De la dynamique des structures aux systèmes non-linéaires : enjeux et perspectives pour la réduction de modèle

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Atelier de la SIA : La réduction de modèles
Le 21 avril 2010

Acknowledgements

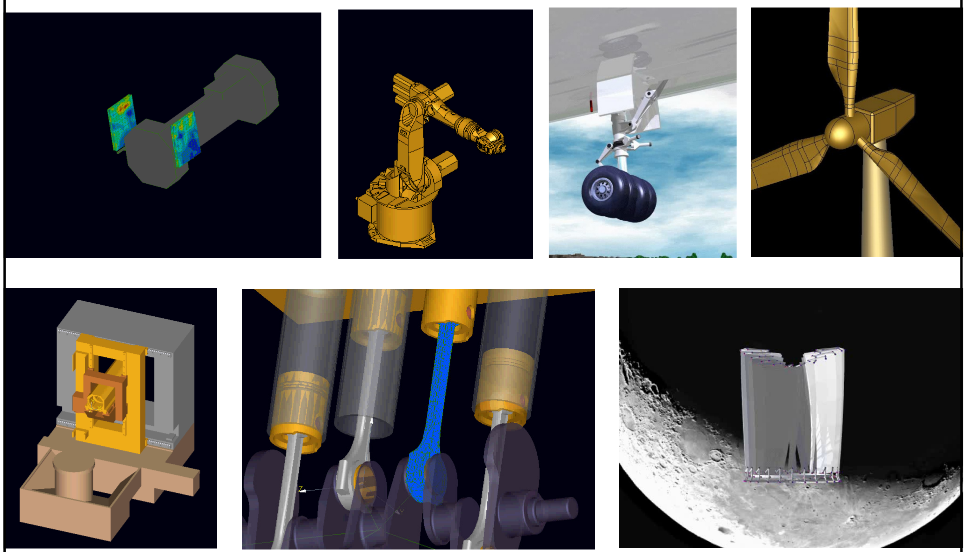
From the University of Liège

- Pierre Duysinx
- Jean-Claude Golinval
- Daryl Hickey
- Sébastien Hoffait
- Gaetan Kerschen

From Open-Engineering S.A.

- Philippe Nachtergaele

Multibody & Mechatronic Systems Lab



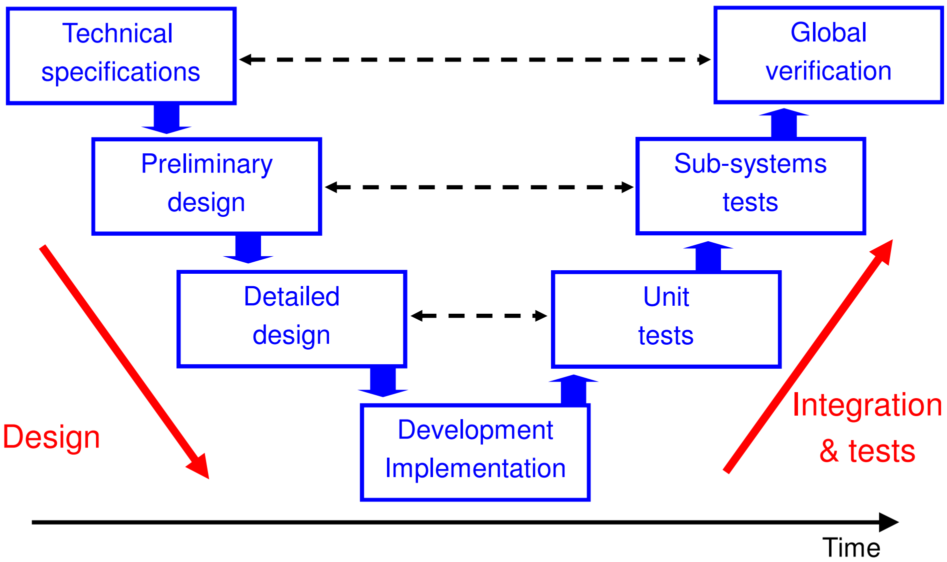
Some models developed using Samcef/Mecano

Outline

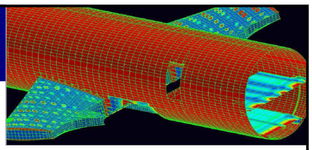
- ❑ Introduction
- ❑ Linear model reduction techniques
- ❑ Nonlinear model reduction techniques

