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Sensitivity analysis for flexible multibody systems formulated on a Lie group

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The sensitivity of the dynamic response of a multibody system is a key information in gradient-based design optimization and optimal control problems. The present contribution addresses the computation of the sensitivities for systems which naturally evolve on a Lie group and not on a linear space.

The Lie group framework offers a number of advantages for the analysis of systems with large rotations variables [4, 5], e.g. for finite element models of systems with rigid bodies, kinematic joints, beams and shells. Firstly, the equations of motion are derived and solved directly on the nonlinear manifold, without an explicit parameterization of the rotation variables, which leads to important simplifications in the formulations and algorithms. Secondly, displacements and rotations are represented as increments with respect to the previous configuration, and those increments can be expressed in the material (body-attached) frame. Therefore, geometric nonlinearities are automatically filtered from the relationship between incremental displacements and elastic forces, which strongly reduces the fluctuations of the iteration matrix during the simulation [3].

Classical sensitivity analysis methods include finite difference methods, semi-analytical approaches or automatic differentiation. Semi-analytical approaches have interesting properties in terms of accuracy, robustness and computational cost. They have been successfully exploited for dynamic systems evolving on a linear parameter space, for which classical ODE or DAE solvers are available [1, 2]. However, to the best of our knowledge, the sensitivity analysis of dynamic systems on a Lie group has not been addressed in literature and deserves some particular investigations.

In a previous work [6], a direct differentiation method was proposed for systems evolving on $SO(3)$, the group of finite rotations. Here, the study is extended to a more general class of dynamic systems with kinematic joints, whose equations of motion have the structure of a DAE on a Lie group. It is shown that the nonlinearity of the Lie group and of the time integration formulae need to be carefully treated for the development of accurate sensitivity analysis algorithms.

A broad class of semi-analytical methods, including the direct differentiation method and the adjoint variable method, is discussed in the presentation. The main properties of those methods, which are well-known for problems on a linear space, are also observed for problems on a Lie group. Accurate sensitivity analysis algorithms are established and implemented in a simple way, exploiting the compact and elegant Lie group formalism. Their performance is studied for academic examples as well as for the optimization of a vehicle multi-link suspension mechanism.

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