Recent developments in simulation, optimization and control of flexible multibody systems

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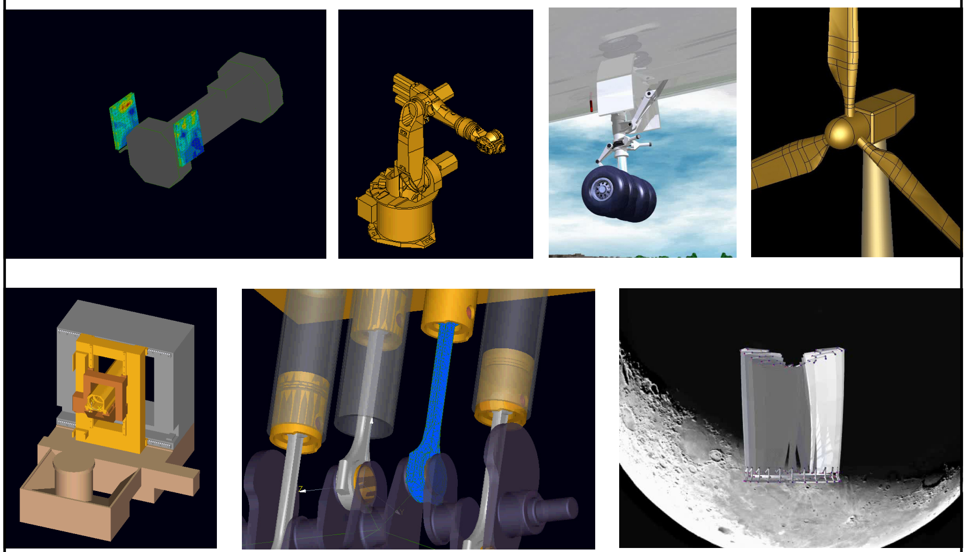
LTAS

Outline

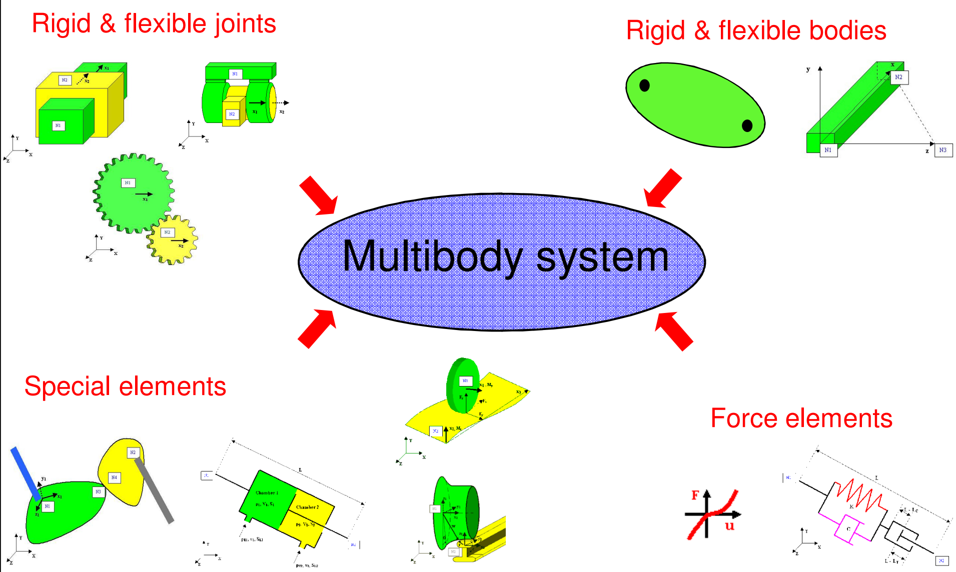
- ❑ Introduction
- ❑ Modelling of multibody & mechatronic systems
- ❑ Time integration algorithms
- ❑ Topology optimization of structural components

Introduction



Some models developed using Samcef/Mecano

Introduction: Typical simulation library



Introduction : Commercial simulation tools

Multibody dynamics approach
- MSC ADAMS
- LMS VIRTUAL LAB MOTION
- SIMPACK
- RECURDYN

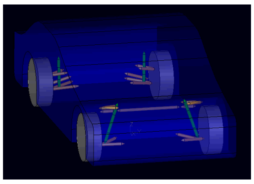
} Linear flexibility effects can be represented

Finite element approach
- SAMCEF MECANO
- OOFELIE

} Most general approach for nonlinear flexible systems

Introduction : Mechatronic design

Active suspension



Mechanism | Actuator Sensor | Control

↓

Mechatronic design

Manipulator (Georgia Tech)

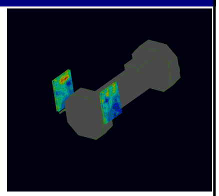
Recent developments in multibody dynamics :
- extension to mechatronic systems
- development of integrated optimization tools

