Finite element modelling and optimization of flexible multibody systems

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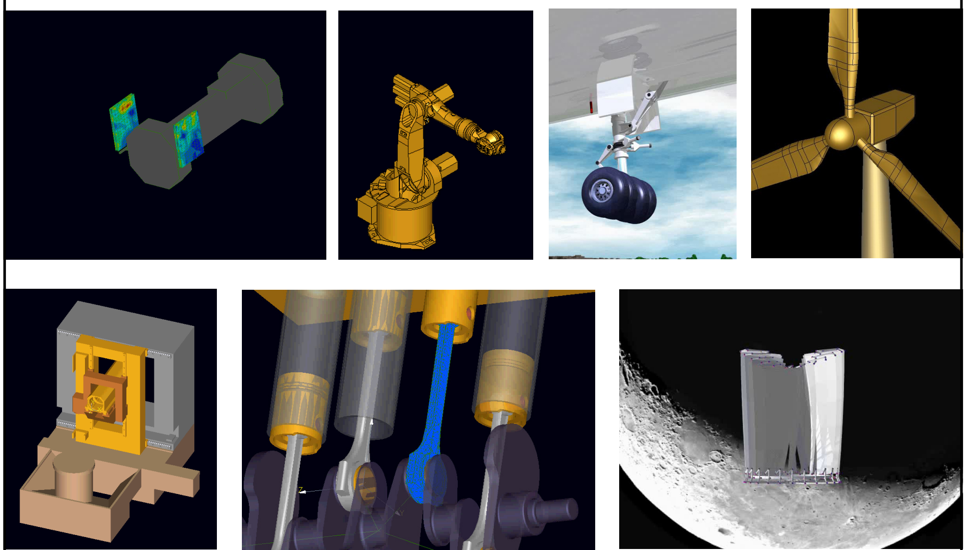
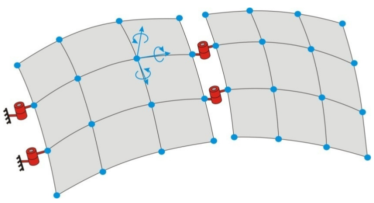
Introduction

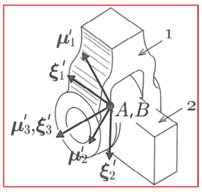


Some SAMCEF/MECANO models

Introduction

Finite element approach for MBS [Géradin & Cardona 2001]





Other elements:
rigid bodies,
flexible beams,
superelements…

Nodal coordinates

➢ translations & rotations

Kinematic joints & rigidity conditions

➢ algebraic constraints

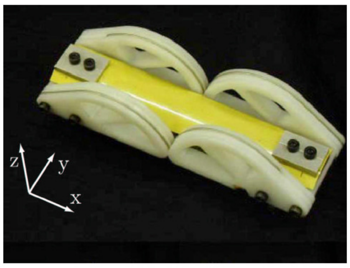
$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t) + \mathbf{B}^T\boldsymbol{\lambda} &= \mathbf{0} \\ \boldsymbol{\Phi}(\mathbf{q}, t) &= \mathbf{0} \end{aligned}$$

index-3 DAE with rotation coordinates

Outline

- ❑ Introduction
- ❑ Modelling of tape-spring hinges for space systems
  - ➢ Single-tape spring study
  - ➢ Full hinge study
- ❑ Advanced solvers for DAEs on Lie group
- ❑ Topology optimization of structural components

Tape-spring hinge



[Watt & Pellegrino 2002]

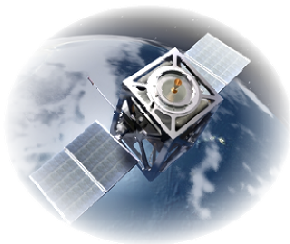
- Deployable space systems
- One or several tape-springs (Carpenter tapes)

MAEVA Hinge

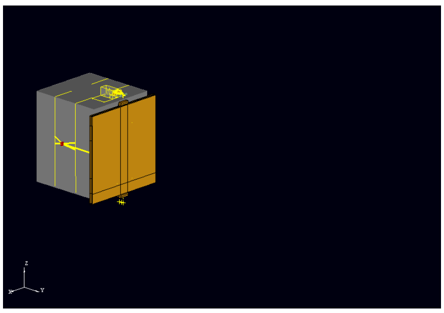
- Developed by METRAVIB & CNES
- Guiding, driving and locking functions
- No contact between sliding surfaces

Prediction of dynamic behaviour?

Design of ESEO educational spacecraft sponsored by ESA



A first modelling attempt



Comparison with experiments:

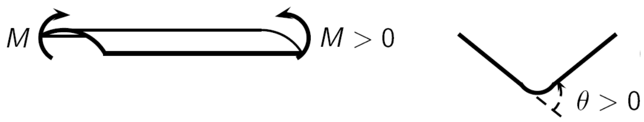
- 3D behaviour?
- Self-locking?

