Fully coupled simulation of mechatronic and flexible multibody systems: An extended finite element approach

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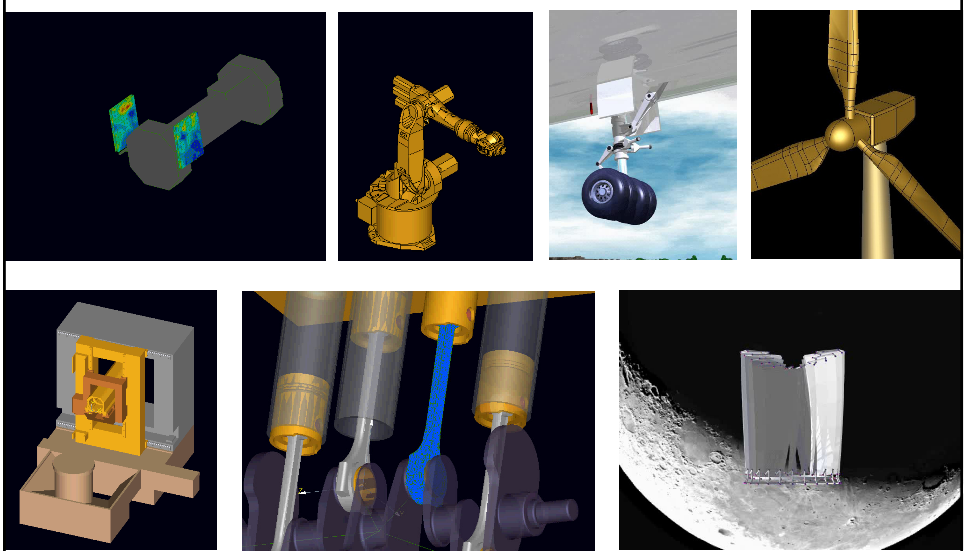
Samtech

Outline

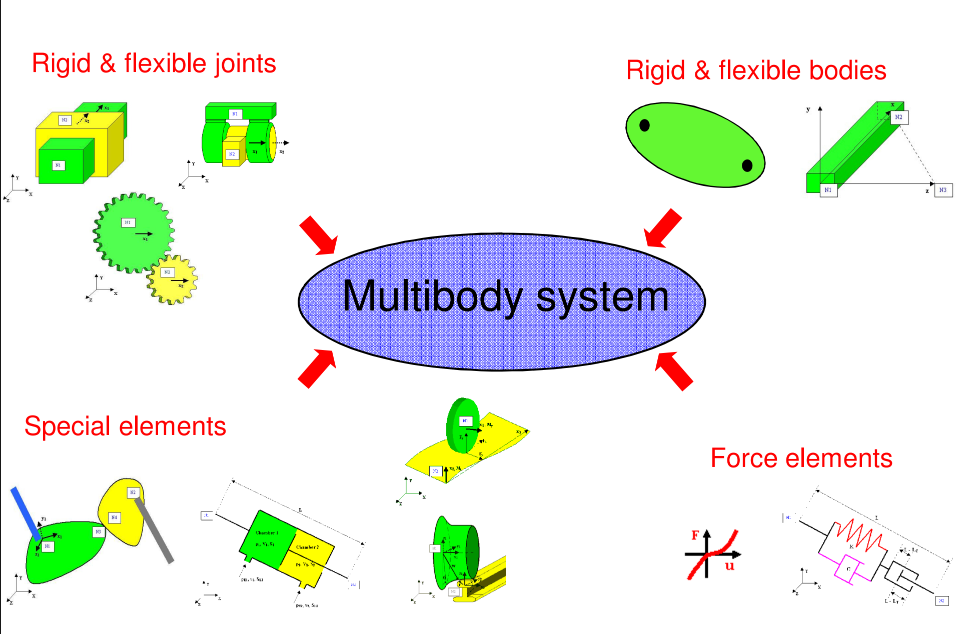
- ❑ Introduction
- ❑ Modelling of multibody & mechatronic systems
- ❑ Time integration algorithms
- ❑ Topology optimization of structural components

Introduction



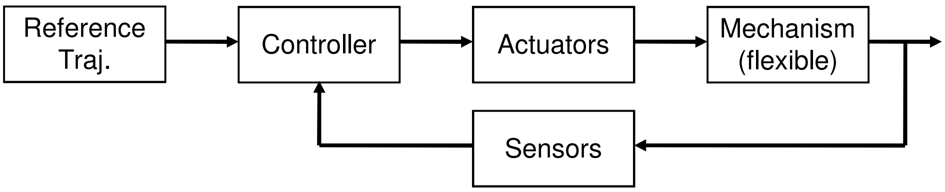
Some models developed using Samcef/Mecano

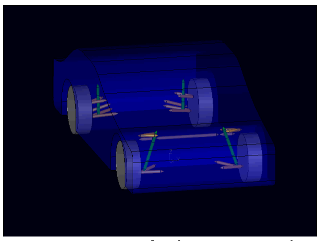
Introduction: Typical simulation library



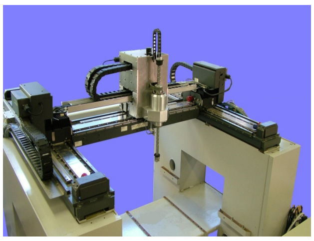
Introduction : Mechatronic systems

Mechatronics is the science of motion control [Van Brussel 1996]





Active suspension



Machine-tool (KULeuven)

Manipulator (Georgia Tech)

