Several years El Niño forecast using a wavelet-based mode decomposition

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From the Fourier series

A Fourier series decomposes a periodic signal into a possibly infinite sum of sines and cosines functions. From a practical point of view, a signal f is decomposed as a sum of K cosines:

$$f(t) \approx \sum_{k=1}^{K} c_k \cos(\omega_k t + \phi_k).$$

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One thus only gets an approximation of the original signal. Such a decomposition often leads to a decomposition with too many terms, i.e. with a too large K.

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One thus tries to have the following decomposition,

$$f(t) \approx \sum_{j=1}^{J} C_j(t) \cos(\omega_j t + \phi_j),$$

where J should be much smaller than K.

The wavelet spectrum

We use the progressive wavelet

$$\psi(t) = \frac{1}{2\sqrt{2\pi}} \exp(i\Omega t) \exp(-\frac{(2\Omega t + \pi)^2}{8\Omega^2})(\exp(\frac{\pi t}{\Omega}) + 1),$$

with $\Omega = \pi\sqrt{2/\ln 2}$.

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with $\Omega = \pi\sqrt{2/\ln 2}$.
The wavelet spectrum associated to the signal f is defined as

$$\Lambda(a) = E|Wf(\cdot, a)|,$$

where Wf is the continuous wavelet transform of f and E denotes the mean over the time t.

The wavelet decomposition

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The signal f is then decomposed in the following way,

$$f(t) = f_0(t) + \sum_{j=1}^{J} f_j(t),$$

with

$$f_j(t) = |Wf(t, a_j)| \cos\big(\arg Wf(t, a_j)\big),$$

if $j \geq 1$ and $f_0 = f - \sum_{j=1}^{J} f_j$.

Approximation of the signal

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If we assume that the mean frequency of a component f_j is constant, the reconstructed signal has the following form,

$$f(t) \approx \hat{f}(t) = \sum_{j=1}^{J} C_j(t) \cos(\omega_j + \phi_j),$$

where ω_j corresponds to the mean (angular) frequency.

Extrapolation

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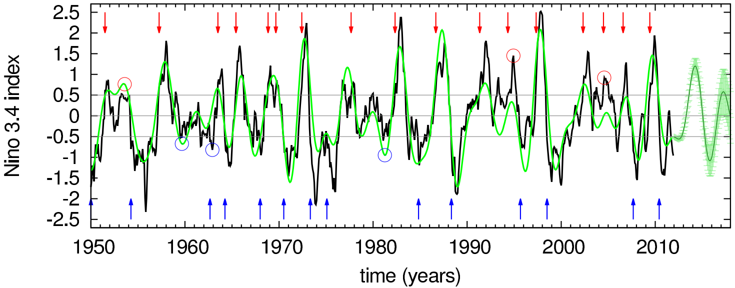
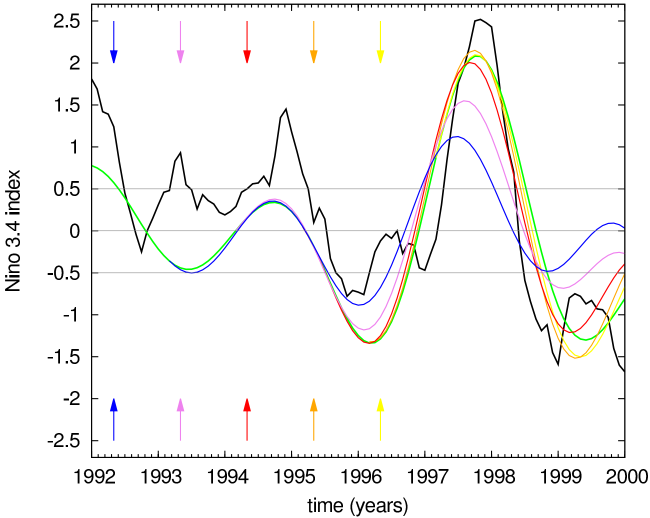
Summing the so-obtained extrapolated values related to each component defines the forecast; more precisely, it corresponds to a forecast of the reconstructed signal $\hat{f}$.
The efficiency of the predictions was tested with probing hindcasts.

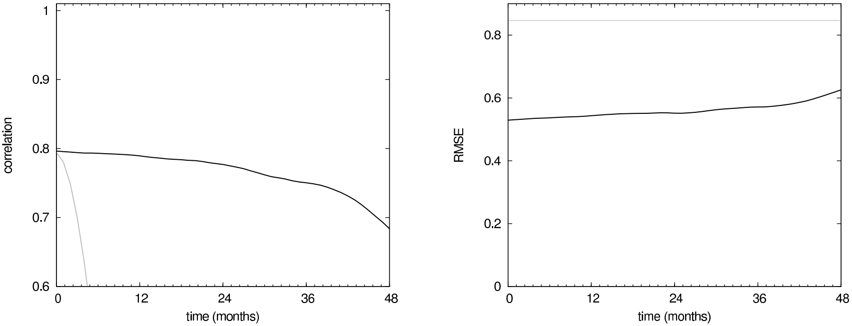
Application to the Nino 3.4 index



Forecast of the 1997–1998 El Niño event



Anomaly correlation and RMSE



The end

Thanks for your attention!