A tape-spring hinge cannot be modelled as an ideal hinge with equivalent springs and dampers…

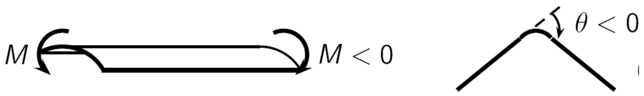
Preamble: Single tape-spring analysis

Sign convention:

(a) opposite-sense bending



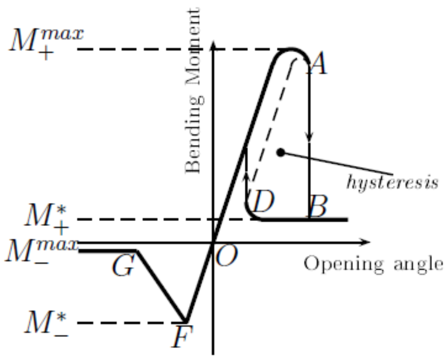
(b) equal-sense bending



State of the art

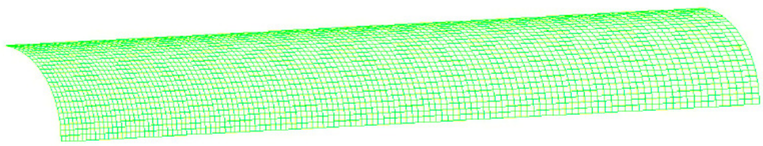
Static behaviour: bending moment vs. opening angle

- Variational method and large deformation shell theory [Mansfield 1973]
- FE study & experimental tests [Seffen et al 1997]



Single tape-spring study

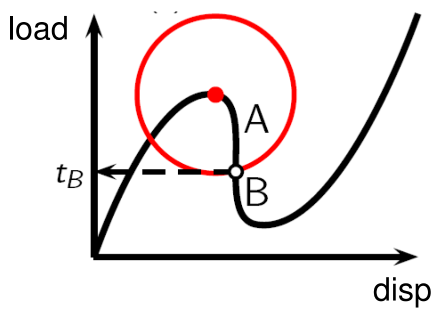
- Second order Mindlin shell elements
- Higher mesh densities in the center
- Symmetry is exploited



Methods

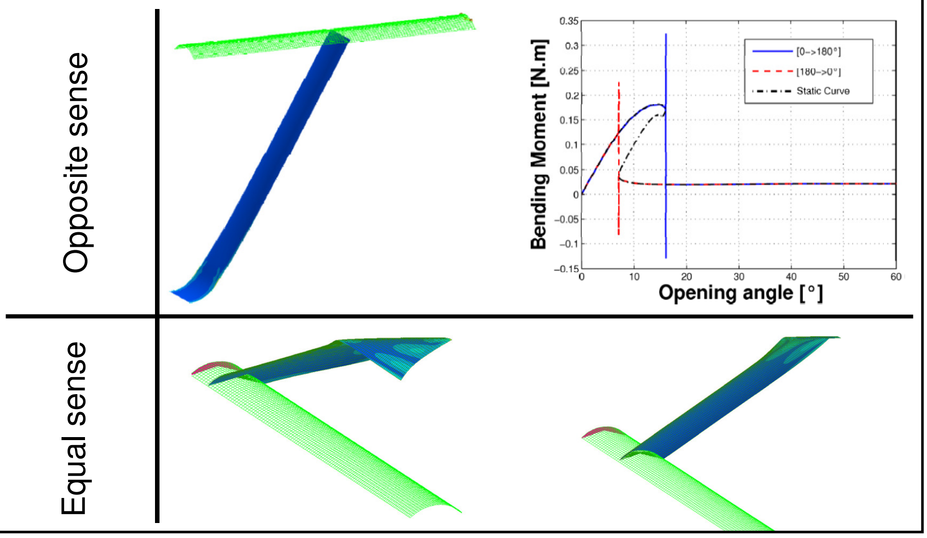
Strongly nonlinear behaviour with a limit point

- Static analysis
  - Continuation method
  - Pseudo-dynamic method
- Dynamic analysis
  - Generalized-α, HHT



Static behaviour of a single tape-spring

- ➢ Continuation vs. pseudo-dynamics
- ➢ Agreement with Mansfield’s results



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  - ➢ Full hinge study
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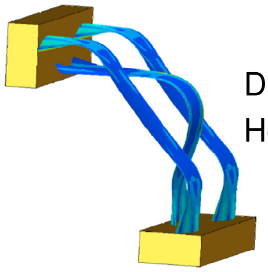
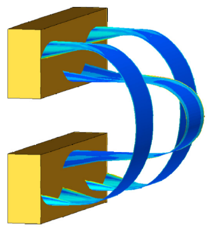
- ❑ Topology optimization of structural components