Introduction : Mechatronic design

From sequential to integrated design methods

1. Mechanical design

2. Actuator/sensor design

3. Control design

Mechanism | Actuator Sensor | Control

Mechatronic design

Integrated software tools

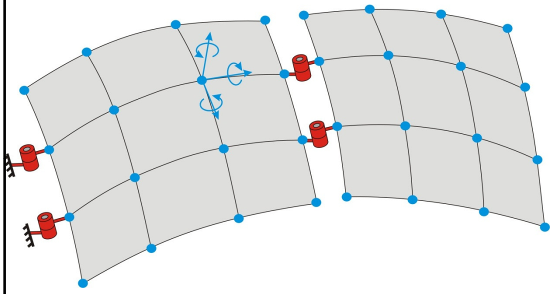
- Modelling
- Simulation
- Optimization

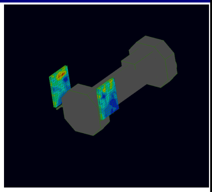
Outline

- Introduction
- Modelling of multibody & mechatronic systems
  - Modelling of flexible multibody systems
  - Modelling of coupled mechatronic systems
  - Application to a semi-active car suspension
- Time integration algorithms
- Topology optimization of structural components

Modelling of flexible multibody systems

Finite element approach [Géradin & Cardona 2001]





Nodal coordinates

- translations & rotations
- geometric nonlinearities

Kinematic joints

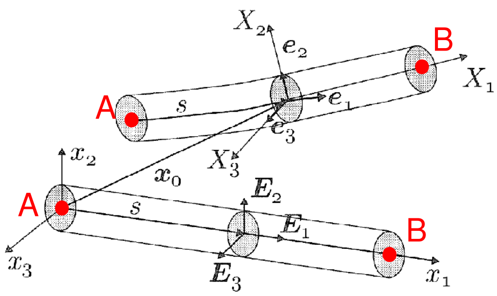
- algebraic constraints

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{g}_{gyr}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}_{int}(\mathbf{q}) + \mathbf{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda} &= \mathbf{g}_{ext} \\ \mathbf{\Phi}(\mathbf{q},t) &= \mathbf{0} \end{aligned}$$

index-3 DAE with rotation variables

Modelling of flexible multibody systems

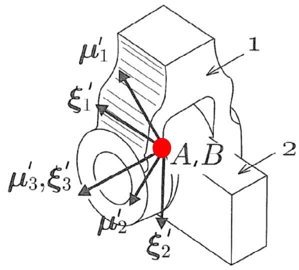
Flexible beam element



- Timoshenko-type geometrically exact model
- Two nodes A and B
- Nodal translations $(\mathbf{x}_A, \mathbf{x}_B)$ and rotations $(\mathbf{R}_A, \mathbf{R}_B)$
- Strain energy : bending, torsion, traction and shear
- Kinetic energy : translation and rotation

Modelling of flexible multibody systems

Hinge element

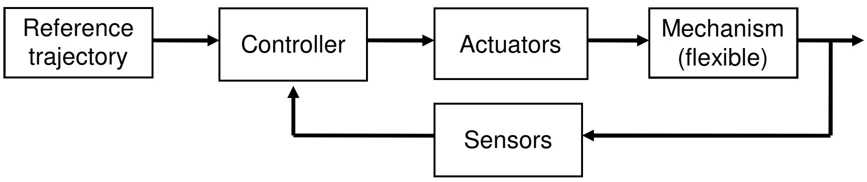


- Two nodes A (on body 1) and B (on body 2)
- Nodal translations $(\mathbf{x}_A, \mathbf{x}_B)$ and rotations $(\mathbf{R}_A, \mathbf{R}_B)$
- 5 kinematic constraints

$$
\begin{aligned}
\mathbf{x}_A - \mathbf{x}_B &= \mathbf{0} \\
\mu_1'(\mathbf{R}_A) \cdot \xi_3'(\mathbf{R}_B) &= 0 \\
\mu_2'(\mathbf{R}_A) \cdot \xi_3'(\mathbf{R}_B) &= 0
\end{aligned}
$$

Modelling of coupled mechatronic systems

Extension to an integrated mechatronic framework



Co-simulation using discipline-oriented software

- ✓ Modularity
- ➢ Software interface
- ➢ Advanced algorithms (stiff problems, algebraic loops)