Toward system-level simulation

V-model of the development cycle



Component-level models



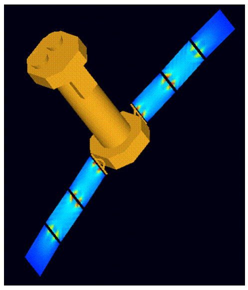
- Structural mechanics, fluid mechanics, electromagnetics, thermal, multiphysics, etc
- Off-the-shelf simulation toolboxes
- ODE/DAE models obtained by spatial discretization
- Large number of states (related to the mesh refinement)
- “Not-so-interesting” high frequency modes
- Linear vs. nonlinear models

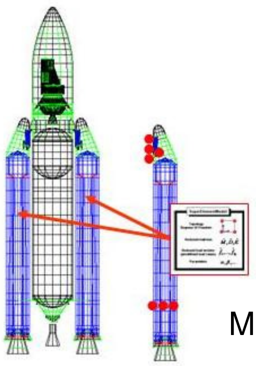
How to exploit those detailed models at system-level ?

Typical applications

Mechatronic system (MEMS)

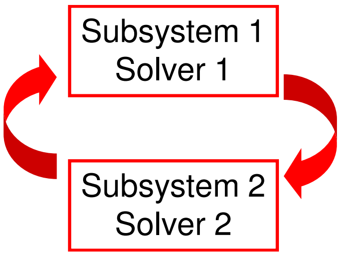
Flexible multibody system





Modular structure

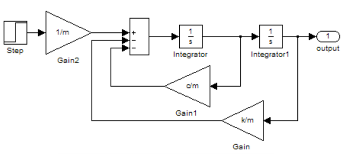
Coupled modelling approaches (I)



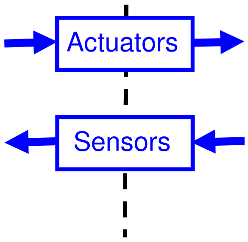
Co-simulation

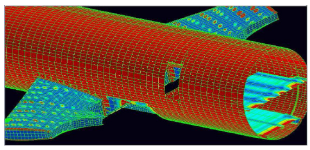
- Reuse specialized software
- Software interface
- Advanced scheduling algorithms

Example:



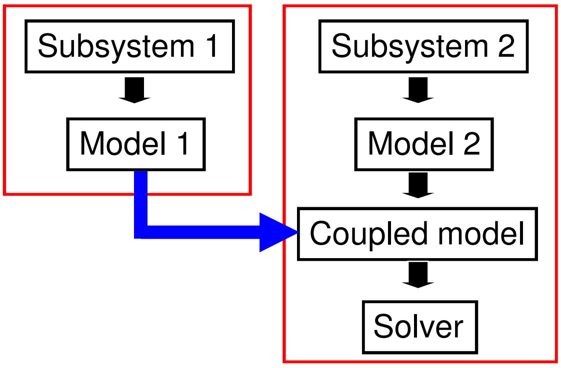
$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$





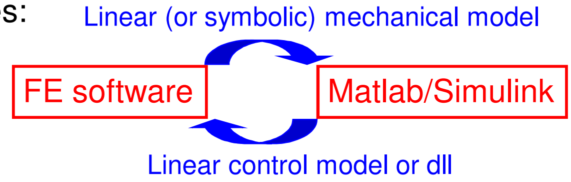
$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{g}$$

Coupled modelling approaches (II)

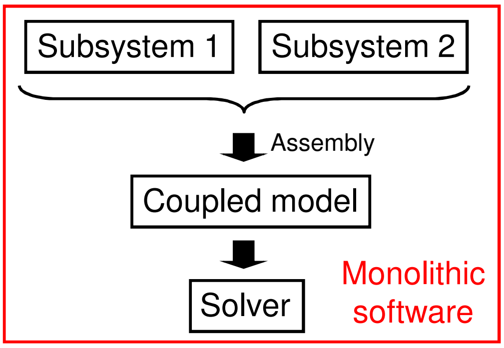


Hybrid approach
- Strong coupling
- Model exchange

Examples:

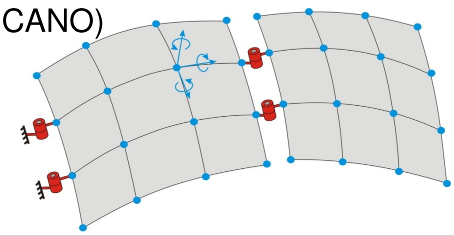


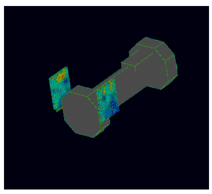
Coupled modelling approaches (III)