Static behaviour of a full hinge



Driving torque : 0.152 Nm
Holding torque : 6.67 Nm

Driving torque : 0.194 Nm
Holding torque : 6.67 Nm

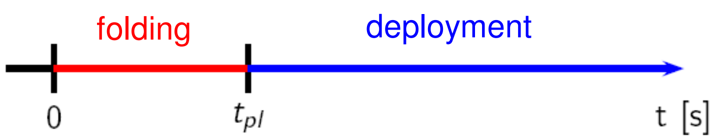


Experimental tests (Metravib) :
Driving torque > 0.15 Nm
Holding torque > 4.5 Nm

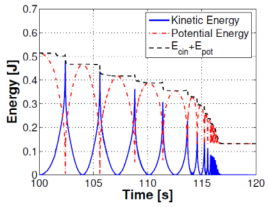
Dynamic behaviour of a full hinge

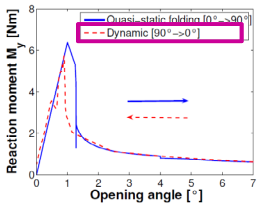
Method :

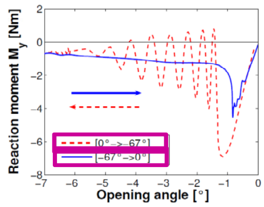
- Detailed hinge model
- The mass of the appendix (solar panel) is considered
- No structural damping but numerical damping
- Analysis procedure
  1. Quasi-static folding
  2. Dynamic deployment