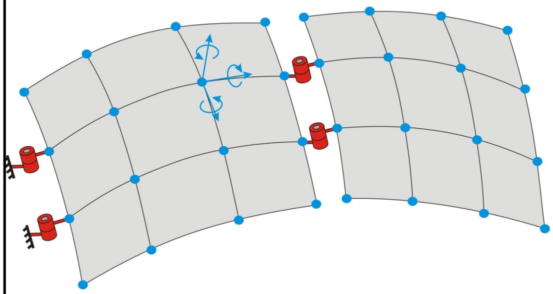
Outline

- ❑ Introduction
- ❑ Modelling of multibody & mechatronic systems
  - ➢ Modelling of flexible multibody systems
  - ➢ Modelling of coupled mechatronic systems
  - ➢ Application to a semi-active car suspension
- ❑ Time integration algorithms
- ❑ Topology optimization of structural components

Modelling of flexible multibody systems

Finite element approach [Géradin & Cardona 2001]





Nodal coordinates

- ➢ translations & rotations
- ➢ geometric nonlinearities

Kinematic joints

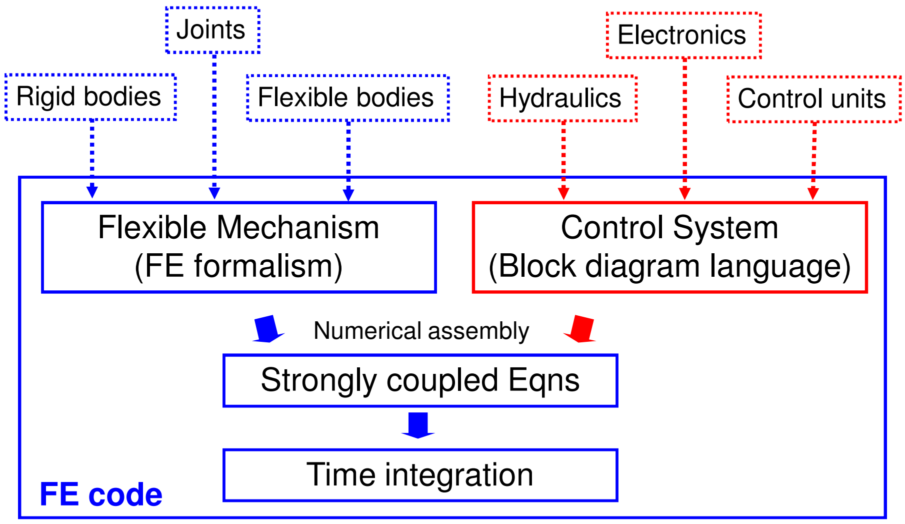
- ➢ algebraic constraints

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{g}_{gyr}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}_{int}(\mathbf{q}) + \boldsymbol{\Phi}_{\mathbf{q}}^{T}\boldsymbol{\lambda} &= \mathbf{g}_{ext} \\ \boldsymbol{\Phi}(\mathbf{q},t) &= \mathbf{0} \end{aligned}$$

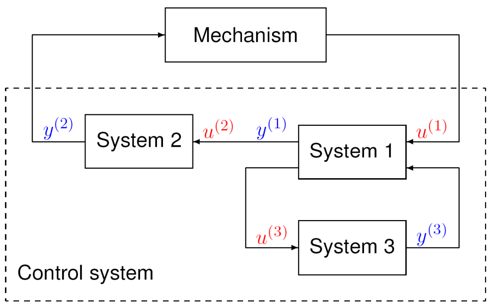
index-3 DAE with rotation variables

Modelling of coupled mechatronic systems

Modular and monolithic FE approach



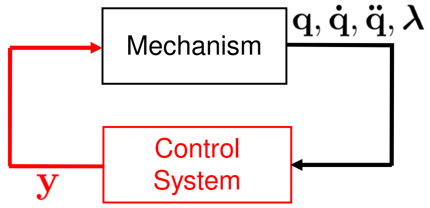
Modelling of coupled mechatronic systems



Block diagram language in a FE code

- Generic blocks : gain, integrator, transfer function…
  $\Rightarrow$ "special" elements
- Control state/output variables $\Rightarrow$ "special" dofs
- Numerical assembly according to the FE procedure

Modelling of coupled mechatronic systems



Coupled equations:

$$
\begin{aligned}
\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} &= \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t) - \boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} + \mathbf{L}\mathbf{y} \\
\mathbf{0} &= \boldsymbol{\Phi}(\mathbf{q}, t) \\
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\lambda}, \mathbf{x}, \mathbf{y}, t) \\
\mathbf{y} &= \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\lambda}, \mathbf{x}, \mathbf{y}, t)
\end{aligned}
$$

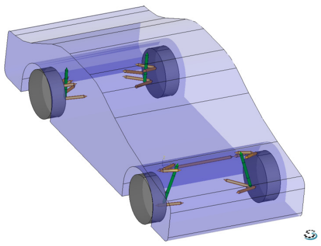
Time-integration scheme for coupled 1st/2nd order DAE ?