Monolithic approach
- No software interface
- Strong (tight) coupling

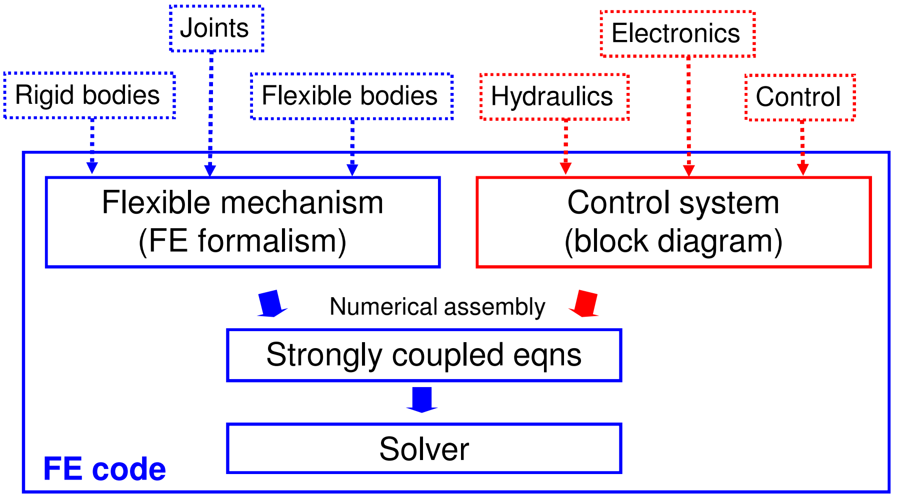
Example:
(SAMCEF-MECANO)





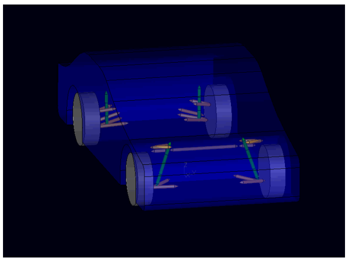
Coupled modelling approaches (IV)

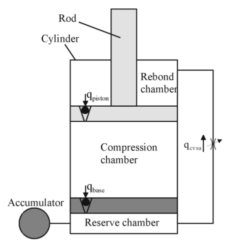
- Functional simulation: bond graph, linear graph, block diagram
- Integrated FE/block diagram formalisms

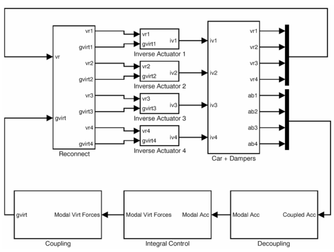


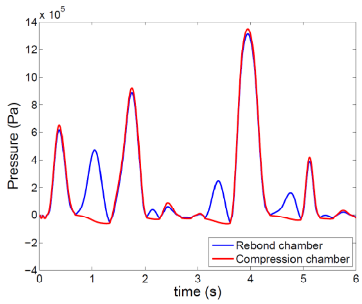
Semi-active car suspension

Monolithic mechatronic simulation (with KULeuven and UCL)

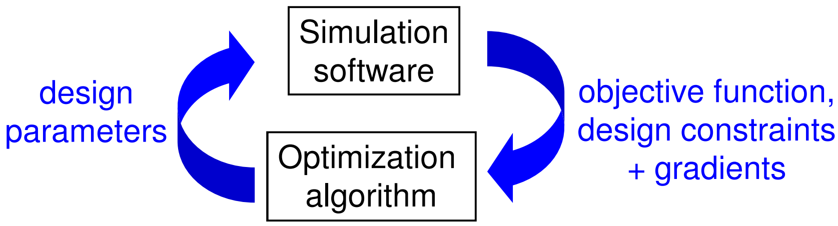








Other applications of model reduction



Optimization

- Component level
- System level
- Simulation/optimization software interface

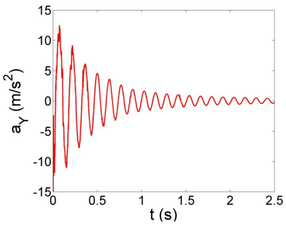
Real-time online applications (model-based control, HIL)

