\}^{0.5} \quad (21)$$

where  $M_1$  is the partial milk yield from day 1 to day 100,  $M_2$  is the partial milk yield from day 101 to day 200,  $M_3$  is the partial milk yield from day 201 to 305 and  $M_T$  is the total milk yield from day 1 to day 305. Gengler (1995) modified slightly the definition replacing the 0.5 by 0.25. He used a suggestion made earlier (Gengler, 1990) and expressed persistency on a relative scale intra-lactation.

## 5. Influence of milk yield on persistency

The problem of links between persistency of lactation and milk yields has interested scientists for many years (e.g., Gaines, 1927; Mahadevan, 1951). There are at least two reasons for this link. Total yield is the area below the curve, and the yield at every moment of the lactation is a function of the curve. On the other hand it is clear that an animal with very high production at peak have likely a steeper slope than another that is low producing (Gengler, 1990). Therefore we can assume that the milk yield has an important influence on persistency.

Many scientists have analyzed the relationships between persistency, measured through different parameters, and initial, peak, partial, total, etc. milk yields. The results found does not show a consistent pattern. Early studies found a negative relationship between persistency and total yield (Gaines, 1927). Later Mahadevan (1951) found a positive relationship. This result is verified by Schneeberger (1981). Now it is clear that the relationship between total yield and persistency depends essentially on the persistency measure used (Sölkner and Fuchs, 1987). Persistency measures based on ratios showed a positive relationship, but the methods based on variation a negative relationship. Gengler (1990) showed that this later relationship is close to a linear one. If we take the definition given by Grossman et al. (1986), persistency should be independently measured, or corrected after, for milk yields. Therefore a good persistency measure should be independent from yields, or corrected for the influence of yields.

As a conclusion we can state that persistency is dependent on yields, especially total yields, but the direction of the relationship depends on the measures used. The ratio measures show a positive one, whereas the variation measures show a negative relationship. The reason for this could be that the first are highly affected by the level of production and the second are influenced by variation in production, with this variation more important for high producing cows.

The influence of total yields has two components, a genetic and a non genetic, therefore if phenotypic relationships are forced to zero through the definition of real persistency, genetic relationships exists. Table 1. gives results obtained by Gengler (1995) using a persistency measure based on a function of the variation of yields.

## 6. Environmental influences on persistency

### 6.1. Parity and age at calving

The effect of age and parity on persistency has been studied by several scientists. Certain scientists separate these effects or worked with separate lactations (e.g., Sölkner and Fuchs, 1987; Gengler, 1990), while others considered them together (Sanders, 1930).

All authors agreed that the first lactation is more persistent than the others (e.g., Shanks et al., 1981; Danell, 1982; Keown et al., 1986; Sölkner and Fuchs, 1987). The most common explanation is based on a lower level of development of the mammary gland for first parity cows (e.g., Sölkner and Fuchs, 1987).

The results concerning the influence of age within parity are not so similar. Mahadevan (1951) for example did not find an influence, Danell (1982) and Grossman et al. (1986) found a very small influence that is more important for younger cows. Grossman et al. (1986) did not find it significant. Others such as Smith and Legates (1962) described a reduction in persistency with age during the first lactation and an augmentation during later ones. Gengler (1990) found results that supported the later hypothesis. All authors agree that it would be important to study effects on persistency of parity and age separately.

### 6.2. Season of calving

Most research that has been done confirms the influence of season of calving on persistency (e.g., Mahadevan, 1951; Cady and Mc Dowell, 1980; Danell, 1982; Ferris et al., 1985; Grossman et al., 1986; Sölkner and Fuchs, 1987; Gengler, 1990). Only Schneeberger (1981) did not confirm this. But the different studies did not find the same influence. Gengler (1990) explained that obviously such differences are due to climatic and management differences among the population studied. Some papers described studies done in North America (e.g., Ferris et al., 1985; Keown et al., 1986) who found that the most persistent lactations begins at end summer. But European studies as Mahadevan (1951), Danell (1982) and Sölkner and Fuchs (1987) found that the most persistent lactations begin in fall-winter. Gengler (1990)

explained that these results are also affected by the length of the measured period. As a matter of fact the way persistency is measured also affects the time of the apparent maximum persistency (Sölkner and Fuchs, 1987). Longer intervals tend to shift this maximum to earlier dates in the year.