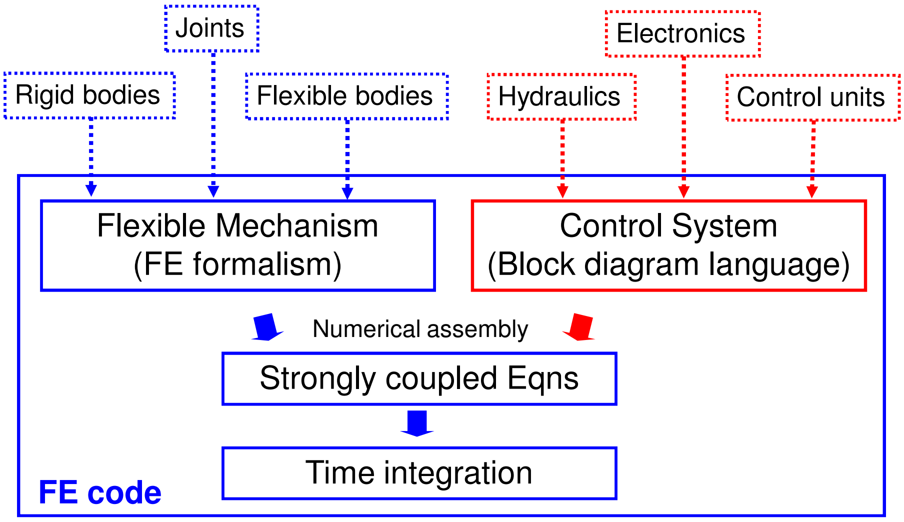
Monolithic approach

- ✓ Integrated simulation
- ✓ Strongly coupled modelling

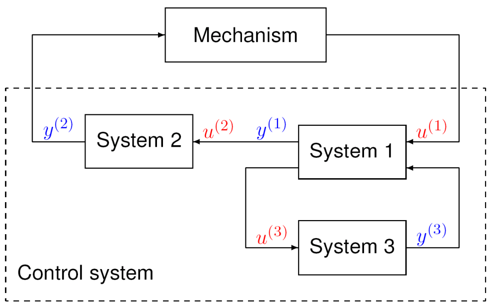
} $\Rightarrow$ Modularity ?

Modelling of coupled mechatronic systems

Modular and monolithic approach



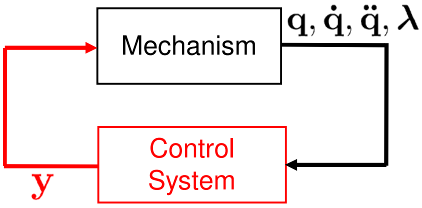
Modelling of coupled mechatronic systems



Block diagram language in a FE code

- Generic blocks : gain, integrator, transfer function…
  $\Rightarrow$ “special” elements
- State variables $\Rightarrow$ “special” dofs
- Numerical assembly according to the FE procedure

Modelling of coupled mechatronic systems



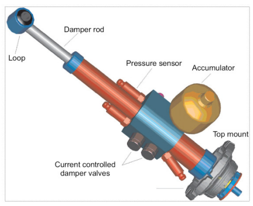
Coupled equations:

$$
\begin{aligned}
\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} &= \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t) - \boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} + \mathbf{L}\mathbf{y} \\
\mathbf{0} &= \boldsymbol{\Phi}(\mathbf{q}, t) \\
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\lambda}, \mathbf{x}, \mathbf{y}, t) \\
\mathbf{y} &= \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\lambda}, \mathbf{x}, \mathbf{y}, t)
\end{aligned}
$$

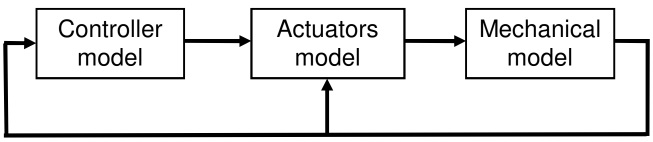
Time-integration scheme for coupled 1st/2nd order DAE ?

- Classical ODE solvers : multistep & Runge-Kutta methods
- Generalized-α time integration scheme

Semi-active car suspension



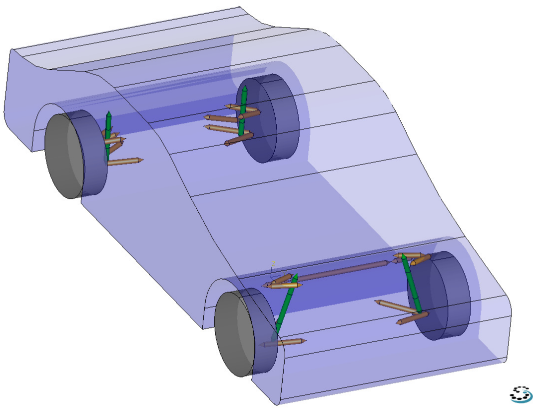
- Hydraulic actuators with electrical valves
- Accelerometers on the car body



Semi-active car suspension

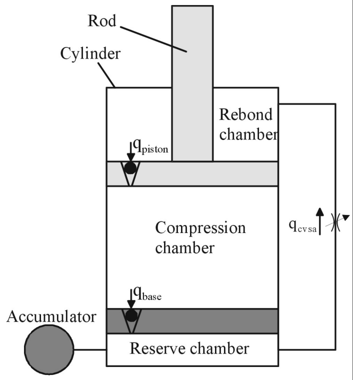
Mechanical model

(rigid bodies)



Actuator model

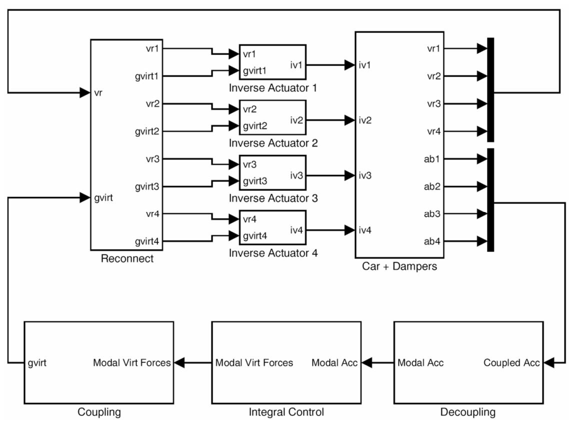
(state variable = hydraulic pressures)