- Compatibility with RT hardware ⇒ model portability
- RT software constraint ⇒ no full implicit solution…

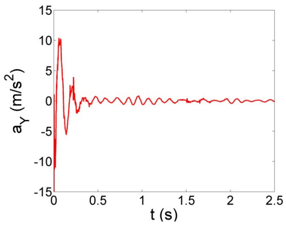
Real-time application of model reduction

- Large & flexible 2-link mechanism
- Dynamics depend on configuration!
- Hydraulic actuators
- Linear sensors & tip accelerometers
- Model-based motion & vibration control

Vibration control off



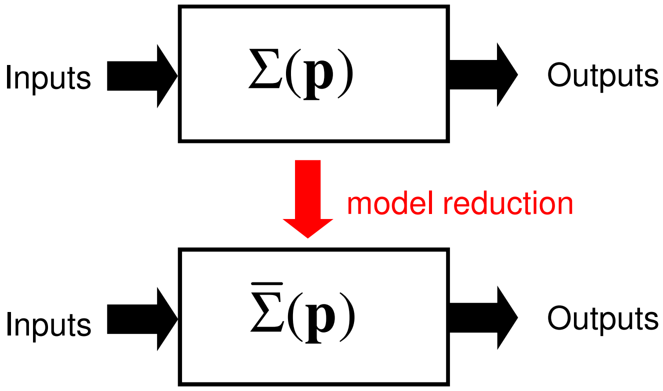
Vibration control on



RALF

with W.J. Book (Georgia Tech)

Principle of model reduction



- Construction stage: using numerical or experimental results
- Exploitation stage: frequency or transient response,…
- Validity: limited excitation & parameter ranges

Summary

The aim of Model Order Reduction (MOR) is not only to reduce the number of states but also to ensure:

- Portability / compatibility with software interface
- Small computational time at exploitation stage
- Reasonable computational time at construction stage
- Limited memory storage
- Limited loss in accuracy
- Appropriate validity domain
- Preservation of important properties of the system

Outline

- ❑ Introduction
- ❑ Linear model reduction techniques
- ❑ Nonlinear model reduction techniques

Linear reduction methods

- In general, linear dynamic equations can be formulated as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

- In structural dynamics (n dimensional)

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{g}$$

Symmetric definite positive matrices + energy conservation

- Most linear reduction methods are based on a projection of the dynamics onto a linear subspace

ROM ($\overline{n}$ dimensional, with $\overline{n} << n$) : $\overline{\mathbf{M}}\ddot{\eta} + \overline{\mathbf{K}}\eta = \overline{\mathbf{g}}$

Galerkin projection method

Back to the weak form (d´Alembert principle)

$$\delta\mathbf{y}^T\left(\mathbf{M}\,\ddot{\mathbf{y}} + \mathbf{K}\,\mathbf{y} = \mathbf{g}\right)$$

Galerkin ↓

$$\mathbf{y} = \boldsymbol{\Psi}\,\boldsymbol{\eta}$$

$$\delta\boldsymbol{\eta}^T\left(\overline{\mathbf{M}}\,\ddot{\boldsymbol{\eta}} + \overline{\mathbf{K}}\,\boldsymbol{\eta} = \overline{\mathbf{g}}\right)$$

Galerkin projection method

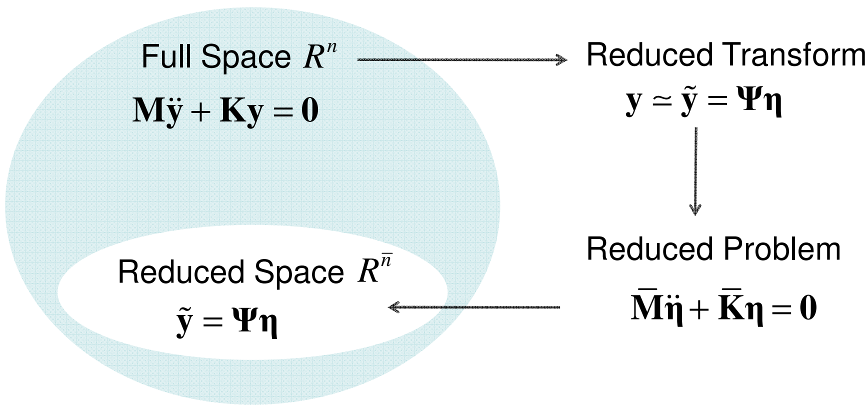
$$\overline{\mathbf{M}} = \boldsymbol{\Psi}^T\,\mathbf{M}\,\boldsymbol{\Psi}$$

$$\overline{\mathbf{K}} = \boldsymbol{\Psi}^T\,\mathbf{K}\,\boldsymbol{\Psi}$$

$$\overline{\mathbf{g}} = \boldsymbol{\Psi}^T\,\mathbf{g}$$

- Sparse structure of $\mathbf{M}$ and $\mathbf{K}$ is lost
- Essential role of $\boldsymbol{\Psi}$