### 6.3. Influence of gestation

The new calf that a cow is carrying during the last part of its lactation is able to influence the potential milk production of a cow. The influence on daily yields during this period is direct and creates an indirect influence on persistency. Several ways have been described to measure this influence. Some scientists used the days open approach (e.g., Schneeberger, 1981; Danell, 1982; Grossman et al., 1986; Sölkner and Fuchs, 1987), others used days carried calf (e.g., Keown et al., 1986). Gengler (1990) took calving interval as the parameter. Animals were often grouped in classes (e.g., Schneeberger, 1981; Danell, 1982; Keown et al., 1986; Gengler, 1990). Sölkner and Fuchs (1987) used a linear and quadratic regression on days open.

The results found were not similar. Some people found no influence (e.g., Danell, 1982; Grossman et al., 1986), others such as Sölkner and Fuchs (1987), Schneeberger (1981), and Smith and Legates (1962) showed that gestation influenced persistency. Sölkner and Fuchs (1987) observed a linear relationship. They also showed that the persistency measures covering a long period are more affected than those covering short periods. Gengler (1990) found a relationship of gestation with persistency only in the first lactation and it had a quadratic component.

## 7. Heritability of persistency

The literature on persistency measures shows that the different measures have also different levels of heritability. This additional source of variation for the heritability makes the interpretation of the literature even more

difficult. Gengler (1990) gave a review of the literature. Results were between under 0.01 (Shanks et al., 1981) and over 0.30 (Smith and Legates, 1962). Another important point is that holding the milk yield constant reduced the heritability (Danell, 1982). According to Sölkner and Fuchs (1987), heritabilities were rather constant from lactation to lactation, but rather different between the persistency measures they studied. They found that longer measures including the whole lactation and variation methods had the highest heritabilities, PSD<sub>3</sub> had a heritability of 0.21, 0.22 and 0.22 in the three first lactations. Gengler (1995) found that values for apparent and real persistency are close if persistency is defined as variation of partial yields. Heritability of real persistency for milk yields was with 0.14 higher than fat (0.06) and protein (0.04) (Table 1 and Table 2). Similar results were reported by Swalve (1994) and Kandzi and Glodek (1990).

#### **8. Repeatability of persistency**

Another important question is what is the repeatability of persistency from lactation to lactation. Most research has been done lactation by lactation (Sölkner and Fuchs, 1987; Gengler, 1990), but it is interesting to assess how similar are two persistency records are for the same animal. Results from Gengler (1995) show that real persistencies for milk, fat and protein yields had slightly higher repeatabilities (Table 2) compared to apparent persistencies (Table 1). The rather low results for repeatability indicate that at least persistency for first and later lactations can be considered different traits.

#### **9. Correlations between milk, fat and protein persistencies**

Very few results are known showing the correlations between milk, fat and protein persistencies. Table 1 and Table 2 show the values obtained by Gengler (1995). They show that genetic correlations are higher than

phenotypic correlations. Correlations are also higher between protein and persistency than between milk and fat or protein.

#### **10. Economic importance of persistency**

Economic importance of persistency is linked to the reduction of costs obtained by better persistency. Two types of reductions exists. First better persistency reduces feed costs as shown by Sölkner and Fuchs (1987) and Gengler (1995) who linked persistency to the replacement of concentrates by roughage. Gengler (1995) found that this reduction gives persistency a relative weight compared to yield of around 3 %. Better persistency reduces also health and reproductive costs. Here he found around 7 % of the relative economic value of yield.

#### **11. Conclusion**

In the literature persistency is mostly defined as lactation or milk yield persistency, and persistencies of fat and protein yield are seldom considered. Different persistency measures were described in the literature. Three great types of measures can be defined: 1) measures based on ratios between total, partial or other yields; 2) measures based on variation of test-day yields and 3) measures based on mathematical models. Environmental influences on persistency are well described in the literature, even if there are differences between persistency measures and population studied. The most important effects are milk yields and seasonal effects. The heritabilities of persistency measures reported in the literature range from under 0.05 to over 0.30.

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Table 1

Genetic correlations (above), heritabilities and repeatabilities (on, repeatabilities between brackets) and phenotypic correlations (below the diagonal) among apparent milk, fat and protein persistencies

Traits	Traits		
	Milk persistency	Fat persistency	Protein persistency
Milk persistency	0.14 (0.24)	0.83	0.89
Fat persistency	0.52	0.06 (0.14)	0.88
Protein persistency	0.52	0.70	0.05 (0.10)

Table 2

Genetic correlations (above), heritabilities and repeatabilities (on, repeatabilities between brackets) and phenotypic correlations (below the diagonal) among real milk, fat and protein persistencies

Traits	Traits		
	Milk persistency	Fat persistency	Protein persistency
Milk persistency	0.14 (0.26)	0.81	0.90
Fat persistency	0.51	0.06 (0.15)	0.86
Protein persistency	0.52	0.72	0.04 (0.10)