- Classical ODE solvers : multistep & Runge-Kutta methods
- Generalized-α time integration scheme

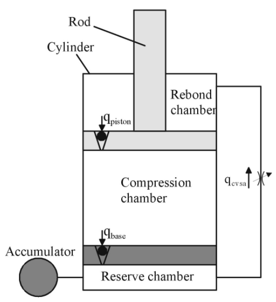
Semi-active car suspension

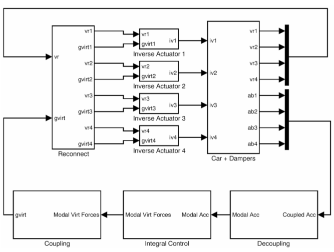
Work in collaboration with KULeuven-PMA and UCL-CEREM (PAI5/6)

Mechanical model



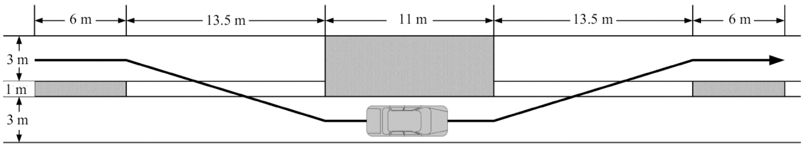
Actuator model

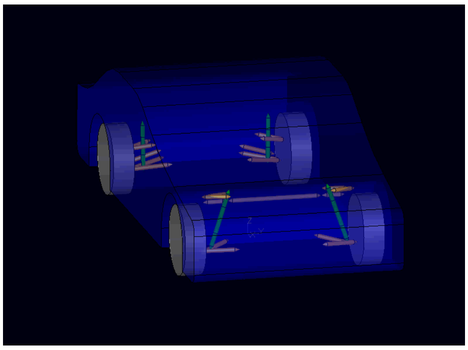


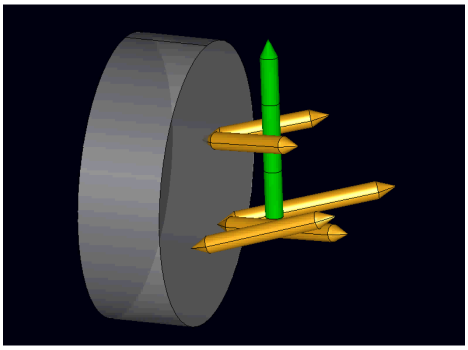


Controller model

Semi-active car suspension

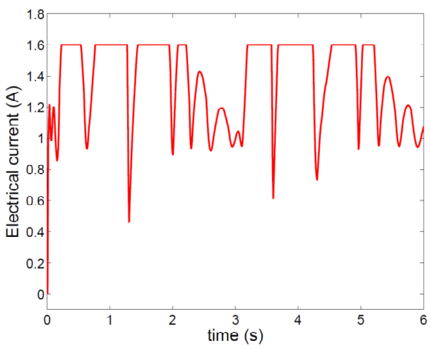




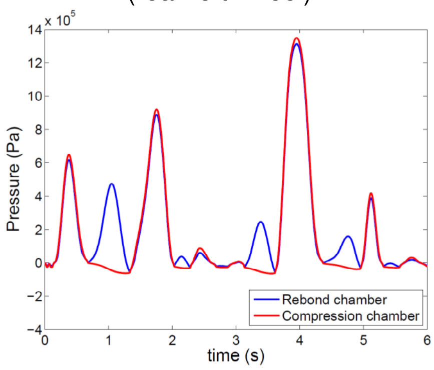


Semi-active car suspension

Electrical current in the valves (A)
(rear-left wheel)



Hydraulic pressures (Pa)
(rear-left wheel)



Summary

Strongly coupled simulation of mechatronic systems:

- ❑ Mechanical equations are obtained using the finite element technique (rigid bodies, elastic bodies & kinematic joints)
- ❑ Control equations are formulated in the FE code using the block diagram language
- ❑ The generalized-α time integrator is used to solve the strongly coupled problem

Outline

- ❑ Introduction
- ❑ Modelling of multibody & mechatronic systems
- ❑ Time integration algorithms
  - ➢ Generalized-α method
  - ➢ Kinematic constraints
  - ➢ Treatment of rotation variables
  - ➢ Controller dynamics
- ❑ Topology optimization of structural components

Generalized-α method

Numerical time-integration methods

- Standard integrators: multistep, Runge-Kutta
- Methods from structural dynamics (Newmark, HHT, g-α)
- Energy conserving schemes

Generalized-α method [Chung & Hulbert 1993]

- One step method for 2nd ODEs
- 2nd order accuracy
- Unconditional stability (A-stability) for linear problems
- Controllable numerical damping at high frequencies
- Computational efficiency for large and stiff problems

$\Rightarrow$ Extensions of the g-α method to deal with kinematic constraints, rotation variables and controller dynamics?