Galerkin projection method



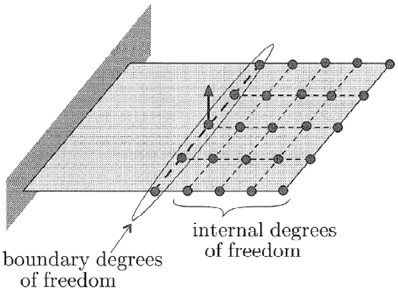
Construction of the modal basis

Selection of $\mathbf{\Psi}$?

- Craig-Bampton :
  static boundary modes + internal modes
- McNeal / Rubin :
  free-free modes
- Master/slave methods (Guyan, Serep)
- Balanced truncation :
  maximize controlability / observability
- Krylov subspace methods :
  interpolation of FRF
- Proper orthogonal decomposition (POD)
  statistical treatment of simulation results

Construction of the modal basis

Craig-Bampton method



[Géradin & Rixen, 1997]

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_B \\ \mathbf{y}_I \end{bmatrix}$$

➡ n_B boundary dofs ➡ n_B static modes

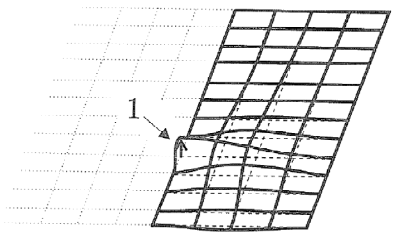
➡ n_I internal dofs ➡ $\overline{n_I}$ internal modes

$\overline{n_I} < n_I$

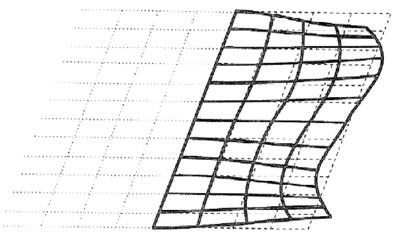
Construction of the modal basis

Craig-Bampton method

Static boundary mode



Internal mode

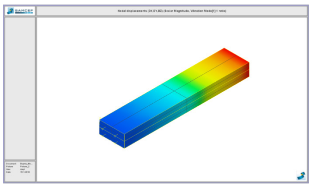


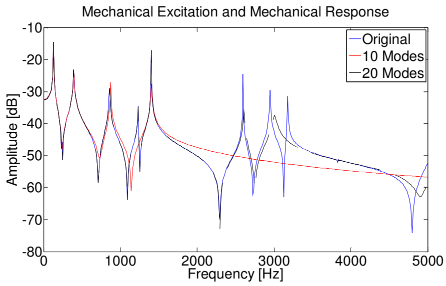
$\Rightarrow$ Exact representation of static boundary response

$$\boldsymbol{\eta} = \begin{bmatrix} \mathbf{y}_B \\ \boldsymbol{\eta}_I \end{bmatrix}$$

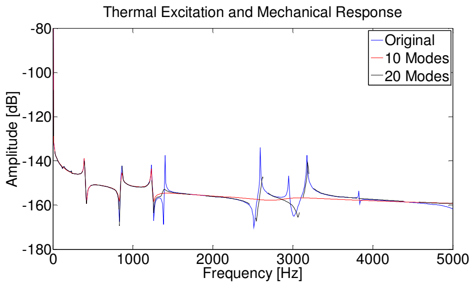
Linear thermomechanical benchmark

Cantilever two-layer beam





$$\begin{bmatrix} M_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} C_{uu} & 0 \\ C_{\theta u} & C_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} K_{uu} & K_{u\theta} \\ 0 & C_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u \\ \theta \end{Bmatrix} = \begin{Bmatrix} F \\ Q \end{Bmatrix}$$



Linear ROMs for systems with large motions

Nonlinear formulations are required to deal with large rotation problems in structural mechanics