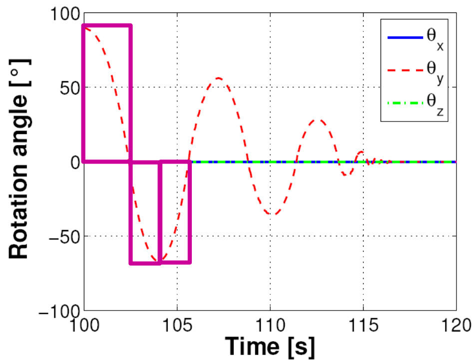


Full hinge - Torsional mode blocked

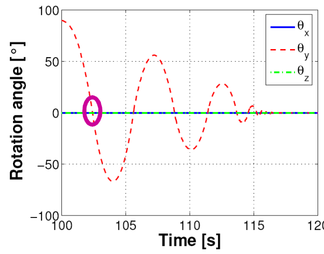


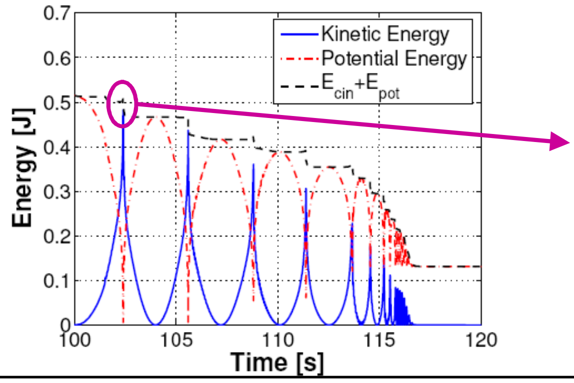






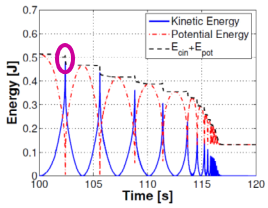
Full hinge - Torsional mode blocked

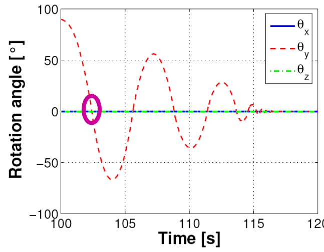


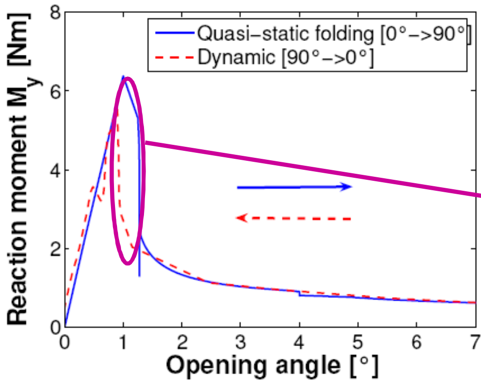


ΔE_{tot} = -0.0457 J

Full hinge - Torsional mode blocked



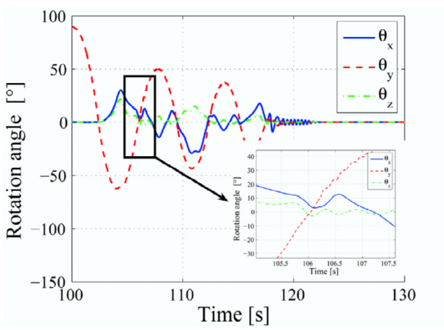


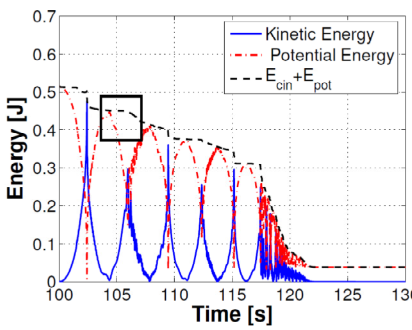


ΔE_{tot} = -0.0457 J

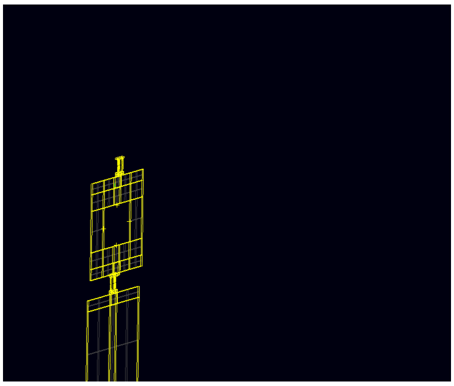
ΔE_{hyst} = -0.0414 J

Full hinge - Torsional mode free





Application to the ESEO satellite



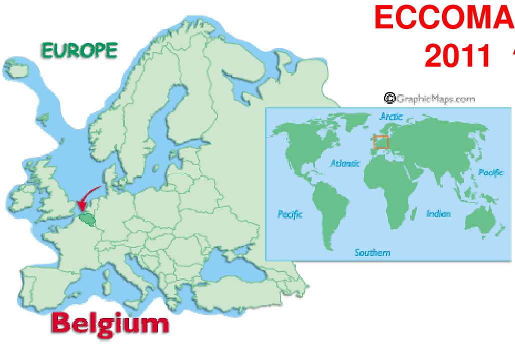
Summary

- FE model of a tape-spring hinge using SAMCEF/MECANO
- Validation for a single tape-spring
- Detailed model of a full hinge
- Self-locking is caused by the hysteresis phenomenon
- Numerical vs. physical dissipation

Motivation for simplified and robust FE-MBS solvers:

- sensitivity analysis
- structural optimization
- optimal control
- nonlinear model reduction (POD)
- RT simulation

Liège - Belgium



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train station

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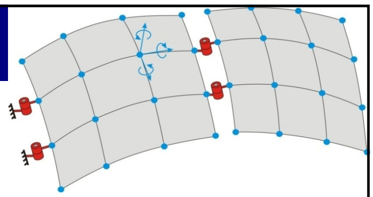
Generalized-α method

- Solution of stiff 2nd order ODEs [Chung & Hulbert 1993]
- Includes Newmark & HHT as special cases
- 2nd order accuracy
- Unconditional stability (A-stability) for linear problems
- Controllable numerical damping at high frequencies
- Direct integration of index-3 DAEs [Cardona & Géradin 1989; Bottasso, Bauchau & Cardona 2007; Arnold & B. 2007]
- Reduced index formulations for DAEs [Lunk & Simeon 2006; Jay & Negrut 2007; Arnold 2009]

About rotations…



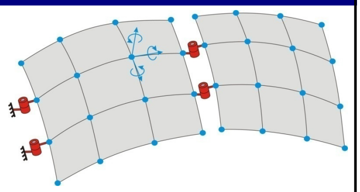
How to avoid parameterization singularities?

- 3-dimensional parameterization + updated Lagrangian point of view [Cardona & Géradin 1989]
- Higher dimensional parameterization + kinematic constraints [Betsch & Steinmann 2001]
- Rotationless formulation, e.g. ANCF [Shabana]
- Lie group time integrator: no parameterization of the manifold is required *a priori* [Crouch & Grossmann 1993; Munthe-Kaas 1995; B. & Cardona 2010; B., Cardona & Arnold 2010]

Lie group formalism for flexible MBS

Nodal configuration variables

$$q = (\mathbf{x}_1, \ldots, \mathbf{x}_N, \mathbf{R}_1, \ldots, \mathbf{R}_N)$$



The configuration evolves on the k-dimensional Lie group

$$G = \mathbb{R}^3 \times \ldots \times \mathbb{R}^3 \times SO(3) \times \ldots \times SO(3)$$

with the composition $q_{tot} = q_1 \circ q_2$ such that

$$\mathbf{x}_{i,tot} = \mathbf{x}_{i,1} + \mathbf{x}_{i,2} \quad \text{and} \quad \mathbf{R}_{i,tot} = \mathbf{R}_{i,1}\mathbf{R}_{i,2}$$

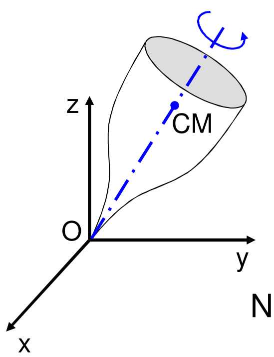
Constrained equations of motion (DAEs on a Lie group)

$$\begin{aligned} \dot{q} &= DL_q(e) \cdot \widetilde{\mathbf{v}} \\ \mathbf{M}(q)\dot{\mathbf{v}} + \mathbf{g}(q, \mathbf{v}, t) + \mathbf{B}^T(q)\boldsymbol{\lambda} &= \mathbf{0} \\ \boldsymbol{\Phi}(q) &= \mathbf{0} \end{aligned}$$

Lie group formalism for flexible MBS

$$\begin{aligned} \dot{q} &= DL_q(e) \cdot \widetilde{\mathbf{v}} \\ \mathbf{M}(q)\dot{\mathbf{v}} + \mathbf{g}(q, \mathbf{v}, t) + \mathbf{B}^T(q)\boldsymbol{\lambda} &= \mathbf{0} \\ \boldsymbol{\Phi}(q) &= \mathbf{0} \end{aligned}$$

