Making Europe more attractive for researchers

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MARIE CURIE ACTIONS

Mathematics: Discrete Optimization

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An example of a discrete problem

What is the shortest possible tour among european capitals?

THE TRAVELING SALESMAN PROBLEM.

25 cities = 15511210043330985984000000 possible tours

Enumerate all the possible tours with a fast computer (1 000 000 000 tours per second) would take approximately

500 000 000 years !!

Phenomenon known as the combinatorial explosion!
Improving hardware does not suffice.

One needs good algorithmic solutions

Today one can solve the traveling salesman with 20 000 cities.

A Mathematical Model

Formulation = Variables + Objective + Constraints

VARIABLES = Choice of decisions

$$x_{city\ 1}^{city\ 2} = \begin{cases} 1 & \text{if city 1 and 2 are joined in the tour} \\ 0 & \text{otherwise} \end{cases}$$

Example: $x_{Lisbon}^{Dublin}, x_{Lisbon}^{London}, x_{Paris}^{Berlin}, \dots$
In total: 300 variables

OBJECTIVE = mathematical quantity to maximize (or min.)

Ex: sum of every variable multiplied by the corresponding distance

$$\min 2870\, x_{Lisbon}^{Dublin} + 1442\, x_{Lisbon}^{London} + 4530\, x_{Lisbon}^{Athens} + 677\, x_{Paris}^{Berlin} + \cdots$$

CONSTRAINTS = conditions that define a feasible solution

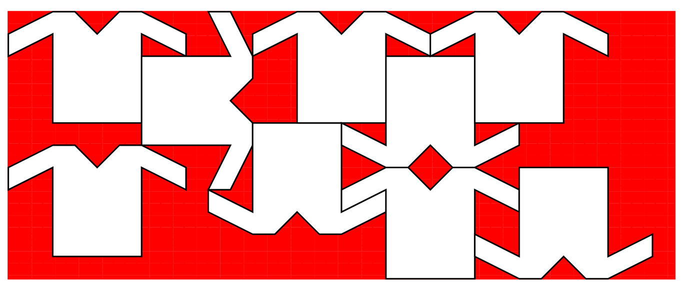
In each city, one in- and out- arc.

$$x_{Berlin}^{London} + x_{Berlin}^{Prague} + x_{Berlin}^{Paris} + \cdots = 2$$

The tour cannot contain subtours.

$$x_{Cop}^{Stock} + x_{Stock}^{Hels} + x_{Hels}^{Cop} \le 2$$

Another Example



How to arrange as many as possible patterns on a roll of material?
This is the cutting stock problem.

Other Applications

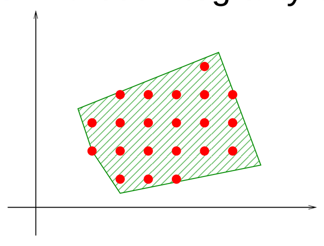
- Sequencing of airplanes landings, GPS systems
- DNA sequencing, Biomedicine
- Electronic chip design, Network design
- Metallurgic or chemical industry, Production planning
- Vehicle routing, Postal tours
- and a lot more ...

State-of-the-art solving techniques

Today's techniques are based on three ingredients.

RELAXATION

Try to solve an easier variant of the problem ⟶ Obtain information.
Example Without integrality constraints: the linear relaxation.



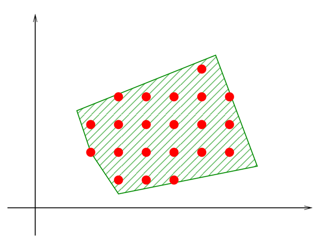
- Green area is a linear relaxation.
- In red : discrete feasible solutions.
- Optimizing over the green area easier than over the red dots.

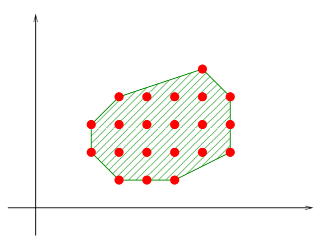
INTELLIGENT ENUMERATION

Enumerate using the information provided by relaxations.

REFORMULATION

Change the reformulation to make the relaxation more meaningful.
Often: Add redundant constraints.





To the right: every optimal point (vertex) of the green area is a red dot.

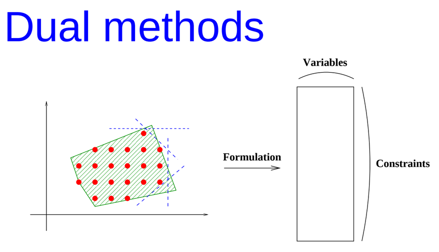
Solving the relaxation = solving the original problem

BRANCH-AND-CUT : Technique present in most solvers

Combine enumeration, linear relaxation and adding constraints (cuts).

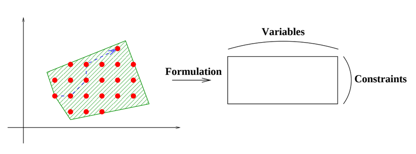
Future and Current Research

Dual methods



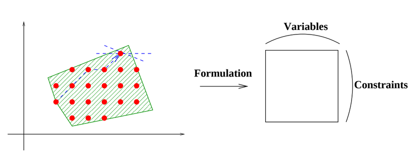
- Add constraints ⟶ better formulation
- “Many” constraints and “few” variables
- Basic method of most softwares
- Successful on many instances but not all.

Primal methods



- Add variables ⟶ improving solutions
- Progress from solutions to solutions
- “Few” constraints and “many” variables
- New method with encouraging success.

Primal-Dual methods



- Add variables **and** constraints
- Good but more compact formulation
- Brand-new idea that opens up the possibility of new algorithms.

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- Magdeburg University, Germany
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- IASI Roma, Italy
- University of Grenoble, France
- University of Cologne, Germany
- EPFL Lausanne, Switzerland
- Lisbon University, Portugal
- Dash associates, UK
- Vienna University, Austria
- University of Budapest, Hungary
- Klagenfurt University, Austria