De la dynamique des structures aux systèmes non-linéaires : enjeux et perspectives pour la réduction de modèle

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Atelier de la SIA : La réduction de modèles
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Some models developed using Samcef/Mecano

Outline

- Introduction
- Linear model reduction techniques
- Nonlinear model reduction techniques
Toward system-level simulation

V-model of the development cycle

- Technical specifications
- Preliminary design
- Detailed design
- Development Implementation
- Unit tests
- Sub-systems tests
- Global verification

Design
Integration & tests

Component-level models

- Structural mechanics, fluid mechanics, electromagnetics, thermal, multiphysics, etc
- Off-the-shelf simulation toolboxes
- ODE/DAE models obtained by spatial discretization
- Large number of states (related to the mesh refinement)
- “Not-so-interesting” high frequency modes
- Linear vs. nonlinear models

How to exploit those detailed models at system-level?
Typical applications

- Mechatronic system (MEMS)
- Flexible multibody system
- Modular structure

Coupled modelling approaches (I)

- Subsystem 1
  - Solver 1
- Subsystem 2
  - Solver 2

**Co-simulation**
- Reuse specialized software
- Software interface
- Advanced scheduling algorithms

Example:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

\[M\ddot{y} + Ky = g\]
Coupled modelling approaches (II)

Hybrid approach
- Strong coupling
- Model exchange

Examples:
- Linear (or symbolic) mechanical model
- FE software
- Matlab/Simulink
- Linear control model or dll

Coupled modelling approaches (III)

Monolithic approach
- No software interface
- Strong (tight) coupling

Example:
(SAMCEF-MECANO)
Coupled modelling approaches (IV)

- Functional simulation: bond graph, linear graph, block diagram
- Integrated FE/block diagram formalisms

Flexible mechanism (FE formalism) — Control system (block diagram)

Rigid bodies — Flexible bodies — Hydraulics — Control

Joints

Electronics

Strongly coupled eqns

Numerical assembly

FE code — Solver

Semi-active car suspension

Monolithic mechatronic simulation (with KULeuven and UCL)
Other applications of model reduction

Optimization
- Component level
- System level
- Simulation/optimization software interface

Real-time online applications (model-based control, HIL)
- Compatibility with RT hardware ⇒ model portability
- RT software constraint ⇒ no full implicit solution…

Real-time application of model reduction

- Large & flexible 2-link mechanism
- Dynamics depend on configuration!
- Hydraulic actuators
- Linear sensors & tip accelerometers
- Model-based motion & vibration control

Vibration control off

Vibration control on

RALF
with W.J. Book
(Georgia Tech)
Summary

The aim of Model Order Reduction (MOR) is not only to reduce the number of states but also to ensure:

- Portability / compatibility with software interface
- Small computational time at exploitation stage
- Reasonable computational time at construction stage
- Limited memory storage
- Limited loss in accuracy
- Appropriate validity domain
- Preservation of important properties of the system
Outline

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Linear reduction methods

- In general, linear dynamic equations can be formulated as:
  \[
  \begin{align*}
  \dot{x} &= Ax + Bu \\
  y &= Cx + Du 
  \end{align*}
  \]

- In structural dynamics (\(n\) dimensional)
  \[
  M\ddot{y} + Ky = g
  \]
  Symmetric definite positive matrices + energy conservation

- Most linear reduction methods are based on a **projection** of the dynamics onto a linear subspace

  ROM (\(\bar{n}\) dimensional, with \(\bar{n} << n\)):
  \[
  \bar{M}\ddot{\eta} + \bar{K}\eta = \bar{g}
  \]
Galerkin projection method

Back to the weak form (d’Alembert principle)

\[
\delta y^T \begin{pmatrix} M \ddot{y} + K y = g \\ \end{pmatrix} = 0
\]

\[
\delta \eta^T \begin{pmatrix} \bar{M} \ddot{\eta} + \bar{K} \eta = \bar{g} \end{pmatrix} = 0
\]

Galerkin projection method

\[
\begin{align*}
\bar{M} &= \Psi^T M \\
\bar{K} &= \Psi^T K \\
\bar{g} &= \Psi^T g
\end{align*}
\]

- Sparse structure of \( M \) and \( K \) is lost
- Essential role of \( \Psi \)
Galerkin projection method

Full Space $\mathbb{R}^n$

$M\ddot{y} + Ky = 0$

Reduced Transform

$y = \tilde{y} = \Psi \eta$

Reduced Problem

$\tilde{M}\eta + \tilde{K}\eta = 0$

Construction of the modal basis

Selection of $\Psi$ ?