Example: an unconstrained system



$$\dot{\mathbf{R}} = \mathbf{R}\widetilde{\boldsymbol{\Omega}}$$

$$\mathbf{J}\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times \mathbf{J}\boldsymbol{\Omega} = \mathbf{C}$$

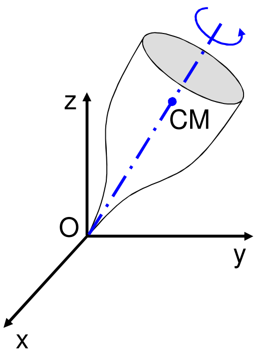
No rotation parameterization is required!

Lie group generalized-α method

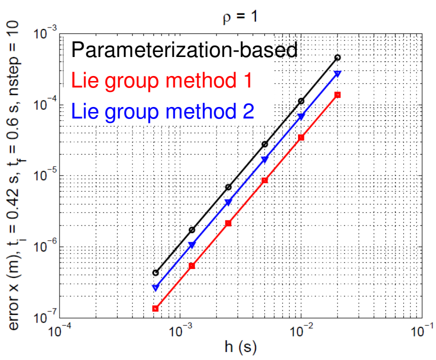
$$
\begin{aligned}
\mathbf{M}(q_{n+1})\dot{\mathbf{v}}_{n+1} &= -\mathbf{g}(q_{n+1}, \mathbf{v}_{n+1}, t_{n+1}) - \mathbf{B}^T(q_{n+1})\boldsymbol{\lambda}_{n+1} \\
\boldsymbol{\Phi}(q_{n+1}) &= \mathbf{0} \\
\boldsymbol{\Delta}\mathbf{q}_n &= \mathbf{v}_n + (0.5-\beta)h^2\mathbf{a}_n + \beta h^2 \mathbf{a}_{n+1} \\
\boxed{q_{n+1} = q_n \circ \exp\left(h\widetilde{\boldsymbol{\Delta}\mathbf{q}_n}\right)} \\
\mathbf{v}_{n+1} &= \mathbf{v}_n + (1-\gamma)h\mathbf{a}_n + \gamma h \mathbf{a}_{n+1} \\
(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n &= (1-\alpha_f)\dot{\mathbf{v}}_{n+1} + \alpha_f \dot{\mathbf{v}}_n
\end{aligned}
$$

1. Non-parameterized equations of motion at time $n+1$
2. Nonlinear integration formulae (composition & exponential)
3. For a vector space ⇒ classical generalized-α algorithm
4. Newton iterations involve (only) $k+m$ unknowns
5. Second-order convergence is proven [B., Cardona & Arnold 2010]

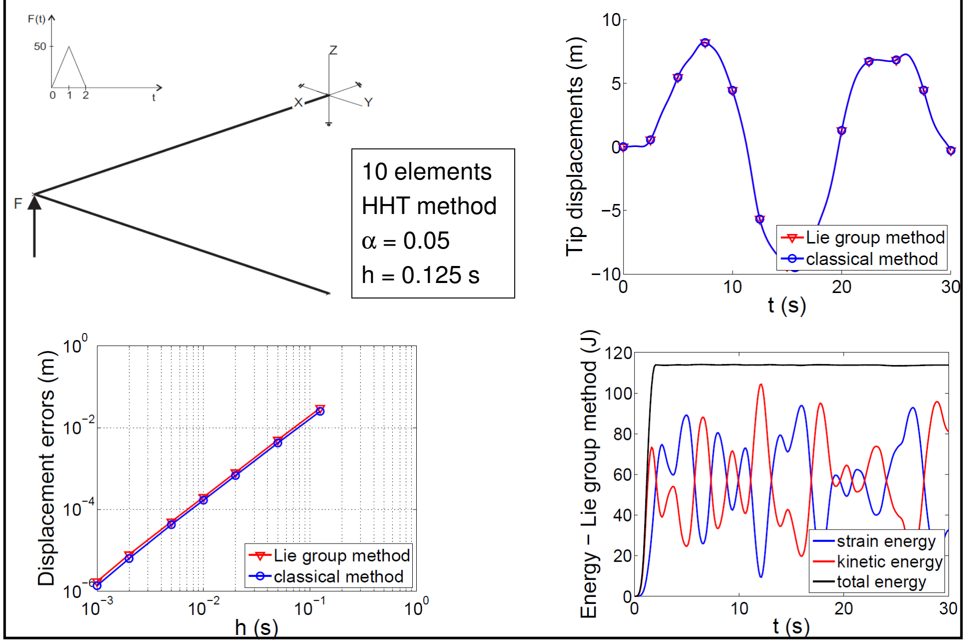
Example 1: Spinning top



spherical ellipsoid of inertia and constant follower torque
⇒ analytical solution [Romano 2008]



Example 2: Rightangle flexible beam



Summary

The generalized-α method combines

- Second-order accuracy (demonstrated for ODEs)
- Adjustable numerical damping
- Computational efficiency for large and stiff problems

Formulation for coupled DAEs on Lie groups:

- Kinematic constraints
- Rotational variables (no parameterization is required)
- Control state variables
- Proven convergence properties!