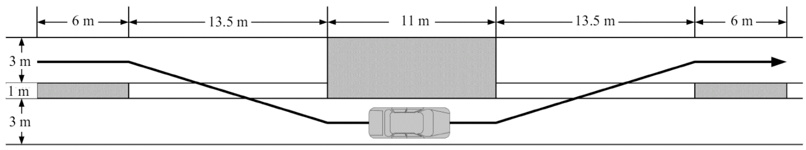
Semi-active car suspension

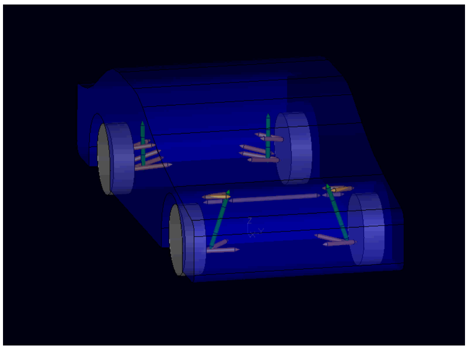
Controller model [Lauwerys et al. 2004]

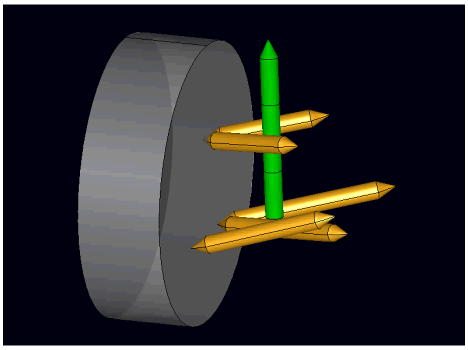
- Feedback linearization
- Transformation into modal space
- Linear integral control



Semi-active car suspension

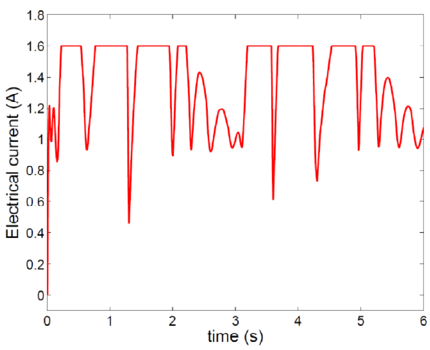




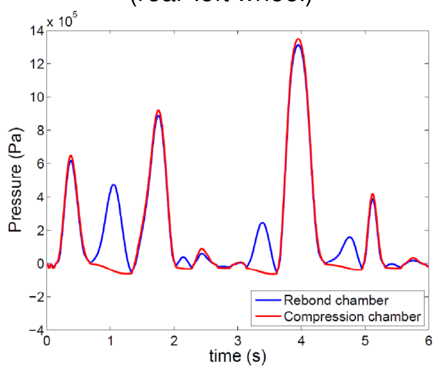


Semi-active car suspension

Electrical current in the valves (A)
(rear-left wheel)



Hydraulic pressures (Pa)
(rear-left wheel)



Summary

- Coupled simulation of mechatronic systems
- The equations of motion are obtained using the block diagram language and the finite element technique
- The generalized-α time integrator is used to solve the strongly coupled problem
- Application to a non-academic mechatronic system

Liège - Belgium

Outline

- ❑ Introduction
- ❑ Modelling of multibody & mechatronic systems
- ❑ Time integration algorithms
  - ➢ Generalized-α method
  - ➢ Kinematic constraints
  - ➢ Treatment of rotation variables
  - ➢ Controller dynamics
- ❑ Topology optimization of structural components

Generalized-α method

Numerical integration methods

- ➢ Standard integrators: multistep, Runge-Kutta
- ➢ Methods from structural dynamics (Newmark, HHT, g-α)
- ➢ Energy conserving schemes

Generalized-α method [Chung & Hulbert 1993]

- ➢ One step method for 2^{nd} ODEs
- ➢ 2^{nd} order accuracy
- ➢ Unconditional stability (A-stability) for linear problems
- ➢ Controllable numerical damping at high frequencies
- ➢ Computational efficiency for large and stiff problems

$\Rightarrow$ Extensions of the g-α method to deal with kinematic constraints, rotation variables and controller dynamics?

Generalized-α method

2nd order ODE system: $\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t)$

Newmark implicit formulae:

$$\begin{aligned}\mathbf{q}_{n+1} &= \mathbf{q}_n + h\dot{\mathbf{q}}_n + h^2(0.5-\beta)\mathbf{a}_n + h^2\beta\mathbf{a}_{n+1} \\ \dot{\mathbf{q}}_{n+1} &= \dot{\mathbf{q}}_n + h(1-\gamma)\mathbf{a}_n + h\gamma\mathbf{a}_{n+1}\end{aligned}$$

Generalized-α method [Chung & Hulbert, 1993]

$$(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m\mathbf{a}_n = (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f\ddot{\mathbf{q}}_n$$

To be solved with : $\mathbf{M}(\mathbf{q}_{n+1})\ddot{\mathbf{q}}_{n+1} = \mathbf{g}(\mathbf{q}_{n+1}, \dot{\mathbf{q}}_{n+1}, t_{n+1})$

- Two kinds of acceleration variables: $\mathbf{a}_n \neq \ddot{\mathbf{q}}_n$
- Algorithmic parameters: $\gamma,\ \beta,\ \alpha_f,\ \alpha_m$
  2nd order accuracy & numerical damping

Kinematic constraints

$$\begin{aligned}\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} &= \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t) - \mathbf{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda} \\ \mathbf{0} &= \mathbf{\Phi}(\mathbf{q}, t)\end{aligned}$$

Direct integration of the index-3 DAE problem

- Linear stability analysis demonstrates the importance of numerical damping [Cardona & Géradin 1989]
- Scaling of equations and variables reduces the sensitivity to numerical errors [Bottasso, Bauchau & Cardona 2007]
- Global convergence is demonstrated [Arnold & B. 2007]

Reduced index formulations

[Lunk & Simeon 2006; Jay & Negrut 2007; Arnold 2009]