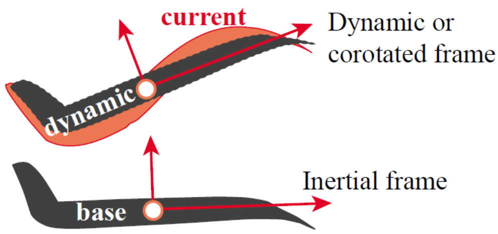


Image source: [Felippa 2001]

However, elastic forces are often linear when computed in a corotated frame (Fraeijs de Veubeke's approach)

Superelement concept: linear ROM + corotational formulation

Summary: linear reduction techniques

- Efficient and mature
- Large toolbox of methods
- Small matrices are obtained
- Sparsity is destroyed $\Rightarrow$ gain in efficiency only if order reduction is significant
- Structure of the problem may be destroyed
- Galerkin projection preserves some structure
- Superelement technique for ROMs with large motions

Outline

- Introduction
- Linear model reduction techniques
- Nonlinear model reduction techniques

Galerkin method for nonlinear models

Full Space R^n → Reduced Transform

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{g}(\mathbf{y}, \dot{\mathbf{y}}, t) = \mathbf{0}$$

$$\mathbf{y} \simeq \tilde{\mathbf{y}} = \mathbf{\Psi}\boldsymbol{\eta}$$

↓

Reduced Problem

$$\bar{\mathbf{M}}\ddot{\boldsymbol{\eta}} + \underbrace{\mathbf{\Psi}^T}_{\bar{n}\times n}\underbrace{\mathbf{g}(\mathbf{\Psi}\boldsymbol{\eta}, \mathbf{\Psi}\dot{\boldsymbol{\eta}}, t)}_{n\times 1} = \mathbf{0}$$

← Reduced Space $R^{\bar{n}}$

$$\tilde{\mathbf{y}} = \mathbf{\Psi}\boldsymbol{\eta}$$

Cost of the linearized problem at each Newton iteration is significantly reduced. However,

- Projection should be repeated at each iteration
- CPU cost of the ROM still depends on n
- No portability

POD provides optimal projection basis

POD = Proper Orthogonal Decomposition

$\mathbf{\Psi}$ is computed from the full nonlinear response (and not from a linearized model)

Basis definition

$$\min_{\mathbf{\Psi}} \sum_i \left\| \mathbf{y}(t_i) - \tilde{\mathbf{y}}(t_i) \right\|^2$$

with $\mathbf{\Psi}^T\mathbf{\Psi} = \mathbf{I}_{\bar{n}}$

Properties

- data-driven method
- error is minimized

POD basis depends on the snapshots

Simulation

↓

Pre-process

$$\mathbf{y}(t_i) = \mathbf{y}(t_i) - \mathbf{y}_{mean}$$

↓

Eigenvalue problem

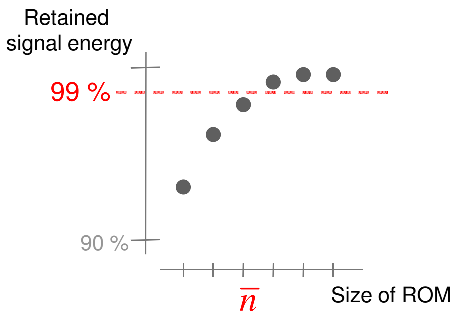
$$\frac{1}{T}\mathbf{Y}^T\mathbf{Y}\mathbf{\Psi} = \lambda\mathbf{\Psi}$$