Department of Aerospace & Mechanical Eng.

Campus of Sart-Tilman

10 km

Liège city center

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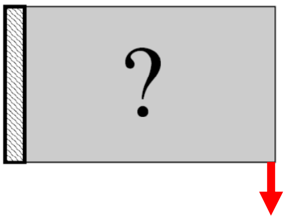
- ❑ Topology optimization of structural components
  - ➢ Motivation
  - ➢ Method
  - ➢ Two-dofs robot arm

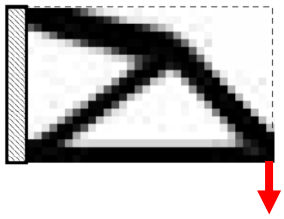
Motivation

Structural topology optimization :

[Bendsøe & Kikuchi 1988]
[Sigmund 2001]



FE discretization
& optimization



- Design volume
- Material properties
- Boundary conditions
- Applied loads
- Objective function
- Design constraints

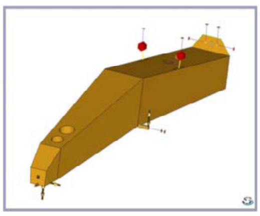
Large scale problem !

Motivation

Achievements in structural topology optimization

- Gradient-based algorithms (CONLIN, MMA, GCMMA…)
- Relevant problem formulations (SIMP penalization…)

A powerful design tool:

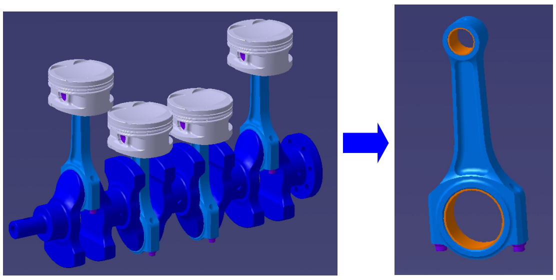


[Poncelet et al. 2005]

Motivation

Our objective: Topology optimization for the design of components of multibody systems

Equivalent static load approach, see e.g. [Kang & Park 2005]



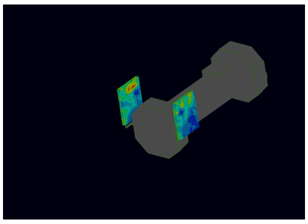
Equivalent static problem
- boundary conditions ?
- load case(s) ?
- objective function ?

⇒ experience and intuition are required
⇒ optimal solution for a wrong problem!

Motivation

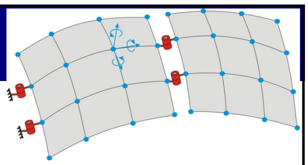
Topology optimization based on the actual dynamic response
[B., Lemaire, Duysinx & Eberhard 2010]

- Flexible multibody model (FE)
- Time integrator (g-α)
- Sensitivity analysis
- Coupling with an optimizer



Advantages:
⇒ Systematic approach
⇒ More realistic objective function

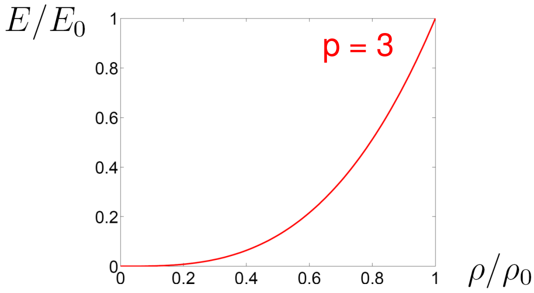
Topology optimization



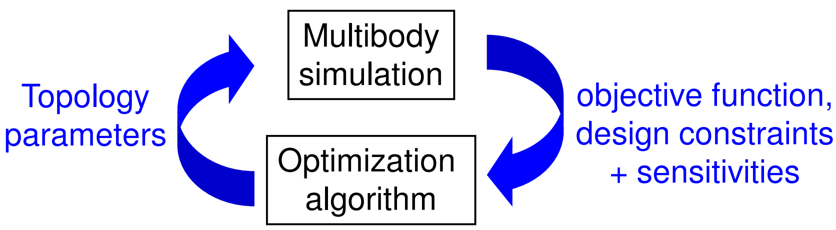
Parameterization of the topology: for each element,

- one density variable is defined $x = \rho/\rho_0, \quad x \in [0, 1]$
- the Young modulus is computed according to the SIMP law

$$E = x^p E_0$$



Global optimization framework



Coupled industrial software

- OOFELIE (simulation and sensitivity analysis)
- CONLIN (gradient-based optimization) [Fleury 1989]

Efficient and reliable sensitivity analysis ?