Treatment of rotations

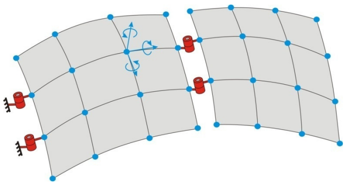
It is impossible to have a global 3-dimensional parameterization of rotations without singular points. [Stuelpnagel 1964]

Possible strategies

- 3-dimensional parameterization + reparameterization to avoid singularities [Cardona & Géradin 1989]
- Higher dimensional parameterization + kinematic constraints [Betsch & Steinmann 2001]
- Rotationless formulation, e.g. ANCF [Shabana]
- Lie group time integrator: no parameterization of the manifold is required *a priori* [Simo & Vu-Quoc 1988; B. & Cardona 2010]

Treatment of rotations

Nodal coordinates



$$q = (\mathbf{x}_1, \ldots, \mathbf{x}_N, \mathbf{R}_1, \ldots, \mathbf{R}_N)$$

The configuration evolves on the n-dimensional Lie group

$$G = \mathbb{R}^3 \times \ldots \times \mathbb{R}^3 \times SO(3) \times \ldots \times SO(3)$$

with the composition $q_{tot} = q_1 \circ q_2$ such that

$$\mathbf{x}_{i,tot} = \mathbf{x}_{i,1} + \mathbf{x}_{i,2} \quad \text{and} \quad \mathbf{R}_{i,tot} = \mathbf{R}_{i,1}\mathbf{R}_{i,2}$$

Constrained equations of motion :

$$\begin{aligned} \dot{q} &= DL_q(e) \cdot \widetilde{\mathbf{v}} \\ \mathbf{M}(q)\dot{\mathbf{v}} + \mathbf{g}(q, \mathbf{v}, t) + \mathbf{B}^T(q)\boldsymbol{\lambda} &= \mathbf{0} \\ \boldsymbol{\Phi}(q) &= \mathbf{0} \end{aligned}$$

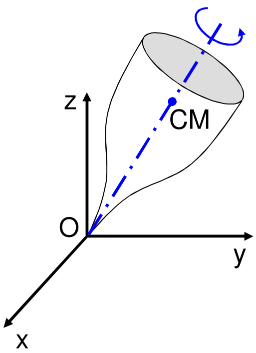
Treatment of rotations

$$\begin{aligned} \mathbf{M}(q_{n+1})\dot{\mathbf{v}}_{n+1} + \mathbf{g}(q_{n+1}, \mathbf{v}_{n+1}, t_{n+1}) + \mathbf{B}^T(q_{n+1})\boldsymbol{\lambda}_{n+1} &= \mathbf{0} \\ \boldsymbol{\Phi}(q_{n+1}) &= \mathbf{0} \\ q_{n+1} &= \varphi_h(q_n, \mathbf{v}_n, \mathbf{a}_n, \mathbf{a}_{n+1}) \\ \mathbf{v}_{n+1} &= \mathbf{v}_n + (1-\gamma)h\mathbf{a}_n + \gamma h \mathbf{a}_{n+1} \\ (1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n &= (1-\alpha_f)\dot{\mathbf{v}}_{n+1} + \alpha_f \dot{\mathbf{v}}_n \end{aligned}$$

with, e.g., $\varphi_h^{(1)}(q_n, \mathbf{v}_n, \mathbf{a}_n, \mathbf{a}_{n+1}) \;=\; q_n \circ \exp\left(h\widetilde{\mathbf{v}_n} + h^2(0.5-\beta)\widetilde{\mathbf{a}_n} + \beta h^2 \widetilde{\mathbf{a}_{n+1}}\right)$

1. No rotation parameterization is explicitly involved
2. The solution inherently remains on the manifold
3. The integration formulae are nonlinear
4. The classical generalized-α algorithm is a special case when G is a linear vector space
5. Second-order accuracy + numerical damping
6. Much simpler than parameterization-based methods

Treatment of rotations: ODE benchmark



Spinning top with spherical ellipsoid of inertia and constant follower torque

Equations of motion (in body frame)

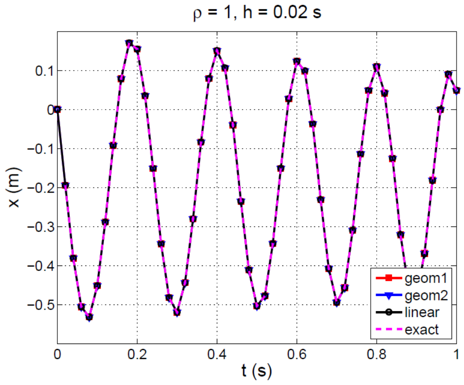
$$\dot{\mathbf{R}} = \mathbf{R}\widetilde{\boldsymbol{\Omega}}$$

$$\mathbf{J}\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times \mathbf{J}\boldsymbol{\Omega} = \mathbf{C}$$

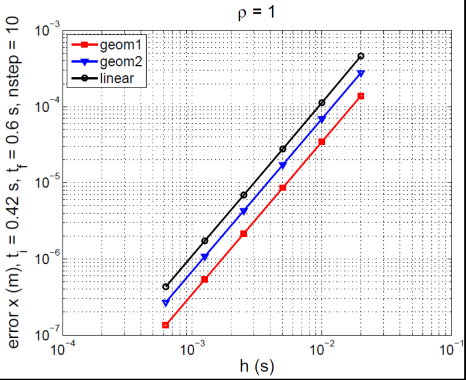
w.r.t. O

An analytical solution is available [Romano 2008]