Generalized-α method

2nd order ODE system: $\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t)$

Newmark implicit formulae:

$$\begin{aligned} \mathbf{q}_{n+1} &= \mathbf{q}_n + h\dot{\mathbf{q}}_n + h^2(0.5-\beta)\mathbf{a}_n + h^2\beta\mathbf{a}_{n+1} \\ \dot{\mathbf{q}}_{n+1} &= \dot{\mathbf{q}}_n + h(1-\gamma)\mathbf{a}_n + h\gamma\mathbf{a}_{n+1} \end{aligned}$$

Generalized-α method [Chung & Hulbert, 1993]

$$(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m\mathbf{a}_n = (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f\ddot{\mathbf{q}}_n$$

To be solved with : $\mathbf{M}(\mathbf{q}_{n+1})\ddot{\mathbf{q}}_{n+1} = \mathbf{g}(\mathbf{q}_{n+1}, \dot{\mathbf{q}}_{n+1}, t_{n+1})$

- Two kinds of acceleration variables: $\mathbf{a}_n \neq \ddot{\mathbf{q}}_n$
- Algorithmic parameters: $\gamma,\ \beta,\ \alpha_f,\ \alpha_m$

  2nd order accuracy & numerical damping

Kinematic constraints

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} &= \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t) - \mathbf{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} \\ \mathbf{0} &= \mathbf{\Phi}(\mathbf{q}, t) \end{aligned}$$

Direct integration of the index-3 DAE problem using g-α

- Linear stability analysis demonstrates the importance of numerical damping [Cardona & Géradin 1989]
- Scaling of equations and variables reduces the sensitivity to numerical errors [Bottasso, Bauchau & Cardona 2007]
- Global convergence is demonstrated [Arnold & B. 2007]

Reduced index formulations

[Lunk & Simeon 2006; Jay & Negrut 2007; Arnold 2009]

Treatment of rotations

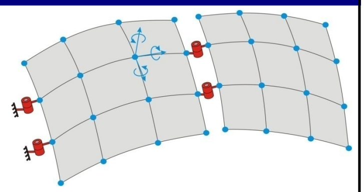
It is impossible to have a global 3-dimensional parameterization of rotations without singular points. [Stuelpnagel 1964]

Possible strategies

- 3-dimensional parameterization + reparameterization to avoid singularities [Cardona & Géradin 1989]
- Higher dimensional parameterization + kinematic constraints [Betsch & Steinmann 2001]
- Rotationless formulation, e.g. ANCF [Shabana]
- Lie group time integrator: no parameterization of the manifold is required *a priori* [Simo & Vu-Quoc 1988; B. & Cardona 2010]

Treatment of rotations

Nodal coordinates



$$q = (\mathbf{x}_1, \ldots, \mathbf{x}_N, \mathbf{R}_1, \ldots, \mathbf{R}_N)$$

The configuration evolves on the n-dimensional Lie group

$$G = \mathbb{R}^3 \times \ldots \times \mathbb{R}^3 \times SO(3) \times \ldots \times SO(3)$$

with the composition $q_{tot} = q_1 \circ q_2$ such that

$$\mathbf{x}_{i,tot} = \mathbf{x}_{i,1} + \mathbf{x}_{i,2} \quad \text{and} \quad \mathbf{R}_{i,tot} = \mathbf{R}_{i,1}\mathbf{R}_{i,2}$$

Constrained equations of motion :

$$\begin{aligned} \dot{q} &= DL_q(e) \cdot \widetilde{\mathbf{v}} \\ \mathbf{M}(q)\dot{\mathbf{v}} + \mathbf{g}(q, \mathbf{v}, t) + \mathbf{B}^T(q)\boldsymbol{\lambda} &= \mathbf{0} \\ \boldsymbol{\Phi}(q) &= \mathbf{0} \end{aligned}$$

Treatment of rotations

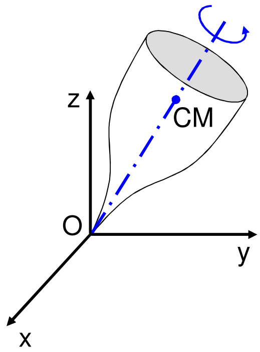
$$\begin{aligned} \mathbf{M}(q_{n+1})\dot{\mathbf{v}}_{n+1} + \mathbf{g}(q_{n+1}, \mathbf{v}_{n+1}, t_{n+1}) + \mathbf{B}^T(q_{n+1})\boldsymbol{\lambda}_{n+1} &= \mathbf{0} \\ \boldsymbol{\Phi}(q_{n+1}) &= \mathbf{0} \\ q_{n+1} &= \varphi_h(q_n, \mathbf{v}_n, \mathbf{a}_n, \mathbf{a}_{n+1}) \\ \mathbf{v}_{n+1} &= \mathbf{v}_n + (1-\gamma)h\mathbf{a}_n + \gamma h \mathbf{a}_{n+1} \\ (1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n &= (1-\alpha_f)\dot{\mathbf{v}}_{n+1} + \alpha_f \dot{\mathbf{v}}_n \end{aligned}$$

with, e.g., $\varphi_h^{(1)}(q_n, \mathbf{v}_n, \mathbf{a}_n, \mathbf{a}_{n+1}) = q_n \circ \exp\left(h\widetilde{\mathbf{v}_n} + h^2(0.5-\beta)\widetilde{\mathbf{a}_n} + \beta h^2 \widetilde{\mathbf{a}_{n+1}}\right)$