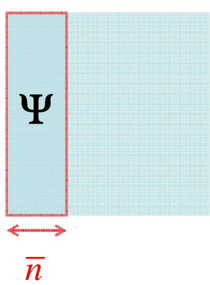
↓

$$\tilde{\mathbf{y}} = \mathbf{y}_{mean} + \mathbf{\Psi}\boldsymbol{\eta}$$

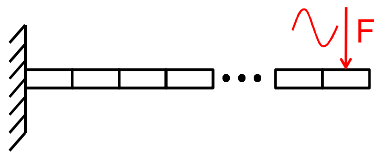
POD allows selection of ROM size

Truncation error is connected to the eigenvalues

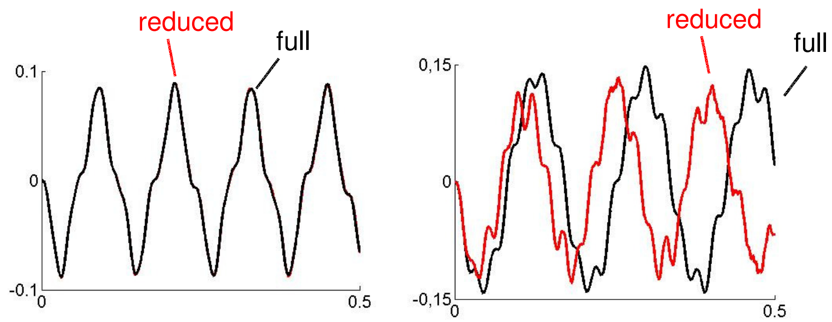




Robustness issues



Reduced model results with different thicknessess



Construction : thick1
Exploitation : thick1

Construction : thick1
Exploitation : thick2

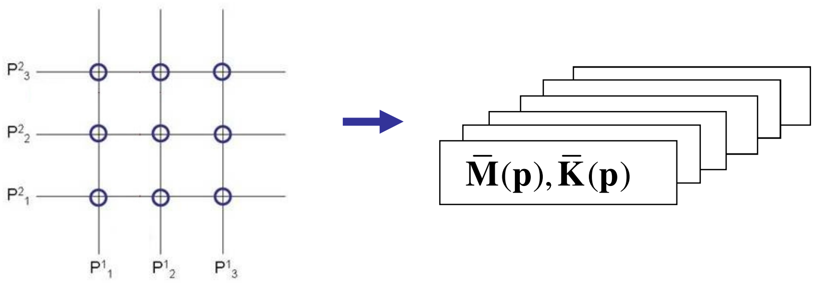
Parametric linear model reduction

Construction

- Sample the parameter space
- Compute & store reduced order models for each vertex

Exploitation

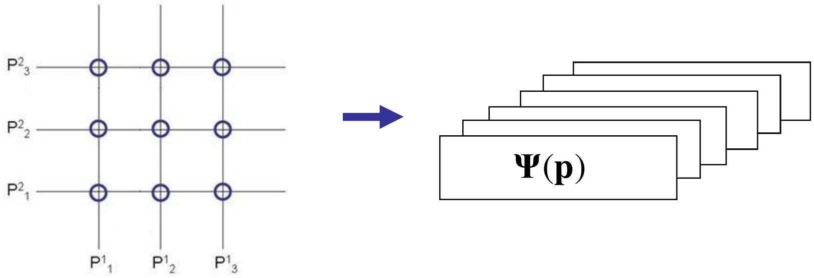
- Interpolate the reduced matrices
- Simulate the linear ROM



Parametric nonlinear model reduction (I)

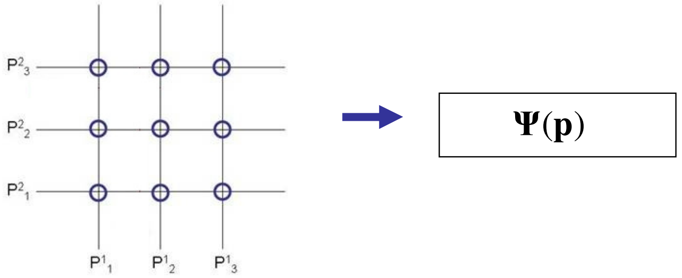
Construction
- Sample the parameter space
- Compute & store the POD basis at each vertex

Exploitation
- Interpolate the POD basis
- Simulate the nonlinear ROM



Parametric nonlinear model reduction (II)

Single but enriched (larger) POD basis
- combine the POD bases from several grid points
- exploit sensitivities of POD basis w.r.t. parameters



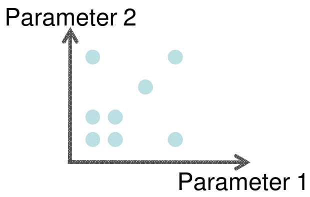
Current research

Reduction the complexity of the non-linear term :

Empirical Interpolation method

Discretization of parameter space :

Greedy method



Error estimator without the full response:

Dual-Weigthed-Residual

Conclusion

Linear model reduction methods are mature

Nonlinear model reduction

- formulation of ROMs closely related to the formulation of initial problem
- difficulty to develop fully generic procedures

Parametric model reduction

- expensive offline computation
- high memory storage requirements

Merci de votre attention !

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