Treatment of rotations: ODE benchmark

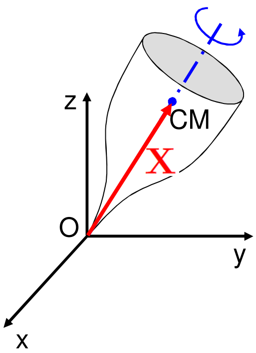


- geom1 = Lie group method 1
- geom2 = Lie group method 2
- linear = parameterization-based
- exact = analytical solution



$$\text{error} = \frac{1}{n_e}\sum_{i=1}^{n_e} \|\mathbf{x}(t_i) - \mathbf{x}^{ref}(t_i)\|$$

Treatment of rotations: DAE benchmark



Top in the gravity field

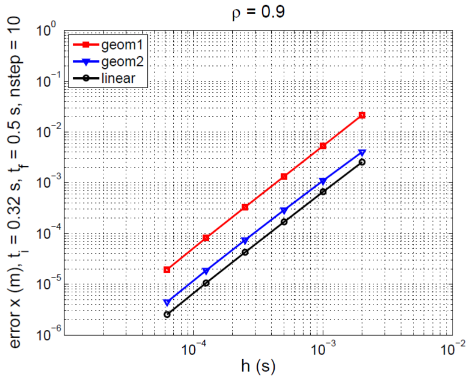
Constrained equations of motion:

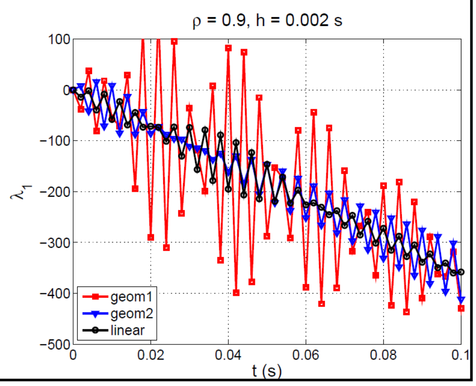
$$\dot{\mathbf{R}} = \mathbf{R}\widetilde{\boldsymbol{\Omega}}$$

$$\begin{aligned} m\ddot{\mathbf{x}} - \boldsymbol{\lambda} &= m\boldsymbol{\gamma} \\ \mathbf{J}\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega}\times\mathbf{J}\boldsymbol{\Omega} + \widetilde{\mathbf{X}}\mathbf{R}^T\boldsymbol{\lambda} &= \mathbf{0} \\ -\mathbf{x} + \mathbf{R}\mathbf{X} &= \mathbf{0} \end{aligned}$$

w.r.t. CM ($\mathbf{J}$)

Treatment of rotations: DAE benchmark





Controller dynamics

Coupled dynamic equations: $\left\{ \begin{array}{lcl} \ddot{\mathbf{q}} & = & \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{x}, t) \\ \dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{x}, t) \end{array} \right.$

$$\begin{array}{lcl} \mathbf{q}_{n+1} & = & \mathbf{q}_n + h\dot{\mathbf{q}}_n + h^2(0.5-\beta)\mathbf{a}_n + h^2\beta\mathbf{a}_{n+1} \\ \dot{\mathbf{q}}_{n+1} & = & \dot{\mathbf{q}}_n + h(1-\gamma)\mathbf{a}_n + h\gamma\mathbf{a}_{n+1} \\ \mathbf{x}_{n+1} & = & \mathbf{x}_n + h(1-\theta)\mathbf{w}_n + h\theta\mathbf{w}_{n+1} \end{array}$$

$$\left\{ \begin{array}{lcl} (1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m\mathbf{a}_n & = & (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f\ddot{\mathbf{q}}_n \\ (1-\delta_m)\mathbf{w}_{n+1} + \delta_m\mathbf{w}_n & = & (1-\delta_f)\dot{\mathbf{x}}_{n+1} + \delta_f\dot{\mathbf{x}}_n \end{array} \right.$$

To be solved with : $\left\{ \begin{array}{lcl} \ddot{\mathbf{q}}_{n+1} & = & \mathbf{g}(\mathbf{q}_{n+1}, \dot{\mathbf{q}}_{n+1}, \mathbf{x}_{n+1}, t_{n+1}) \\ \dot{\mathbf{x}}_{n+1} & = & \mathbf{f}(\mathbf{q}_{n+1}, \dot{\mathbf{q}}_{n+1}, \mathbf{x}_{n+1}, t_{n+1}) \end{array} \right.$

Order conditions: $\left\{ \begin{array}{lcl} \gamma & = & 0.5 + \alpha_f - \alpha_m \\ \theta & = & 0.5 + \delta_f - \delta_m \end{array} \right.$

Summary

The generalized-α method combines

- Second-order accuracy (demonstrated for ODEs)
- Adjustable numerical damping
- Computational efficiency for large and stiff problems

Extension to coupled DAEs on Lie groups with a consistent treatment of:

- Kinematic constraints
- Rotational variables (with or without parameterization)
- Control state variables

Department of Aerospace & Mechanical Eng.