Lie group generalized-α solver [B. & Cardona 2010]

1. No rotation parameterization is explicitly involved
2. The solution inherently remains on the manifold
3. The integration formulae are nonlinear
4. The classical generalized-α algorithm is a special case when G is a linear vector space
5. Second-order accuracy + numerical damping
6. Much simpler than parameterization-based methods

Treatment of rotations: benchmark



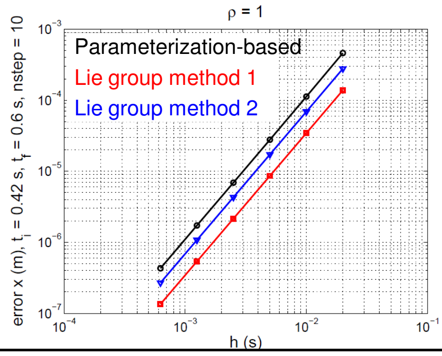
Spinning top with spherical ellipsoid of inertia and constant follower torque

$$\dot{\mathbf{R}} = \mathbf{R}\widetilde{\mathbf{\Omega}}$$

$$\mathbf{J}\dot{\mathbf{\Omega}} + \mathbf{\Omega} \times \mathbf{J}\mathbf{\Omega} = \mathbf{C}$$

$\Rightarrow$ analytical solution [Romano 2008]

$$\text{error} = \frac{1}{n_e} \sum_{i=1}^{n_e} \|\mathbf{x}(t_i) - \mathbf{x}^{ref}(t_i)\|$$



Controller dynamics

Coupled dynamic equations: $\begin{cases} \ddot{\mathbf{q}} = \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{x}, t) \\ \dot{\mathbf{x}} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{x}, t) \end{cases}$

$$\begin{aligned} \mathbf{q}_{n+1} &= \mathbf{q}_n + h\dot{\mathbf{q}}_n + h^2(0.5-\beta)\mathbf{a}_n + h^2\beta\mathbf{a}_{n+1} \\ \dot{\mathbf{q}}_{n+1} &= \dot{\mathbf{q}}_n + h(1-\gamma)\mathbf{a}_n + h\gamma\mathbf{a}_{n+1} \\ \mathbf{x}_{n+1} &= \mathbf{x}_n + h(1-\theta)\mathbf{w}_n + h\theta\mathbf{w}_{n+1} \end{aligned}$$

$$\begin{cases} (1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m\mathbf{a}_n = (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f\ddot{\mathbf{q}}_n \\ (1-\delta_m)\mathbf{w}_{n+1} + \delta_m\mathbf{w}_n = (1-\delta_f)\dot{\mathbf{x}}_{n+1} + \delta_f\dot{\mathbf{x}}_n \end{cases}$$

To be solved with : $\begin{cases} \ddot{\mathbf{q}}_{n+1} = \mathbf{g}(\mathbf{q}_{n+1}, \dot{\mathbf{q}}_{n+1}, \mathbf{x}_{n+1}, t_{n+1}) \\ \dot{\mathbf{x}}_{n+1} = \mathbf{f}(\mathbf{q}_{n+1}, \dot{\mathbf{q}}_{n+1}, \mathbf{x}_{n+1}, t_{n+1}) \end{cases}$

Order conditions: $\begin{cases} \gamma = 0.5 + \alpha_f - \alpha_m \\ \theta = 0.5 + \delta_f - \delta_m \end{cases}$

Summary

The generalized-α method combines

- Second-order accuracy (demonstrated for ODEs)
- Adjustable numerical damping
- Computational efficiency for large and stiff problems

Extension to coupled DAEs on Lie groups with a consistent treatment of:

- Kinematic constraints
- Rotational variables
- Control state variables

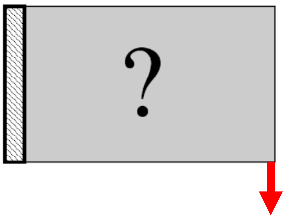
Outline

- Introduction
- Modelling of multibody & mechatronic systems
- Time integration algorithms
- Topology optimization of structural components
  - Motivation
  - Method
  - Two-dofs robot arm

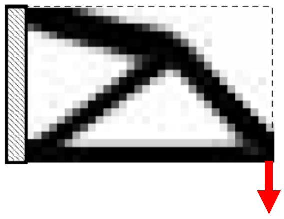
Motivation

Structural topology optimization :

[Bendsøe & Kikuchi, 1988]
[Sigmund, 2001]



FE discretization
& optimization



- ➢ Design volume
- ➢ Material properties
- ➢ Boundary conditions
- ➢ Applied loads
- ➢ Objective function
- ➢ Design constraints

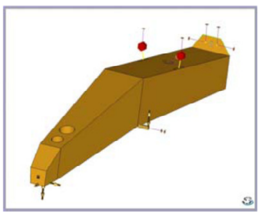
Large scale problem !