➡ Direct differentiation technique

Sensitivity analysis

For one design variable x, direct differentiation leads to

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}}' + \mathbf{C}_t\dot{\mathbf{q}}' + \mathbf{K}_t\mathbf{q}' + \mathbf{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda}' + \mathbf{r}_{,x} &= 0 \\ \mathbf{\Phi}_{\mathbf{q}}\mathbf{q}' + \mathbf{\Phi}_{,x} &= 0 \end{aligned}$$

pseudo-loads

Inertia forces $\propto \rho$
Elastic forces $\propto E$
$\Rightarrow$ Analytical expressions for $\mathbf{r}_{,x}$

Integration of the sensitivities
- iteration matrix already computed and factorized
- one linear pseudo-load case for each design variable

$\mathbf{q}_{n+1}, \dot{\mathbf{q}}_{n+1}, \ddot{\mathbf{q}}_{n+1}, \boldsymbol{\lambda}_{n+1}$

$\mathbf{q}'_{n+1}, \dot{\mathbf{q}}'_{n+1}, \ddot{\mathbf{q}}'_{n+1}, \boldsymbol{\lambda}'_{n+1}$

$n := n+1$

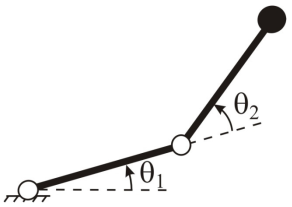
Sensitivity analysis

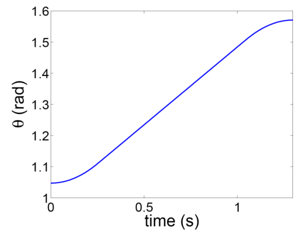
Importance of an efficient sensitivity analysis :
- Test problem with (only) 60 design variables
- Finite difference (61 simulations)
  $\Rightarrow$ **CPU time = 141 s**
- Direct differentiation (1 extended simulation)
  $\Rightarrow$ **CPU time = 16 s**

Moreover, the direct differentiation method leads to higher levels of accuracy

[B. & Eberhard 2008]

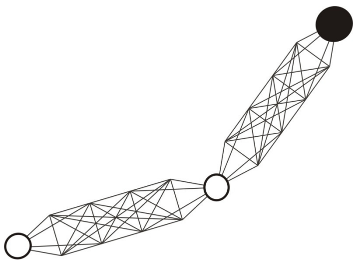
Two dofs robot arm

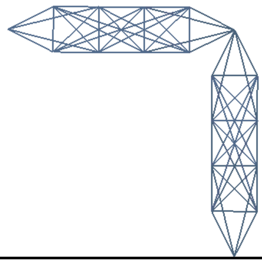




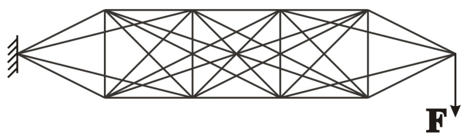
Point-to-point joint trajectory

Initial structural universe of beams:



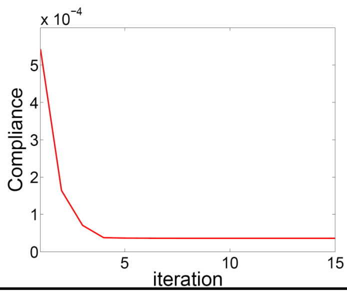


Equivalent static case



Minimization of the compliance $c = \frac{1}{2} \int_V \boldsymbol{\epsilon}^T \mathbf{H} \boldsymbol{\epsilon} \, dV$

subject to a volume constraint $V \leq 0.4 V_{full}$

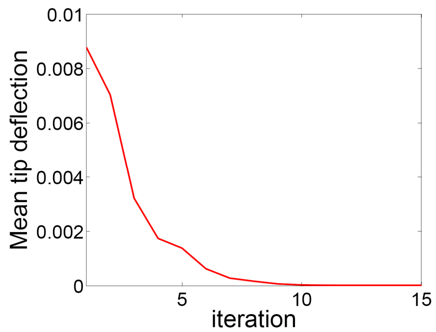


Final design:

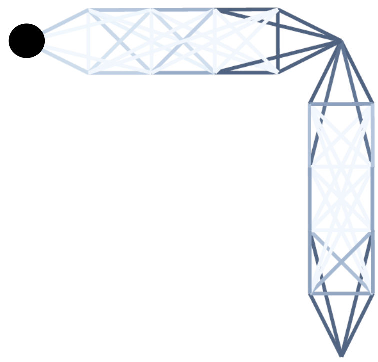
Optimization based on multibody simulations

Minimization of the tip deflection $\frac{1}{t_f}\int_0^{t_f} \|\mathbf{r}-\mathbf{r}_{rigid}\|^2\, dt$

subject to a volume constraint $V_{(i)} \leq 0.4\, V_{full,(i)}$



Final design:



Summary

- ❑ Topology optimization of mechanisms components
- ❑ Equivalent static load ⇒ multibody dynamics approach
  - ➢ flexible multibody simulation
  - ➢ semi-analytical sensitivity analysis
  - ➢ coupling with an optimization code
- ❑ Application to a two dofs robot arm with truss linkages
  - ➢ importance of problem formulation
- ❑ Perspectives
  - ➢ 3D mechanisms
  - ➢ Mechatronic systems

Danke für Ihre Aufmerksamkeit!