Campus of Sart-Tilman

1 km

10 km

Liège
city center

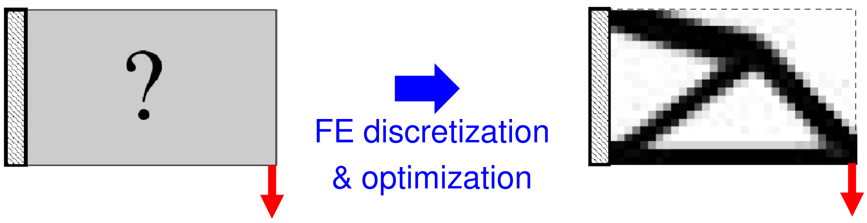
Outline

- ❑ Introduction
- ❑ Modelling of multibody & mechatronic systems
- ❑ Time integration algorithms
- ❑ Topology optimization of structural components
  - ➢ Motivation
  - ➢ Method
  - ➢ Two-dofs robot arm

Motivation

Structural topology optimization :

[Bendsøe & Kikuchi, 1988]
[Sigmund, 2001]



- ➢ Design volume
- ➢ Material properties
- ➢ Boundary conditions
- ➢ Applied loads
- ➢ Objective function
- ➢ Design constraints

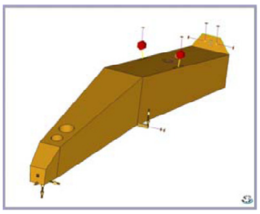
Large scale problem !

Motivation

Achievements in structural topology optimization

- ➢ Gradient-based algorithms (CONLIN, MMA, GCMMA…)
- ➢ Relevant problem formulations (SIMP penalization…)

A powerful design tool:

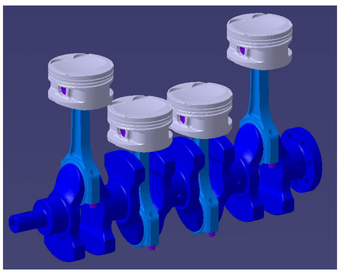


[Poncelet et al., 2005]

Motivation

Our objective : Topology optimization for the design of components of multibody systems

Equivalent static load approach, see e.g. [Kang & Park, 2005]



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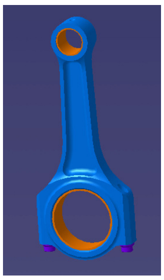


Equivalent static problem

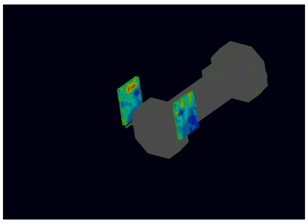
- ➢ boundary conditions ?
- ➢ load case(s) ?
- ➢ objective function ?

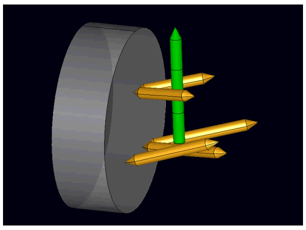
⇒ experience and intuition are required

⇒ optimal solution for a wrong problem !

Motivation

Full dynamic simulation for topology optimization





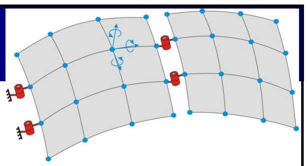
- Flexible multibody model (FE)
- Time integrator (g-α)
- Sensitivity analysis
- Coupling with an optimizer

Advantages:

⇒ Systematic approach

⇒ More realistic objective function

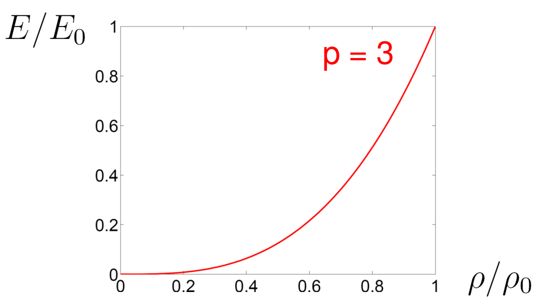
Topology optimization



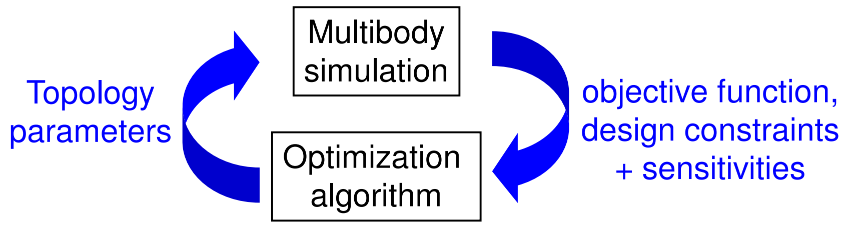
Parameterization of the topology: for each element,

- one density variable is defined $x = \rho/\rho_0, \quad x \in [0, 1]$
- the Young modulus is computed according to the SIMP law

$$E = x^p E_0$$



Global optimization framework



Coupled industrial software

- OOFELIE (simulation and sensitivity analysis)
- CONLIN (gradient-based optimization) [Fleury 1989]

Efficient and reliable sensitivity analysis ?

➡ Direct differentiation technique

Sensitivity analysis

For one design variable x, direct differentiation leads to

$$\begin{aligned}\mathbf{M}\ddot{\mathbf{q}}' + \mathbf{C}_t\dot{\mathbf{q}}' + \mathbf{K}_t\mathbf{q}' + \boldsymbol{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda}' + \mathbf{r}_{,x} &= 0 \\ \boldsymbol{\Phi}_{\mathbf{q}}\mathbf{q}' + \boldsymbol{\Phi}_{,x} &= 0\end{aligned}$$

pseudo-loads

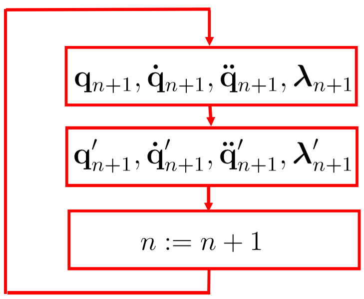
Inertia forces $\propto \rho$

Elastic forces $\propto E$

$\Rightarrow$ Analytical expressions for $\mathbf{r}_{,x}$

Integration of the sensitivities

- iteration matrix already computed and factorized
- one linear pseudo-load case for each design variable