Motivation

Achievements in structural topology optimization

- ➢ Gradient-based algorithms (CONLIN, MMA, GCMMA…)
- ➢ Relevant problem formulations (SIMP penalization…)

A powerful design tool:

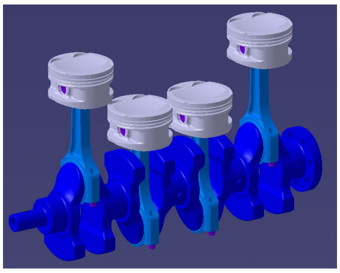


[Poncelet et al., 2005]

Motivation

Our objective : Topology optimization for the design of components of multibody systems

Equivalent static load approach, see e.g. [Kang & Park, 2005]

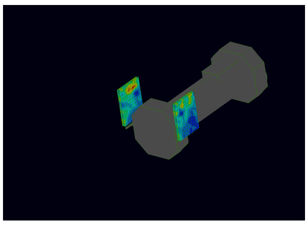


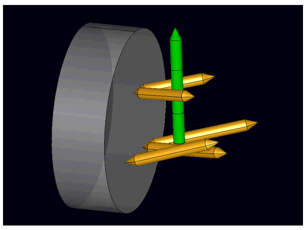
Equivalent static problem
- boundary conditions ?
- load case(s) ?
- objective function ?

⇒ experience and intuition are required
⇒ optimal solution for a wrong problem !

Motivation

Topology optimization based on the actual dynamic response



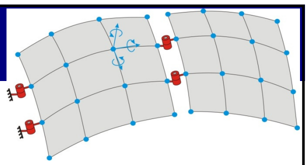


- Flexible multibody model (FE)
- Time integrator (g-α)
- Sensitivity analysis
- Coupling with an optimizer

Advantages:

⇒ Systematic approach
⇒ More realistic objective function

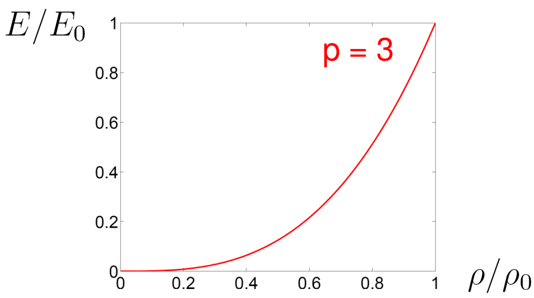
Topology optimization



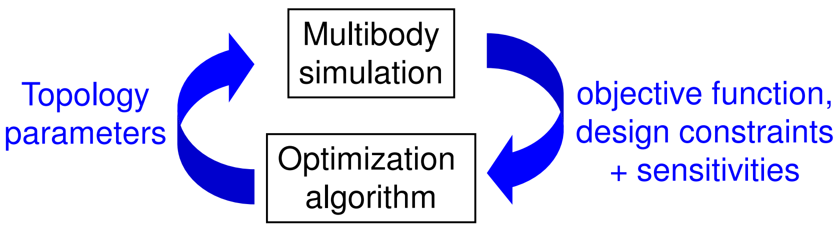
Parameterization of the topology: for each element,

- one density variable is defined $x = \rho/\rho_0, \quad x \in [0,1]$
- the Young modulus is computed according to the SIMP law

$$E = x^p E_0$$



Global optimization framework



Coupled industrial software

- OOFELIE (simulation and sensitivity analysis)
- CONLIN (gradient-based optimization) [Fleury 1989]

Efficient and reliable sensitivity analysis ?

➡ Direct differentiation technique

Sensitivity analysis

For one design variable x, direct differentiation leads to

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}}' + \mathbf{C}_t\dot{\mathbf{q}}' + \mathbf{K}_t\mathbf{q}' + \boldsymbol{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda}' + \mathbf{r}_{,x} &= \mathbf{0} \\ \boldsymbol{\Phi}_{\mathbf{q}}\mathbf{q}' + \boldsymbol{\Phi}_{,x} &= \mathbf{0} \end{aligned}$$

pseudo-loads

Inertia forces $\propto \rho$
Elastic forces $\propto E$
$\Rightarrow$ Analytical expressions for $\mathbf{r}_{,x}$

Integration of the sensitivities
- iteration matrix already computed and factorized
- one linear pseudo-load case for each design variable