- Craig-Bampton:
  - static boundary modes + internal modes
- McNeal / Rubin:
  - free-free modes
- Master/slave methods (Guyan, Serep)
- Balanced truncation:
  - maximize controllability / observability
- Krylov subspace methods:
  - interpolation of FRF
- Proper orthogonal decomposition (POD):
  - statistical treatment of simulation results
Construction of the modal basis

Craig-Bampton method

$y = \begin{bmatrix} y_B \\ y_I \end{bmatrix} \Rightarrow n_B \text{ boundary dofs} \Rightarrow n_B \text{ static modes}
\Rightarrow n_I \text{ internal dofs} \Rightarrow \overline{n_I} \text{ internal modes}
\overline{n_I} < n_I$

$\eta = \begin{bmatrix} y_B \\ \eta_I \end{bmatrix}$

$\Rightarrow$ Exact representation of static boundary response

[Geradin & Rixen, 1997]
Linear thermomechanical benchmark

Cantilever two-layer beam

\[
\begin{bmatrix}
M & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\phi}
\end{bmatrix}
+ \begin{bmatrix}
C_m & 0 \\
0 & C_m
\end{bmatrix}
\begin{bmatrix}
\phi
\end{bmatrix}
+ \begin{bmatrix}
K_m & K_m \\
K_m & K_m
\end{bmatrix}
\begin{bmatrix}
\phi
\end{bmatrix} = \begin{bmatrix}
F
\end{bmatrix}
\]

Linear ROMs for systems with large motions

Nonlinear formulations are required to deal with large rotation problems in structural mechanics.

However, elastic forces are often linear when computed in a corotated frame (Fraeijs de Veubeke’s approach)

Superelement concept: linear ROM + corotational formulation
### Summary: linear reduction techniques

- Efficient and mature
- Large toolbox of methods
- Small matrices are obtained
- Sparsity is destroyed ⇒ gain in efficiency only if order reduction is significant
- Structure of the problem may be destroyed
- Galerkin projection preserves some structure
- Superelement technique for ROMs with large motions

### Outline

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  - Nonlinear model reduction techniques
**Galerkin method for nonlinear models**

- Full Space $\mathbb{R}^n$
  \[ \dot{y} + g(y, \dot{y}, t) = 0 \]

- Reduced Transform
  \[ y = \tilde{y} = \Psi \eta \]

- Reduced Problem
  \[ \tilde{M} \ddot{\eta} + \Psi^T g(\Psi \eta, \dot{\eta}, t) = 0 \]

Cost of the linearized problem at each Newton iteration is significantly reduced. However,

- Projection should be repeated at each iteration
- CPU cost of the ROM still depends on $n$
- No portability

**POD provides optimal projection basis**

- POD = Proper Orthogonal Decomposition
- $\Psi$ is computed from the full nonlinear response (and not from a linearized model)

Basis definition

\[ \min_{\Psi} \sum \|y(t_i) - \tilde{y}(t_i)\|^2 \]

with \( \Psi^T \Psi = I_\pi \)

Properties

- data-driven method
- error is minimized
POD basis depends on the snapshots

Simulation
\[ \downarrow \]
Pre-process
\[ y(t_i) = y(t_i) - y_{mean} \]
\[ \downarrow \]
Eigenvalue problem
\[ \frac{1}{T} Y^T Y \Psi = \lambda \Psi \]
\[ \downarrow \]
\[ \tilde{y} = y_{mean} + \Psi \eta \]

POD allows selection of ROM size

Truncation error is connected to the eigenvalues

Retained signal energy

99 %

90 %

Size of ROM

\[ \Psi \]

\[ \tilde{\Pi} \]
Robustness issues

Reduced model results with different thicknesses

![Graph showing reduced and full models with different thicknesses.]

Parametric linear model reduction

Construction
- Sample the parameter space
- Compute & store reduced order models for each vertex

Exploitation
- Interpolate the reduced matrices
- Simulate the linear ROM

![Diagram illustrating parameter space and models.]
Parametric nonlinear model reduction (I)

Construction
- **Sample** the parameter space
- Compute & store the POD basis at each vertex

Exploitation
- **Interpolate** the POD basis
- Simulate the nonlinear ROM

Parametric nonlinear model reduction (II)

**Single** but **enriched** (larger) POD basis
- combine the POD bases from several grid points
- exploit sensitivities of POD basis w.r.t. parameters
Current research

Reduction the complexity of the non-linear term:

Empirical Interpolation method

Discretization of parameter space:

Greedy method

Error estimator without the full response:

Dual-Weighted-Residual

Conclusion

Linear model reduction methods are mature

Nonlinear model reduction

- formulation of ROMs closely related to the formulation of initial problem
- difficulty to develop fully generic procedures

Parametric model reduction

- expensive offline computation
- high memory storage requirements
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Merci de votre attention !