Sensitivity analysis

Importance of an efficient sensitivity analysis :

- Test problem with (only) 60 design variables

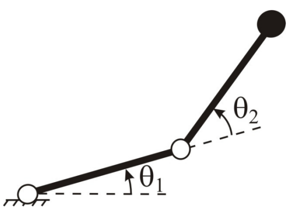
- Finite difference (61 simulations)
  - $\Rightarrow$ **CPU time = 141 s**

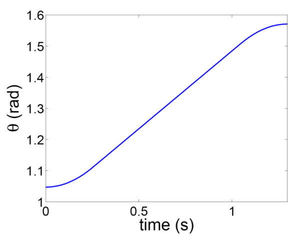
- Direct differentiation (1 extended simulation)
  - $\Rightarrow$ **CPU time = 16 s**

Moreover, the direct differentiation method leads to higher levels of accuracy

[B. & Eberhard 2008]

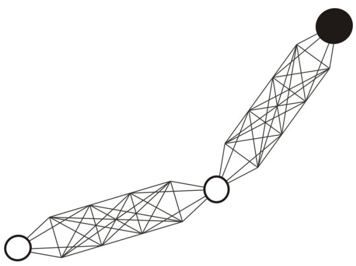
Two dofs robot arm

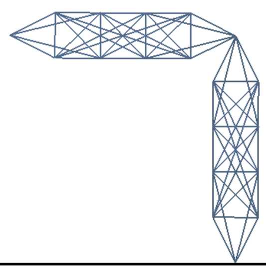




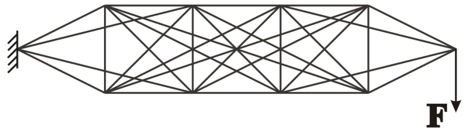
Point-to-point joint trajectory

Initial structural universe of beams:



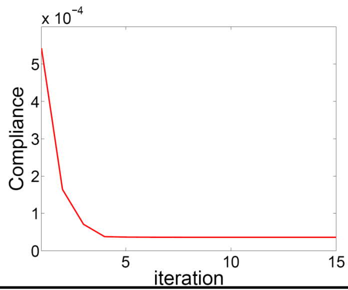


Equivalent static case



Minimization of the compliance $c = \frac{1}{2}\int_V \boldsymbol{\epsilon}^T \mathbf{H} \boldsymbol{\epsilon} \, dV$

subject to a volume constraint $V \leq 0.4 V_{full}$

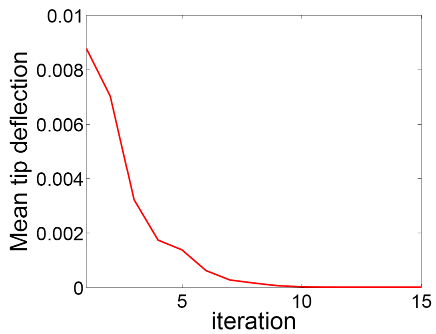


Final design:

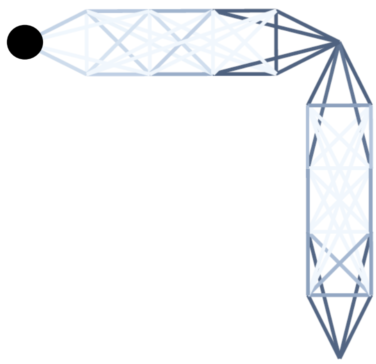
Optimization based on multibody simulations

Minimization of the tip deflection $\frac{1}{t_f}\int_0^{t_f} \|\mathbf{r} - \mathbf{r}_{rigid}\|^2 \, dt$

subject to a volume constraint $V_{(i)} \leq 0.4 \, V_{full,(i)}$



Final design:



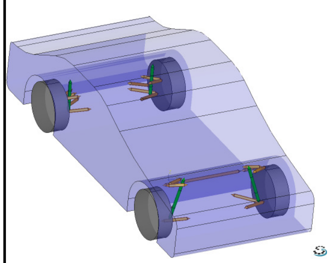
Summary

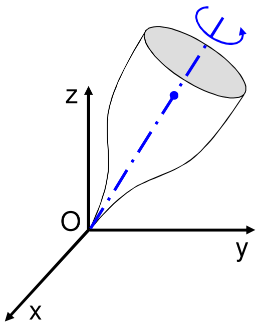
- ❑ Topology optimization of mechanisms components
- ❑ Equivalent static load ⇒ multibody dynamics approach
  - ➢ flexible multibody simulation
  - ➢ sensitivity analysis
  - ➢ coupling with an optimization code
- ❑ Application to a two dofs robot arm with truss linkages
  - ➢ importance of problem formulation
- ❑ Perspectives
  - ➢ Complex 3D mechanisms
  - ➢ Mechatronic systems

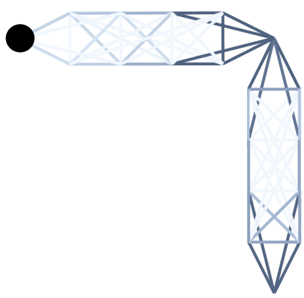
Thank you for your attention!

Fully coupled simulation of mechatronic and flexible multibody systems: An extended finite element approach

Olivier Brüls







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