$\mathbf{q}_{n+1}, \dot{\mathbf{q}}_{n+1}, \ddot{\mathbf{q}}_{n+1}, \boldsymbol{\lambda}_{n+1}$
$\downarrow$
$\mathbf{q}'_{n+1}, \dot{\mathbf{q}}'_{n+1}, \ddot{\mathbf{q}}'_{n+1}, \boldsymbol{\lambda}'_{n+1}$
$\downarrow$
$n := n+1$

Sensitivity analysis

Importance of an efficient sensitivity analysis :
- Test problem with (only) 60 design variables

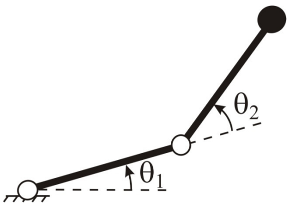
- Finite difference (61 simulations)
  $\Rightarrow$ **CPU time = 141 s**

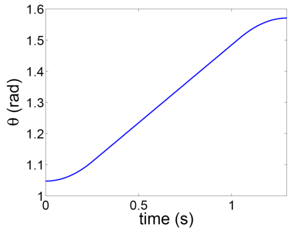
- Direct differentiation (1 extended simulation)
  $\Rightarrow$ **CPU time = 16 s**

Moreover, the direct differentiation method leads to higher levels of accuracy

[B. & Eberhard 2008]

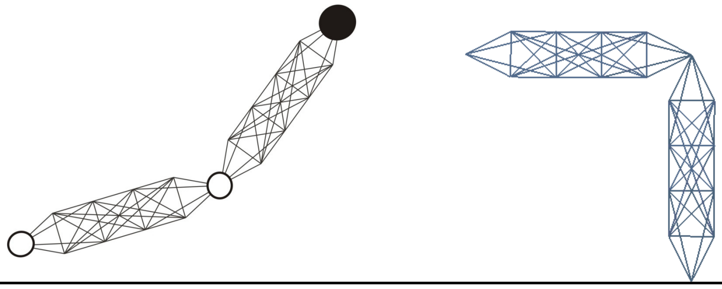
Two dofs robot arm



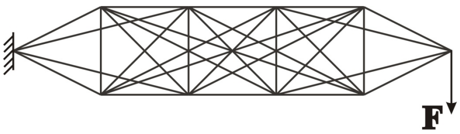


Point-to-point joint trajectory

Initial structural universe of beams:

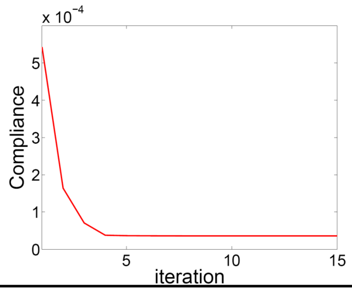


Equivalent static case



Minimization of the compliance $c = \frac{1}{2}\int_V \boldsymbol{\epsilon}^T \mathbf{H} \boldsymbol{\epsilon}\, dV$

subject to a volume constraint $V \leq 0.4 V_{full}$

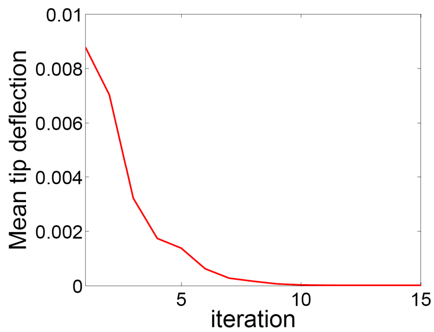


Final design:

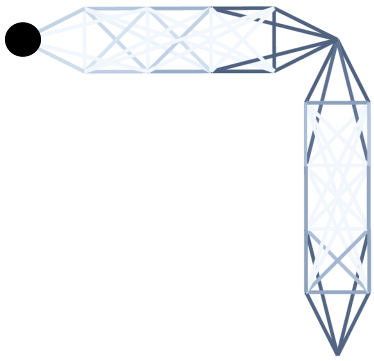
Optimization based on multibody simulations

Minimization of the tip deflection $\frac{1}{t_f}\int_0^{t_f} \|\mathbf{r}-\mathbf{r}_{rigid}\|^2\, dt$

subject to a volume constraint $V_{(i)} \leq 0.4\, V_{full,(i)}$



Final design:



Summary

- Topology optimization of mechanisms components
- Equivalent static load ⇒ multibody dynamics approach
  - flexible multibody simulation
  - sensitivity analysis
  - coupling with an optimization code
- Application to a two dofs robot arm with truss linkages
  - importance of problem formulation
- Perspectives
  - Complex 3D mechanisms
  - Mechatronic systems

Future work

Optimal control methods?