Finite element modelling and optimization
of flexible multibody systems

Olivier Brüls

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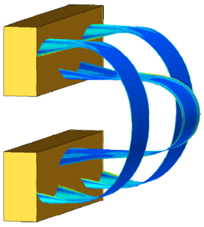
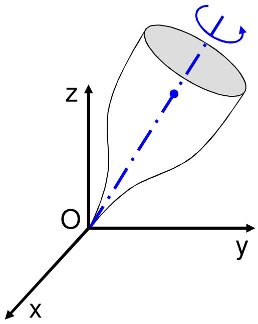
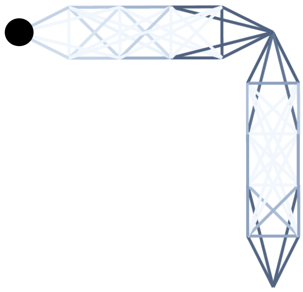
M. Arnold, A. Cardona, P. Eberhard





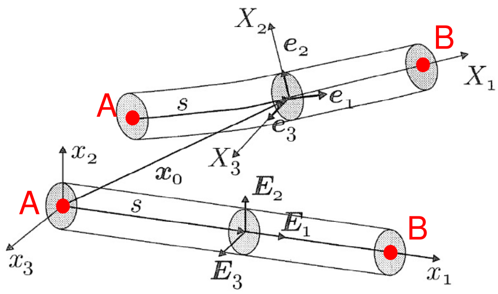


Université de Liège

University of Stuttgart, June 8, 2010

Modelling of flexible multibody systems

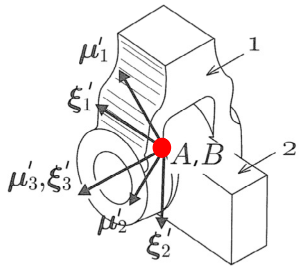
Flexible beam element



- Timoshenko-type geometrically exact model
- Two nodes A and B
- Nodal translations $(\mathbf{x}_A, \mathbf{x}_B)$ and rotations $(\mathbf{R}_A, \mathbf{R}_B)$
- Strain energy : bending, torsion, traction and shear
- Kinetic energy : translation and rotation

Modelling of flexible multibody systems

Hinge element



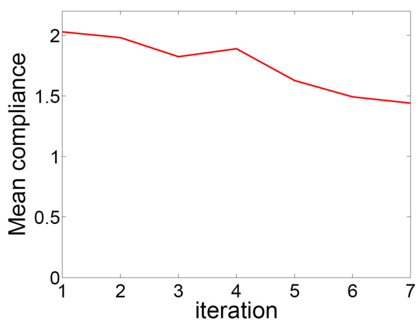
- Two nodes A (on body 1) and B (on body 2)
- Nodal translations $(\mathbf{x}_A, \mathbf{x}_B)$ and rotations $(\mathbf{R}_A, \mathbf{R}_B)$
- 5 kinematic constraints

$$\begin{aligned} \mathbf{x}_A - \mathbf{x}_B &= \mathbf{0} \\ \boldsymbol{\mu}_1'(\mathbf{R}_A) \cdot \boldsymbol{\xi}_3'(\mathbf{R}_B) &= 0 \\ \boldsymbol{\mu}_2'(\mathbf{R}_A) \cdot \boldsymbol{\xi}_3'(\mathbf{R}_B) &= 0 \end{aligned}$$

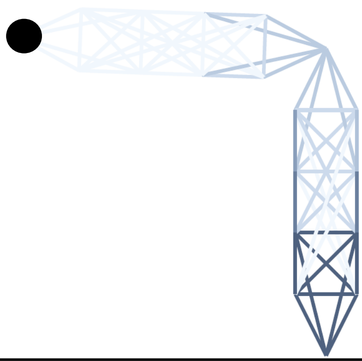
Multibody dynamics approach

Minimize the mean compliance: $\frac{1}{t_f}\int_0^{t_f}\sum_{i=1}^{n_c} c_{(i)}\,dt$

subject to a volume constraint $V_{(i)} \leq 0.4\,V_{full,(i)}$

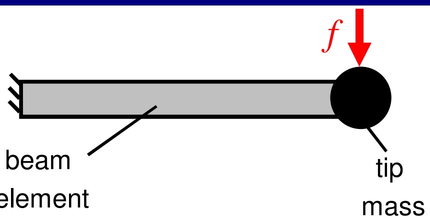


Final design:



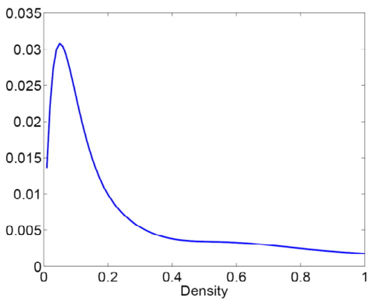
Choice of the objective function

A one-element test-case :



Objective functions :

Mean compliance



Mean square tip deflection