A set $E \subseteq \mathbb{N}$ is called $k$-recognizable if the set of base-$k$ expansions of elements of $E$ forms a regular language (recognizable by means of a finite automaton). A classical theorem of Cobham [2] states that a set is simultaneously $k$- and $\ell$-recognizable ($k, \ell$ being multiplicatively independent, i.e., $\log k / \log \ell$ is irrational) if and only if it is the finite union of arithmetic progressions. Roughly speaking it means that the only sets of integers whose representations in all bases have “simple” syntactical properties are exactly the ultimately periodic sets.

This result has led to many developments during the last forty years. The aim of this talk is to present to non-specialists some of these directions linking formal language theory and number theory: connections with first order logic, automatic sequences and combinatorics on words, introduction of non-standard and abstract numeration systems. For instance, the following result has been completely settled by F. Durand [3]. Let $\alpha$ and $\beta$ be two multiplicatively independent Perron numbers. An infinite sequence taking values in a finite set $A$ is both $\alpha$-substitutive and $\beta$-substitutive if and only if it is ultimately periodic. With any morphism of the free monoid $A^*$ is associated a square matrix $M$ with non-negative integer entries and such a matrix has a real eigenvalue $\alpha$ larger or equal to the modulus of any other eigenvalue of $M$. Therefore the notion of $\alpha$-substitutive word over the finite alphabet $B$ can easily be derived (it includes a projection on $B$ given by a coding $\Phi : A \to B$). We also aim at presenting some related decision problems in connection with Emilie Charlier’s talk and studied extensively by the community. To that end, we will briefly present the notion of state complexity.

References