- inverse dynamics problems:
  identification of applied loads from kinematic data
- control-structure design optimization:
  the trajectory can be tuned for a given structural design
  the structural design can be tuned for a given trajectory

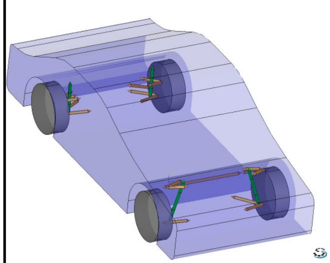
How to exploit our background for OC? [Bottasso et al, 2004]

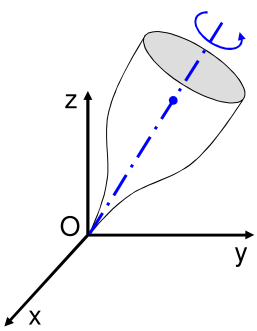
- Indirect OC approach: an existing simulation framework cannot be reused easily
- Direct OC approach: reuse of an existing simulation framework is more feasible, but one needs to solve large NLP problems

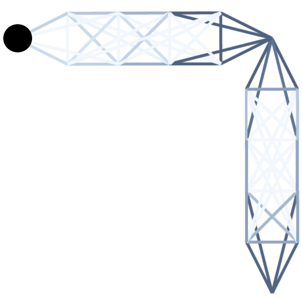
Thank you for your attention!

Recent developments in simulation, optimization and control of flexible multibody systems

Olivier Brüls



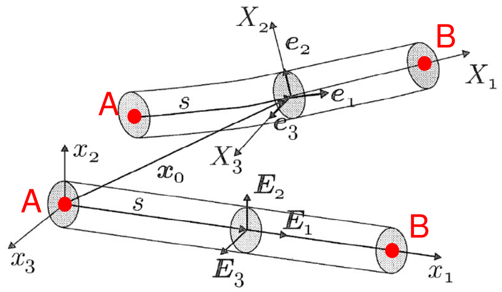




Katholieke Universiteit Leuven, January 28, 2010

Modelling of flexible multibody systems

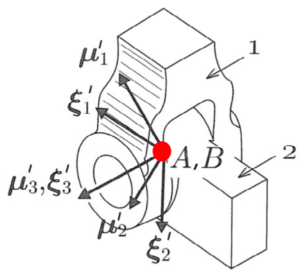
Flexible beam element



- Timoshenko-type geometrically exact model
- Two nodes A and B
- Nodal translations $(\mathbf{x}_A, \mathbf{x}_B)$ and rotations $(\mathbf{R}_A, \mathbf{R}_B)$
- Strain energy : bending, torsion, traction and shear
- Kinetic energy : translation and rotation

Modelling of flexible multibody systems

Hinge element



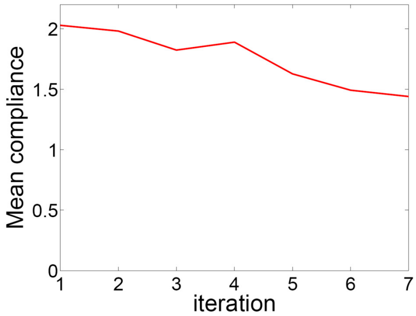
- Two nodes A (on body 1) and B (on body 2)
- Nodal translations $(\mathbf{x}_A, \mathbf{x}_B)$ and rotations $(\mathbf{R}_A, \mathbf{R}_B)$
- 5 kinematic constraints

$$\begin{aligned} \mathbf{x}_A - \mathbf{x}_B &= \mathbf{0} \\ \boldsymbol{\mu}_1'(\mathbf{R}_A) \cdot \boldsymbol{\xi}_3'(\mathbf{R}_B) &= 0 \\ \boldsymbol{\mu}_2'(\mathbf{R}_A) \cdot \boldsymbol{\xi}_3'(\mathbf{R}_B) &= 0 \end{aligned}$$

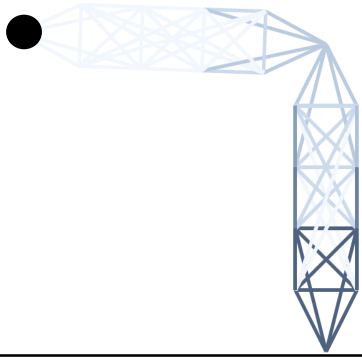
Multibody dynamics approach

Minimize the mean compliance: $\frac{1}{t_f} \int_0^{t_f} \sum_{i=1}^{n_c} c_{(i)} \, dt$

subject to a volume constraint $V_{(i)} \leq 0.4 \, V_{full,(i)}$

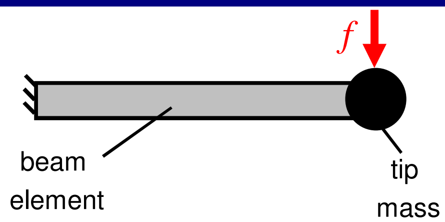


Final design:



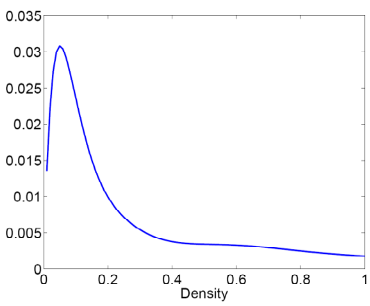
Choice of the objective function

A one-element test-case :



Objective functions :

Mean compliance



Mean square tip deflection